SHOW ME THE MATH

GRAVITY VERSUS PRESSURE

In order for a cloud to collapse, the strength of gravity pulling in must exceed the strength of pressure pushing out. Let's see how these forces battle each other mathematically.

Thermal pressure is directly proportional to both the temperature T and number density n within a cloud.

$$P = nkT$$

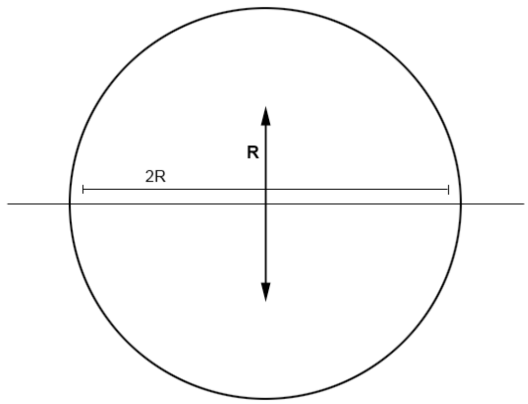
is known as the ideal gas law. P represents gas pressure, number density n is equal to the number of particles contained within each cubic centimeter, k is Boltzmann's constant and T represents temperature measured in Kelvin. Let's start by finding out more about the number density. If each gas particle has a mass m, then $\frac{M}{m}$ gives the total number of particles within a cloud of mass M. The number density is then found by dividing this number by the volume of the cloud. For simplicity, we will use a spherical cloud. The volume of a spherical cloud of mass M and radius r is $\frac{4}{3}\pi r^3$.

$$n \approx \frac{M}{m} \div \left(\frac{4}{3}\pi r^3\right) \approx \frac{3M}{4\pi m r^3}$$

The force on a spherical cloud due to pressure is equal to the thermal pressure times the area of the cloud.

$$\begin{aligned} F_P &= pressure \times area \\ &\approx nkT \times \pi r^2 \\ &\approx \frac{3MkT}{4\pi m r^3} \times \pi r^2 \\ &\approx \frac{3MkT}{4mr} \end{aligned}$$

This is the expression we will use as the force on the cloud from thermal pressure. Now, in order to find an equation for the force due to gravity we have to think about the cloud in a slightly different manner. Imagine slicing the same spherical cloud into halves, so that each half exerts a gravitational force on the other half. The mass of each half is $\frac{M}{2}$, and the separation between their centers is approximately equal to r.



Newton's law of gravity says that the force due to gravity is $G\frac{M_1 M_2}{r^2}$. With the mass of each half set to $\frac{M}{2}$, the force due to gravity is:

$$F_G \approx G\frac{(\frac{M}{2})(\frac{M}{2})}{r^2}$$

$$\approx G\frac{M^2}{4r^2}$$

where G is the gravitational constant.

These equations for the force due to pressure and the force due to gravity show how each depend differently on the mass, temperature and density of the cloud. An increase in the mass of the cloud increases both the gravity and the pressure. However, the force of gravity increases more since it is proportional to the mass squared.

The mass at which the forces of gravity and pressure are equal is called Jean's mass, after the British physicist Sir James Jeans.

$$M_{Jeans}=18M_{Sun}\sqrt{\frac{T^3}{n}}$$

where temperature T is measured in Kelvin and number density n is stated in units of particles per cubic centimeter. In a cloud with mass M greater than Jean's mass, gravity is stronger than thermal pressure and the cloud will collapse. The opposite is true for a cloud with mass M less than Jean's mass. This is the method scientists used to find the minimum mass of newborn stars.