

ANALYSIS OF DATA
FROM THE VOYAGER PLASMA SCIENCE EXPERIMENT
USING THE FULL CUP RESPONSE

by

ALAN SETH BARNETT
S.B. Massachusetts Institute of Technology
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ABSTRACT

The Voyager Plasma Science experiment consists of four modulated grid Faraday cups which measure positive ions and electrons in the energy-per-charge range of 10-5950 volts. A formula for the full response function of each of the cups is derived from the solution to the equations of motion for a charged particle inside the cup. The current in each of the energy-per-charge channels of the detector can be expressed as the difference between two integrals over velocity space of the product of the response function and the distribution function which describes the plasma. The integrals are performed for two special cases when the distribution function is a convected Maxwellian.

For the case when the sonic Mach number of the flow is large ("cold plasma" approximation), we approximated the dependence of the distribution function on the components of the flow perpendicular to the cup normal by a Dirac delta function, permitting the integrals over those components to be performed trivially. The remaining integral must be performed numerically.

For the more general case when the sonic Mach number is not large, we approximated the analytic expressions for the response function by a functional form which permits the integration over the components of velocity transverse to the cup normal to be performed analytically. Once again, the integral over the normal component of the velocity must be performed numerically. We developed a special integration scheme which greatly reduces the computer time required for the numerical computation of the remaining integral.

The formula for the response function was tested by using the solar wind as a test beam when the spacecraft was rotating during a cruise maneuver. Analysis of the data taken during the maneuver using the "cold plasma" approximation confirmed the accuracy of our response function for angles of incidence of up to 70° for the main sensor cups and up to at least 55° for the side sensor cup. By using data from all four cups simultaneously, we are able to determine the solar wind direction with a precision of better than 0.5° .

We then present analysis of data taken in the vicinity of the Jovian satellite Io. The interaction between Io and the Jovian magnetosphere is a topic of considerable interest due to the fact that the decametric radio emission from Jupiter appears to be modulated by the satellite. Unfortunately, the problem of determining the plasma parameters near Io is extremely difficult due to the low Mach number of the flow, the

large angle of the flow with respect to the cup look directions, and the presence of several different ionic species. Under the assumption that the only ions present are O^+ , O^{++} , S^+ , and SO_2^+ and that all of the ionic species have the same thermal speed, we obtain estimates of the plasma parameters by fitting seven spectra taken in the vicinity of Io. We interpret our results in terms of a model of the interaction between Io and the Jovian ionosphere due to Neubauer (1980). The results indicate qualitative agreement with the model that the flow is analogous to the potential flow of an incompressible fluid around an infinitely long cylinder.

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Title: Professor of Physics

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Table of Contents

| | | |
|---|---|----|
| Title Page | p | 1 |
| Abstract | p | 2 |
| Acknowledgements | p | 4 |
| Table of Contents | p | 6 |
| Chapter 1 Introduction | p | 8 |
| Chapter 2 Derivation of the Response Function of the Voyager Plasma Science Experiment | p | 13 |
| Section 2.1 Location and Orientation of the Instrument | p | 14 |
| Section 2.2 Structure and Operation of the Main Sensor Cups | p | 15 |
| Section 2.3 Response Function of the Main Sensor | p | 17 |
| Section 2.3a The Grid Transparency | p | 18 |
| Section 2.3b The Sensitive Area | p | 20 |
| Section 2.4 Structure and Operation of the Side Sensor | p | 24 |
| Section 2.4a The Grid Transparency | p | 25 |
| Section 2.4b The Sensitive Area | p | 26 |
| Section 2.5 The "Cold Plasma" Approximation | p | 29 |
| Section 2.6 The "Hot Plasma" Approximation for the Main Sensor | p | 32 |
| Section 2.7 The "Hot Plasma" Approximation for the D-Cup | p | 37 |
| Chapter 3 Experimental Test of the Response Function | p | 38 |
| Section 3.1 The Cruise Maneuver | p | 39 |
| Section 3.2 Analysis of Data Taken During the Cruise Maneuver | p | 41 |
| Section 3.3 Discussion and Conclusions | p | 45 |

| | | | |
|-----------------|---|---|-----|
| Chapter 4 | The Theory of the Interaction Between Io and the | | |
| | Jovian Magnetosphere | p | 49 |
| | Section 4.1 Introduction | p | 50 |
| | Section 4.2 The Equations Which Describe the Plasma at Io | p | 52 |
| | Section 4.3 The Io-Magnetosphere Interaction | p | 59 |
| Chapter 5 | Analysis of Plasma Data Taken in the Vicinity of | | |
| | Io by Voyager I | p | 73 |
| | Section 5.1 The Io Flyby | p | 74 |
| | Section 5.2 Analysis of the Data | p | 75 |
| | Section 5.3 The Flow Around the Alfvén Wing | p | 82 |
| | Section 5.4 Summary | p | 89 |
| References | | p | 92 |
| Tables | | p | 94 |
| Figure Captions | | p | 100 |
| Figures | | p | 113 |
| Appendix A | | p | 166 |
| Appendix B | | p | 178 |
| Appendix C | | p | 182 |
| Appendix D | | p | 186 |

Chapter I

Introduction

On 20 August 1977, a spacecraft named Voyager II was launched from the Kennedy Space Center in Florida, bound for the outer solar system. Voyager I, its sister ship, was launched two weeks later on September 5. Both spacecraft were targeted for close encounters with Jupiter and Saturn. Voyager II will also fly close to Uranus and Neptune.

One of the experiments carried by both of these spacecraft was a set of four modulated grid Faraday cups called the Plasma Science Experiment (PLS). The PLS experiment measures positive ions and electrons in the energy range of 10-5950 eV. It was designed and constructed at MIT, and includes several novel features (Bridge et al). Three of its four cups are very shallow, resulting in an extremely wide field of view. These same three cups are arrayed about an axis of symmetry such that their fields of view overlap. This region of overlap includes the direction of the solar wind flow throughout most of the mission. By analyzing positive ion data taken by all three of these cups simultaneously, it is possible to determine the direction of the solar wind flow to better than one-half degree and its magnitude to within a few km/sec.

The fourth cup is more conventional in design, and it looks in a direction perpendicular to the symmetry axis of the main cluster. During the interplanetary, or cruise, phase of the mission, this cup is used to measure electrons. In addition, during the inbound pass of planetary encounters, this cup looks in the direction of the corotating plasma, measuring both electrons and positive ions.

The main sensor cups were designed in such a way that for a wide range of sonic Mach numbers and flow directions, including all of the situations which one expects to encounter during the cruise phases of the mission, almost all

of the particles which enter the aperture and are not stopped by the modulator voltage do reach the collector. During planetary encounters, however, the facts that the Mach number is frequently low and that the flow is highly oblique to the cups makes the "unity response" approximation not valid. In order to analyze data taken at these times, a knowledge of the full response function of the cups is required.

This thesis presents a derivation of the full response function based on a calculation of the trajectories of charged particles inside the cups. The current measured by the sensors is given by the integral over velocity space of the product of the plasma distribution function and the cup response function. A computer program which performs this integration numerically has already been written by V. Vasyliunas, one of the designers of the instrument. Unfortunately, his algorithm is very slow. Not only is it impossible to use it to perform any iterative fitting procedure to determine the macroscopic plasma parameters, but it is even too slow to use at all for simulating the high resolution (M-mode) energy-per-charge spectra. We have overcome this difficulty by approximating the analytic expression for the response function by a functional form which permits the integrations over the components of velocity perpendicular to the cup normal to be performed analytically for the case where the distribution function is a convected Maxwellian. This derivation is the topic of Chapter 2.

In order to test the response function derived in Chapter 2, it is necessary to have an extremely narrow test beam. Since such a beam is very difficult to make in the laboratory and the quiet solar wind has just these properties, we have tested the response function by analyzing data taken by Voyager I during a cruise maneuver. During the cruise maneuver, the

spacecraft performed a series of rotations which changed the orientation of the spacecraft, causing the solar wind to enter the cups from a wide variety of angles. The use of the data taken during the cruise maneuver to test our cup response functions is the topic of Chapter 3.

On the outbound pass of its Jupiter encounter, Voyager I flew about 20,000 km above the south pole of the satellite Io. As the decametric radio burst from Jupiter are known to be correlated with the phase angle of Io, the interaction between Io and the magnetospheric plasma is a topic of considerable interest. During the Io flyby, the sonic Mach number of the plasma was low (about 2) and the flow direction was perpendicular to the main sensor symmetry axis. This situation makes knowledge of the full response function necessary for the analysis of the data taken during this period. This stretch of data was chosen for the first use of the full response function in analyzing data.

The satellite Io appears to have a high electrical conductivity. Drell, Foley and Ruderman (1965) have shown that any conductor which moves through a magnetized plasma will be a source of Alfvén waves. If the velocity of the conductor with respect to the ambient medium does not change with time, there will be a standing wave pattern in the rest frame of the conductor consisting of a pair of Alfvén "wings" which extend away from the conductor in the direction of the Alfvén characteristics. Neubauer (1980) has shown that the plasma flow around each of these wings is analogous to the potential flow of an incompressible fluid around an infinite cylinder.

The final two chapters of the thesis are concerned with Io's interaction with the Jovian magnetospheric plasma. Chapter 4 contains a discussion and critique of Neubauer's theory, while Chapter 5 consists of analysis of the

data taken by Voyager I during the Io flyby. We conclude that the data is consistent with the overall picture that the plasma flows around the Alfvén wing as if the wing were a long, cylindrical obstacle.

Chapter 2

Derivation of the Response Function

of the

Voyager PLS Experiment

2.1 Location and Orientation of the Instrument

The Voyager Plasma Science Experiment consists of four modulated grid Faraday cups. A sketch of the instrument is shown in Figure 2.1. Three of the cups, called the A-cup, B-cup, and C-cup, comprise the main sensor. These three cups have the same pentagonal shape and are arrayed with their cup normals 20° from an axis of symmetry. The fourth cup, called the side sensor or D-cup, has a circular aperture. The normal to the D-cup aperture points in a direction 88° from the main sensor symmetry axis.

Figure 2.2 shows the location and orientation of the plasma instrument on the spacecraft. The instrument is mounted on the science boom, a metal support structure which extends away from the main body of the spacecraft. Also on the science boom are the cosmic ray, imaging, UV spectrometry, IR spectrometry, photopolarimetry, and low energy charged particle experiments.

The system of coordinates called spacecraft coordinates is defined as follows: The spacecraft center of mass is taken as the origin. The unit vector \hat{z}_{sc} points along the axis of the main antenna, with $+\hat{z}_{sc}$ pointing into the antenna. The unit vector \hat{y}_{sc} lies in the plane containing the \hat{z}_{sc} and the axis of symmetry of the science boom. It is perpendicular to the \hat{z}_{sc} and makes an acute angle with the science boom. The unit vector \hat{x}_{sc} is defined so as to make a right-handed system (see Fig. 2.2).

The outward pointing symmetry axis of the PLS main sensor is parallel to $-\hat{z}_{sc}$. As this axis is also parallel to the axis of the main antenna, it is pointed at the earth during most of the mission. Since the angular separation between the earth and the sun as seen from the outer solar

system is small, the solar wind flow direction is substantially into the main sensor. The D-cup is oriented such that it looks into corotating flow during the inbound pass of a planetary flyby. The relative orientations of the cup apertures as viewed from along the main sensor symmetry axis is shown in Fig. 2.3.

During the interplanetary, or cruise, phase of the mission, the main sensor measures positive ions and the side sensor measures electrons. During planetary encounters, the D-cup is also used to measure positive ions.

2.2 Structure and Operation of the Main Sensor Cups

A vertical cross section of a main sensor cup is shown in Figure 2.4, and a top view is shown in Figure 2.5. The collector of the cup has the shape of home plate on a baseball field with the corners smoothed out. The aperture is similar in shape, differing in that it is smaller and its parallel sides are shorter with respect to its other sides. The aperture area (A_{ap}) is 102 cm^2 . Around the edge of the collector is a rim of metal. The collector plane is considered to be the plane defined by the top of this rim. The distance (h) from the aperture to this plane is 4.1 cm.

The cup has nine parallel grids. Each grid consists of a woven mesh of two perpendicular sets of parallel wires. When measuring ions, the suppressor grid is held at -95V relative to the collector, and the same positive voltage square wave is impressed on all three modulator grids. The rest of the grids and the collector are grounded to the spacecraft.

When a square wave voltage is impressed on the modulator grids, the

collector current is a square wave differing in phase by 180° , as shown in Figure 2.6. The amplitude of this square wave is the information which is telemetered back to Earth. During operation, a sequence of such square waves is used. The frequency of the wave is 400hz, and the limiting voltages are changed every 0.240 secs. The voltages are changed such that the higher voltage of any one square wave is the lower voltage of the next. In this way the voltage range of 10-5950 volts is divided into contiguous channels. In the low resolution (L-)mode there are 16 channels; in the high resolution (M-)mode there are 128 channels. Hereafter, we will label the channels with a subscript k. Appendix A includes a table which lists the threshold voltage ϕ_k , the voltage width $\Delta\phi_k$ ($\phi_{k+1} - \phi_k$), and the average voltage $\bar{\phi}_k$ ($(\phi_{k+1} + \phi_k)/2$) for each channel in each of the positive ion modes (L and M). A more thorough description of the instrument and its operation is given in Bridge et al (1977).

In order to define the threshold speeds v_k we will use a coordinate system called cup coordinates. We take as the origin the center of the long side of the aperture (point O in Figure 2.5). The unit vector \hat{z}_{cup} is perpendicular to the aperture plane and points into the cup, the unit vector \hat{x}_{cup} is defined by the vector cross product of \hat{z}_{sc} with \hat{z}_{cup} , and the unit vector \hat{y}_{cup} is defined to make a right-handed coordinate system.

Consider a parallel beam of positive monoenergetic ions incident on the cup. The only particles to reach the collector will be those with a z-component of velocity greater than v_k given by

$$v_k = (2Z^* e\phi_k / Am_p)^{1/2} \tag{2.1a}$$

where Z^* is the charge state of the ion, e is the elementary charge, ϕ_m is the modulator voltage, A is the atomic weight and m_p is the proton mass. It is

convenient to define the average proton speed \bar{v}_k and the channel width Δv_k as

$$\bar{v}_k = (v_{k+1} + v_k)/2 \quad 2.1b$$

$$\Delta v_k = v_{k+1} - v_k \quad 2.1c$$

The values of \bar{v}_k and Δv_k for each channel are included in Appendix A.

In terms of the distribution function $f(\vec{v})$ which describes the plasma environment of the spacecraft, the current in the k-th channel is given by Equations 2.2a and 2.2b.

$$I_k = I_k^* - I_{k+1}^* \quad 2.2a$$

$$I_k^* = A_0 Z e \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dv_x dv_y \int_{v_k}^{\infty} dv_z v_z f(\vec{v}) R(\vec{v}, v_k) \quad 2.2b$$

where A_0 is the aperture area times the transparency at normal incidence and $R(\vec{v}, v_k)$ is the cup response function, to be derived hereinafter. The quantity $f(\vec{v})$ in the above equation is the distribution function of a single species; if the plasma contains more than one species the current will be given by a sum of terms like Equation 2.2b, one for each type of ion.

The quantity I_k can be thought of as a function of \bar{v}_k . For the remainder of this thesis, the quantity $I_k(\bar{v}_k)/\Delta\phi_k$ will be referred to as the reduced distribution function. This is because when the unity response approximation is valid, $I_k/\Delta\phi_k$ is in fact proportional to the object which generally goes by that name (see McNutt et al (1981)).

2.3 Response Function of the Main Sensor

Our problem is to determine the function R. A particle incident on the aperture will reach the collector unless it is stopped by the modulator

voltage, it collides with a grid, or its trajectory is such that it misses the collector and collides with the side of the cup. The first effect is taken care of by the lower limit of the integration over v_z ; the latter two effects are included in R . R can therefore be written as a product of two terms and a normalization constant,

$$R = TA/A_0 \quad 2.3$$

where T is the transparency of the grids and A is the "sensitive" area of the collector. Note that R is normalized to unity at normal incidence. T and A are functions of velocity and channel number, while A_0 is a constant. We will first consider the effect of collisions with the grids (transparency).

2.3a The Grid Transparency

The transparency of a single grid is defined as the probability of an incident particle traversing the plane of the grid without colliding with the wires. We model a grid as a planar structure consisting of two perpendicular sets of parallel cylindrical wires. In the main sensor, the sets of wires of all of the grids are parallel to the x - and y - axes in cup coordinates. The transparency of the grid will be the product of the transparencies of each set of wires considered separately.

Consider the wires which run in the \hat{x} -direction. Since the transparency of these wires does not depend upon v_x , we only need to consider the projection of the motion into the y - z plane. The probability of a particle colliding with one of the wires is simply the ratio of the area of the wires to the area of the gaps between the wires projected into a plane perpendicular to the particle velocity vector (see Figure 2.7). Per unit length along the

wire, this ratio is

$$\text{Probability of collision} = \frac{d}{L \cos \alpha} = c \sec \alpha \quad 2.4$$

where d is the wire diameter, L is the grid spacing, $c(=1/42)$ is their ratio, and α is the angle between the particle velocity and the grid normal. The probability of a particle not colliding with the grid, is

$$\text{Probability of no collision} = 1 - (c \sec \alpha) \quad 2.5$$

If \vec{v}^* is the velocity of the particle as it crosses the grid plane, we see that the transparency of one grid is

$$T = [1 - c(1 + (\frac{v_x^*}{v_z^*})^2)^{1/2}] [1 - c(1 + (\frac{v_y^*}{v_z^*})^2)^{1/2}] \quad 2.6$$

Under the assumption that the potential inside the cup depends only on z , \vec{v}^* depends solely upon the velocity of the particle before it enters the cup and the voltage on the grid. The validity of this approximation will be discussed in the next section. An expression relating \vec{v}^* with \vec{v} is now found from energy conservation to be

$$v_x^* = v_x \quad 2.7a$$

$$v_y^* = v_y \quad 2.7b$$

$$v_z^* = (v_z^2 - 2e\phi / Am_p)^{1/2} \quad 2.7c$$

The transparency of all of the grids is the product of the transparencies of the individual grids. It is given by

$$T = \prod_i [1 - c(1 + \frac{v_x^2}{v_z^2 - \frac{2e\phi_i}{Am_p}})^{1/2}] [1 - c(1 + \frac{v_y^2}{v_z^2 - \frac{2e\phi_i}{Am_p}})^{1/2}] \quad 2.8$$

where ϕ_i is the voltage on the i -th grid

Each cup has three modulator grids, one suppressor grid, and five grounded grids (see Fig. 2.4). The transparency is therefore given explicitly

by

$$T = \left[1 - c \left(1 + \frac{v_x^2}{v_z^2}\right)^{1/2}\right]^5 \left[1 - c \left(1 + \frac{v_x^2}{v_z^2 \left(1 - \frac{v_k^2}{v_z^2}\right)}\right)^{1/2}\right]^3 \left[1 - c \left(1 + \frac{v_x^2}{v_z^2 \left(1 + \frac{v_s^2}{v_z^2}\right)}\right)^{1/2}\right] \times$$

$$\left[1 - c \left(1 + \frac{v_y^2}{v_z^2}\right)^{1/2}\right]^5 \left[1 - c \left(1 + \frac{v_y^2}{v_z^2 \left(1 - \frac{v_k^2}{v_z^2}\right)}\right)^{1/2}\right]^3 \left[1 - c \left(1 + \frac{v_y^2}{v_z^2 \left(1 + \frac{v_s^2}{v_z^2}\right)}\right)^{1/2}\right]$$
2.9

The subscript s refers to the suppressor grid; v_s is defined in a manner analogous to the definition of v_k in Equation 2.1

$$v_s = (2Z^* e \phi_s / A m_p)^{1/2}$$
2.10

where ϕ_s is the voltage on the suppressor grid.

Note that for normal incidence $T = T_0 = (1 - c)^{18} = 0.65$ and $A_0 = 66 \text{ cm}^2$.

2.3b The Sensitive Area

The second factor in Equation 2.3 is the "sensitive area". This is defined to be the overlap of the area of the collector with the area of the image of the aperture in the plane of the collector.

Consider a beam of particles incident at an angle α to the cup normal. In the collector plane, the beam will have the shape of the aperture, but its position will be displaced because of the components of the particle velocity transverse to the cup normal direction, as shown in Figure 2.8. First we will compute the amount of the shift, and then we will discuss the functional dependence of the sensitive area on the shift vector.

We define a two dimensional vector \vec{S} , also shown in Figure 2.8, to be the displacement of the aperture image from a perpendicular projection of the

aperture into the plane of the collector. Figure 2.9 shows the projection into the x-z plane of a possible trajectory of a positive ion as it moves from the aperture to the collector of one of the main sensor cups. If the particles were not deflected by the electrostatic fields inside the cup, the shift would be given by

$$S_x = \frac{v_x}{v_z} h \quad 2.11a$$

$$S_y = \frac{v_y}{v_z} h \quad 2.11b$$

The effect of the fields is to bend the beam, thereby changing the amount of the shift. To compute the amount of the shift, we will solve the equations of motion for a charged particle moving in the electric field of one of the cups.

We will assume that the potential between the grids depends only upon z. This neglects the fine structure of the fields close to grids as well as the fringing fields near the edges of the grids.

The fine structure of the fields in the vicinity of the wires decays in a distance comparable to the mesh size of the grid. Since this distance is much less than the spacing between grids, the ripple in the fields near the wires can safely be ignored.

The fringing fields are important only in a region around the edge of the grids which has a width comparable to the grid spacing. Because of the cup geometry, any particle whose trajectory includes the region where the fringing fields are important will miss the collector. As this will occur whether or not the fringing fields are used in computing the trajectory, these fields can be ignored.

The potential is therefore well approximated by a linear function of distance between any two neighboring grids. Figure 2.10a shows the potential

as a function of z for a typical channel. Using energy conservation and the fact that the fields are entirely in the z -direction the equation of motion is easily solved. The equations of motion of the particle are

$$v_x^* = \frac{dx}{dt} = v_x \quad 2.12a$$

$$v_y^* = \frac{dy}{dt} = v_y \quad 2.12b$$

$$v_z^* = \frac{dz}{dt} = \left(v_z^2 - \frac{2z^* e \phi(z)}{A m_p} \right)^{1/2} \quad 2.12c$$

with $\phi(z)$ given by

$$\phi(z) = \frac{z}{.762} \phi_k \quad 0 < z < .762 \quad 2.13a$$

$$\phi(z) = \phi_k \quad .762 < z < 1.143 \quad 2.13b$$

$$\phi(z) = \frac{1.905 - z}{.762} \phi_k \quad 1.143 < z < 1.905 \quad 2.13c$$

$$\phi(z) = 0 \quad 1.905 < z < 2.286 \quad 2.13d$$

$$\phi(z) = \frac{z - 2.286}{.414} \phi_s \quad 2.286 < z < 2.700 \quad 2.13e$$

$$\phi(z) = \frac{3.089 - z}{.389} \phi_s \quad 2.700 < z < 3.089 \quad 2.13f$$

$$\phi(z) = 0 \quad 3.089 < z < 4.100 \quad 2.13g$$

Equations 2.12a and 2.12b can be solved by inspection. For a particle which crosses the aperture plane at the origin of the cup coordinate system, the result is

$$x = v_x t \quad 2.14a$$

$$y = v_y t \quad 2.14b$$

The components of the shift vector are simply the values of x and y evaluated for t equal to the transit time of the particle from the aperture to the collector plane. This can be evaluated with the aid of Equation 2.12c, which can be rewritten as

$$\int_0^t dt = \int_0^h \frac{dz}{\left(v_z^2 - \frac{2z^* e \phi(z)}{A m_p} \right)^{1/2}} \quad 2.15$$

Inserting the expressions for $\phi(z)$ from Equations 2.13a-2.13g and performing the integrations we find

$$t = t_a + t_b + t_c + t_d + t_e + t_f + t_g \quad 2.16$$

$$t_a = \left(\frac{2}{v_z}\right)(.762) \frac{[1 - (1 - \frac{v_k^2}{v_z^2})^{1/2}]}{(v_z^2/v_k^2)} \quad 2.17a$$

$$t_b = \left(\frac{.381}{v_z}\right) \left(\frac{1}{(1 - \frac{v_k^2}{v_z^2})}\right) \quad 2.17b$$

$$t_c = t_a \quad 2.17c$$

$$t_d = \frac{.381}{v_z} \quad 2.17d$$

$$t_e = \left(\frac{2}{v_z}\right)(.414) \frac{[(1 + \frac{v_s^2}{v_z^2})^{1/2} - 1]}{(v_z^2/v_s^2)} \quad 2.17e$$

$$t_f = \left(\frac{2}{v_z}\right)(.389) \frac{[(1 + \frac{v_s^2}{v_z^2})^{1/2} - 1]}{(v_z^2/v_s^2)} \quad 2.17f$$

$$t_g = \frac{1.011}{v_z} \quad 2.17g$$

Using Equations 2.14a, 2.14b, 2.16, and 2.17a-2.17g the shift vector can be written as

$$S_x = S \left(\frac{v_x}{v_z}\right) h \quad 2.18a$$

$$S_y = S \left(\frac{v_y}{v_z}\right) h \quad 2.18b$$

with S , the shift function, defined by

$$S = .743 \left(\frac{[1 - (1 - \frac{v_k^2}{v_z^2})^{1/2}]}{(v_k^2/v_z^2)} \right) + .093 \left(\frac{1}{1 - (v_k^2/v_z^2)} \right)^{1/2} + .392 \left(\frac{[(1 + \frac{v_s^2}{v_z^2})^{1/2} - 1]}{(v_s^2/v_z^2)} \right) + .340 \quad 2.19$$

We shall now very briefly consider the sensitive area as a function of the shift vector, which we will denote by $A(\vec{S})$. Because of the shapes of the aperture and the collector, this functional dependence is complicated. As there are 16 separate regions where the dependence is different (see Figure 2.11), an analytic representation is cumbersome. $A(S_x, S_y)$ is given in tabular form in Table 1. A plot of the sensitive area as a function of S_y , with S_x as a parameter, is shown in Figure 2.15.

2.4 Structure and Operation of the Side Sensor

The geometry of the D-cup is quite different from that of the main sensor cups. Figure 2.12 shows its cross section. Its aperture and collector are circular, and it has a metal annulus called the guard ring which is located 1.4 cm above the collector. The outer edge of the guard ring is connected to the side of the cup, while its inner diameter is smaller than the diameter of the aperture. The radii of the aperture, guard ring, and collector are 5.64 cm, 5.13 cm, and 6.35 cm, respectively. The distance from the collector to the guard ring is 1.413 cm, and from the collector to the aperture is 6.000 cm. It has eight grids, two of which are suppressor grids and only one of which is the modulator grid for the positive ion mode. The potential as a function of position for a typical channel is shown in Figure 2.10b.

In addition to measuring positive ions, the D-cup is also used to measure electrons. A different modulator grid is used for the electron mode. The potential as a function of position for a typical channel in the electron mode is shown in Figure 2.10c. The voltage thresholds for the two electron modes are included in appendix A.

2.4a The Grid Transparency

We will first consider the transparency function for the D-cup. The only complication not found in the main sensor cups is caused by the fact that the wires meshes in the different grids are not parallel to each other, but are rotated relative to each other by a specific angle. Also, the Voyager I and Voyager II instruments are different from each other. The mesh orientations for both spacecraft are given in Table 2.2.

We define cup coordinates for the D-cup a fashion analagous to that used for main sensor cups; the z_{cup} -axis is parallel to the cup normal and points into the cup, $\hat{x}_{\text{cup}} = \hat{z}_{\text{sc}} \times \hat{z}_{\text{cup}}$, and $\hat{y}_{\text{cup}} = \hat{z}_{\text{cup}} \times \hat{x}_{\text{cup}}$. The origin of the coordinate system is defined to be the center of the aperture. The transparency of any grid is still given by Equation 2.6 provided v_x and v_y are interpreted as the components of velocity along the directions of the grid wires. The grid orientation as given in Table 2.2 can then be used to rotate from cup coordinates to "grid" coordinates. The resulting expression for the

transparency of the D-cup is

$$\begin{aligned}
 T = \prod_{i=1}^6 & \left[1 - c \left(1 + \frac{v_t^2}{v_z^2} \cos^2(\xi - \alpha_i) \right)^{1/2} \right] \left[1 - c \left(1 + \frac{v_t^2}{v_z^2} \sin^2(\xi - \alpha_i) \right)^{1/2} \right] \times \\
 & \left[1 - c \left(1 + \frac{v_t^2 \cos^2(\xi - \alpha_m)}{v_z^2 \left(1 - \frac{v_m^2}{v_z^2} \right)} \right)^{1/2} \right] \left[1 - c \left(1 + \frac{v_t^2 \sin^2(\xi - \alpha_m)}{v_z^2 \left(1 - \frac{v_m^2}{v_z^2} \right)} \right)^{1/2} \right] \times \\
 & \left[1 - c \left(1 - \frac{v_t^2 \cos^2(\xi - \alpha_s)}{v_z^2 \left(1 - \frac{v_s^2}{v_z^2} \right)} \right)^{1/2} \right] \left[1 - c \left(1 - \frac{v_t^2 \sin^2(\xi - \alpha_s)}{v_z^2 \left(1 - \frac{v_s^2}{v_z^2} \right)} \right)^{1/2} \right]
 \end{aligned} \tag{2.20}$$

where the α 's are the angles from Table 2.2 which describe the grid orientation, and ξ and v_t are defined by

$$v_t = (v_x^2 + v_y^2)^{1/2} \tag{2.21a}$$

$$\xi = \arctan(v_y/v_x) \tag{2.21b}$$

At normal incidence, $T = T_0 = (1 - c)^{16} = 0.68$ and $A_0 = 56.2 \text{ cm}^2$.

2.4b The Sensitive Area

We now proceed to the sensitive area calculation. Because of the guard ring, the sensitive area is the mutual overlap of three circles of different sizes, the centers of which lie on the same line. There are now two independent shift vectors, one for the collector-guard ring shift and one for the collector-aperture shift. Because of the circular symmetry, the sensitive area does not depend upon the direction of the shift vector, only its magnitude. We therefore assume, without loss of generality, that the velocity vector of the incident beam lies in the x-z plane. By assumption, the y-components of the shift vectors are zero. We will denote by S_{ac} and S_{gc} the

magnitude of the (x-directed) shifts between the aperture and collector and between the guard and collector, respectively. It is also convenient to define S_{ag} as the relative shift between the aperture and the guard ring, $S_{ag} = S_{ac} - S_{gc}$.

We will now derive an expression for the overlap area of two circles with radii R_s and R_L ($R_L > R_s$) whose centers are separated by a distance S , as shown in Figures 2.13a, 2.13b and 2.13c. Let the coordinates of the centers of the circles be $(0,0)$ and $(s,0)$. The equations of the two circles are

$$x^2 + y^2 = R_L^2 \quad 2.22a$$

$$(x-s)^2 + y^2 = R_s^2 \quad 2.22b$$

An important parameter, which we shall call X , is the x-coordinate of the points where the two circles intersect (see Figure 2.13b). X can be determined as a function of s , R_L , and R_s by subtracting Equation 2.22b from 2.22a and solving for x . The result is

$$X = \frac{R_L^2 - R_s^2 + s^2}{2s} \quad 2.23$$

Note that X as defined by Equation 2.23 is real and well defined even when the circles do not intersect. In this case $X > R_L$ and the corresponding value of y which simultaneously satisfies Equations 2.22a and 2.22b is imaginary. We shall find it convenient to define X by Equation 2.23 even when the geometrical interpretation of it no longer holds.

Now consider the following three cases, as shown in Figures 2.13a, 2.13b, and 2.13c.

$$\text{Case I} \quad s \leq R_L - R_s$$

In this case the overlap area is obviously just the area of the smaller circle.

$$A = \pi R_s^2 \quad 2.24a$$

$$\text{Case II} \quad R_L - R_s < s < R_L + R_s$$

The overlap area is given by the following sum of two integrals.

$$A = \int_{s-R_s}^X [(x-s)^2 - R^2]^{1/2} dx + \int_X^{R_L} [x^2 - R^2]^{1/2} dx \quad 2.25$$

This integral is elementary, yielding

$$A = R^2 \left[\frac{\pi}{2} + \arcsin Q_1 + Q_1 (1 - Q_1)^{1/2} \right] + R^2 \left[\frac{\pi}{2} - \arcsin Q_2 - Q_2 (1 - Q_2)^{1/2} \right] \quad 2.24b$$

$$Q_1 = \frac{R_L^2 - R_s^2 - s^2}{2sR_s} \quad 2.26a$$

$$Q_2 = \frac{R_L^2 - R_s^2 + s^2}{2sR_L} \quad 2.26b$$

$$\text{Case III} \quad s \geq R_L + R_s$$

In this case the overlap area is zero.

$$A = 0 \quad 2.24c$$

We shall denote by X_{ac} and X_{gc} the x-coordinates of the points of intersection of the image of the aperture in the plane of the collector and the collector and of the image of the guard in the plane of the collector and the collector, respectively (see Figure 2.14). These can be evaluated with Equation 2.23 by substituting the radius of the collector for R_L , the radius of the aperture (guard ring) for R_s , and S_{ac} (S_{gc}) for s .

To determine the overlap area of the three circles, one must consider the following two cases;

$$\text{Case A:} \quad X_{gc} > X_{ac}$$

This is the most frequently encountered case, and it covers several apparently different situations. These are all shown in Figures 2.14a-2.14c. For all of these cases the sensitive area is given by

$$A = A_{ag} + A_{gc} - A_g \quad 2.27a$$

where A_{ag} and A_{gc} are the overlap areas of the aperture and the guard ring and

of the guard ring and the collector, respectively, as given by Equations 2.24a-2.24c, and A_g is the area of the guard ring.

$$\text{Case B: } X_{gc} \leq X_{ac}$$

For this case, shown in Figure 2.14d, the sensitive area is the overlap area of the aperture and the collector.

$$A = A_{ac} \tag{2.27b}$$

This completes the derivation of the cup response function.

2.5 The "Cold Plasma" Approximation

The problem of data reduction now formally reduces to inverting a set of integral equations like Equations 2.2 to solve for the distribution function. Unfortunately, this task is very difficult, and a unique solution may not exist. The approach which we have adopted involves parameterizing the distribution function and then searching parameter space for the "best fit" to the data.

From statistical mechanics we know that the distribution function which describes a gas in thermodynamic equilibrium is the Maxwell-Boltzmann distribution

$$f(\vec{v}) = \frac{N_0}{\pi^{3/2} w^3} \exp\{-\frac{(\vec{v}-\vec{V})^2}{w^2}\} \tag{2.28}$$

where N_0 is the particle number density, \vec{V} is the bulk flow velocity, and w is the thermal speed. Although neither the solar wind nor the Jovian magnetospheric plasma is in local thermodynamic equilibrium, there is some empirical justification for using this form for the distribution function. It can be shown that distributions with more than one peak are unstable, and in

the vicinity of the peak one expects the distribution function to be bell shaped. We shall therefore assume that the distribution functions can be approximated by expressions like Equation 2.28.

We now must evaluate the integrals in Equation 2.2b. This cannot be done analytically without further approximations. For the case of a "cold" plasma, i.e. when $|\vec{V}|/w \gg 1$, we can approximate the dependence of $f(\vec{v})$ on the components of velocity transverse to the cup normal by a product of two delta functions

$$f(\vec{v}) \approx \frac{N_0}{w\sqrt{\pi}} \delta(v_x - V_x) \delta(v_y - V_y) \exp\{-(v_z - V_z)^2/w^2\} \quad 2.29$$

This permits the integrations over v_x and v_y to be performed trivially. The result is

$$I_k^* = \frac{N_0}{w\sqrt{\pi}} \int_{v_k}^{\infty} \exp\{-(v_z - V_z)^2/w^2\} R(V_x, V_y, v_z) dv_z \quad 2.30$$

For the D-cup, Equation 2.2b can be evaluated numerically using Equations 2.20, 2.21 and 2.23-2.27. For the main sensor Table 2.1 must be used in addition to Equation 2.9. The use of the lookup table can be eliminated by fitting the area overlap with an easily evaluated function. This approach will be particularly important for the "hot plasma" approximation described hereinafter.

The family of curves representing the sensitive area is shown in Figure 2.15, which shows A plotted as a function of S_y/h , with S_x/h as a parameter.

These functions will be fit by the "trapezoidal approximation"

$$A = A_x(S_x/h)A_y(S_x/h, S_y/h) = \hat{A}_x(S_x/h)\xi(S_x/h)A_y(S_x/h, S_y/h) \quad 2.31$$

$$A_x = \frac{(S_x/h) + X'_r}{X'_r - X_r} \xi(S_x/h) \quad -X'_r < S_x/h < X_r \quad 2.32a$$

$$A_x = 1 \quad -X_r < S_x/h < X'_r \quad 2.32b$$

$$A_x = \frac{(S_x/h) - X'_r}{X'_r - X_r} \xi(-S_x/h) \quad X_r < S_x/h < X'_r \quad 2.32c$$

$$A_x = 0 \quad \text{Otherwise} \quad 2.33d$$

$$A_y = \frac{(S_y/h) + Y'_d}{Y'_d - Y_d} \quad Y'_d < S_y/h < Y_d \quad 2.33e$$

$$A_y = 1 \quad -Y_d < S_y/h < Y'_d \quad 2.33f$$

$$A_y = \frac{(S_y/h) - Y'_u(S_x)}{Y'_u(S_x) - Y_u(S_x)} \quad Y_u(S_x) < S_y/h < Y'_u(S_x) \quad 2.33g$$

$$A_y = 0 \quad \text{Otherwise} \quad 2.33h$$

with

$$X_r = 1.10 \quad 2.34a$$

$$X'_r = 4.94 \quad 2.34b$$

$$Y_d = -2.02 \quad 2.34c$$

$$Y'_d = -3.62 \quad 2.34d$$

$$Y_u = \frac{.762 \cos\{1.018(S_x/h) + .247\}}{1 + 0.25(S_x/h)} \quad 2.34e$$

$$Y'_u = 2.50 - 0.125[(S_x/h) - 1]^2 \quad 2.34f$$

$$\xi(S_x/h) = 1.257 - 0.063(S_x/h) - .126 / \{(S_x/h)^2 - 5.1(S_x/h) + 6.612\} \quad 2.34g$$

All of the quantities defined by Equations 2.34a-2.34f are dimensionless.

Y_u and Y'_u are plotted in Figure 2.16. Figure 2.17 shows the trapezoidal approximation for A plotted versus S_y/h with S_x/h as a parameter, while Figure 2.18 shows A plotted versus S_x/h for S_y equal to 0. Figure 2.19 shows a 3D plot of $A(\vec{S})$. The values of X_r , X'_r , Y_d , Y'_d , Y_u , and Y'_u were chosen so as to

match the volume of the solid of Fig. 2.19 as closely as possible with the volume of the solid representing the true area overlap. Figure 2.20 shows a 3D plot of $R(S_x/h, S_y/h)$ for which the "trapezoidal" approximation was used to evaluate the sensitive area and Eq. 2.9 was used to compute the transparency. Appendices B and C contains listings of Fortran programs which compute the integrals in Equation 2.2b for the main sensor cups using the "trapezoidal" approximation and for the D-cup, respectively.

The "cold plasma" approximation was used to test our theoretical response function by analyzing data taken during a cruise maneuver, as described in the next chapter.

2.6 The "Hot Plasma" Approximation for the Main Sensor

When the thermal speed of the plasma is comparable to the magnitude of the bulk streaming velocity Equation 2.30 is no longer valid. This is the case during Voyager I's pass near the Io flux tube. To perform the integral of Equation 2.2b in this case we have fit the previously derived expression for the transparency, Equation 2.9, by an expression with a functional form which permits the integrals over v_x and v_y to be performed analytically. The expression we have used is a sum of two Gaussians

$$T=T_0 \sum_{i=1}^2 \sum_{j=1}^2 c_i c_j \exp\left(-a_i \left(\frac{v_x}{v_z}\right)^2 - a_j \left(\frac{v_y}{v_z}\right)^2\right) \quad 2.35$$

where c_i , c_j , a_i , and a_j are functions of v_z^2/v_k^2 and of v_z^2/v_s^2 , and T_0 is the transparency at normal incidence. The values of the c's and a's were determined from Equation 2.9 by the following procedure: We reduced the number of independent parameters to 2 by requiring that $c_1+c_2=1.0$ and that the

integral over v_x and v_y of Eq. 2.2b give the correct answer for an infinitely hot plasma, and then did a nonlinear least-squares fit to determine the best values for the a's and c's.

Using a Maxwell-Boltzmann distribution (Equation 2.28) for $f(\vec{v})$, the "trapezoidal approximation" (Equations 2.31-2.34) for A, and Equation 2.35 for T, Equation 2.2b becomes

$$I_k^* = \frac{T_0 z^* e N_0}{\pi^{3/2} w^3} \int_{v_k}^{\infty} dv_z v_z \exp\left(-\frac{(v_z - V_z)^2}{w^2}\right) \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{i,j} c_i c_j \exp\left\{-a_i \frac{v_x^2}{v_z^2} - a_j \frac{v_y^2}{v_z^2} - \frac{(v_x - V_x)^2}{w^2} - \frac{(v_y - V_y)^2}{w^2}\right\} dv_x dv_y \quad 2.36$$

It will be convenient to perform the following change of variables

$$X = S v_x / v_z = S_x / h \quad 2.37a$$

$$Y = S v_y / v_z = S_y / h \quad 2.37b$$

After some algebra Equation 2.36 becomes

$$I_k^* = \frac{N_0 e z^* T_0 R_0}{\pi^{3/2} w^3} \int_{v_k}^{\infty} dv_z \frac{v_z^3}{S^2} \exp\left(-\frac{(v_z - V_z)^2}{w^2}\right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dX dY A(X, Y) \sum_{i,j} C_{ij} G_{ix} G_{jy} \quad 2.38$$

$$C_{ij} = c_i c_j \exp\left\{-\left[\mu_x^2 \left(\frac{a_i w^2}{v_z^2 + a_i w^2}\right) + \mu_y^2 \left(\frac{a_j w^2}{v_z^2 + a_j w^2}\right)\right]\right\} \quad 2.39a$$

$$G_{ix} = \exp\{-\alpha_i (X - \sigma_x)^2\} \quad 2.39b$$

$$G_{jy} = \exp\{-\alpha_j (Y - \sigma_y)^2\} \quad 2.39c$$

$$\alpha_i = \frac{a_i w^2 + v_z^2}{S^2 w^2} \quad 2.39d$$

$$\sigma_x = \frac{S v_z \mu_x w}{v_z^2 + a_i w^2} \quad 2.39e$$

$$\sigma_y = \frac{S v_z \mu_y w}{v_z^2 + a_j w^2} \quad 2.39f$$

$$\vec{\mu} = \vec{V} / w \quad 2.39g$$

with $A(X, Y)$ defined by Equations 2.31-2.34

We are now ready to carry out the necessary integrations. We will perform the integral over Y first. Regrouping terms in Equation 2.36 we find

$$I_k^* = \frac{N_0 z^* e T_0 A_0}{\pi^{3/2} w^3} \sum_{i,j} \int_{-\infty}^{\infty} \frac{v_z^3}{v_k} D_{ij} \left[\int_{-\infty}^{\infty} dX (A_X(X) G_{iX}(X)) \int_{-\infty}^{\infty} dY (A_Y(X,Y) G_{jY}(Y)) \right] \quad 2.40a$$

$$D_{ij} = c_i c_j \exp \left\{ - \left[\left(\frac{v_z}{w} - \mu_z \right)^2 + \mu_x^2 \left(\frac{a_i w^2}{v_z^2 + a_i w^2} \right) + \mu_y^2 \left(\frac{a_j w^2}{v_z^2 + a_j w^2} \right) \right] \right\} \quad 2.40b$$

The Y-integration can be expressed in terms of elementary functions and the Gaussian error function. The result is

$$H_j(X) = \int_{-\infty}^{\infty} dY (A_Y(X,Y) G_{jY}) = \frac{\sqrt{\pi}}{2\alpha_j} \left\{ \frac{\phi(Z'_u) - \phi(Z_u)}{Z'_u - Z_u} - \frac{\phi(Z'_d) - \phi(Z_d)}{Z'_d - Z_d} \right\} \quad 2.41a$$

$$\phi(Z) = Z \operatorname{erf}(Z) + (1/\sqrt{\pi}) \exp(-Z^2) \quad 2.41b$$

$$Z_d = \alpha_j (Y_d(X) - \sigma_j) \quad 2.41c$$

$$Z'_d = \alpha_j (Y'_d(X) - \sigma_j) \quad 2.41d$$

$$Z_u = \alpha_j (Y_u(X) - \sigma_j) \quad 2.41e$$

$$Z'_u = \alpha_j (Y'_u(X) - \sigma_j) \quad 2.41f$$

Equation 2.36 now becomes

$$I_k^* = \frac{N_0 z^* e T_0 A_0}{\pi^{3/2} w^3} \sum_{i=1}^2 \sum_{j=1}^2 \int_{-\infty}^{\infty} \frac{v_z^2}{v_k} D_{ij} \left[\int_{-\infty}^{\infty} dX (A_X(X) G_{iX}(X) H_j(X)) \right] \quad 2.42$$

Since the integral over X is now very complicated, it cannot be done without further approximations. If $H_j(X)$ is a slowly varying function of its

argument, we can use the saddle point method to write the integral over X as

$$\int_{-\infty}^{\infty} dX A_X(X) G_{iX}(X) H_j(X) \cong H_j(\bar{X}_i) \xi(\bar{X}_i) \int_{-\infty}^{\infty} dX A_X(X) G_{iX}(X) \quad 2.43a$$

$$\bar{X}_i = \frac{\int_{-\infty}^{\infty} |X| A_X(X) G_{iX}(X) dX}{\int_{-\infty}^{\infty} A_X(X) G_{iX}(X) dX} \quad 2.43b$$

Again, all the integrals can be expressed in terms of elementary functions and the Gaussian error function. The results are

$$F_i = \int_{-\infty}^{\infty} A_X(X) G_{iX}(X) dX = \frac{\sqrt{\pi}}{2\alpha_i} \left\{ \frac{\phi(Z'_{ri}) + \phi(Z'_{li}) - \phi(Z_{ri}) - \phi(Z_{li})}{Z'_r - Z_r} \right\} \quad 2.44a$$

$$Z_{ri} = \alpha_i (X_r - \sigma_i) \quad 2.44b$$

$$Z'_{ri} = \alpha_i (X'_r - \sigma_i) \quad 2.44c$$

$$Z_{li} = \alpha_i (X_l - \sigma_i) \quad 2.44d$$

$$Z'_{li} = \alpha_i (X'_l - \sigma_i) \quad 2.44e$$

$$X_l = -X_r = -1.10 \quad 2.44f$$

$$X'_l = -X'_r = -4.94 \quad 2.44g$$

$$\int_{-\infty}^{\infty} |X| A_X(X) G_{iX}(X) dX = \frac{1}{2\alpha_i} \left\{ \frac{\psi(Z_l) + \psi(Z'_r) - \psi(Z'_l) - \psi(Z_r) + \frac{2}{\sqrt{\pi}} (Z'_r - Z_r) \phi(\sigma/\alpha_i)}{Z'_r - Z_r} \right\} \quad 2.44h$$

$$\psi(Z) = (\sigma_i/\alpha_i Z - 1/2) \sqrt{\pi} \operatorname{erf}(Z) + \sigma_i/\alpha_i \exp(-Z^2) \quad 2.44i$$

We now summarize the results of this section by rewriting Eq. 2.2 in terms of the functions defined by Eqs. 2.19 and 2.39-2.44 as

$$I_k^* = \frac{z^* e^T A_0 N_0}{4\sqrt{\pi} w^3} \sum_{i=1}^2 \sum_{j=1}^2 \int_{-\infty}^{\infty} dv_z \frac{v_z^3 D_{ij}}{S^2 \sqrt{(\alpha_i \alpha_j)}} H_j(X_i) F_i \quad 2.45$$

The integral over v_z in Eq. 2.45 must be performed numerically. To devise an efficient scheme for calculating I_k for many adjacent channels, we

shall rewrite Eqs. 2.2a and 2.2b as

$$I_k = \int_{v_k}^{v_{k+1}} dv_z Q(v_z, v_k) + \int_{v_{k+1}}^{\infty} dv_z (Q(v_z, v_k) - Q(v_z, v_{k+1})) \quad 2.46$$

where Q denotes the integrand of Eq. 2.45. The contribution from the second integral on the right side of Eq. 2.46 is called the feedthrough current. The dependence of Q upon v_k is implicit in its dependence on the a 's and c 's.

Since the a 's and c 's are slowly varying functions of v_k for $v_z \ll v_k$, we can expand the second integrand in Eq. 2.46 in a Taylor series in the a 's and c 's.

$$\begin{aligned} Q(v_z, v_k) - Q(v_z, v_{k+1}) = & Q(v_z, c_1, c_2, a_1, a_2) + (\partial Q / \partial c_1)(c_1(v_z, v_k) - c_1(v_z, v_{k+1})) + \\ & (\partial Q / \partial c_2)(c_2(v_z, v_k) - c_2(v_z, v_{k+1})) + (\partial Q / \partial a_1)(a_1(v_z, v_k) - a_1(v_z, v_{k+1})) + \\ & (\partial Q / \partial a_2)(a_2(v_z, v_k) - a_2(v_z, v_{k+1})) \end{aligned} \quad 2.47$$

This enables us to use compute one value of $Q(v_z)$ and use it in the numerical integration of several channels, greatly reducing the amount of computer time required to simulate an M-mode. Appendix D contains the listing of a Fortran program which utilizes this technique to simulate an M-mode spectrum. We expect that this program can simulate the response of the main sensor cups to a convected Maxwellian of any temperature or flow direction to within about 7%.

2.7 The "Hot Plasma" Approximation for the D-Cup

Except for a small effect due to the alignment of the grids, the D-cup response is azimuthally symmetric about its z-axis. Therefore, the response can only depend upon $v_x^2 + v_y^2$ and cannot depend upon v_x and v_y individually. We have fit the the full response function by a function of the form

$$R(\vec{v}, v_k, v_s) = \sum_{i=1}^3 c_i \exp\left\{-a_i \frac{v_x^2 + v_y^2}{v_z^2}\right\} \quad 2.48$$

where once again the a_i 's and c_i 's are functions of v_z^2/ϕ_k and ϕ_s . Using a Maxwellian for the distribution function and Equation 2.48 for R in Equation 2.2b we obtain

$$I_k^* = \sum_{i=1}^3 c_i \frac{A_0 T_0 N_0 z^* e}{\pi^{3/2} w^3} \int_{-\infty}^{\infty} dv_z v_z \exp\left\{-\frac{(v_z - V_z)^2}{w^2}\right\} \times \int_{-\infty}^{\infty} \exp\left\{-\frac{(v_x - V_x)^2}{w^2} - \frac{a_i v_x^2}{v_z^2}\right\} dv_x \int_{-\infty}^{\infty} \exp\left\{-\frac{(v_y - V_y)^2}{w^2} - \frac{a_i v_y^2}{v_z^2}\right\} dv_y \quad 2.49$$

The integration over the transverse components of the velocity are easily done, yielding

$$I_k^* = \sum_{i=1}^3 c_i \frac{A_0 T_0 N_0 z^* e}{\sqrt{\pi} w} \int_{-\infty}^{\infty} \frac{v_z dv_z}{v_k (1 + a_i w^2/v_z^2)} \exp\left\{-\left(\frac{v_z - V_z}{w}\right)^2 - \frac{a_i (v_x^2 + v_y^2)}{a_i w^2 + v_z^2}\right\} \quad 2.50$$

As in the case of the main sensor cups, the integration over v_z must be done numerically.

This completes the discussion of the response function of the Voyager PLS experiment.

Chapter 3

Experimental Test of the Response Function

3.1 The Cruise Maneuver

Before the formulas derived in the preceding chapter can be used with confidence to analyze data, they should be tested. Attempts to measure the response of the cups as a function of angle were made prior to launch. This was done by placing the instrument in an evacuated chamber in the path of a beam of charged particles. Data were then taken with the instrument in different orientations.

Unfortunately, it was not possible to test the cup response function to the desired accuracy. Tests were performed with both a proton beam and an electron beam. Since the quantity which is measured is an integral over velocity space of the product of the response function and the distribution function (Eqs. 2.2a,b), it is desirable to have a beam with a small thermal velocity dispersion. The proton beam had too great a velocity dispersion for the desired measurement. The electron beam, although sufficiently narrow, caused the emission of a large number of secondary electrons which contaminated the data. In particular, the response at large angles of incidence was different from what was expected.

The quiet solar wind, on the other hand, has ideal properties for use as a test beam. At 1 AU, the magnitude of its bulk streaming velocity is typically eight times its thermal speed, and this ratio increases with distance from the sun. In order to test the response function, however, the direction of the test beam must be varied. Since the direction of the solar wind is steady to within a few degrees, this requirement can only be met by rotating the spacecraft.

On 14 September 1978, Voyager I, then 4.1 AU from the sun, executed a series of rotations called a cruise maneuver. Data from this particular

maneuver were selected for analysis because during it the solar wind was quiet and the Mach number of the flow was high (~20). The maneuver consisted of ten rotations about the spacecraft z-axis(rolls), ten rotations about the spacecraft y-axis(yaws), and ten more rolls. Each rotation took about 33 minutes. Since the symmetry axis of the main sensor is aligned with the spacecraft z-axis, only the yaws are useful for the purpose of determining the cup response because the angle of the solar wind to the cup normals does not change appreciably during the rolls.

During the cruise maneuver the PLS experiment was taking one M-mode measurement every 96 seconds. Due to telemetry rate constraints, the data from all 128 channels of each spectrum was not sent back to Earth. On alternate spectra, the data from channels 1-72 and 57-128 were transmitted. During the maneuver the solar wind speed was about 380 km/sec, so the peaks in the spectra were never in a channel higher than about 68. Thus, only the spectra containing channels 1-72 could be used for analysis when the beam was oblique to one or more of the cups.

Due to a coincidence, the period between PLS M-mode spectra and the rotation period of the spacecraft during the maneuver were almost commensurate, resulting in the spacecraft having almost the same orientation at the times of corresponding spectra taken in different rotations. Since an odd number (21) of M-mode spectra were taken during each rotation, an upper-half spectrum was taken with the spacecraft in the same orientation as a lower-half spectrum from the previous rotation. We therefore did not lose any angular coverage due to the fact that only 72 of the 128 channels were available from each spectrum.

The angular coverage of the cruise maneuver is shown in Figure 3.0, which is a polar plot of a unit vector which points radially away from the

sun. This vector, the nominal solar wind direction, is plotted in cup coordinates at the times of the start of the M-modes for each of the four cups. The polar angle θ and the azimuth angle ϕ of a vector \vec{V} are defined by Equations 3.1a and 3.1b

$$\theta = \cos^{-1}(V_z / \sqrt{(V_x^2 + V_y^2 + V_z^2)}) \quad 3.1a$$

$$\phi = \tan^{-1}(V_y / V_x) \quad 3.1b$$

where V_x , V_y , and V_z are the cartesian components of \vec{V} . The numbered points in Fig. 3.1 correspond to the spectra which were analyzed as described hereinafter.

3.2 Analysis of Data Taken During the Cruise Maneuver

To test the response function we adopted the following strategy. We analyzed data from all of the orientations for which there were signals in at least three of the four cups. We then did a simultaneous fit to these data using the "cold" plasma approximation described in Section 2.5. The derived macroscopic plasma parameters were then compared with each other. We also fit an additional spectrum taken when the plasma flow direction was almost aligned with the main sensor symmetry axis. In this case we expect the "unity" response approximation to be valid. Since we have confidence in the parameters we derive at these times we can estimate how much the solar wind is changing between the times of the other measurements and see if the parameters derived from the other measurements stay reasonably steady. If they are, this fact and the goodness of the fits indicates how well we understand the response function. In addition, comparison of the parameters derived from the fits of the solar wind using the "unity" response approximation, where appropriate, with parameters derived from

fits to these same spectra using the "cold beam" approximation indicate the systematic error, if any, which the former approximation introduces.

Nine spectra taken during the cruise maneuver were analyzed. The numerical integrations in Equation 2.30 were done using Simpson's rule. For a term with velocity threshold v_k , we chose a step size of $\Delta v_k/10$. Since the integrand is formally undefined at $v_z = v_k$, the proper limiting value of zero was used.

The spectra consist of a background and one or two peaks. The main peak, due to protons, was fitted. In each cup, about twelve channels around the peak were included. The fits have five parameters; the three components of bulk velocity in spacecraft coordinates, the density, and the thermal speed. The velocity was then rotated into cup coordinates and the currents were computed.

The criterion used to define the "best" fit was the minimization of χ^2 , defined by

$$\chi^2 = \sum_i (D_i - A_i)^2 / (.04 D_i)^2 \quad 3.2$$

where each of the D_i 's is the measured current divided by the channel voltage width and the A_i 's are the simulated reduced distribution functions. The solution to the extremum problem was found using a gradient search algorithm similar to that described in Bevington (1969). The derivatives with respect to the velocity components and thermal speed were computed numerically, while the derivative with respect to the density was computed analytically.

The results of the analysis of the cruise maneuver are shown in Table 3.1 and Figures 3.2-3.10. Figure 3.2 corresponds to point 1 in Fig. 3.1, Fig. 3.3 to point 2, etc. Figures 3.2-3.10 are plots of reduced

distribution function versus velocity for the nine spectra used. The data are represented by the "staircase" while the fit is represented by the smooth curve. Table 3.1 lists the time, the wind velocity in RTN coordinates, the density, and the thermal speed derived from the fits for each of the spectra. (RTN coordinates are a sun centered orthogonal system. \hat{r} points radially away from the sun; \hat{t} lies in a plane parallel to the ecliptic and points in a positive sense when viewed from the North, and \hat{n} completes a right-handed system.)

The variation in these parameters is about what one expects to find in a quiet solar wind, and the fits are quite good. The fits correctly reproduce the relative heights, positions, and shapes of the spectra in the oblique cups.

The question of choosing the proper criterion for determining the "best fit" deserves more discussion. If one chooses to minimize the square of the difference between the data and the fitting function, there remains the problem of choosing the proper statistical weights. This choice must be made by analyzing the sources of error in the measurement. In our case, there are two main sources of error; electrical noise and digitization error. Since the logarithm of the currents is digitized, this error is a fixed percentage of the signal. This accounts for the weight factor of $1/0.04D_i^2$ in Eq. 3.2. The electrical noise is a more difficult problem. We expect thermal fluctuations in the amplifiers to be seen as fluctuations in the current. The rms power in these current fluctuations should be about the same in all of the channels. Since the reduced distribution function is the current divided by the channel width and the channel width increases with increasing channel number, this component of the noise should be most pronounced in the low channels.

An example of spectra which we expect to be entirely noise are the D-cup spectrum in Fig. 3.4 and the lower channels in the main sensor cups in the same figure. The predominately smooth trace is due to the signal being less than the minimum that can be digitized with our coding scheme; the fluctuations are noise. The predominant smoothness of the curve, especially the lower channels in the B-cup, indicate that the true thermal noise is below the threshold of the detector. We conclude that the noise which we see has another origin. Other sources of noise, such as cosmic rays or interference with other instruments on the spacecraft, are more difficult to estimate. I have accounted for them in my selection of which channels to include in the fits. Unless specifically mentioned, all of the graphs of the results of fits to spectra include all of the data in a particular spectrum and the simulations for all of the channels used in determining the best fit, and no others.

The question of estimating the uncertainty in the macroscopic plasma parameters derived from the previously described fitting procedure deserves discussion. The formal uncertainty in the fit parameters is contained in the so-called error matrix. The properties of this matrix are described in Bevington (1969). The uncertainties which I quote throughout the remainder of this chapter are defined by

$$\sigma_i^2 = \epsilon_{ii} \chi^2 / n_f \quad 3.3$$

where σ_i is the formal uncertainty in the determination of the i -th parameter, ϵ_{ij} is the error matrix, and n_f is the number of degrees of freedom (number of data points/number of parameters). In the case of linear parameters and Gaussian statistics, σ is simply the standard deviation one would expect to find in the value of the i -th parameter if the same experiment were done many times.

If the uncertainty in the individual data points can be accurately estimated, σ_i^2 is simply equal to ϵ_{ii} . Unfortunately, the values of χ^2 we obtained in the fits of the cruise maneuver were sufficiently large to convince us that we had underestimated the uncertainty in the measurement. The factors which multiply ϵ_{ii} in Eq. 3.3 are an attempt to compensate for this underestimate.

In addition to random errors estimated by σ , there is always the possibility of some systematic error. There are many possible causes for systematic errors. They can be related to the detector, such as uncertainty in the values of the threshold voltages, or they can be related to the plasma itself, such as the presence of suprathermal tails to the distribution functions or thermal anisotropy. In general, systematic errors are more difficult to estimate than random errors.

3.3 Discussion and Conclusions

An examination of Table 3.1 shows that the formal errors in the density and thermal speed are about 3%. In addition, the magnitude of the bulk flow velocity is determined with a precision of less than one percent, and the direction is determined to within 10 arcseconds. We expect that the systematic errors in the determination of these quantities is considerably larger.

The quality of the fits, even at large angles of incidence, gives us great confidence in the accuracy of the response function. For example, compare Figures 3.1, 3.1a, 3.1b, and 3.5. (The following numbering scheme is used; the spectrum in Figure 3.n was taken when the orientation of the spacecraft was such that the nominal solar wind direction corresponded to

the points marked with the number n in Figure 3.0.) In the spectrum of Fig. 3.1, the angles of the solar wind velocity to the A-, B-, and C-cup normals were 35° , 70° , and 55° , respectively. Note that, although the flow is highly oblique, the fit successfully fits the relative heights, locations, and shapes of the peaks.

In Figure 3.1a, the results of simulating the low energy channels in the B-cup, which were not used in the fit, are shown. The simulation shows that the currents in these channels are not all noise, but include a real signal.

Figure 3.1b shows a simulation of what a hypothetical cup with unity response would see in the conditions of Fig. 3.1. Notice that only the location of the peaks remains the same, as all of the peaks are the same height and shape if all of the incident particles reach the collector.

For the case of the B-cup, the effect of the response function is to reduce the height of the peak by a factor of about 5, to broaden it, and to make it skewed. All of these effects can be easily understood qualitatively. The reduction of the height of the peak is caused by the effects of both the sensitive area and the grid transparency. The broadening and skewness of the peak are both caused by the "feedthrough" effect; the contribution to the signal by modulation of particles with $v_z > v_{k+1}$. This effect tends to widen the left side of the peak; there are very few particles which satisfy the above condition on the right side of the peak!

The actual reduction in the size of the signal is much more than the factor of five mentioned in the preceding paragraph. Consider the spectrum shown in Fig. 3.5, taken when the solar wind was flowing down the main sensor symmetry axis. Although the heights of the peaks is about the same

as those in the simulation of Fig. 3.1b, one must remember that the quantity which is actually measured is the current, not the reduced distribution function. Since the current is equal to the reduced distribution function times the channel voltage width, and the channel widths increase with increasing channel number, the ratio of the current in the peak channel in the B-cup of Fig. 3.5 to the same quantity for the spectrum of Fig. 3.1 is about 25. Our response is thus good even when only a few percent of the particles in the beam reach the collector. The accuracy of our expression for the response function over such a wide range of angles and signal strengths is important since in the I_0 flux tube flyby the flow is highly oblique to the cups.

The sensitivity of the array of four detectors to the direction of the flow is dramatically shown in Figs. 3.8 and 3.8a. The fit for Fig. 3.8a was done assuming that the flow velocity in spacecraft coordinates was the same for all four cups. This neglects effects due to the fact that the spacecraft was spinning while the measurements were being taken. We did another fit to the same data in which we compensated for the change in the spacecraft orientation between the times of the measurements of the channel with the largest current in each cup. The result is shown in Fig. 3.8. This is clearly a much better fit to the data. The angle that the spacecraft rotated through between the time of the measurement of channel 24 (A-cup peak) and channel 46 (C, D-cup peaks) was only about 0.9° .

Since the two spectra shown in Figs. 3.5 and 3.6 were taken when the solar wind velocity was almost parallel to the main sensor symmetry axis, we also fit these with the assumption of unity response. A comparison of the results of these fits with the results of the fits of the corresponding spectra using the full response function is included in Table 3.1. The

quality of the fit, as measured by χ^2 , was significantly better for the fits which used the full response function. The x- and y- components of the velocity in spacecraft coordinates agreed to within statistical error, while the z-components of the full response fit exceeded the z-components of unity response fits by about 1%. In addition, the unity response results overestimated the thermal speeds by 7% and underestimated the density by 5%. All of these discrepancies are easily understood. The density difference is caused by the fact that when the flow velocity makes an angle of about 20° to the cup normal, the grid transparency is already slightly less than it is for normal incidence. The other two effects are both caused by the "feedthrough" contribution. The effect of feedthrough is to widen the peak on the low energy side, resulting in a lower estimate of the normal component of the velocity and a higher estimate of the thermal speed.

We must conclude, however, that the unity response approximation is quite good at small angles of incidence, as expected. Uncertainty in the gain of the amplifiers probably introduces a larger error into the density determination than the effect due to the response. Since the response by its nature widens and introduces a certain amount of skewness into the peaks, its effect on the determination of the thermal anisotropy and the heat flux should be considered.

In summary, we have shown that the analytic formulas describing the response function derived in Chapter 2 accurately describe the response of the cups for angles up to 70° for the main sensor cups and 55° for the D-cup. The expressions also correctly predict the complete absence of a signal for angles of incidence $>80^\circ$ for the main sensor cups and $>65^\circ$ for the D-cup.

Chapter 4

The Theory of the Interaction

Between Io and the Jovian Magnetosphere

4.1 Introduction

In Chapter 5 we will present analysis of Voyager I data taken in the vicinity of the satellite Io, shortly after the spacecraft's closest approach to Jupiter. In the present chapter, a description of the plasma environment at Io and a theory of the Io-magnetosphere interaction will be presented. The model plasma parameters which we use in this discussion are the result of a combination of a large number of ground based and in situ measurements. In particular, on its inbound pass the symmetry axis of Voyager I's PLS experiment was pointed in the direction of the corotating flow when the spacecraft flew past Io's orbit, enabling it to make measurements which could be analyzed using the unity response approximation.

Io itself is a large satellite, only slightly smaller than the Moon. Its physical properties are summarized in Table 4.1. Orbiting Jupiter at a distance of 424 thousand kilometers ($5.9 R_J$), Io is deep within the magnetosphere, which extends to 50-100 R_J . Since the Jovian magnetic field is very strong at Io's orbit (~ 0.2 gauss) and Jupiter's ionosphere is highly conducting, we expect the plasma to rigidly corotate with the planet. Voyager I observed this on its inbound pass (Bagenal and Sullivan (1981)). Because Io's orbital period of 42.8 hours is longer than Jupiter's rotation period of 10 hours, the bulk velocity of the plasma in Io's vicinity is greater than Io's orbital velocity by ~ 58 km/sec.

One of the great discoveries made by Voyager I was the volcanoes on Io. Io is the most volcanically active body known, and its volcanoes are believed to be the ultimate source of not only the neutral cloud around Io

(which has been observed from Earth) and the plasma torus which surrounds Jupiter at the radius of Io's orbit, but also most of the plasma in the entire Jovian magnetosphere. Voyager I inbound observed two distinct regions in the torus; the inner or cold torus and the outer, hot torus. In the cold torus, where individual peaks in the spectra were resolved, several different ion species, primarily different ionization states of oxygen and sulfur, were observed. The chemical complexity was also present in the hot torus, although the merging of the peaks in the spectra made the analysis more difficult and the results somewhat ambiguous.

To estimate the magnitude of the relevant time and distance scales in the vicinity of Io, we have used the model plasma torus of Johnson and Strobel, consisting of the following species: O^+ , O^{++} , S^+ , S^{++} , S^{+++} , and electrons. The values of many characteristic plasma parameters for this model torus is given in Table 4.2. A magnetic field strength of .019 Gauss, the value measured by the Voyager I magnetometer experiment (Acuna et al (1981)), was used to estimate the gyroradii and Larmor frequencies. Examination of Table 4.2 indicates that the characteristic time for the plasma to flow past Io (30 sec) is long compared with the Ion gyroperiod (0.5 sec), and short compared with all of the collision times. In addition, the Debye length (65 cm) is very small compared with the radius of Io (1820 km). We therefore expect that the ideal MHD approximation, described hereinafter, is valid and that collisional effects are negligible.

4.2 The Equations Which Describe the Plasma at Io

In section 4.3, we shall present a review and critique of a theory of the Io-magnetosphere interaction which is based on the ideal MHD equations. In the present section we review the derivation of the those equations for the purpose of examining the approximations required for their validity and justifying their use for the purpose of describing the interaction between Io and the Jovian magnetospheric plasma.

We take as our starting point the set of fluid equations which are obtained by taking moments of the kinetic (Boltzmann) equation and summing over the different ionic species. The Boltzmann equation is valid whenever the statistical correlation between the phase space position of two particles can be neglected. This approximation is quite good for the conditions which exist in the Jovian ionosphere, as it is for almost all space plasmas.

The fundamental equations are those of conservation of mass, momentum, and energy, and the Maxwell equations, supplemented by the equation of state, equations for the anisotropic structure of the pressure tensor, the generalized Ohm's law, and an equation for the heat flux.

The equation of mass conservation is

$$\frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial x_i} (\rho V_i) \quad 4.1$$

where ρ is the mass density and V_i is the i -th component of the bulk velocity of the plasma as a whole. For the remainder of this chapter, we shall use the Einstein summation convention, i.e. all repeated indices are summed over.

The equation of momentum conservation is

$$\frac{\partial}{\partial t}(\rho V_i + \frac{S_i}{c^2}) = -\frac{\partial}{\partial x_k}(P_{ik} + \rho V_i V_k - T_{ki}) + g_i \quad 4.2$$

where P_{ik} is the pressure tensor, g_i is the local gravitational field, and T_{ik} and S_i are the Maxwell stress tensor and the Poynting vector, respectively, defined by

$$T_{ik} = \frac{1}{4\pi}(B_i B_k + E_i E_k - \frac{1}{2} \delta_{ik}(E^2 + B^2)) \quad 4.2a$$

$$S_i = \frac{c}{4\pi}(\vec{E} \times \vec{B})_i \quad 4.2b$$

If the equations are written in a non-inertial frame of reference, \vec{g} should include the D'Alembertain forces. If the frame of reference is rotating, the the coriolus force should be included by adding a term $-2\vec{\Omega} \times \vec{v}$ to Eq. 4.2, where $\vec{\Omega}$ is the angular velocity of the reference frame.

The equation of energy conservation is

$$\frac{\partial}{\partial t}(\rho V^2 + \epsilon_T + \epsilon_{EM}) = -\frac{\partial}{\partial x_k}(V_k(\rho V^2 + \epsilon_T) + P_{kj} V_j + S_k + q_k) + \rho g_k V_k \quad 4.3$$

where ϵ_T is the thermal energy per unit volume, q_k is the heat flux vector, and ϵ_{EM} is the electromagnetic energy density, defined by

$$\epsilon_{EM} = \frac{1}{8\pi}(E^2 + B^2) \quad 4.3a$$

Maxwell's equations are Gauss's law

$$\nabla \cdot \vec{E} = 4\pi\eta \quad 4.4$$

where η is the electric charge density; the Faraday induction law

$$\nabla \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = 0 \quad 4.5$$

Ampere's law

$$\nabla \times \vec{B} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c} \vec{J} \quad 4.6$$

and the divergence-free condition on \vec{B} .

$$\nabla \cdot \vec{B} = 0 \quad 4.7$$

The equation of state relates ϵ_T to p and ρ , $\epsilon_T = \epsilon_T(p, \rho)$, where p , the scalar pressure, is one-third of the trace of P_{ik} . For a gas of non-relativistic particles with no internal degrees of freedom or long-range interactions the equation of state is

$$\epsilon_T = \frac{3}{2} p \quad 4.8$$

For any plasma which can be accurately described by the Boltzmann equation, the contribution of long-range forces to the internal energy is small and can be neglected. Therefore, a plasma consisting only of electrons and protons, particles which have no internal degrees of freedom, is accurately described by Eq. 4.8. For a plasma of partially ionized heavy ions, the condition of no internal degrees of freedom is violated if the thermal kinetic energy of the ions or electrons is comparable to the excitation energies. In the hot plasma torus, we do not expect Eq. 4.8 to apply, since the thermal kinetic energy of the plasma is comparable to the excitation energy of the ions.

For a plasma in which the negative ions are electrons, the generalized

Ohm's law is formally given by (see Rossi and Olbert (1970), pp 347-350)

$$\begin{aligned} \frac{\partial J_i}{\partial t} = & - \frac{\partial}{\partial x_k} (J_k V_i + J_i V_k - \eta V_i V_k) + n g_i + \\ & + \frac{e}{m_e} \frac{\partial P_{e,ik}}{\partial x_k} + \frac{e^2 n_e}{m_e} (\vec{E} + \frac{\vec{V}}{c} \times \vec{B})_i - \frac{e}{m_e} \{ (\vec{J} - \eta \vec{V}) \times \frac{\vec{B}}{c} \}_i + (\frac{\delta J}{\delta t})_{\text{coll}} \end{aligned} \quad 4.9$$

where n_e is the number density of electrons, m_e is the electron mass, and $P_{e,ik}$ is the partial pressure tensor for the electron component of the plasma, and $(\delta J / \delta t)_{\text{coll}}$ is the time rate of change of current density due to collisions.

Unfortunately, this set of equations is not closed; there are more unknowns than equations. This situation is usually remedied by making some additional assumption regarding q_i and P_{ik} and neglecting some of the terms in the generalized Ohm's law. One commonly made set of assumptions is that $q_i = 0$, P_{ik} is isotropic ($P_{ik} = p \delta_{ik}$), and $\vec{E} = -\vec{V} \times \vec{B} / c$.

In the vicinity of I_0 , we expect P_{ik} to be gyrotropic; that is, P_{ik} will have cylindrical symmetry about the local direction of \vec{B} . This condition must be satisfied whenever the ion gyroradius is small compared to the length scale of gradients of any of the macroscopic plasma parameters. In this case, the pressure tensor can be written as

$$P_{ik} = p_{\perp} \delta_{ik} + (p_{\parallel} - p_{\perp}) \frac{B_i B_k}{B^2} \quad 4.10$$

where p_{\parallel} and p_{\perp} are called the parallel pressure and the perpendicular pressure, respectively. The equations can then be closed by introducing an equation for the heat flux, appropriate approximations in the generalized Ohm's law and supplementary equations, if necessary, and an equation relating the parallel and perpendicular pressures. The effect of the

pressure anisotropy in the resulting equations can be expressed in terms proportional to $p_{\parallel} - p_{\perp}$. Since these terms are usually small corrections, they do not qualitatively effect the solutions. We shall therefore neglect them and assume a scalar pressure.

The equations as they now stand are very difficult to solve, and they describe an incredible richness of phenomena which are not of primary importance in understanding the interaction between Io and the Jovian magnetospheric plasma. In particular, we shall be concerned only with changes on time scales long compared to the proton gyroperiod. The characteristic time for the flow of the plasma past Io is of the order of R_{Io}/V , or about 30 sec, while the ion gyroperiods are of the order of 0.5 sec. This enables us to make many further approximations which will greatly simplify the equations.

We start making our approximations with the generalized Ohm's law. For the conditions which we expect to encounter in the flux tube, dimensional analysis (Rossi and Olbert) shows that all of the terms are negligible compared to the term proportional to $\vec{V} \times \vec{B}$. We therefore can write

$$\vec{E} = - \frac{\vec{V}}{c} \times \vec{B} \quad 4.11$$

Using Eq. 4.11 and the conservation of mass equation (Eq. 4.1), we can rewrite Faraday's law as

$$\left(\frac{\partial}{\partial t} + \vec{V} \cdot \nabla \right) \frac{\vec{B}}{\rho} = \left(\frac{\vec{B}}{\rho} \cdot \nabla \right) \vec{V} \quad 4.12$$

This is called the MHD "frozen-in" law, because it is equivalent to the statement that the magnetic flux linked by any closed curve which moves

along with the plasma does not change with time; hence the field is "frozen-in" to the plasma.

Maxwell's displacement current can be omitted from Ampere's law, yielding

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} \quad 4.13$$

Turning now to the equation of momentum conservation, we can safely neglect \vec{S} and the terms in the Maxwell stress tensor which contain \vec{E} . While the gravitational term is important in the overall dynamics of the Jovian magnetosphere, it can safely be neglected for a treatment of the interaction with Io. Incorporating these approximations, assuming a scalar pressure, and using the equation of conservation of mass (Eq. 4.1) and some vector identities, we rewrite Eq. 4.2 as

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right)\vec{v} = -\frac{1}{\rho} \nabla p + \frac{1}{4\pi\rho} \{(\vec{B} \cdot \nabla)\vec{B} - \frac{1}{2} \nabla B^2\} \quad 4.14$$

Using Eq. 4.11 and assuming an ideal gas equation of state and a scalar pressure, the energy equation takes the particularly simple form

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right)\left(\ln\left(\frac{p}{\rho^{5/3}}\right)\right) = -\frac{2}{3p} \nabla \cdot \vec{q} \quad 4.15$$

Noting that $\ln(p/\rho^{5/3})$ is proportional to the specific entropy of an ideal gas, we interpret this equation saying that if we move along with the gas, the time rate of change of the specific entropy is determined by the divergence of the heat flux. For the case of a gas with a different equation of state, the functional form of the entropy density will be different from $\ln(p/\rho^{5/3})$, but it will still be expressible as a function

of p and ρ . If we further assume that $\nabla \cdot \vec{q} = 0$, the energy equation becomes

$$\left(\frac{\partial}{\partial t} + \vec{V} \cdot \nabla\right) f(p, \rho) = 0 \quad 4.16$$

where the functional form of $f(p, \rho)$ depends upon the equation of state of the gas.

Since only the gradient of the pressure enters into the momentum equation, and not the pressure itself, Eq. 4.16 can be used to eliminate p in favor of ρ in Eq. 4.14. The result is

$$\left(\frac{\partial}{\partial t} + \vec{V} \cdot \nabla\right) \vec{V} = -c_s^2 \nabla(\ln \rho) + \frac{1}{4\pi\rho} \{(\vec{B} \cdot \nabla)\vec{B} - \frac{1}{2} \nabla B^2\} \quad 4.17$$

where c_s , the local 'sound' speed, is defined by

$$c_s^2 = \left(\frac{\partial p}{\partial \rho}\right)_f \quad 4.17a$$

Equations 4.1, 4.7, 4.12, 4.17 and 4.17a are now a closed set of equations which, with the proper boundary conditions, uniquely determine ρ , \vec{V} , and \vec{B} . \vec{E} can then be determined Eq. 4.11, η from Gauss's law (Eq. 4.4), and \vec{J} from Ampere's law (eq. 4.13). These equations, with the ideal gas equation of state, are sometimes called the ideal MHD equations. While much simpler than the kinetic equations or the set of moment equations, they are still nonlinear and, therefore, difficult to solve. In fact, they are considerably more complicated than the equations which describe a perfect nonconducting fluid. In the next section we shall apply these equations to the problem of the flow around I_0 .

4.3 The Io-Magnetosphere Interaction

Since the discovery that the decametric radio bursts from Jupiter are correlated with the phase of Io, the interaction of Io with the surrounding plasma has been a subject of much theoretical interest. Unlike the Earth's Moon, whose only interaction with the solar wind is to cast a geometrical shadow, Io interacts very strongly with the plasma in which it is embedded. It is believed that this interaction is caused by the high electrical conductivity of Io or its ionosphere.

According to the MHD "frozen-in" law, an observer who is moving along with a plasma sees no electric field. Assuming a uniform background flow, an observer moving along with Io would see an electric field given by $\vec{E} = -(\vec{V} \times \vec{B})/c$, where \vec{V} is the velocity of the background flow with respect to Io. But since Io is a good electrical conductor, the electric field in its rest frame must vanish. If we model Io as perfectly conducting sphere, we conclude that there must be a layer charge on its surface to shield out the external electric field. But since the magnetic flux tube linking Io is "frozen" to the satellite, there is effectively a cylindrical obstacle that the plasma must flow around which extends all the way to Jupiter.

In an early discussion of this phenomenon, Goldreich (1969) hypothesized that the Io flux tube would carry field aligned currents (see Figure 4.1). These currents would flow toward Io on one side of the flux tube, cross the magnetic field by flowing through Io or its ionosphere, and flow away from Io on the other side of the flux tube. This current loop would then be closed in Jupiter's ionosphere. The magnitude of the

currents in this model is determined by the electrical conductivity of Io and that of the Jovian ionosphere.

This early model has been superseded by a theory first proposed to explain the anomalously large amount of drag on the Echo weather satellites by Drell (1965) and later extended and applied to Io by Neubauer (1980). We will now discuss this theory in detail and, in the next chapter, use it to interpret in situ measurements taken by Voyager I.

Drell (1965) realized that the effect of a conductor moving through a magnetized plasma is to generate hydromagnetic waves. If the velocity of the conductor is greater than the fast mode phase velocity in the direction of the motion, a bow shock will form. If the velocity of the conductor is slower than the fast mode phase velocity, the result will be a standing system of Alfvén waves (see Figure 4.2), consisting of two wings, one along each of the two Alfvén characteristics (described hereinafter) which pass through the conductor. In this case one would also expect to find a disturbance associated with the MHD slow mode. The generation of these waves is analogous to the emission of Cerenkov radiation by a particle in a medium which is moving faster than the speed of light in that medium.

The current associated with the Alfvén wings is determined primarily by the wave impedance of the plasma as long as the height-integrated conductivity of the conductor is much smaller than the speed of light squared divided by the Alfvén speed. Thus, for Io, the magnitude of the current, which can be calculated using the linearized MHD equations, is independent of the conductivity of the Jovian ionosphere.

It is generally believed that Goldreich's model would be valid if the Alfvén speed were sufficiently large that the Alfvén wave, after reflection

from the Jovian ionosphere, were to again encounter Io. Since estimates show that the round trip travel time of the Alfvén wave from Io to Jupiter and back to the latitude of Io's orbit is sufficiently long for Io to move a distance greater than its own diameter, the Goldreich model does not apply Neubauer (1980).

Neubauer (1980) noted that there is an exact analytical solution to the MHD equations which describes a finite amplitude standing Alfvén wave. Our presentation follows Neubauer.

It has long been known that there exists an exact solution of the ideal MHD equations which corresponds to a finite amplitude Alfvén wave. To find this solution, we consider the special case where the density does not change with time and the density and the magnitude of the magnetic field have the same value everywhere in space; that is

$$\frac{\partial \rho}{\partial t} = 0 \quad 4.18$$

$$\frac{\partial \rho}{\partial x_i} = 0 \quad 4.19$$

$$\frac{\partial B^2}{\partial x_i} = 0 \quad 4.20$$

This solution is also valid without the previous assumptions about the relation between the density and the pressure (see Eq. 4.17) if we independently require the pressure to have the same value everywhere in

space ($\nabla p = 0$). Eqs. 4.12 and 4.17 can now be written as

$$\frac{\partial \vec{V}_A^{**}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V}_A^{**} = (\vec{V}_A^{**} \cdot \nabla) \vec{V} \quad 4.21$$

$$\left(\frac{\partial}{\partial t} + \vec{V} \cdot \nabla \right) \vec{V} = (\vec{V}_A^{**} \cdot \nabla) \vec{V}_A^{**} \quad 4.22$$

where \vec{V}_A^{**} is the Alfven velocity, defined by

$$\vec{V}_A^{**} = \frac{\vec{B}}{\sqrt{4\pi\rho}} \quad 4.23$$

By adding or subtracting Eqs. 4.18 and 4.19, they can be written elegantly in the canonical form

$$\left(\frac{\partial}{\partial t} + (\vec{V}_A^{**} + \vec{V}) \cdot \nabla \right) (\vec{V}_A^{**} - \vec{V}) = 0 \quad 4.24$$

$$\left(\frac{\partial}{\partial t} + (\vec{V}_A^{**} - \vec{V}) \cdot \nabla \right) (\vec{V}_A^{**} + \vec{V}) = 0 \quad 4.25$$

In Eqs. 4.24 and 4.25 we see that $d\vec{x}/dt = \vec{V}_A^{**+} = \vec{V} + \vec{V}_A^{**}$ is the equation which defines a characteristic along which the Reimann invariant $\vec{V}_A^{**} - \vec{V}$ stays constant, while $d\vec{x}/dt = \vec{V}_A^{** -}$ defines a characteristic along which the Riemann invariant \vec{V}_A^{**+} stays constant. (A characteristic is a line along which small amplitude wave packets propagate.) If \vec{V}_A^{**+} is not only constant along the characteristic defined by Eq. 4.24, but also has the same value on all such characteristics, then Eq. 4.24 and 4.25 imply

$$\left(\frac{\partial}{\partial t} + \vec{V}_A^{**+} \cdot \nabla \right) \vec{B} = 0 \quad 4.26$$

$$\left(\frac{\partial}{\partial t} + \vec{V}_A^{**+} \cdot \nabla \right) \vec{V} = 0 \quad 4.27$$

This is the finite amplitude Alfven wave. If we further require a

stationary solution, we find

$$(\vec{V}_A^{\rightarrow+} \cdot \nabla) \vec{B} = 0 \quad 4.28$$

$$(\vec{V}_A^{\rightarrow+} \cdot \nabla) \vec{V} = 0 \quad 4.29$$

$$\vec{V}_A^{\rightarrow+} = \frac{\vec{B}}{\sqrt{4\pi\rho}} + \vec{V} = \text{const} \quad 4.30$$

\vec{V} and \vec{B} , and hence \vec{E} , are all constant along lines in the direction of $\vec{V}_A^{\rightarrow+}$.

If we think of \vec{B} (\vec{V}) as consisting of the sum of a background field and a (possibly finite) perturbation field, $\delta\vec{B}$ ($\delta\vec{V}$), according to Eq. 4.30 we have

$$\delta\vec{V} = -\frac{\delta\vec{B}}{\sqrt{4\pi\rho}} \quad 4.31$$

The realization that Eq. 4.28-4.31 can be used to describe the interaction between I_0 and the magnetosphere is due to Neubauer (1980).

We shall now examine this solution in detail. It is important to note that since the characteristics are directional, that is, information can propagate along them in only one direction, this solution can only describe one of the Alfvén wings. We can try to construct a solution describing both wings by dividing space into two semi-infinite half spaces, with I_0 lying in the surface which divides them. In one of the half spaces, we will use the above solution, while in the other, its analog, with \vec{V}_A^- taken to have the same value throughout the entire region. Unfortunately, the solutions which we require for our physical model of the Alfvén wing, described hereinafter, cannot be patched together at a boundary surface in such a way as to keep both the tangential component of \vec{E} and the normal component of \vec{B} continuous, except in the special case where \vec{V}_0 and \vec{B}_0 are

perpendicular to each other. This implies that the nature of the interaction directly upstream of I_0 is not described by this solution. Furthermore, the assumption that \vec{V}_A^{++} has the same value everywhere in a large region of space cannot be true when the "background" magnetic field is the dipole field of a planet. We therefore conclude that this solution can accurately describe the fields only in a region near the Alfvén wing, close enough to I_0 so that the gradients in the background magnetic field can be neglected, yet sufficiently far from the satellite that the effect of the other wing is small. We will discuss other limitations of this model after we have developed it more fully.

We now continue our development of this special solution. The following important result relating the component of the current density along \vec{V}_A^{++} to the charge density can be obtained by using Eq. 4.24, the MHD "frozen-in" law, and Gauss's law:

$$\eta = (\vec{V}_A^{++} \cdot \vec{J})/c^2 \quad 4.32$$

Since charge density and current density transform like a four-vector under Lorentz transformation, this implies that in a frame of reference moving with velocity \vec{V}_A^{++} with respect to the frame in which the pattern is stationary the charge density vanishes.

It is convenient to write down the solution in a coordinate system in which one of the axes points along \vec{V}_A^{++} . We shall call this direction \hat{z} . We will further assume that \vec{V} lies in the x-z plane. Once $\vec{E}(x,y)$ is known for all values of x and y, all of the other quantities are determined. B_x and B_y are computed by taking the cross product of \vec{V}_A^{++} with Eq. 4.11, and B_z can

be computed from Eq. 4.23. The results are

$$B_x = - \frac{cE_y}{|V_A^+|} \quad 4.33a$$

$$B_y = \frac{cE_x}{|V_A^+|} \quad 4.33b$$

$$B_z = \sqrt{(B_0^2 - B_x^2 - B_y^2)} \quad 4.33c$$

\vec{V} is then found from Eq. 4.31.

$$V = V_0 - \frac{(\vec{B} - \vec{B}_0)}{\sqrt{(4\pi\rho)}} \quad 4.34$$

J_x and J_y are computed using Ampere's law.

$$J_x = \frac{c}{4\pi} \frac{\partial B_z}{\partial y} \quad 4.35a$$

$$J_y = - \frac{c}{4\pi} \frac{\partial B_z}{\partial x} \quad 4.35b$$

We will now model Io as a perfectly conducting sphere moving through a plasma which, in the absence of the sphere would be at rest and contain a uniform and constant background magnetic field. We will use a frame of reference which is at rest with respect to Io, and take the center of Io as the origin. We define the z-axis to point along \vec{V}_A^+ , and we seek a solution which is valid for $z > 0$. An example of an appropriate set of boundary conditions is the specification of $\vec{E}(x,y)$ everywhere in the $z=0$ plane. Unfortunately, our physical model only gives us information about \vec{E} far from the sphere and in the Alfvén wing.

Far from the sphere and the Alfvén wing, we require that the velocity and the magnetic field be given by

$$\vec{V} = V_0 (\cos(\zeta) \hat{x}_1 + \sin(\zeta) \hat{x}_3) = V_{0\perp} \hat{x}_1 + V_{0\parallel} \hat{x}_3 \quad 4.36$$

$$\vec{B} = B_0 \hat{x}_3 \quad 4.37$$

where ζ is the angle between the background flow and the \hat{x}_1 -axis, $V_{0\parallel}$ and $V_{0\perp}$ are the components of \vec{V} parallel and perpendicular to \vec{B} , and \hat{x}_1 - and \hat{x}_3 - are related to \hat{z} by the definition of \vec{V}_A^+ :

$$V_A^+ \hat{z} = V_0 \hat{x}_1 + \left(\frac{B_0}{\sqrt{4\pi\rho}} + V_0 \right) \hat{x}_3 \quad 4.38$$

The electric field far from the sphere is found from Eq. 4.11 to be

$$\vec{E} = \hat{x}_2 E_0 = \hat{y} E_0 \quad 4.39$$

with E_0 defined by

$$E_0 = \frac{V_{0\perp} B_0}{c} \quad 4.40$$

Since the sphere is assumed to be a perfect conductor, the electric field in its rest frame must vanish. But, since all quantities are constant along z , we conclude that the electric field must vanish in a semi-infinite cylinder of radius R_{I0} , and that a polarization charge must be present on the surface of the cylinder on which the lines of the electric field in the moving plasma can terminate.

In order to enable us to determine the fields in the vicinity of the cylinder, we must make several further assumptions. First, we assume that the electric charge density vanishes everywhere except at infinity and on the surface of the cylindrical Alfvén wing. This, along with our previous assumption that there is no time variation, implies that the electric field

is the gradient of a potential which satisfies Laplace's equation. The boundary conditions are that the potential vanish at the surface of the Alfvén wing, and that the electric field far from the wing is that given by Eq. 4.39. The electric field which satisfies Laplace's equation and these boundary conditions is

$$E_x = \frac{2xy}{(x^2 + y^2)^2} R_c^2 E_0 \quad 4.41a$$

$$E_y = \left(1 - \frac{R_c^2}{(x^2 + y^2)^2} (x^2 - y^2)\right) E_0 \quad 4.41b$$

The velocity and the magnetic field, from Eq. 4.33 and 4.34, are

$$B_x = -B_0 \sin \theta_A \left(1 - \frac{R_c^2}{(x^2 + y^2)^2} (x^2 - y^2)\right) \quad 4.42a$$

$$B_y = B_0 \sin \theta_A \left(\frac{2xy}{(x^2 + y^2)^2} R_c^2\right) \quad 4.42b$$

$$B_z = \sqrt{B_0^2 - B_x^2 - B_y^2} \quad 4.42c$$

$$V_x = V_{ox} - \left(\frac{B_0 \sin \theta_A}{\sqrt{4\pi\rho}}\right) \left(\frac{R_c^2 (x^2 - y^2)}{(x^2 + y^2)^2}\right) \quad 4.43a$$

$$V_y = - \left(\frac{B_0 \sin \theta_A}{\sqrt{4\pi\rho}}\right) \left(\frac{2xy}{(x^2 + y^2)^2}\right) \quad 4.43b$$

$$V_z = V_{oz} - \frac{B_z}{\sqrt{4\pi\rho}} \quad 4.43c$$

where we have introduced the Alfvén angle θ_A , defined as the angle between the the Alfvén characteristic \vec{V}_A^+ and the background magnetic field \vec{B} . This velocity field is very similar to the potential flow of an incompressible

fluid around an infinite cylinder. The streamlines for this flow are shown in Fig. 4.3.

The surface charge density σ and the surface current density \vec{K} , from Gauss's law and Eq. 4.32, are

$$\sigma_s = (2V_{O1} B_O \sin \phi) / c \quad 4.44$$

$$K_z = \frac{cV_{O1} B_O \sin \phi}{(V_O^2 + V_A^2 + 2V_O V_A^+ \sin \zeta)} \quad 4.45$$

where ϕ is the polar angle in the x-y plane. Eq. 4.41-4.45 constitute the complete solution for the fields in the vicinity of I_0 and its Alfvén wing.

Far enough from the wing for $\delta\vec{B}$ and $\delta\vec{V}$ to be considered small perturbations, Eqs. 4.42c and 4.43c can be rewritten as

$$B_z = B_O (1 + \sin(\theta_A) \tan(\theta_A)) R_c^2 \frac{(x^2 - y^2)}{(x^2 + y^2)^2} \quad 4.42c'$$

$$V_z = V_{Oz} - \left(\frac{B_O R_c^2 \sin(\theta_A)}{\sqrt{4\pi\rho}} \right) \tan(\theta_A) \frac{(x^2 - y^2)}{(x^2 + y^2)^2} \quad 4.43c'$$

We can also treat the case where the conductivity of I_0 is large, but finite. In this case, we simply assume that the electric field inside the wing is uniform and y-directed. The electric field outside the wing is still the linear superposition of a uniform field and the field of a line dipole, but the dipole moment is reduced. The fields are still given by Eqs. 4.41-4.43 if R_c is interpreted as the "effective radius" of the Alfvén wing. For the case where the currents across \vec{B} flow through I_0 's ionosphere, rather than through the satellite itself, the same equations are still valid, only now the "effective radius" is greater than R_{I_0} .

We shall now examine the validity of the approximations used in deriving Eq. 4.41-4.45 in more detail. If we relax the restrictive assumptions of Eqs. 4.18-4.20, we find that Eqs. 4.24 and 4.25 become

$$\left(\frac{\partial}{\partial t} + \vec{V}_A^{++} \cdot \nabla\right) \vec{V}_A^{-} = -c_s^2 \nabla(\ln \rho) - \frac{vB^2}{8\pi\rho} + \vec{V}_A^{*} \left(\frac{\partial}{\partial t} + \vec{V}_A^{-} \cdot \nabla\right) (\ln \rho) + \vec{g} \quad 4.46$$

$$\left(\frac{\partial}{\partial t} + \vec{V}_A^{-} \cdot \nabla\right) \vec{V}_A^{++} = -c_s^2 \nabla(\ln \rho) - \frac{vB^2}{8\pi\rho} + \vec{V}_A^{*} \left(\frac{\partial}{\partial t} + \vec{V}_A^{++} \cdot \nabla\right) (\ln \rho) + \vec{g} \quad 4.47$$

The errors committed by making those restrictive assumptions can be estimated by dimensional analysis. We will estimate the order of magnitude of each of the terms which we neglected and compare it to the terms which we have included in the subsequent derivation. In the following equations, the left side is the term whose magnitude we wish to estimate, while the right side is its expected size, in terms of characteristic length, time, and velocity scales of the problem.

To estimate the error in the position and properties of the Alfvén wing itself, we consider first Eq. 4.47. For the first term we have

$$\frac{\partial \vec{V}_A^{-}}{\partial t} = \frac{V_A^{*}}{T} = \frac{V_A^{*} V}{\pi R_J} = \frac{V_A^{*} V}{3R_J} \quad 4.48$$

The time in the denominator of the right side of Eq. 4.48 is of the order of one half of the rotation period of Jupiter. We expect changes on this time scale because of the fact that the dipole axis of Jupiter is tilted with respect to the rotation axis. We obtain the term on the far right of Eq. 4.48 by noticing that the corotation velocity is simply the circumference of Io's orbit divided by Jupiter's rotation period. In this

estimate we have neglected the angular velocity of Io compared to the angular velocity of Jupiter.

The second term on the left side of Eq. 4.46 is of order

$$(\vec{V}^+ \cdot \nabla) V_A^- \approx \frac{V_A^{*2}}{L} \quad 4.49$$

The length L in the denominator can be taken to be the distance of the point in question from Io.

In the same spirit, for the first term on the right side of Eq. 4.46 we have

$$c_s^2 \nabla(\ln \rho) \approx \frac{5c_s^2}{R_J} \quad 4.50$$

The characteristic length for change in $\ln \rho$ is very different in different directions. We know from Voyager measurements and theoretical arguments (Bagenal and Sullivan (1981)) that the characteristic length for changes along \vec{B} near Io is about $1 R_J$. In the direction of corotating flow, the characteristic length is on the order of the radius of Io's orbit, about $6 R_J$. Radially in towards Jupiter, the gradients are very steep, with a characteristic length of the order of $0.2 R_J$. In our estimate of the size of this term we have used the shortest of these three distances.

Assuming the magnetic field of Jupiter to be a dipole field, the size of the second term on the right side of Eq. 4.46 is

$$\frac{|\nabla B^2|}{8\pi\rho} = \frac{-6B^2}{8\pi\rho (6R_J)} \approx \frac{V_A^{*2}}{2R_J} \quad 4.51$$

Using the same arguments, the next two terms are seen to be

$$V_A^* \frac{\partial}{\partial t} (\ln \rho) \approx \frac{V_A^* V}{3R_J} \quad 4.52$$

$$V_A^* (\vec{V}_A^- \cdot \nabla) (\ln \rho) \approx \frac{V_A^{*2}}{R_J} \quad 4.53$$

Finally, the gravitational acceleration is primarily due to the centrifugal force, which is of the order of

$$g \approx \frac{V^2}{R_J} \quad 4.54$$

The solution derived herinbefore will be valid only when the term $(\vec{V}_A^+ \cdot \nabla) V_A^-$ is large compared with all of the others. Comparison of Eqs. 4.48-4.54 shows that this is true as long as $L \ll 1 R_J$; that is, much closer than $1 R_J$ to I_0 .

We cannot use Eq. 4.47 to study the region of validity of our solution perpendicular to \vec{V}_A^+ because the term which determines our solution contains spatial derivatives only along \vec{V}_A^+ . We therefore must use Eq. 4.46. Each term in that equation will be of the same order as the corresponding term in Eq. 4.46, with two exceptions. Since we are now concerned with errors made when we integrate along a path perpendicular to \vec{V}_A^+ , the second term on the left side of Eq. 4.46 is of the order of

$$(\vec{V}_A^- \cdot \nabla) V_A^+ \approx \frac{V V_A^*}{L} \quad 4.55$$

Similarly, the term proportional to $(\vec{V}_A^+ \cdot \nabla) \ln \rho$ vanishes. We conclude that our solution is valid for $L \ll (2V/V_A^*) R_J \approx 0.3 R_J$, or within a few I_0

radii of the Alfvén wing. These results define what we meant by "near the Alfvén wing" and "close enough to Io" in the discussion which followed Eq. 4.31.

Chapter 5

Analysis of Plasma Data

Taken in the Vicintiy of Io

By Voyager I

5.1 The Io Flyby

On 5 March 1979, Voyager I flew through the Jovian system. Its orbit is shown in Figures 5.1, 5.2 and 5.3. Figure 5.1 shows the projection of the spacecraft trajectory into the Jovian equatorial plane, in addition to the orientation of the main sensor symmetry axis and the D-cup normal. As can be seen from these figures, the D-cup was looking into the co-rotating plasma throughout the inbound pass. As the spacecraft approached perijove, the corotating flow swung around towards the main sensors, almost coming down the symmetry axis at closest approach. As the spacecraft receded from Jupiter, the flow direction shifted away from the all of the cups. The closest approach to Io occurred on the outbound pass at SCET 1510.

Figures 5.2 and 5.3 are closeups of the Io flyby. Both are in Io-centered coordinates; Fig. 5.2 is also a projection of the orbit into Jupiter's equatorial plane, while 5.3 plots the distance from the origin of that projection against the height above the plane.

The Io flyby occurred on the spacecraft's outbound pass, when Voyager I flew about 20 thousand kilometers due south of the satellite. Though the orbit was planned to fly directly through the flux tube, analysis of the magnetic field data (Acuna et al (1981)) showed that the spacecraft missed the Alfvén wing, passing several thousand kilometers upstream of it. Subsequent examination of the plasma data (Belcher (1981)) revealed a signature which was interpreted as being due to the velocity perturbation associated with the flow around the wing. The approximate position of the Alfvén wing in the plane of the spacecraft orbit is shown in Fig. 5.2.

During the Io flyby, the flow direction of a strictly corotating plasma would have been almost perpendicular to the main sensor symmetry axis. Table 5.1 lists the angle between the flow direction of a strictly corotating plasma, corrected for spacecraft aberration, and the normals to each of the PLS cups for each of the useful spectra taken during the flyby. One lower half M-mode spectrum was taken every 192 seconds during the flyby, and there are no useful data in the upper half spectra. The L-modes taken during this time cannot be analyzed because the signal in the A-cup is saturated; that is, the digitized currents were equal to the maximum value for several channels, indicating that the true currents were higher.

5.2 Analysis of the Data

We have analyzed eight spectra taken in the vicinity of Io. Using the same nonlinear least squares fitting technique described in Chapter 3, we have attempted to determine the macroscopic parameters of the plasma at the times of each of these spectra. The velocity determinations were then compared with the predictions of the theory outlined in the previous chapter. We shall first describe the determination of the plasma parameters.

Figures 5.4-5.11 show the reduced distribution function plotted as a function of equivalent proton velocity in each cup for each of the eight spectra. The spectra were taken at SCET (spacecraft event time) 1424, 1451, 1454, 1457, 1500, 1504, 1507, and 1510, respectively. As in the spectrum plots of Figs 3.3-3.11, the staircases are the data and the smooth

curves are the "best" fits. All of the spectra are smooth and almost featureless, with a single, broad peak. Furthermore, in each spectrum the peak in the A-cup is in a higher channel than the peaks in the B- and C-cups. The D-cup spectra contain mostly noise, and were not included in the fits.

To determine macroscopic plasma parameters from these spectra is a difficult proposition. Since the flow is so very oblique to all of the cups, the results depend critically upon the cup response function at large angles. In addition, the plasma contains many different kinds of ions, which are probably not in thermal equilibrium. In order to get any results at all, we have had to make some assumptions about the chemical composition and the distribution functions of the various ionic species. These assumptions are:

1. The plasma consists of 4 species, with A/Z^* of 8, 16, 32, and 64.
2. The species all have the same bulk velocity.
3. The distribution function which describes each of the species is a convected Maxwellian.
4. The species all have the same thermal speed.

As discussed in the previous chapter, measurements made by Voyager I inbound (Bagenal and Sullivan (1981)) lead us to expect the plasma to consist of various ionization states of Oxygen and Sulfur. The values of A/Z^* for O^{++} , O^+ , S^{+++} , S^{++} , S^+ , and SO_2^+ are 8, 16, $10 \frac{2}{3}$, 16, 32, and 64, respectively. We have not included S^{+++} in our fit in the interest of reducing the number of parameters, and because, since the spectra are so featureless, we do not expect to be able to differentiate between the contribution due to a species with A/Z^* of $10 \frac{2}{3}$ and its neighbors with

A/Z^* of 8 and 16. The justification for the inclusion of the species with A/Z^* of 64 is that we expect that the distribution functions will have suprathermal tails, and we hope to be able to account for this in a rough manner by including a very heavy species. In fact, it is impossible to fit the higher channels without it. In some of the fits, the inclusion of the heavy species resulted in driving one of the other densities negative; in these cases we fit the spectra without the heavy species, omitting some of the higher channels. In all of these cases, the derived bulk velocity was almost unaffected by this change.

The assumption of a common bulk velocity is the least likely to be violated of the four which we have made. The component of the bulk velocity perpendicular to the magnetic field is determined by the ideal MHD generalized Ohm's law (Eq. 4.11). Solving this equation for \vec{V} , we find

$$\vec{V} = c \frac{\vec{E} \times \vec{B}}{B^2} \quad 5.1$$

This is the same as the equation for the $\vec{E} \times \vec{B}$ drift velocity of a single particle moving in crossed electric and magnetic fields (see Rossi and Olbert, chapter 2). This is no coincidence; the bulk velocity can be thought of as the average velocity of a large number of particles gyrating around the magnetic field with the same $\vec{E} \times \vec{B}$ drift velocity. Since the latter velocity is independent of the mass or charge of the particle, all species must have the same value of \vec{V}_\perp . Although the different species may stream along \vec{B} at different speeds when the magnetic field lines are open, in closed line topology we expect \vec{V}_\parallel will be zero. This is in fact what we found from our analysis of these spectra.

The assumption that the distribution functions are Maxwellian has already been discussed in Chapter 1. Since the self-Maxwellization time for the various species are comparable to or greater than the residence time in the torus (see Table 4.1), we must consider the Maxwellian form as no more than a convenient way to parameterize the distribution function. As long as the true distribution functions do not have multiple peaks, which is indicated against by the smoothness of the spectra, a Maxwellian must be a reasonably good approximation to the true distribution function around the maximum. Therefore, our justification for this assumption is that it is a reasonable representation of the true distribution function which enables us to perform the two of the velocity space integrals of Eq. 2.2b analytically.

Unfortunately, the fact that the bulk velocity is so oblique to the cup normals implies that most of the signal comes from particles not near the peak of the distribution function. The simulations indicate that the largest contribution comes from particles about one thermal speed away from the peak. We must conclude that our approximation of Maxwellian distributions is a possible source of error. As mentioned above, the inclusion of a species with a mass to charge ratio of 64 is an attempt to compensate for the presence of suprathermal tails.

The assumption of a common thermal speed for all of the species, like the assumption of the Maxwellian shape for the distribution functions, was motivated partly by physical arguments and partly for computational convenience. Since the time scales for equipartition of energy between the various ion species are comparable to or longer than the residence time in the torus (see table 4.1), there is no reason to expect that the different

ion species share a common temperature. We did try to fit the spectra with the assumption of a common temperature for the various species; that we were unable to do so convinced us that the different species do not have the same temperature. Furthermore, there are many physical processes, such as various wave-particle interactions or "pick-up", the process by which the newly-made ions are accelerated up to corotational speed, which tend to produce equal thermal speeds.

Since we really have no way of knowing a priori what the relationship between the thermal speeds of the various species is, it would be nice to be able to vary them independently. Unfortunately, the lack of detail in the spectra makes this impossible. Furthermore, the extra computer time involved in computing the derivatives with respect to each individual thermal speed for the gradient search makes this approach impractical. We must make some assumption which relates the various thermal speeds; the assumption that they are all the same, in addition to being physically plausible, has the advantage that it enables us to greatly reduce the computer time required to fit each spectrum by taking advantage of an approximate symmetry of the response function.

The quantity which we use in our fits is the "reduced distribution function", defined as the current in a given channel divided by the voltage width of that channel. This quantity can be expressed as

$$\frac{I_k}{\Delta\phi_k} = \frac{Z^2}{A} \frac{n}{w\sqrt{\pi}} \exp\left\{-\left(V_z - v_z\right)^2/w^2\right\} R\left(\frac{v_z}{w}, \Delta\phi_k, \phi_s\right) \Big|_{v_z = \bar{v}_k} \quad 5.2$$

where $R(v_z/w, \Delta\phi_k, \phi_s)$ contains the effect of the response function. It is important to note that to the extent that the dependence of the response

function on the channel width and the suppressor voltage can be ignored, the expression on the right side of Eq. 5.2 depends only on V_z/w and v_z/w , never on v_k . This fact enables us to calculate the value of the "reduced distribution function" in channel k for a species with mass A and charge state Z^* from the value of the "reduced distribution function" in channel k' for a species of mass A' and charge state Z'^* from the formula

$$\frac{I_{k'}(\bar{v}_{k'})}{\Delta\phi_{k'}} = \left(\frac{Z'^*}{Z^*}\right)^2 \left(\frac{A}{A'}\right) \left(\frac{I_k(\bar{v}_k)}{\Delta\phi_k}\right) \quad 5.3$$

if the average proton speeds characterizing channels k and k' are related by

$$\bar{v}_{k'} = \sqrt{\left(\frac{A' Z'^*}{A Z^*}\right)} \bar{v}_k \quad 5.4$$

This approximation was used in calculating the "best fits" to the spectra shown in Figs. 5.4-5.11. The procedure we followed to simulate a spectrum is as follows: for a given bulk velocity and thermal speed we first computed the reduced distribution function in all 128 channels for a species with $A=64$ and $Z^*=1$. We then used Eqs. 5.3 and 5.4 to compute the contribution due to the other species. Since, for a given channel k with average proton speed v_k the value of $v_{k'}$, required by Eq. 5.4 is probably not equal to the average proton speed of any other channel, but rather lies between two channels, we used linear interpolation between the adjacent channels to compute $I_k(v_{k'})/\Delta\phi_{k'}$, for use in Eq. 5.3. For several test cases, the approximate values of the "reduced distribution function" calculated in this way were compared with the values obtained from

calculations done without taking advantage of the scaling law just described. The error introduced never exceeded 6%, and frequently was less than 1%.

The parameters derived from the fits of the spectra of Figs. 5.4-5.11 are given in Table 5.2. In all of these spectra, the geometry is such that the flow is almost perpendicular to the symmetry axis of the main sensor. The A-cup look direction is closest to the flow. We expect the determination of the velocity to be more reliable than the determination of the densities, although the errors in these quantities are correlated. The information about the flow direction is primarily contained in the relative heights of the signals in the three cups; the further the flow direction moves away from the axis of symmetry, the larger the difference between the A-cup signal and the B and C cup signals becomes. The information about the densities is primarily contained in the absolute height of the peaks. If the true flow is closer to the main sensor symmetry axis than the fit indicates, the density will be overestimated, since a larger density is required to produce the same signal in a given cup when the flow is more oblique. Conversely, if the true flow direction is further away from the main sensor symmetry axis than the fit indicates, the density will be underestimated.

There were several spectra taken after the spectrum of 1510 (Fig. 5.11) which we were unable to fit because the difference between the signal in the A-cup and the signals in the B- and C- cups were too large. At about this same time, the magnetometer team noticed a field disturbance which did not fit the model described in Chapter 4 (Acuna et al (1981)). Our difficulty in fitting these spectra might be caused by a change in the

plasma distribution function which is related to this magnetic field disturbance.

5.3 The Flow Around the Alfvén Wing

The velocities derived from the fits to the spectra of SCET 1451 through 1510 were compared with the theory which has been derived in Chapter 4. To do this, we did a five parameter nonlinear least squares fit, described hereinafter.

Equations 4.43a, 4.43b, and 4.43c' describe the velocity at every point in space as a function of the background flow velocity \vec{V}_0 , the magnitude of the magnetic line dipole moment $\mu (B_0 R_c \sin \theta_A)$ of the Alfvén wing, and the Alfvén Mach angle θ_A . The only difficulty is that the orientation of the coordinate system of Eqs. 4.43 also depends upon three of these five parameters. This is because the z-axis of the coordinates of Eqs. 4.43 points along the Alfvén characteristic \vec{V}_A^+ .

To overcome this difficulty, we devised the following fitting procedure. First we define one set of coordinates, called magnetic coordinates, as follows. The center of Io is taken to be the origin of the coordinate system. The unit vector $-\hat{z}_{mag}$ points in the direction of the extrapolated background magnetic field at the location of the spacecraft at SCET 1500, as quoted in Acuna et al (1981). The unit vector \hat{x}_{mag} lies in the plane containing \hat{z}_{mag} and the direction of strictly corotating flow, making an acute angle with the flow direction, and \hat{y}_{mag} is defined so as to

form a right-handed coordinate system. (The unit vector \hat{z}_{mag} was defined to be antiparallel to the background magnetic field direction because \vec{B} points predominantly south near the Jovian equatorial plane. In this way, \hat{z}_{mag} makes an acute angle with Jupiter's angular velocity vector. Compared with a cylindrical coordinate system whose polar axis is Jupiter's spin axis, \hat{z}_{mag} points in the same general direction as \hat{z}_{cyl} , \hat{x}_{mag} points in the same general direction as $\hat{\phi}_{\text{cyl}}$, and \hat{y}_{mag} points radially in toward Jupiter.)

To compare the theory with experiment, still another set of coordinates is required. We call this set Alfvén coordinates. The origin of these coordinates is also the center of Io. The vector $-\hat{z}_{\text{Alfvén}}$ points along \vec{V}_A^+ (note: this is antiparallel to \hat{z} as defined for Equations 4.32-4.45). The unit vector $\hat{x}_{\text{Alfvén}}$ lies in the plane determined by \vec{V}_A^+ and \vec{V}_O , so as to be perpendicular to $\hat{z}_{\text{Alfvén}}$ and make an acute angle with \vec{V}_O . The unit vector $\hat{y}_{\text{Alfvén}}$ is defined so as to complete a right-handed system. The rotation matrix for the transformation from magnetic coordinates to Alfvén coordinates is completely determined by the specification of \vec{V}_O and θ_A . It is given explicitly by

$$\vec{A}_{\text{Alfvén}} = \tilde{R} \cdot \vec{A}_{\text{mag}} \quad 5.5$$

where

$$\tilde{R} = \begin{pmatrix} \cos \theta_A \cos \alpha & \cos \theta_A \sin \alpha & -\sin \theta_A &) \\ \sin \alpha & \cos \alpha & 0 &) \\ \sin \theta_A \sin \alpha & \sin \theta_A \cos \alpha & \cos \theta_A &) \end{pmatrix} \quad 5.5a$$

and

$$\sin \alpha = \frac{\hat{v}_o \cdot \hat{y}_{\text{mag}}}{\{(\hat{v}_o \cdot \hat{y}_{\text{mag}})^2 + (\hat{v}_o \cdot \hat{x}_{\text{mag}})^2\}^{1/2}} \quad 5.5b$$

We are now prepared to outline the fitting procedure. First, the spacecraft position at the time of each of the seven spectra is computed in magnetic coordinates. Then, a first guess for the five parameters is made and used to compute the spacecraft positions in Alfvén coordinates. The theoretical velocity at each of the seven spacecraft positions is then computed using Eqs. 4.43 and compared with the measured velocity obtained from the fits to the spectra. The "best fit" values of the five parameters are then found using a gradient search algorithm. These "best fit" parameters are given in Table 5.3, while the measured components of the flow velocity at each of the points are compared with the theoretical values in Figure 5.12.

Since it is important to compare our results with the magnetic field measurements, we shall now summarize the results of the analysis of the magnetic field data (Acuna et al (1981)). Acuna fit the magnetic field to Neubauer's theory in the following manner. First, they assumed that the background flow was strictly corotational, i.e. they set α equal to zero. Then they assumed a value of θ_A . This defined their coordinate system.

In order to compare their measurements with the theory it was necessary to separate the perturbation magnetic field from the (non-uniform) background field. The resulting values of $\delta\vec{B}$ were then compared with values predicted by Eqs. 4.42.

At this point they introduced three more free parameters into their model. According to Eqs. 4.42, once the orientation of the Alfvén coordinate system has been determined, the only free parameter remaining is the dipole strength. In order to obtain a better fit, the location of the dipole and its direction were varied so as to minimize an appropriately defined χ^2 . This process was then repeated with different assumed values for θ_A . The "best" value of θ_A was chosen by the requirements that the dipole location be close to the origin and that the angle between the dipole direction and the \hat{x} -direction be small. Their procedure was essentially a five parameter nonlinear least squares fit employing a grid search over the values of θ_A and a gradient search through the space spanned by the other four parameters.

Their analysis resulted in the following estimates for the values of the parameters which are of interest to us: Alfvén Mach number 0.15, Alfvén speed 400 km/sec, and "effective radius" of Alfvén wing $1.1 R_{IO}$.

In addition to the analysis using the plasma data alone, we also tested Eq. 4.31. We assumed that the plasma velocity and the magnetic field perturbation are related by

$$\vec{V} = \vec{V}_0 + \frac{V_A}{B_0} \delta\vec{B} \quad 5.6$$

We then performed a linear regression to determine the "best" values for \vec{V}_0 and V_A . The results are tabulated in Table 5.3. The value of the Alfvén speed is large, about 600 km/sec, while the background velocity has a large component radially in toward Jupiter. Figure 5.14 is a plot, similar to Fig. 5.13, of the projections of $\delta\vec{B}$ and $\delta\vec{V}$ (as determined from the linear

regression) into the plane perpendicular \hat{z}_{mag} . As can be seen from this figure, there is a strong anticorrelation between $\delta\vec{B}$ and $\delta\vec{V}$, as expected.

5.4 Discussion and Conclusions

We shall now examine the derived plasma parameters for the eight spectra which we fit (Table 5.3). Figure 5.13 shows, superimposed on the spacecraft trajectory, the projection of the bulk velocity vector into a plane perpendicular to the background magnetic field for each of the seven spectra taken in the vicinity of Io. As the spacecraft passed the flux tube, the radial component of the velocity varied smoothly, decreasing from -4 km/sec at 1451 to -23 km/sec at 1457, then increasing again to 10 km/sec at 1507 before finally decreasing again to 0 in 1510. This is the trend one would expect for the flow around the Alfvén wing; as Voyager approached the wing from the direction of Jupiter, it first saw the plasma flowing toward the planet as it passed through streamlines which passed around the side of the flux tube toward Jupiter. As Voyager continued its outbound journey, it passed through streamlines which passed the flux tube on the side away from the planet, hence the outward flow. As Voyager left the flux tube behind, it entered a region where the flow was undisturbed. We conclude that the flow observed by Voyager qualitatively agrees with what we would expect to see if our model is correct.

Unfortunately, the maximum inward flow is much greater than the maximum outward flow, and there seems to be a systematic inflow of plasma towards Jupiter. In addition, there is a correlation between the derived densities and the amount of inflow. The larger the speed of the inflow, the larger the densities. Since we really do not expect there to be such large density

fluctuations, and there is no theoretical reason for this correlation, we shall attempt to explain this effect in terms of a systematic error in our fitting procedure.

As mentioned hereinbefore, an error in the determination of the velocity which changes the angle between the velocity vector and the main sensor symmetry axis will be associated with a corresponding error in the density determination. This can be caused by a violation of any of the assumptions described in the preceding section. If this is the case, we expect that the density changes are artifacts of the fitting procedure, and that the change in the radial component of the velocity is less than the result of the fit. We expect that the overall trend is real, however. Since a negative radial component of plasma velocity corresponds to the flow moving further away from the main sensor symmetry axis, and the more oblique the flow is the less accurate we expect our approximation to be, we expect that this effect can cause an overestimate of the average radial inflow and a corresponding overestimate of the density. Such a systematic error would also have the effect of reducing the large value of V_A which was determined from the linear regression with the magnetic field data so as to bring this estimate of V_A into agreement with the value determined by Acuna et al.

The thermal speeds derived from the fits are all between 30 and 36 km/sec. This is consistent with there being no change in the thermal speed of the plasma throughout the flyby.

The north-south component of all of the velocity vectors is consistent with zero. This is what we expect, considering that the \hat{B} field points almost directly due south, and we do not expect to see any flow along \hat{B} .

The azimuthal component of the bulk velocity decreases as Voyager approached the Alfvén wing, and then increased again as the spacecraft left the wing. Again, this is just the trend one would expect to see in the flow as it passes around the wing.

We will now proceed to examine the parameters describing the Alfvén wing which we determined from the non-linear least squares fit to the theory that was expounded in Chapter 4. We first consider α , which measures the angle between the background flow and rigid corotation. The derived value of 18° is much larger than the expected value of 0. The systematic error described in the preceding paragraph would account for this discrepancy.

The value of our determination of $\vec{V}_0 \cdot \hat{z}_{\text{mag}}$, the component of the bulk flow along the direction of the background magnetic field, is 0. This is exactly what we expect for a system with closed magnetic field lines. Because the B- and C- cup look directions are symmetric with respect to \vec{B} , our measurements are very sensitive to small deviations of this component of velocity. We are therefore very confident that this result is not affected by any systematic error of any kind.

Our derived value of the magnitude of the component of the background flow perpendicular to \vec{B} is 63 km/sec. This is slightly greater than the expected value of 57 km/sec. We consider these values to be consistent to within experimental uncertainties.

Our value of the Alfvén velocity is 250 km/sec. It should be emphasized that the only magnetic field data used in this determination was the direction of the background field. The values of the Alfvén velocity from the mass density measurements and the measured field strength range from 160-250 km/sec. While the same systematic error would tend to increase the latter of

to be made. These are: only species with A/Z^* of 8, 16, 32, and 64 are present, all of the species have the same streaming velocity, the distribution function of each of the ionic species can be approximated by a convected Maxwellian, and all of the species have the same thermal speed. The velocity determination from seven of these spectra were then used to determine the parameters of a model of the Io-magnetosphere interaction based on the work of Neubauer (1980). Although the quantitative agreement with the theory was not outstanding, the results confirm the overall picture of the plasma flow pattern being analogous to incompressible flow around a cylinder.

The discrepancies between the theory and the observations are probably caused by the violation of one or more of the aforementioned assumptions. We have argued that a systematic error whose primary effect is to cause us to overestimate the angle between the direction of the flow and the main sensor symmetry axis could account for all of the discrepancies. In particular, the assumptions that all of the ion species have the same thermal speed and that the distribution functions can be approximated by convected Maxwellians are certainly violated to some extent and this might be the cause of the systematic error.

An additional complication which we have not discussed before is that the spacecraft potential with respect to the ambient plasma is probably not zero. In a quasi-steady state situation, the net electric current flowing into the spacecraft must be vanish. There are three main contributions to this current; positive ions, electrons, and photoelectrons caused by the sun. In the solar wind, the spacecraft tends to be positively charged, since if the spacecraft were neutral the current due to the photoelectrons would dominate that due to the plasma electrons, the contribution due to the positive ions

being negligible due to their much lower velocities. In the high densities of the Io plasma torus, the situation is reversed; the plasma electron current would overwhelm the photoelectron current, resulting in the spacecraft becoming negatively charged. Analysis of the electron data taken by Voyager I near Io indicate that this is in fact the case (Ed Sittler, private communication).

Unfortunately, the effect of the spacecraft potential is difficult to compensate for. Its effect is largest in the lower channels; in channels for which $\phi_k \gg \phi_s$ the effect is entirely negligible. This effect should be taken into account in future work.

All of these difficulties notwithstanding, we expect that the methods of analyzing the plasma data which we have developed in this thesis can be profitably applied to other spectra, not only those taken at Jupiter, but those from the Saturnian system as well.

References

- Acuna, M.H., and F.M. Ness; Standing Alfvén Wave Current System at Io: Voyager I Observations. JGR 86:8513-8522 30 September 1981
- Bagenal, F. and J.D. Sullivan; Direct Plasma Measurements in the Io Torus and Inner Magnetosphere of Jupiter. JGR 86:8477-8466 30 September 1981
- Belcher, J.W., C.K. Goertz, J.D. Sullivan, and M.H. Acuna; Plasma Observations of the Alfvén Wave Generated by Io. JGR 86:8508-8512 1981
- Bevington, P.R.; Data Reduction and Error Analysis for the Physical Sciences. McGraw-Hill 1969
- Bridge, H.S., J.W. Belcher, R.J. Butler, A.J. Lazarus, A.M. Mauretic, J.D. Sullivan, G.L. Siscoe, and V.M. Vasylunas; The Plasma Experiment on the 1977 Voyager Mission. Sp. Sci. Rev. 21:259-287 1977
- Drell, S.D., H.M. Foley, and M.A. Ruderman; Drag and Propulsion in the Ionosphere: An Alfvén Propulsion Engine in Space. JGR 70:3131 1965
- Goldreich, P., and Lynden-Bell; Io, a Jovian Unipolar Inductor. ApJ 156:59-78 1969
- Hill, T.W., and A.J. Dessler; Magnetospheric Models. Chapter 10 in Physics of the Jovian Magnetosphere. A.J. Dessler, editor Cambridge University Press 1983
- Johnson, R.E. and D.F. Strobel; Charge Exchange in the Io Torus and Exosphere. JGR 87(A10):10385-10393 1 December 1982
- McNutt, R.L., J.W. Belcher, and H.S. Bridge; Positive Ion Observations in the Middle Magnetosphere off Jupiter. JGR 86:8319-43 1981
- Neubauer, F.M.; Nonlinear Standing Alfvén Wave Current System at Io: Theory. JGR 85:1171-1178 1980
- Rossi, B. and S. Olbert; Introduction to the Physics of Space. McGraw-Hill 1970

Tables

Table 2.1

Main Sensor Sensitive Area (Normalized)

| S_y | S_x | 0.0 | 2.0 | 4.0 | 6.0 | 8.0 | 10.0 | 12.0 | 14.0 | 16.0 | 18.0 | 20.0 |
|-------|-------|------|------|------|------|------|-------|------|------|------|------|-------|
| -16.3 | | .019 | .019 | .019 | .019 | .019 | .019 | .013 | .0 | .0 | .0 | .0 |
| -14.3 | | .15 | .15 | .15 | .15 | .15 | .13 | .085 | .034 | .006 | .0 | .0 |
| -12.3 | | .40 | .40 | .40 | .40 | .363 | .305 | .225 | .127 | .062 | .019 | .0008 |
| -10.3 | | .70 | .70 | .70 | .67 | .60 | .50 | .383 | .248 | .144 | .063 | .0065 |
| -8.3 | | 1.00 | 1.00 | 1.00 | .93 | .82 | .69 | .53 | .370 | .227 | .108 | .0123 |
| -6.3 | | 1.00 | 1.00 | 1.00 | .93 | .82 | .69 | .53 | .370 | .227 | .108 | .0123 |
| -4.3 | | 1.00 | 1.00 | 1.00 | .93 | .82 | .69 | .53 | .370 | .227 | .108 | .0123 |
| -2.3 | | 1.00 | 1.00 | 1.00 | .93 | .82 | .67 | .51 | .367 | .227 | .108 | .0123 |
| -0.3 | | 1.00 | 1.00 | 1.00 | .91 | .75 | .60 | .45 | .328 | .212 | .108 | .0123 |
| 1.7 | | 1.00 | 1.00 | .90 | .77 | .62 | .49 | .363 | .257 | .161 | .075 | .0079 |
| 3.7 | | .88 | .80 | .71 | .59 | .46 | .347 | .243 | .156 | .084 | .030 | .0022 |
| 5.7 | | .58 | .55 | .49 | .39 | .285 | .213 | .109 | .049 | .013 | .0 | .0 |
| 7.7 | | .28 | .275 | .252 | .205 | .138 | .069 | .023 | .002 | .0 | .0 | .0 |
| 9.7 | | .076 | .076 | .076 | .068 | .037 | .0072 | .0 | .0 | .0 | .0 | .0 |
| 11.7 | | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 |

Table 2.2

Relative Angles of the Grids in the D-cup

| Grid | α_1 | α_2 | α_3 | α_4 | α_5 | α_6 | α_M | α_S |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Spacecraft | | | | | | | | |
| Voyager I | 0° | 62° | 50° | 55° | 40° | 4° | 68° | 0° |
| Voyager II | 0° | 62° | 80° | 55° | 24° | 0° | 68° | 10° |

Table 3.1
Plasma Parameters From Cruise Maneuver on 14 September 1978

| Time | Number | V_x | V_y | V_z | V_R | V_T | V_N | N_o | w |
|------|--------|-------|-------|-------|----------|----------|-----------|-----------|---------|
| 2345 | 1 | 305. | -8. | 225. | 378.±1.0 | 18.2±1.2 | -23.8±1.4 | .284±.008 | 19.8±.5 |
| 2313 | 2 | 221. | -8. | 297. | 370.±0.4 | 11.8±0.8 | -19.1±0.9 | .300±.007 | 16.5±.3 |
| 2315 | 3 | 118. | -5. | 353. | 372.±0.3 | 9.1±0.8 | -12.9±1.0 | .294±.009 | 17.2±.2 |
| 2316 | 4 | 11.4 | -6.4 | 370. | 370.±0.2 | 10.0±0.8 | -16.2±0.8 | .293±.009 | 16.4±.2 |
| 2316 | 4a | 10.9 | -6.5 | 368. | 368.±0.3 | 9.5±1.1 | -16.0±1.1 | .276±.011 | 17.7±.3 |
| 2243 | 5 | 17.8 | -6.6 | 370. | 371.±0.2 | 7.0±0.9 | -14.1±0.9 | .237±.008 | 17.0±.3 |
| 2243 | 5a | 17.5 | -7.0 | 369. | 369.±0.3 | 6.5±1.1 | -14.3±1.2 | .258±.010 | 18.3±.3 |
| 2211 | 6 | -81.5 | -7.8 | 363. | 372.±0.4 | 8.1±1.0 | -15.3±1.0 | .304±.012 | 15.9±.3 |
| 2212 | 7 | -184. | -4.7 | 323. | 371.±0.7 | 10.8±0.9 | -13.8±1.0 | .291±.009 | 16.4±.3 |
| 2248 | 8 | -276. | -2.1 | 248. | 371.±1.0 | 13.8±0.6 | -13.9±1.0 | .258±.006 | 14.3±.4 |
| 2216 | 9 | -341. | -1.6 | 161. | 377.±1.3 | 9.0±0.7 | -9.8±1.3 | .278±.007 | 18.1±.6 |

Table 4.1

The Physical Properties of Io

| | | |
|-------------------------------------|---------------------------------------|---|
| Mass | 8.9×10^{22} kg | $(4.5 \times 10^{-8} M_E)$ |
| Radius | 1820 km | |
| Volume | 2.53×10^{10} km ³ | |
| Distance from Jupiter | 4.24×10^5 km | $(5.9 R_J)$ |
| Orbital Speed | 17.3 km/sec | |
| Orbital Period | 1.54×10^5 sec | $(42\text{hrs}, 48\text{mins}, 38\text{sec})$ |
| Rate of Mass Injection into Torus * | 6×10^{29} AMU/sec | |

* From Hill et al (1983)

Table 4.2

Parameters of Model Plasma

| | |
|---------------------------------|--|
| Electron number density | 1900 cm ⁻³ |
| Electron temperature | 5 eV |
| S ⁺ density | 165 cm ⁻³ |
| S ⁺⁺ number density | 450 cm ⁻³ |
| S ⁺⁺⁺ number density | 65 cm ⁻³ |
| O ⁺ number density | 600 cm ⁻³ |
| O ⁺⁺ number density | 20 cm ⁻³ |
| Ion thermal speed | 30 km/sec (80 eV) |
| Mass density | 3.2 × 10 ⁴ AMU/cm ³ (5.3 × 10 ⁻²⁰ g/cm ³) |
| Magnetic field strength | 2.1 × 10 ⁻² gauss |
| Alfven velocity | 260 km/sec |
| Bulk speed of corotating plasma | 74.1 km/sec (inertial frame) 56.8 km/sec (Io rest frame) |
| Alfven Mach number | 0.22 |
| Electron plasma frequency | 2.4 v 10 ⁶ sec ⁻¹ |
| Electron gyrofrequency | 3.6 v 10 ⁵ sec ⁻¹ |
| Ion gyrofrequency | 12.6 sec ⁻¹ (O ⁺) |
| Electron gyroradius | 4.24 m |
| Skin depth | 20 km |

Table 4.2 continued

| | |
|-------------------------------------|---|
| Debye length | 65 cm |
| Interparticle distance | 8×10^{-2} cm |
| Number of electrons in Debye sphere | 2.3×10^9 |
| Ion gyroradius | 3 km (O^+ with $w=30$ km/sec) |
| Centrifugal acceleration | 33 cm/sec (Io rest frame) |
| Magnetic energy density | 1.6×10^{-5} ergs/cm ³ |
| Thermal energy density | 6.3×10^{-7} ergs/cm ³ |
| β | 5.2×10^{-2} |
| Self-Maxwellization times | 65 sec (electrons) |
| | 24 days (O^+) |
| | 47 days (O^{++}) |
| | 340 days (S^+) |
| | 8 days (S^{++}) |
| | 12 days (S^{+++}) |
| Thermal relaxation times | 50 hrs (ion-electron) |
| | 1-50 days (ion-ion) |
| Plasma residence time in torus * | 30 days |
| Period of Io | 43 hours |
| Rotation period of Jupiter | 10 hours |
| Plasma time of flight past Io | 30 sec |

* Estimated from mass injection rate of Table 4.1, assuming that the torus has a radius of $1 R_J$.

Table 5.1
Angles Between Cup Normals
and the Flow Direction of a Strictly Corotating Plasma
During Io Flyby on 5 March 1979

| SCET | θ_A | θ_B | θ_C | θ_D |
|------|------------|------------|------------|------------|
| 1422 | 57° | 86° | 89° | 163° |
| 1451 | 64° | 93° | 96° | 169° |
| 1454 | 64° | 93° | 96° | 169° |
| 1457 | 65° | 94° | 97° | 169° |
| 1500 | 66° | 95° | 98° | 170° |
| 1504 | 67° | 95° | 98° | 171° |
| 1507 | 67° | 96° | 99° | 171° |
| 1510 | 68° | 96° | 99° | 171° |

Table 5.2
Plasma Parameters Derived From Spectra Taken Near Io
Difference from Corotation

| SCET | ΔV_r | ΔV_ϕ | ΔV_z | $n_{O^{++}}$ | n_{O^+} | n_{S^+} | ρ | w | V_A |
|------|--------------|-----------------|--------------|--------------|-----------|-----------|--------|----|-------|
| 1422 | -4.9 | 4.5 | -2.9 | 593 | 226 | 801 | 3.9 | 31 | 210 |
| 1451 | -4.2 | 1.2 | -0.7 | 702 | 104 | 875 | 4.1 | 35 | 200 |
| 1454 | -13.7 | -2.9 | -1.4 | 1044 | 289 | 1356 | 6.5 | 36 | 160 |
| 1457 | -22.7 | -6.2 | -0.5 | 1088 | 602 | 1233 | 6.7 | 36 | 160 |
| 1500 | -16.4 | -7.7 | -0.7 | 886 | 338 | 706 | 4.2 | 36 | 200 |
| 1504 | 4.1 | -5.4 | -3.3 | 538 | 228 | 500 | 2.8 | 33 | 250 |
| 1507 | 9.7 | 10.0 | -7.1 | 457 | 253 | 500 | 2.7 | 33 | 250 |
| 1510 | -5.0 | 9.5 | -5.6 | 621 | 338 | 682 | 3.7 | 32 | 210 |

All speeds are in km/sec

n in cm^{-3}

ρ in 10^4 AMU/cm^3

Velocity differences in Jupiter equatorial cylindrical polar coordinates

Table 5.3
Results of Linear Regression to Test Equation 4.31

| $\vec{V}_O \cdot \hat{x}_{\text{mag}}$ | $\vec{V} \cdot \hat{y}_{\text{mag}}$ | $\vec{V} \cdot \hat{z}_{\text{mag}}$ | V_A |
|--|--------------------------------------|--------------------------------------|------------|
| 60 km/sec | 13 km/sec | -2 km/sec | 600 km/sec |

Figure Captions

Figure 2.1 The PLS Experiment. The figure consists of a sketch of instrument, showing the positions of the four cups.

Figure 2.2 The Voyager Spacecraft. The figure consists of a drawing of the spacecraft, showing the location of all of the scientific instruments and the orientation of the axes of the spacecraft coordinate system.

Figure 2.3 The PLS Experiment Aperture Positions. The figure shows the position of the four cups as viewed from along the main sensor symmetry axis, and the directions of the coordinate axes of the cup coordinate system.

Figure 2.4 Main Sensor Cross-section. The figure shows a cross-section of one of the cups in the main cluster, including the positions of the grids and the collector.

Figure 2.5 Main sensor Aperture and Collector Areas. The figure shows the shape and size of the aperture and the collector of a main sensor cup.

Figure 2.6 Modulator Voltage and Collector Current Versus Time. The figure shows the voltage waveform which is impressed on the modulator grid and the resulting collector current waveform for a hypothetical positive ion measurement.

Figure 2.7 Geometry for Grid Transparency Calculation. The figure shows a beam of particles incident on a grid of parallel, cylindrical wires. α is the angle between the beam direction and the normal to the plane of the grid, L is the distance between the centers of two adjacent wires, and d is the wire diameter. The wires run in the \hat{y} -direction, and the \hat{z} -direction is normal to the grid plane, with $+\hat{z}$ making an acute angle with the direction of the incident beam.

Figure 2.8 Definition of the Shift Vector. The figure shows the outline of the collector of one of the main sensor cups, with the image of the aperture in the collector superposed on it. An incident monoenergetic beam of particles will have the shape of the aperture as it travels through the cup. The shift vector \vec{S} is the vector which lies in the collector plane and points from the point directly underneath the center of the long side of the aperture to the corresponding point on the image of the aperture in the incident beam.

Figure 2.9 Particle Trajectory in a Main Sensor Cup. The figure shows the projection into the x - z plane (cup coordinates) of the path of a particle through one of the main sensor cups. The locations of the nine grids are shown, along with identifying numbers which correspond to the grid numbers in Figure 2.4. The x -component of the shift vector is shown as the difference between the x -coordinate of the particle as it crosses the plane of the aperture grid and the x -coordinate of the particle when it reaches the collector.

Figure 2.10 Potential Versus Distance From Aperture. The figure shows the electrostatic potential inside the cup plotted against z (cup coordinates) in the approximation that the potential depends only on z . Three cases are shown; a) the main sensor, b) the side sensor in positive ion mode, and c) the side sensor in the electron mode.

Figure 2.11 The Sensitive Area of a Main Sensor Cup. The figure shows the 16 distinct regions in which there is a different functional dependence of the sensitive area on the shift vector.

Figure 2.12 Side Sensor Cross-Section. The figure consists of a cross-section of the D-cup, showing the locations of the eight grids and the collector. Compare with Figure 2.4.

Figure 2.13 Common Area of Two Circles. The figure shows the common area of two circles of different radius as a function of the distance between their centers. It also defines the parameter X . If the line connecting the centers of the two circles is taken to be the x axis, with the center of the larger circle at $x=0$, then X is the x coordinate of the points of intersection of the two circles.

Figure 2.14 Sensitive Area of the D-cup. The D-cup sensitive area is the common area of three circles of different radii all of whose centers lie on the same line. The figure shows four different cases for which different combinations of arcs from the three circles determine the boundary of the common area. The formula for the common area for the cases shown in a, b, and c is given by Equation 2.27a, while the formula for the case shown in d is given by Equation 2.27b. The relative positions of the points X_{AC} and X_{GC} determine which formula is to be used.

Figure 2.15 Main Sensor Sensitive Area Versus S_y/h . The figure shows a family of curves which represent the main sensor sensitive area as a function of the ratio of the y-component of the shift vector divided by the height of the cup, with the x-component of the shift vector as a parameter.

Figure 2.16 Y_u and Y'_u Versus S_x/h . The figure shows a graph of two of the functions required for the trapezoidal approximation to the sensitive area of the main sensor cups.

Figure 2.17 Main Sensor Sensitive Area Versus S_y/h (Trapezoidal Approximation). The figure shows a family of curves which represent the main sensor sensitive area as a function of the ratio of the y-component of the shift vector divided by the height of the cup, with the x-component of the shift vector as a parameter, in the trapezoidal approximation. Compare with Figure 2.15.

Figure 2.18 Main Sensor Sensitive Area Versus $|S_x|/h$ (Trapezoidal Approximation). The plot is valid for S_y equal to zero.

Figure 2.19 3-D Plot of the Sensitive Area Versus X and Y in the Trapezoidal Approximation.

Figure 2.20 3-D Plot of the Full Response Function. The figure shows the full response function plotted versus X and Y, utilizing the trapezoidal approximation (defined by Equations 2.3, 2.9, and 2.31-2.34).

Figure 3.0 The Cruise Maneuver. The figure consists of four lines, each of which has several dots on it. Each line represents a polar plot of the cup coordinates of a hypothetical purely radial solar wind during the cruise maneuver. The dots correspond to the times of the M-mode spectra. Each dot which corresponds to the time of a spectrum which was analyzed is numbered. The spectrum taken when the spacecraft was in the orientation corresponding to point 1 is plotted in Figure 3.1; that corresponding to point 2 is Figure 3.2; etc. The orientation of the spacecraft was the same for the spectra shown in Figures 3.4 and 3.5.

Figure 3.1 Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 1. The staircases are the data, while the smooth curve is the fit.

Figure 3.1a Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 1. The staircases are the data, while the smooth curve is the fit. The lower channels in the B-cup were not used in determining the plasma parameters; they are included to illustrate that the currents in these channels is not only noise, but actually includes some signal.

Figure 3.1b Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 1. The staircases are the data, while the smooth curve is a simulation done assuming "unity" response using the plasma parameters determined from the fit which is plotted in Figure 3.1. Note how the locations of the peaks in the simulation are correct, but their heights and shapes are wrong.

Figure 3.2 Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 2. The staircases are the data, while the smooth curve is the fit.

Figure 3.3 Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 3. The staircases are the data, while the smooth curve is the fit.

Figure 3.4 Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 4. The staircases are the data, while the smooth curve is the fit. This fit was done using the full response function. The data are the same as were fit using the "unity response" approximation for Figure 3.4a.

Figure 3.4a Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 4. The staircases are the data, while the smooth curve is the fit. This fit was done using the "unity" response approximation. The data are the same as were fit using the full response function for Figure 3.4.

Figure 3.5 Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 5. The staircases are the data, while the smooth curve is the fit. This fit was done using the full response function. The data are the same as were fit using the "unity response" approximation for Figure 3.5a. The orientation of the spacecraft was the same as for Spectrum 4.

Figure 3.5a Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 1. The staircases are the data, while the smooth curve is the fit. This fit was done using the "unity response" approximation. The data are the same as were fit using the full response function for Figure 3.5. The orientation of the spacecraft was the same as for Spectrum 4.

Figure 3.6 Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 6. The staircases are the data, while the smooth curve is the fit.

Figure 3.7 Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 7. The staircases are the data, while the smooth curve is the fit.

Figure 3.8 Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 8. The staircases are the data, while the smooth curve is the fit. The change in the orientation of the spacecraft between the times of the peaks in the various cups was corrected for.

Figure 3.8a Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 8. The staircases are the data, while the smooth curve is the fit. The change in the orientation of the spacecraft between the time of the peaks in the different cups was not compensated for. Compare Figure 3.8.

Figure 3.9 Reduced Distribution Function Versus Velocity for Cruise Maneuver Spectrum 9. The staircases are the data, while the smooth curve is the fit.

Figure 4.1 The Goldreich Picture of the Interaction between Io and the Jovian Magnetosphere. Field aligned currents flow in the fluxtube which links Io. The current system is closed in the Jovian ionosphere. A similar system for the southern hemisphere is not shown.

Figure 4.2 Io's Alfvén Wings. Two views are shown of the Alfvén wing and associated current system. Figure 4.2a shows the Alfvén wings as seen looking along the flow direction of the corotating plasma. Figure 4.2b shows the Alfvén wings as viewed by an observer located along on the line which connects Jupiter and Io. In model shown here, due to Drell, it is not necessary to close the current system in the Jovian magnetosphere, so the currents are determined by the Alfvén wave impedance of the plasma.

Figure 4.3 The Streamlines of Potential Flow Around an Infinite Cylinder.

Figure 5.1 The Voyager I Jupiter Encounter. The figure shows the projection of the spacecraft trajectory into the Jovian Equatorial plane, along with the directions of the main sensor symmetry axis and the D-cup -z axis. Note how the D-cup points into the corotating flow on the inbound pass, while none of the cups points into the flow on the outbound pass.

Figure 5.2 The Voyager I Io Flyby. The figure shows the trajectory in an Io-centered coordinate system, projected into Jupiter's equatorial plane. The y-axis points toward Jupiter, the x-axis points in the direction of the corotating flow, and the z-axis completes a right-handed system. The axes are labeled in units of Io radii. The circle with its center at the origin is Io; the circle with its center displaced from the origin is the probable position of the Io Alfvén wing in the plane of the spacecraft orbit.

Figure 5.3 The Voyager I Io Flyby. The coordinate system used to describe the position of the spacecraft this figure is that defined in the caption for Figure 5.2. The figure shows z plotted against $\sqrt{(x^2 + y^2)}$.

Figure 5.4 Reduced Distribution Function Versus Velocity for the spectrum taken at SCET 1422. The staircases are the data; the smooth curves are the fits. The dropouts in the spectra are caused by interference with another instrument on the spacecraft.

Figure 5.5 Reduced Distribution Function Versus Velocity for the spectrum taken at SCET 1451. The staircases are the data; the smooth curves are the fits. The dropouts in the spectra are caused by interference with another instrument on the spacecraft.

Figure 5.6 Reduced Distribution Function Versus Velocity for the spectrum taken at SCET 1454. The staircases are the data; the smooth curves are the fits. The dropouts in the spectra are caused by interference with another instrument on the spacecraft.

Figure 5.7 Reduced Distribution Function Versus Velocity for the spectrum taken at SCET 1457. The staircases are the data; the smooth curves are the fits. The dropouts in the spectra are caused by interference with another instrument on the spacecraft.

Figure 5.8 Reduced Distribution Function Versus Velocity for the spectrum taken at SCET 1500. The staircases are the data; the smooth curves are the fits. The dropouts in the spectra are caused by interference with another instrument on the spacecraft.

Figure 5.9 Reduced Distribution Function Versus Velocity for the spectrum taken at SCET 1504. The staircases are the data; the smooth curves are the fits. The dropouts in the spectra are caused by interference with another instrument on the spacecraft.

Figure 5.10 Reduced Distribution Function Versus Velocity for the spectrum taken at SCET 1507. The staircases are the data; the smooth curves are the fits. The dropouts in the spectra are caused by interference with another instrument on the spacecraft.

Figure 5.11 Reduced Distribution Function Versus Velocity for the spectrum taken at SCET 1510. The staircases are the data; the smooth curves are the fits. The dropouts in the spectra are caused by interference with another instrument on the spacecraft.

Figure 5.12 Flow Velocity Versus Time. The figure shows the flow velocities determined from the fits to the spectra of Figs. 5.5-5.11. The smooth curves are the velocities determined from a "best fit" to the model of Neubauer. The quantities V_1 , V_2 and V_3 are the components of \vec{V} along \hat{x}_{mag} , \hat{y}_{mag} , and \hat{z}_{mag} respectively.

Figure 5.13 Plasma Bulk Velocity at Different Positions in the Vicinity of Io. The figure shows the projection of the bulk velocity of the plasma into a plane perpendicular to the background magnetic field direction. The points at which the vectors representing the velocities intersect the line which represents the spacecraft orbit corresponds to the position of the spacecraft at the time the velocity measurement was made.

Figure 5.14 $\delta\vec{V}$ and $\delta\vec{B}$ near Io. The figure shows the projection of the deviation from uniform flow $\delta\vec{V}$ and the deviation from a dipole field $\delta\vec{B}$ into a plane perpendicular to the background magnetic field. The points of intersection of the vectors $\delta\vec{V}$ and $\delta\vec{B}$ with the spacecraft trajectory are the positions of the spacecraft at the time of the measurements.

Figures

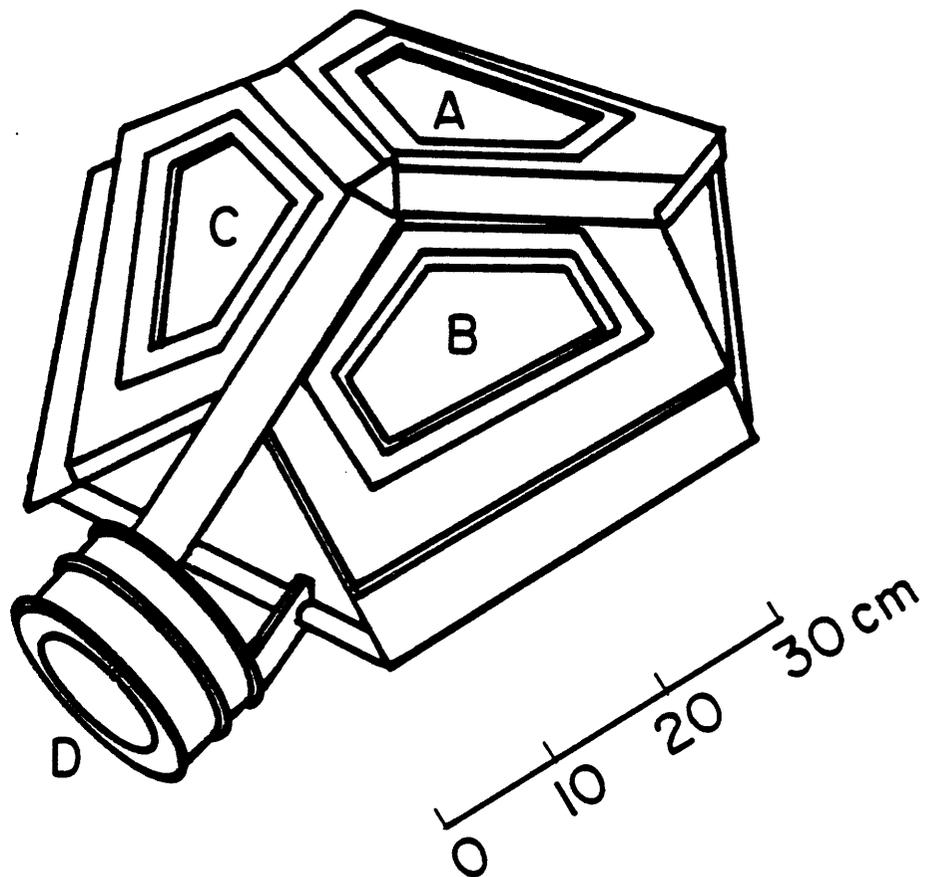


Figure 2.1

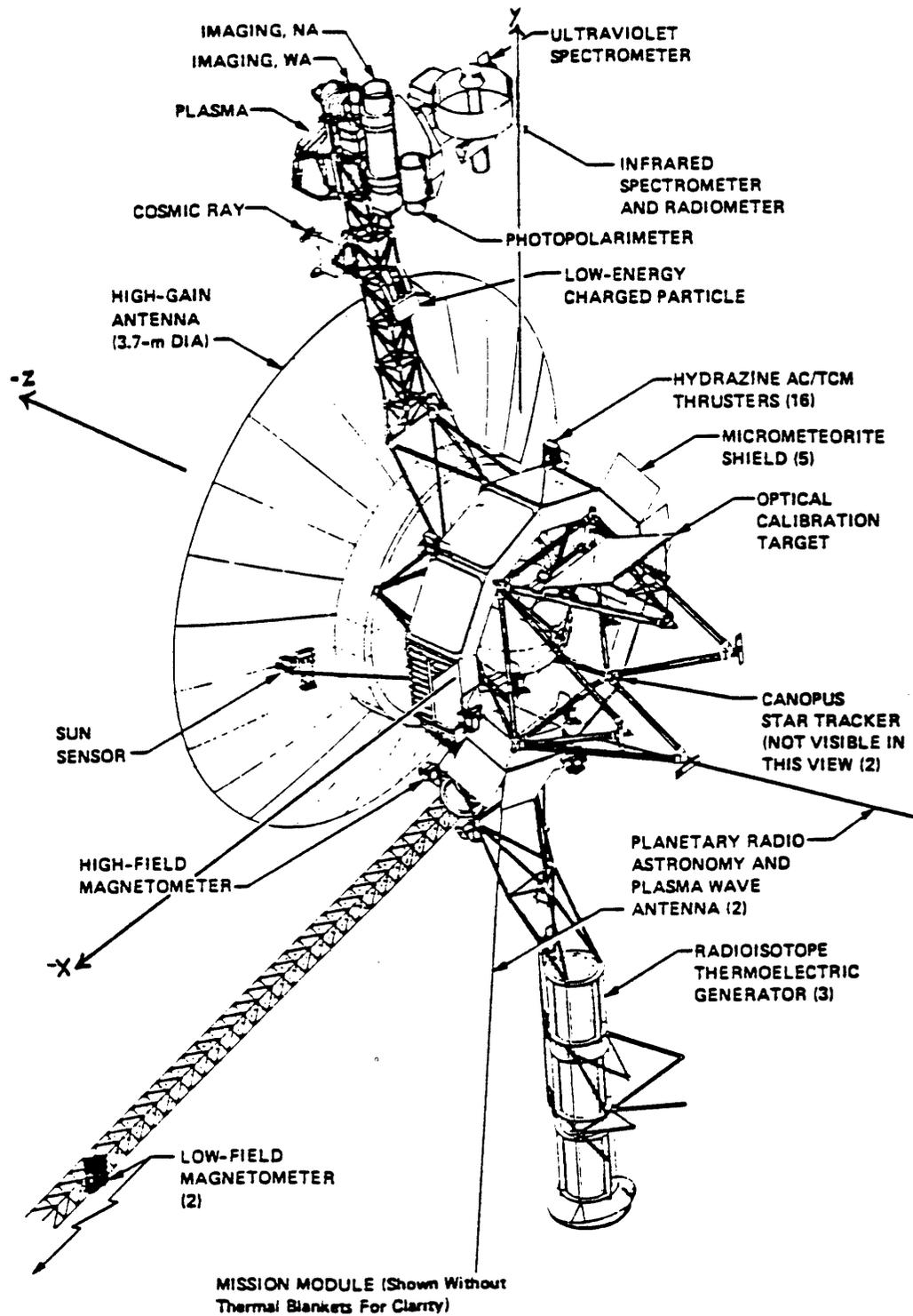


Figure 2.2

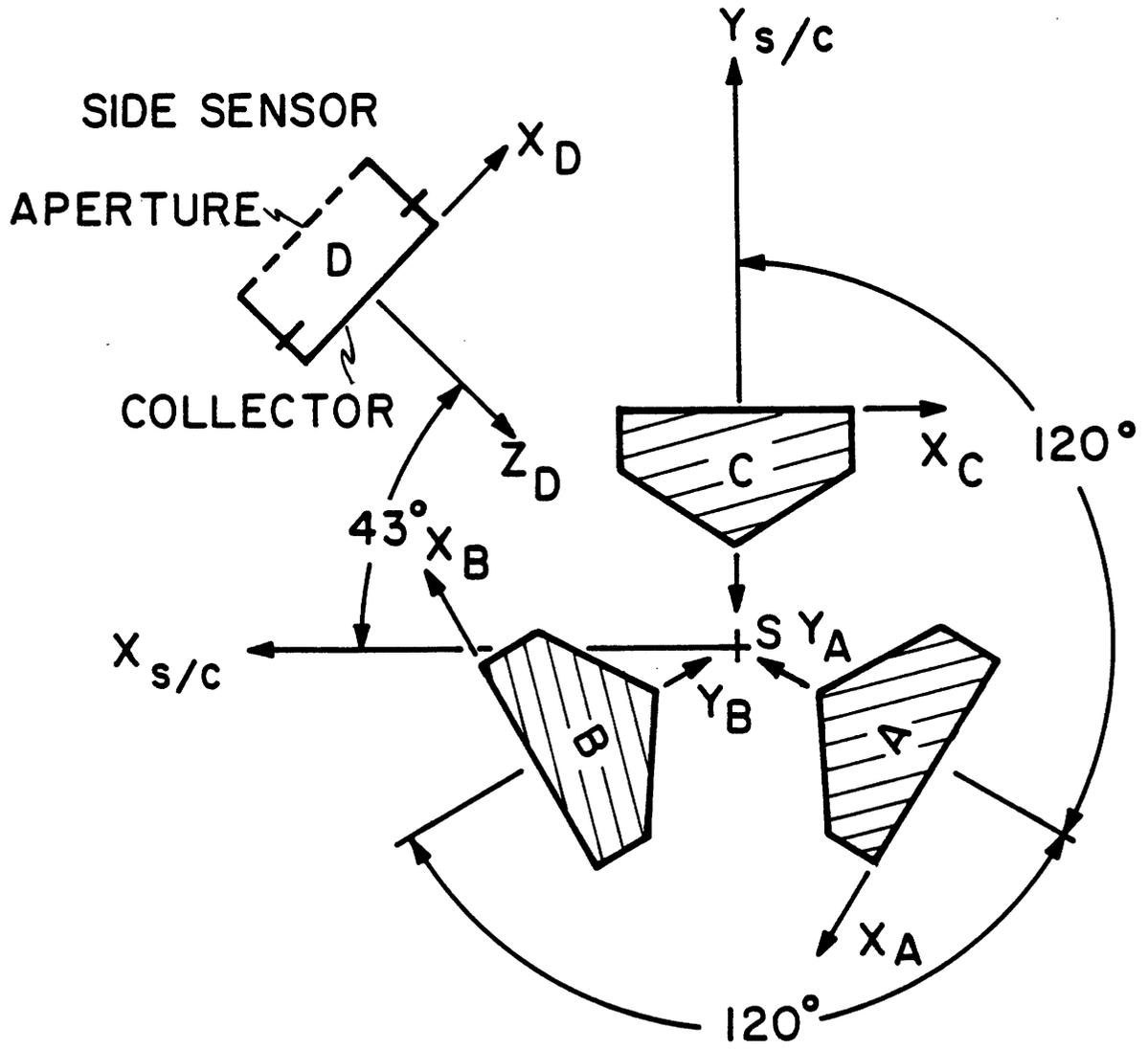
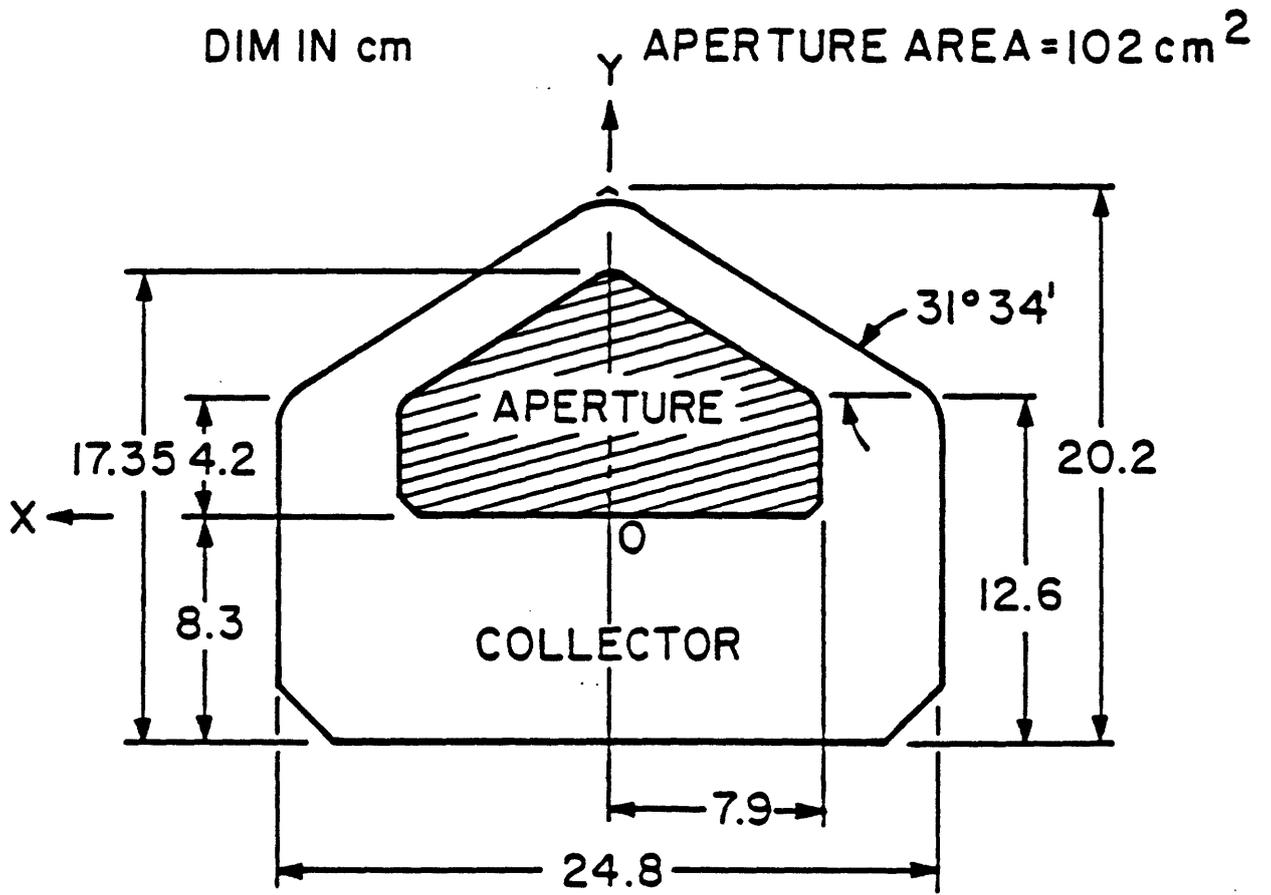
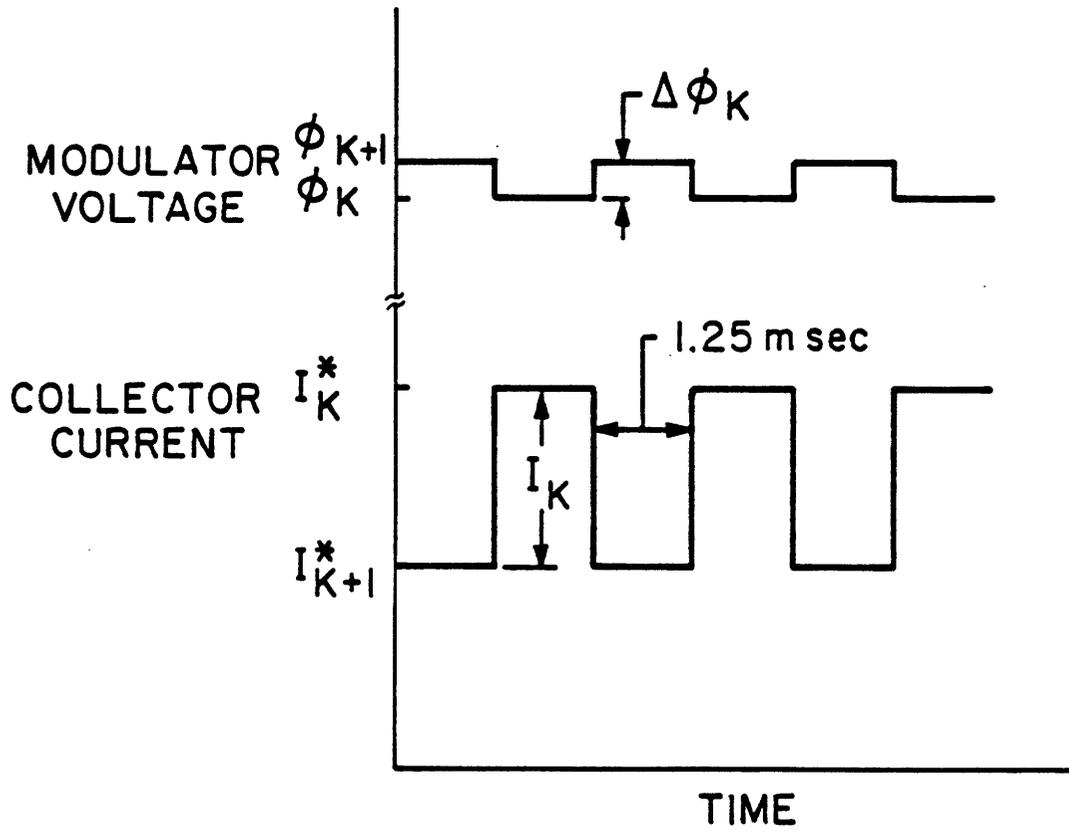


Figure 2.3



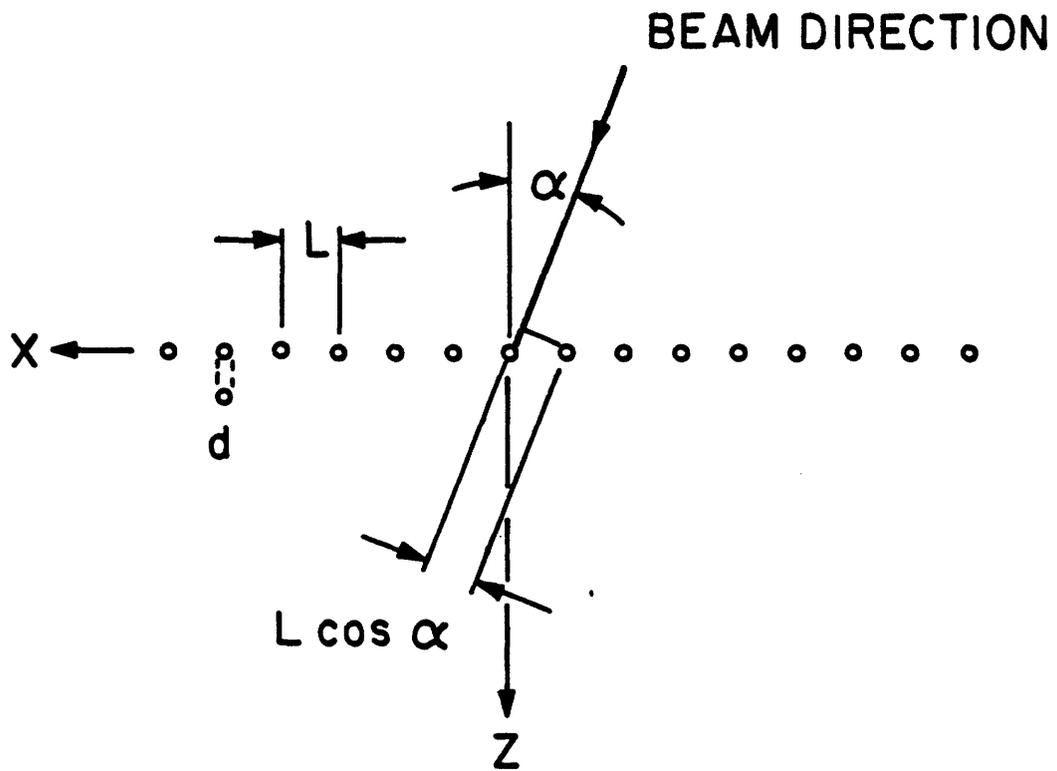
MAIN SENSOR (APERTURE AND COLLECTOR AREAS)

Figure 2.5



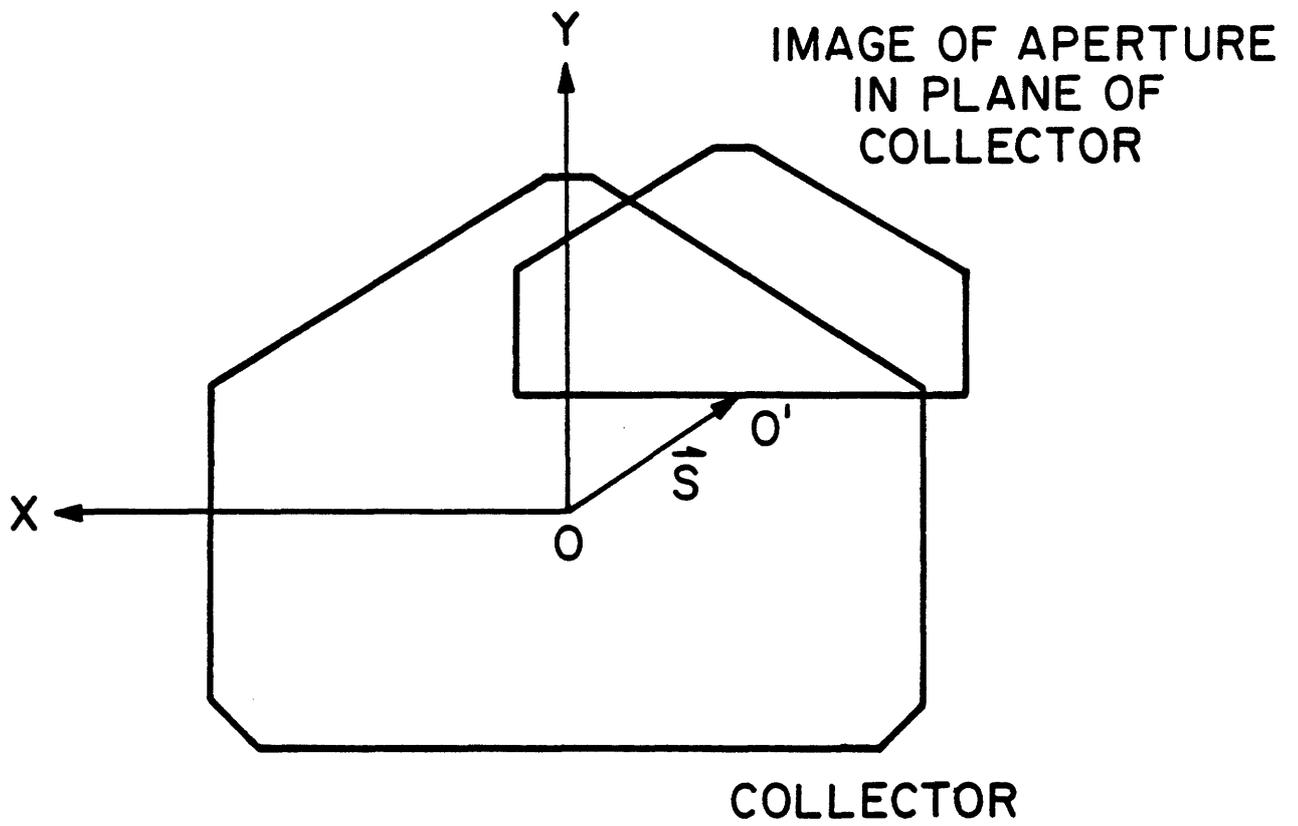
MODULATOR VOLTAGE AND COLLECTOR CURRENT
VS
TIME

Figure 2.6



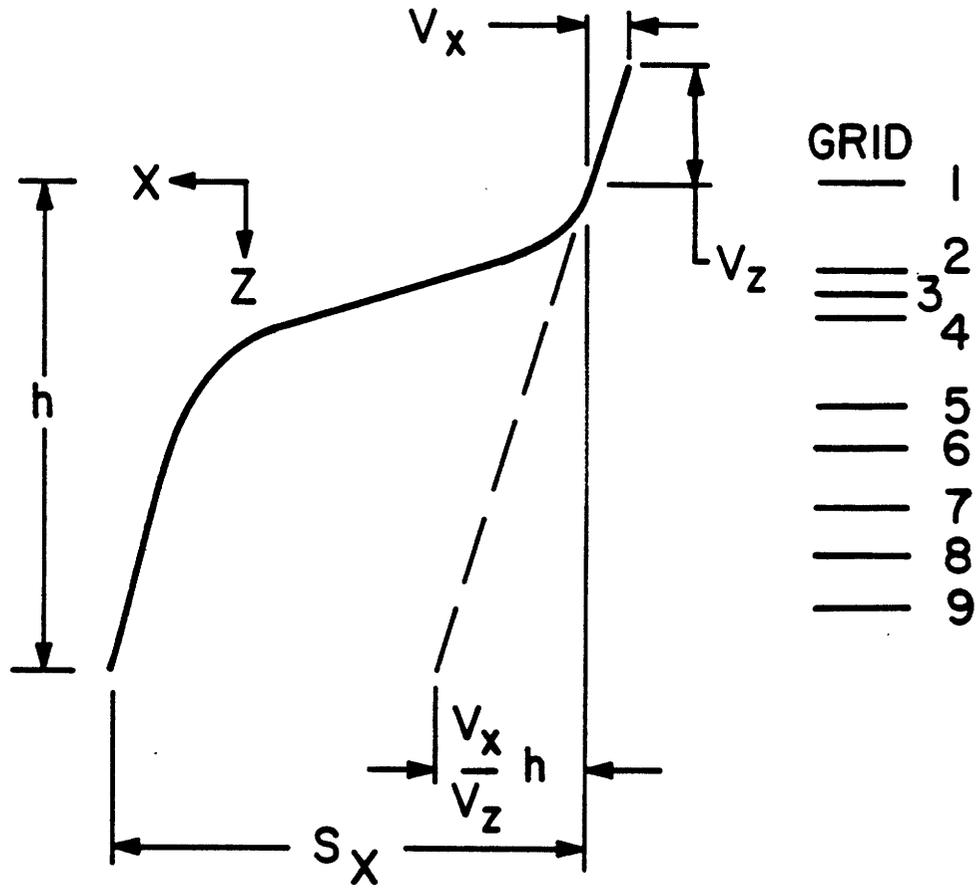
GEOMETRY FOR GRID TRANSPARENCY

Figure 2.7



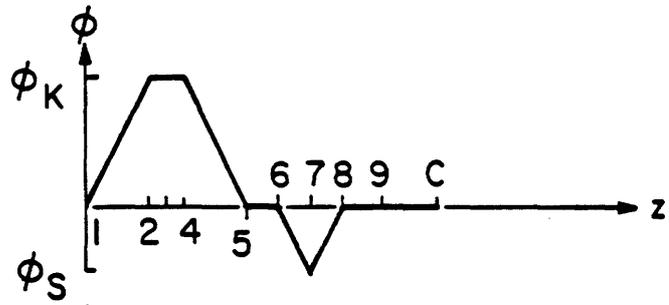
DEFINITION OF SHIFT VECTOR

Figure 2.8

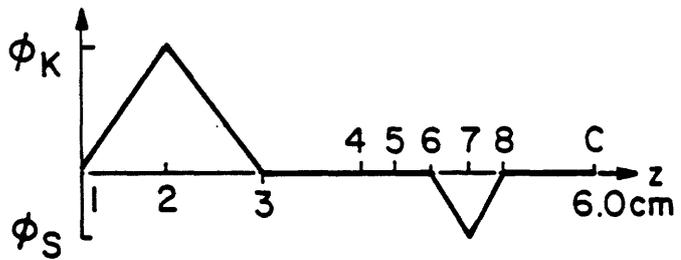


PARTICLE TRAJECTORY
IN A
MAIN SENSOR CUP

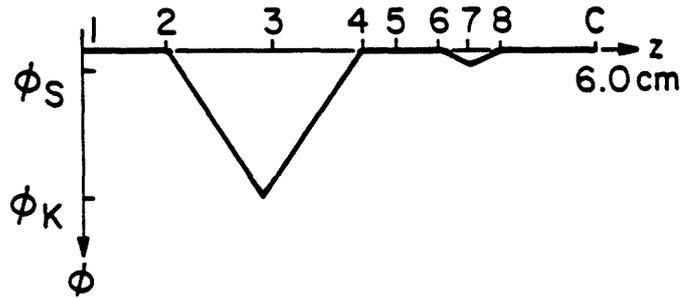
Figure 2.9



A) MAIN SENSOR



B) SIDE SENSOR-POSITIVE Ion MODE



C) SIDE SENSOR-ELECTRON MODE

POTENTIAL vs DISTANCE FROM APERTURE

Figure 2.10

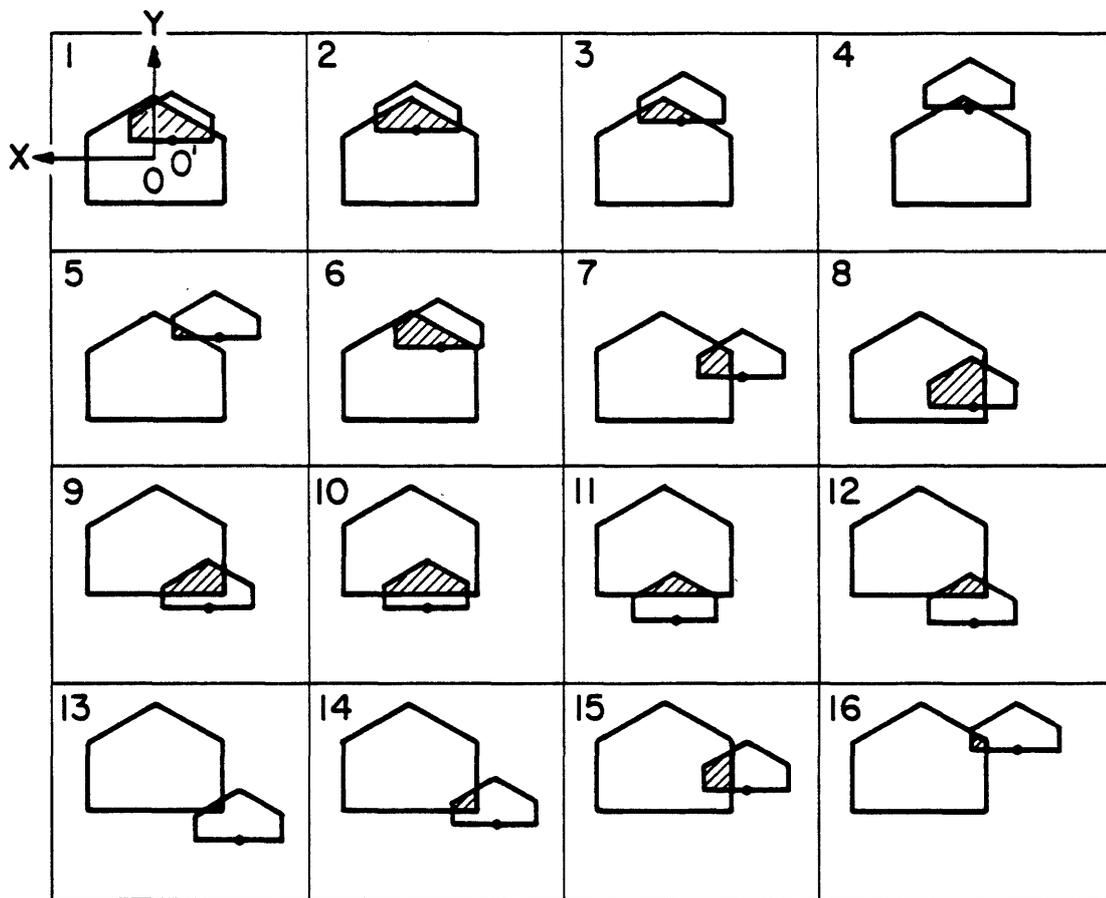
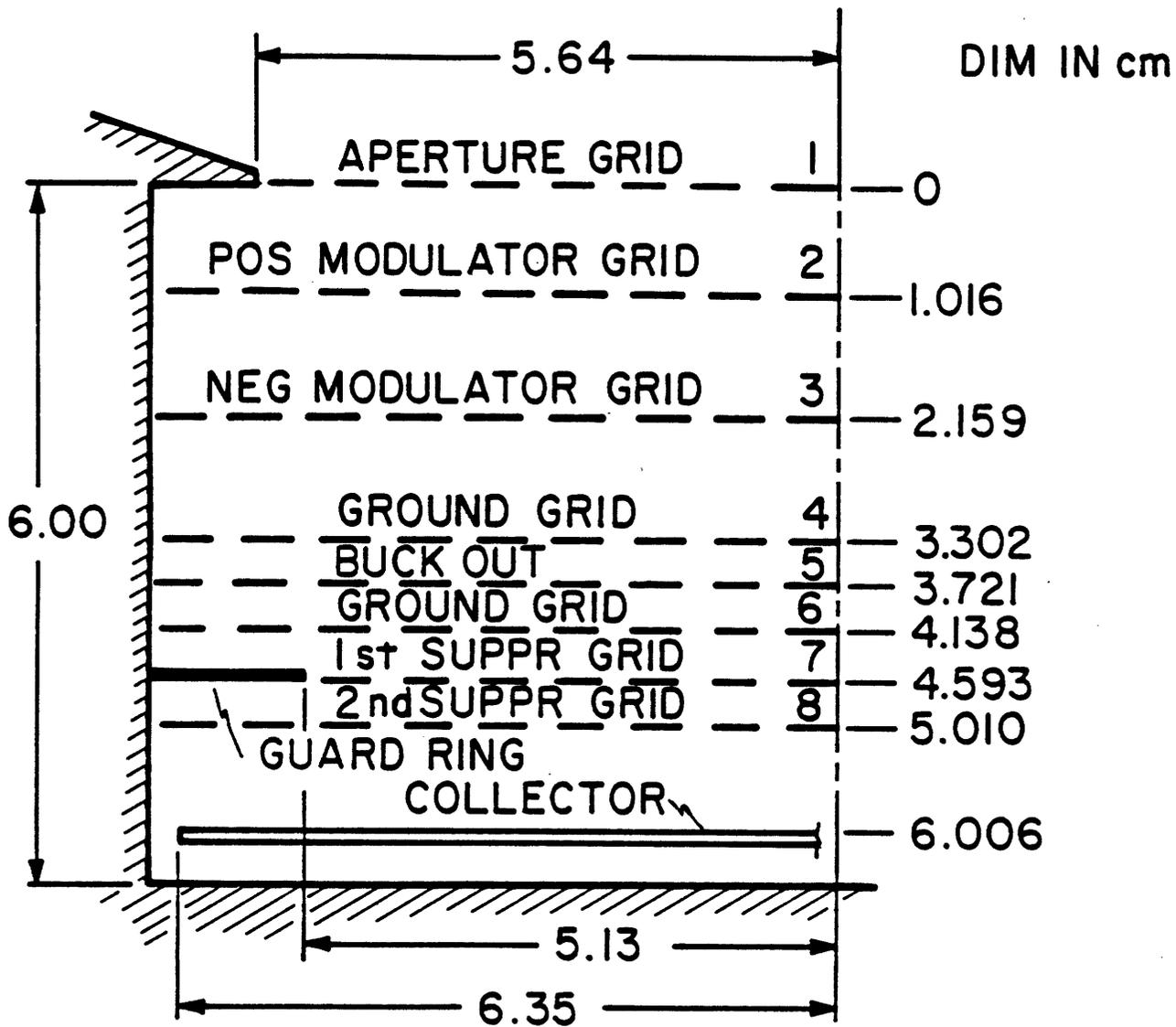
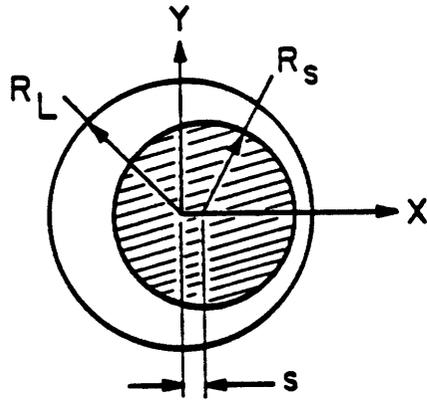


Figure 2.11

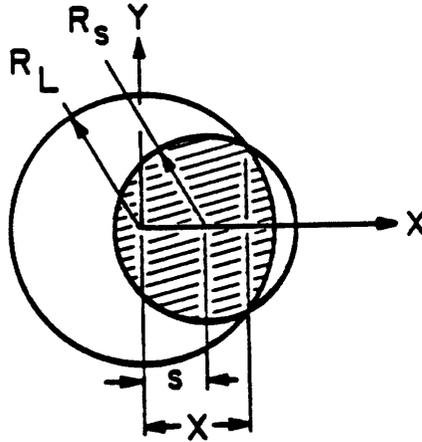


SIDE SENSOR - VERTICAL CUT

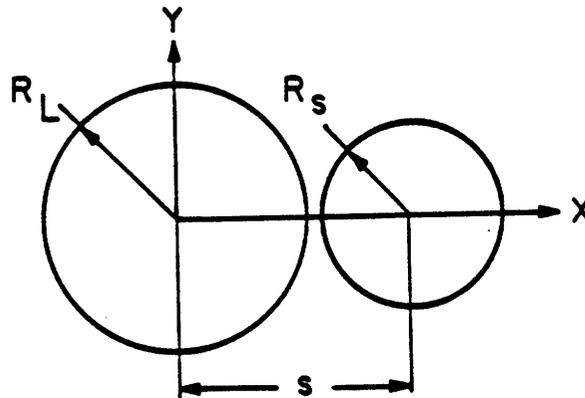
Figure 2.12



A) $s < R_L - R_s$



B) $R_L - R_s < s < R_L + R_s$



C) $s > R_L + R_s$

COMMON AREA OF TWO CIRCLES

Figure 2.13

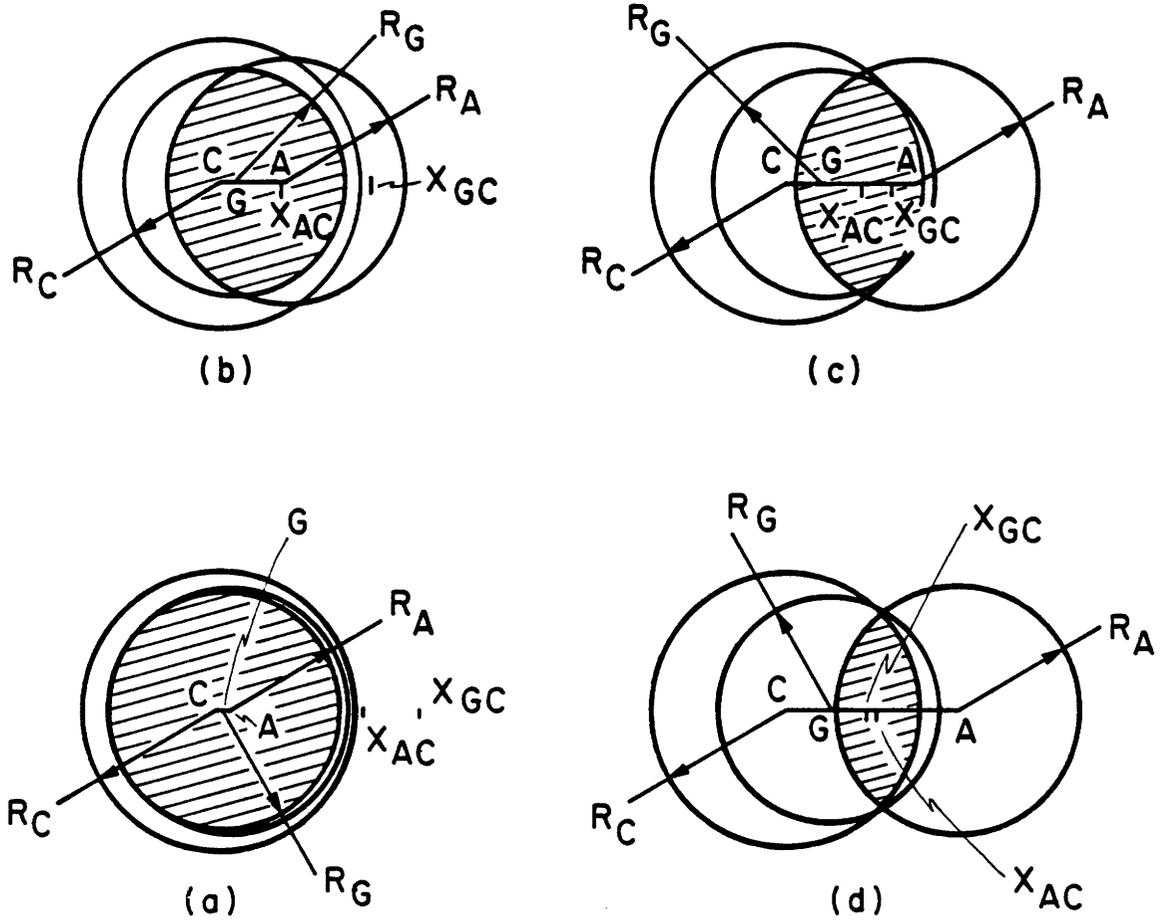
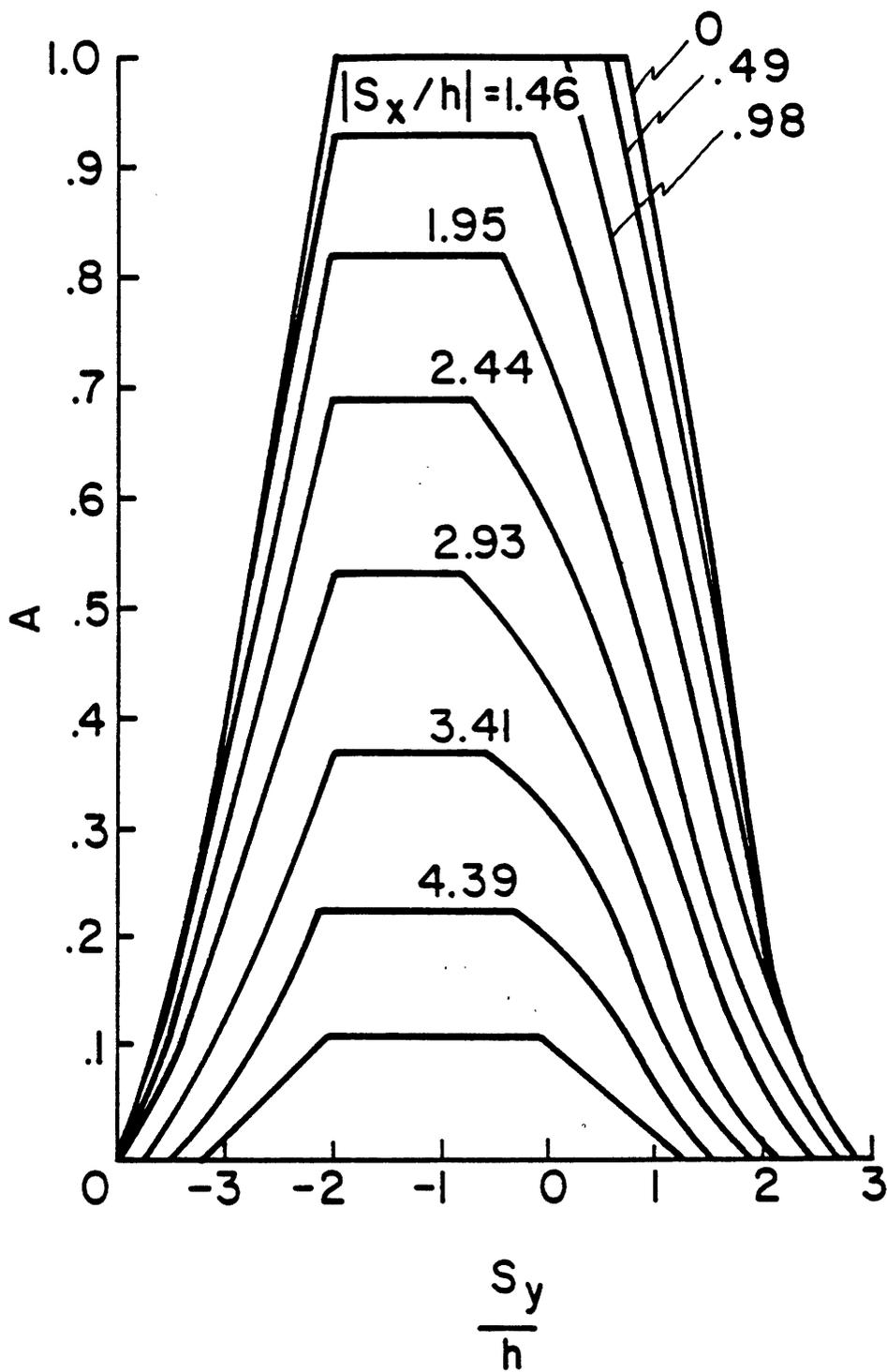


Figure 2.14



MAIN SENSOR SENSITIVE AREA vs S_y/h

Figure 2.15

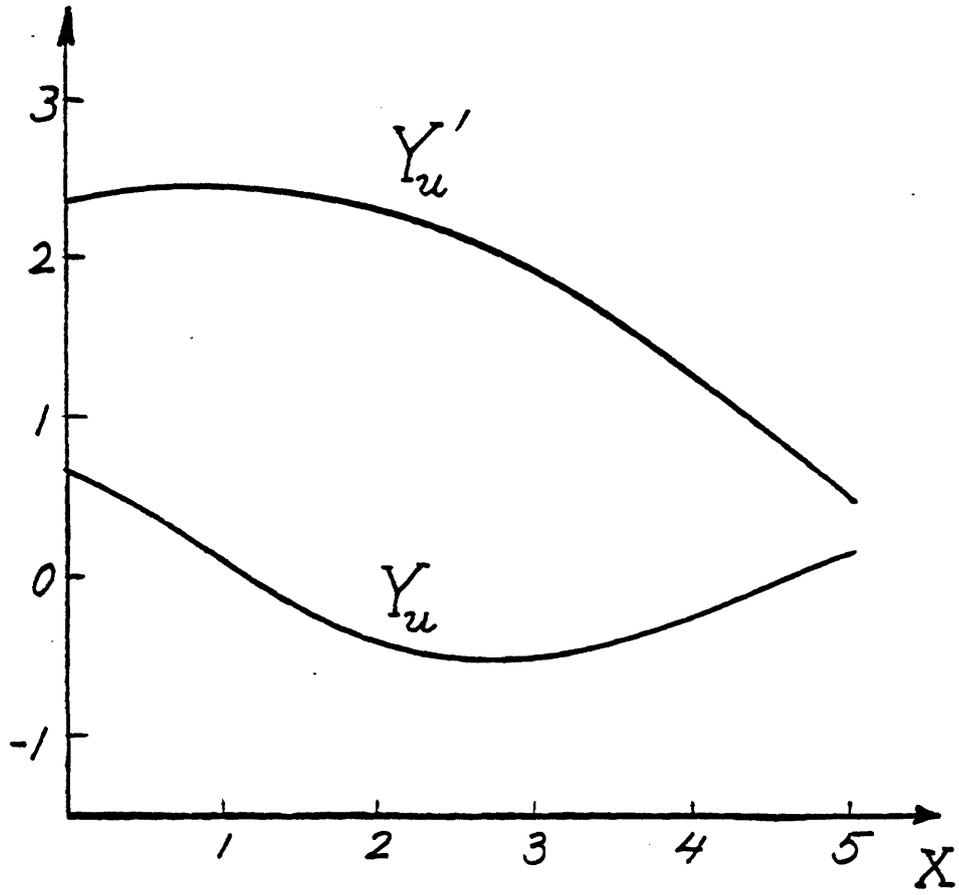
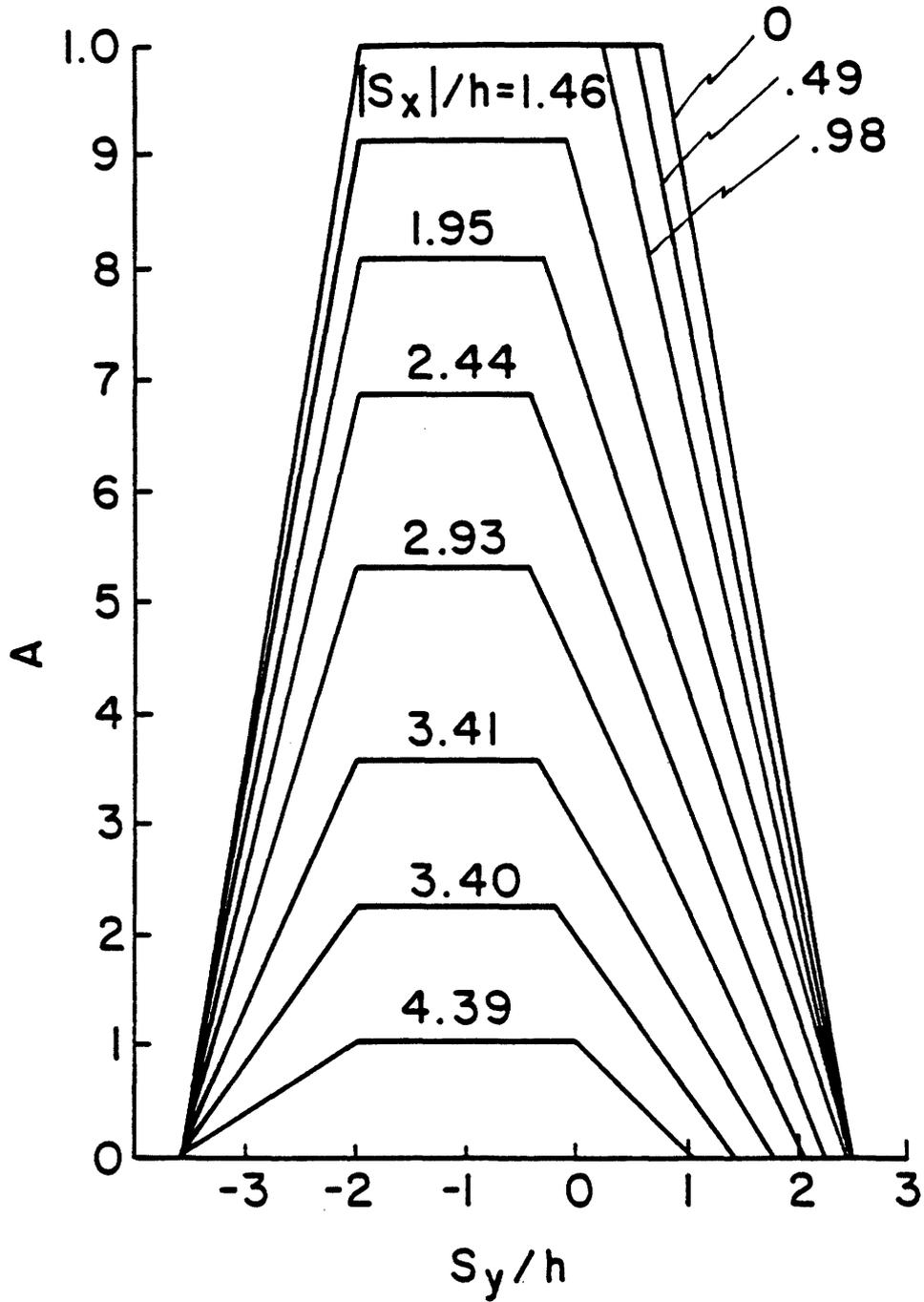


Figure 2.16



MAIN SENSOR SENSITIVE AREA vs S_y/h
(TRAPEZOIDAL APPROXIMATION)

Figure 2.17

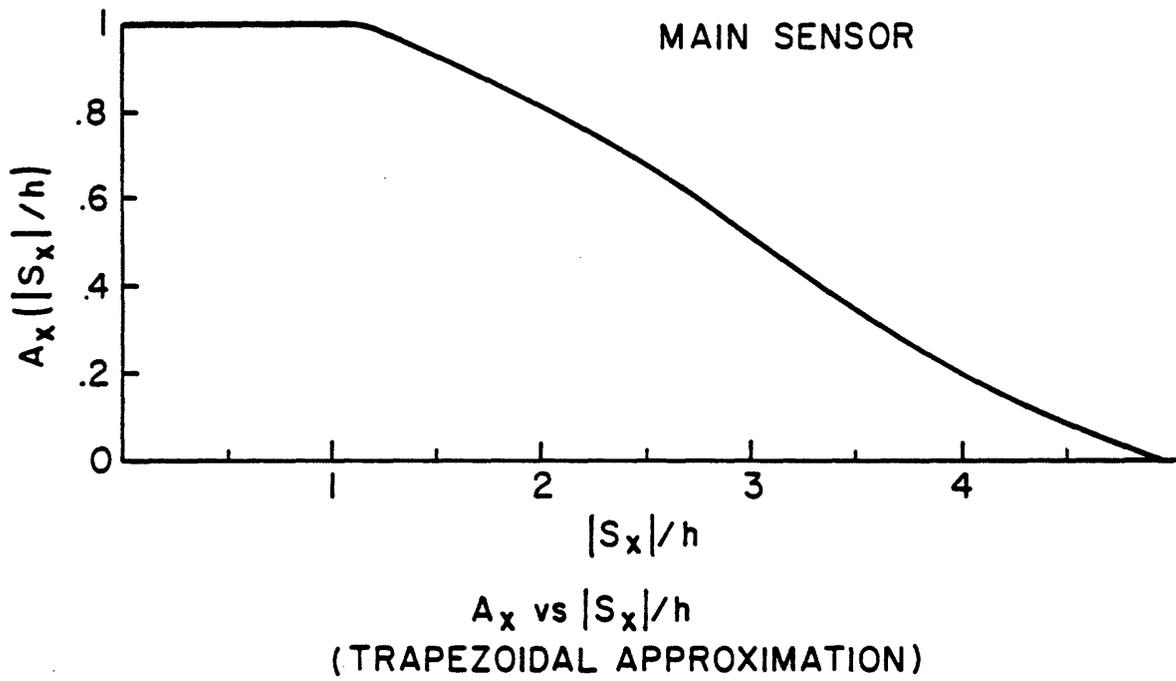


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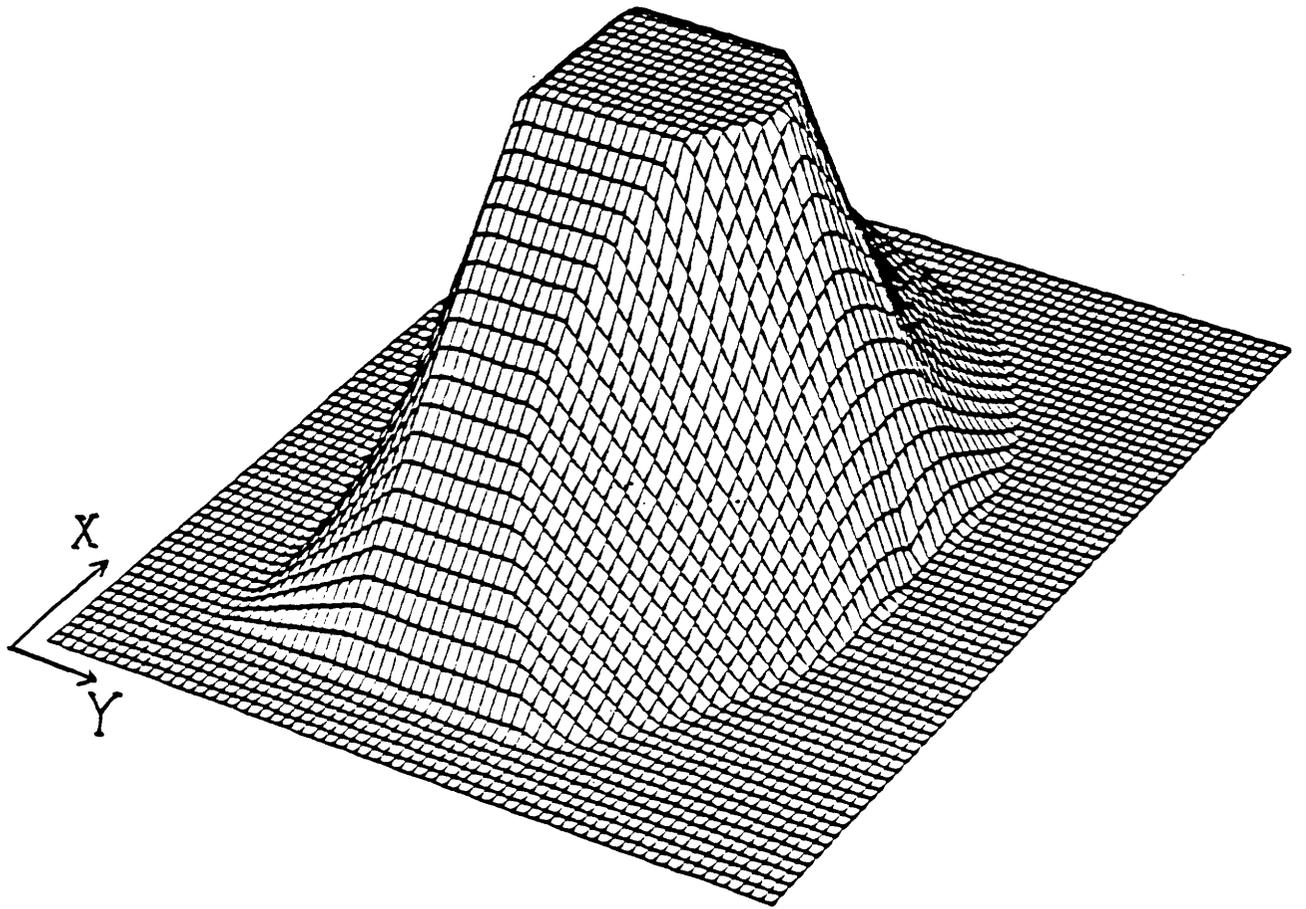


Figure 2.19

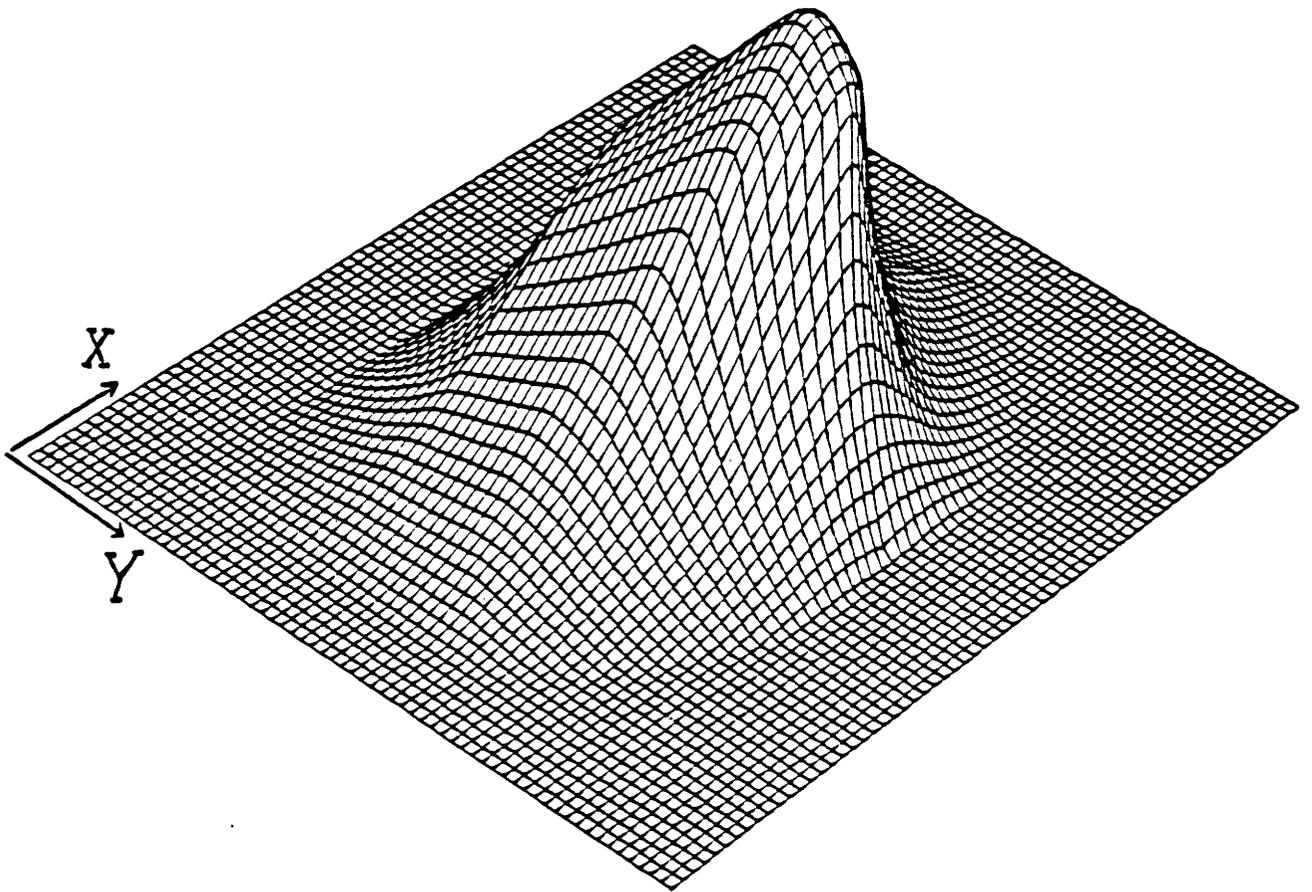


Figure 2.20

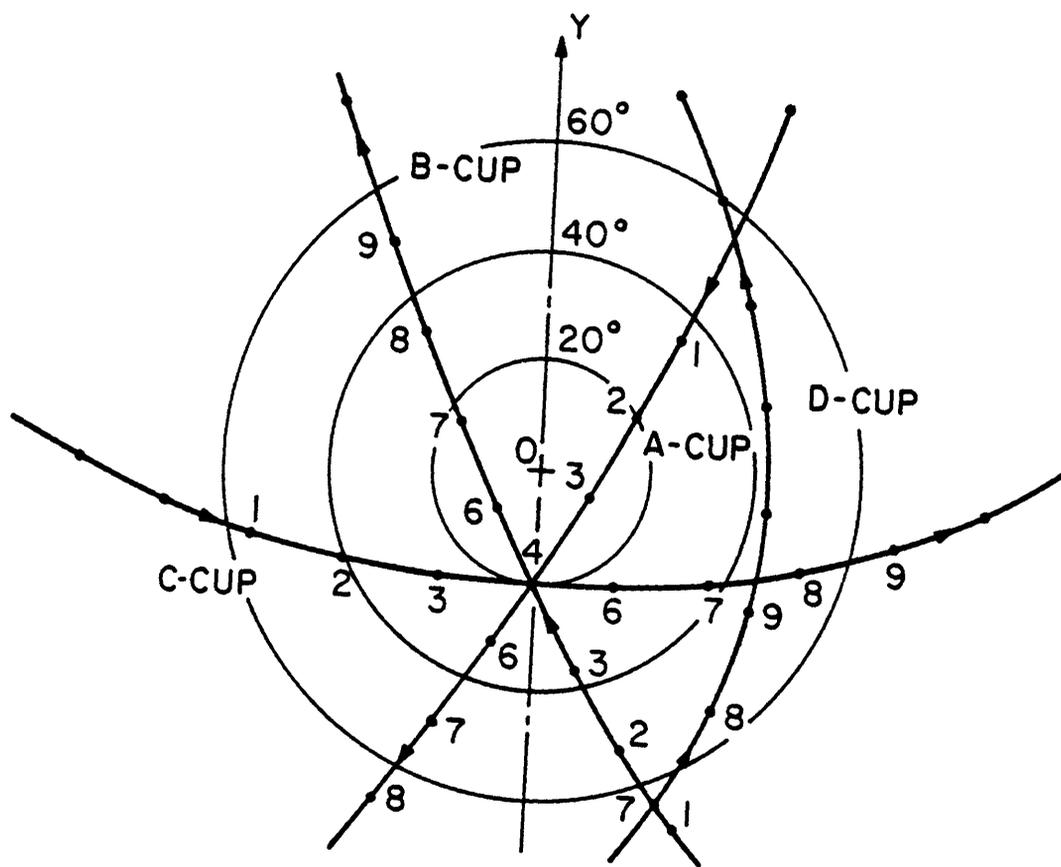


Figure 3.0

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

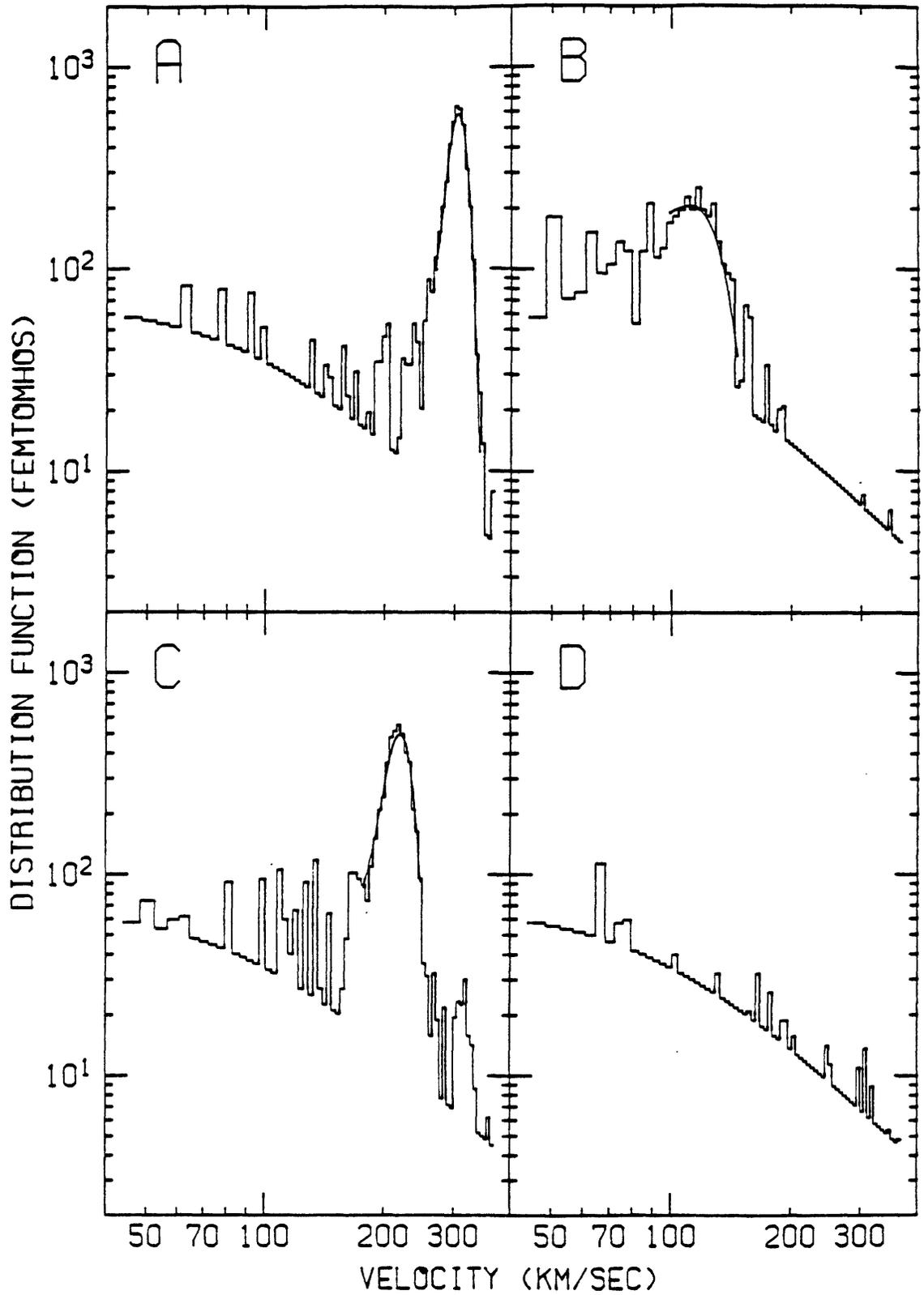


Figure 3.1

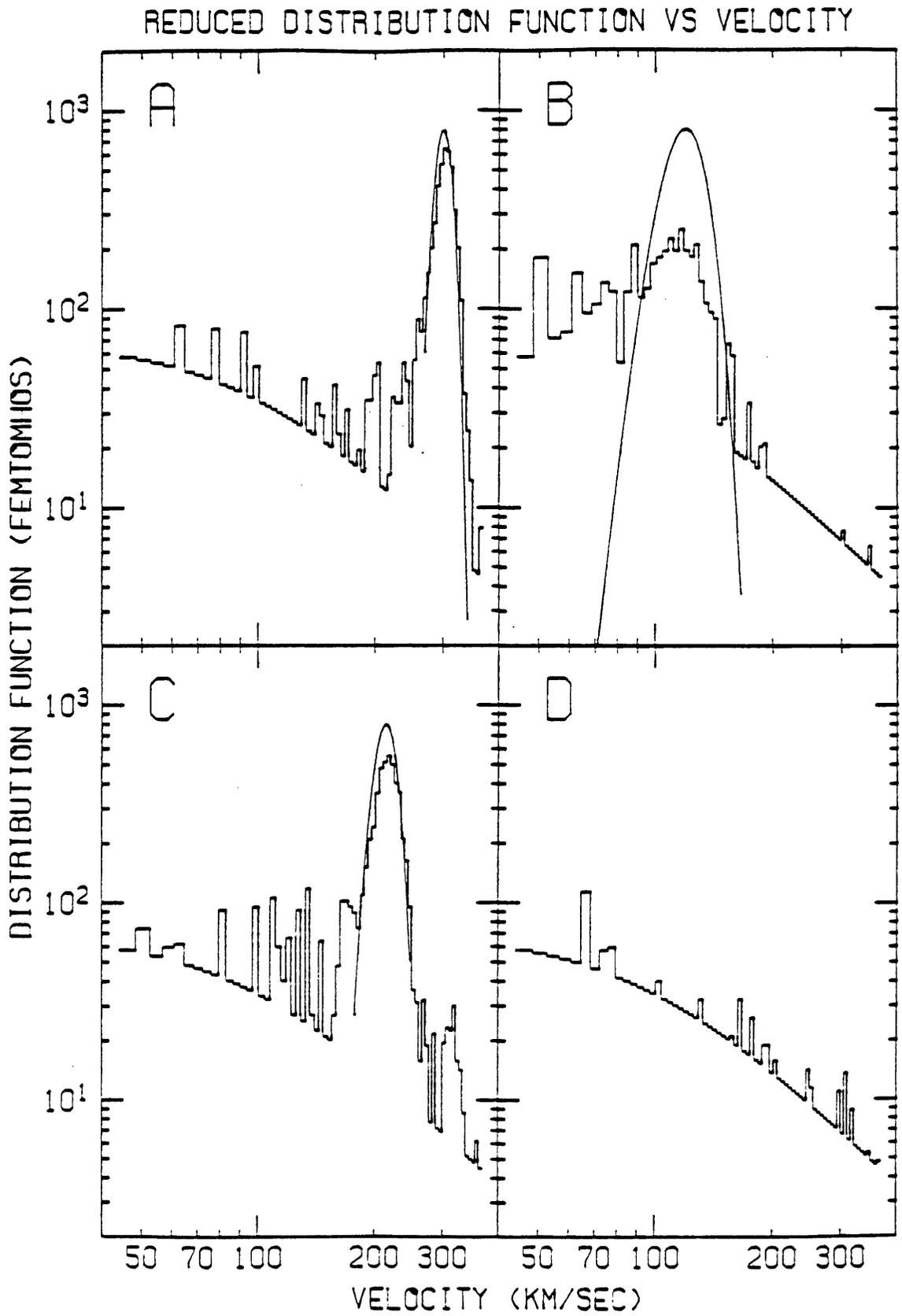


Figure 3.1a

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

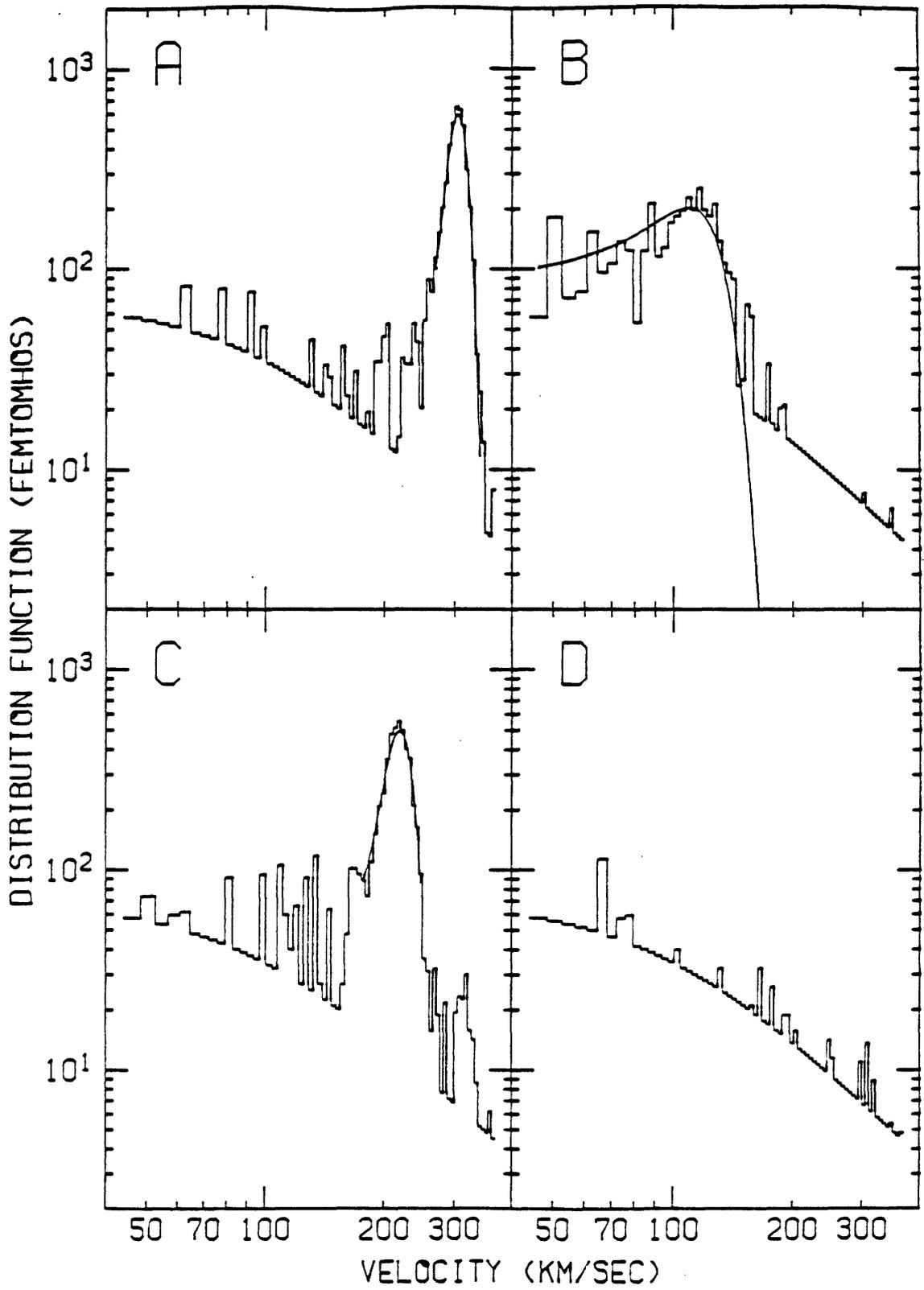


Figure 3.1b

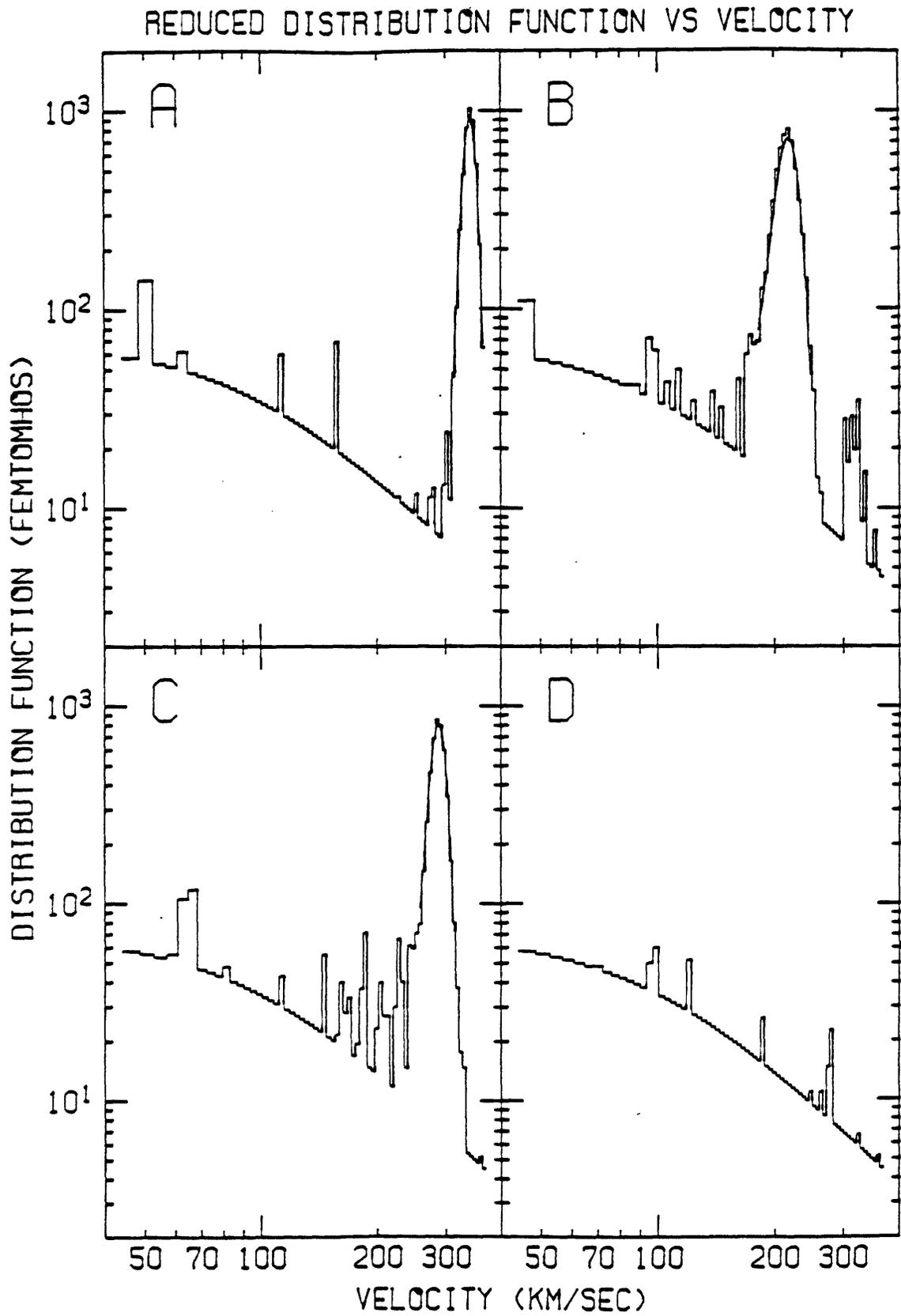


Figure 3.2

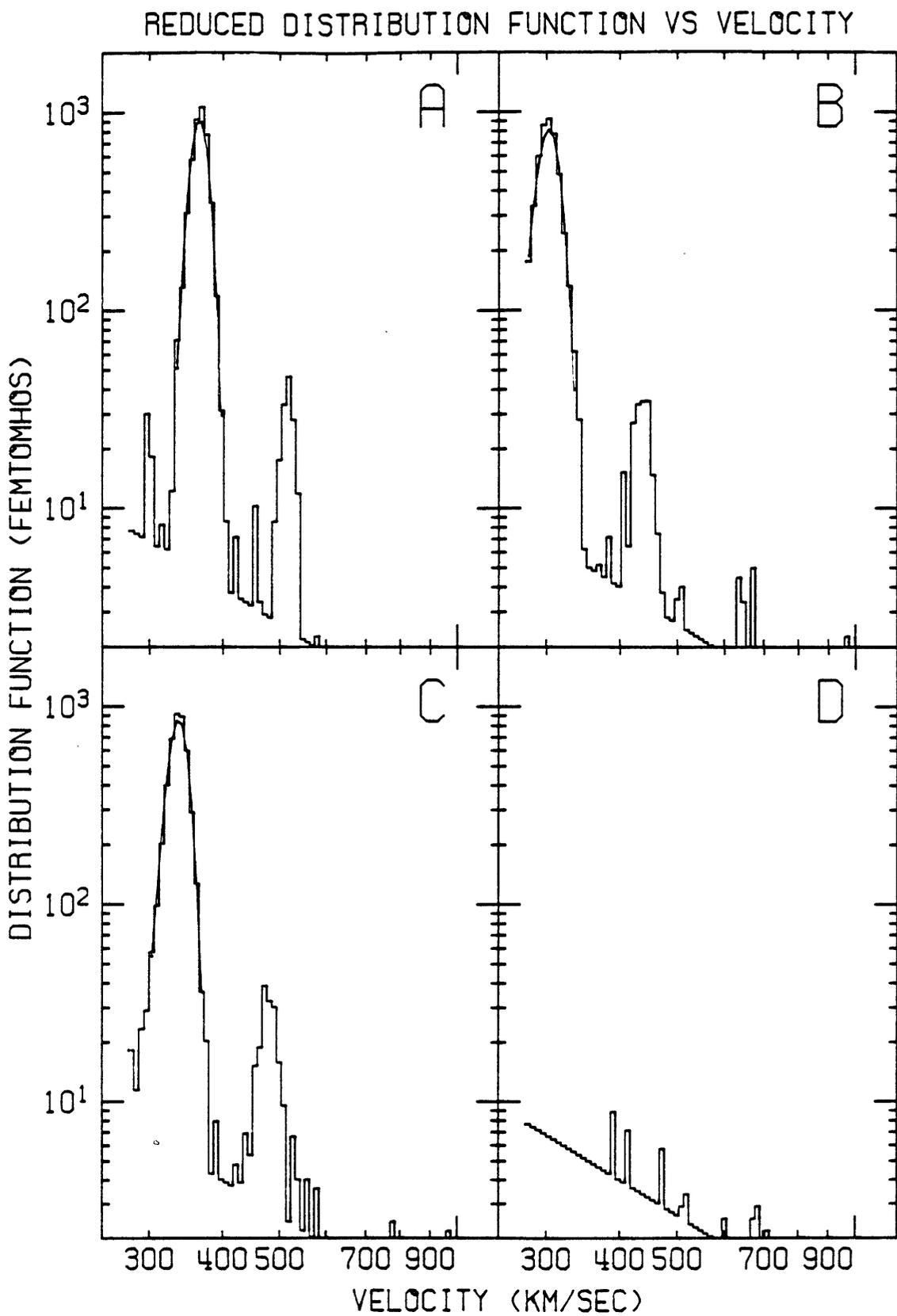


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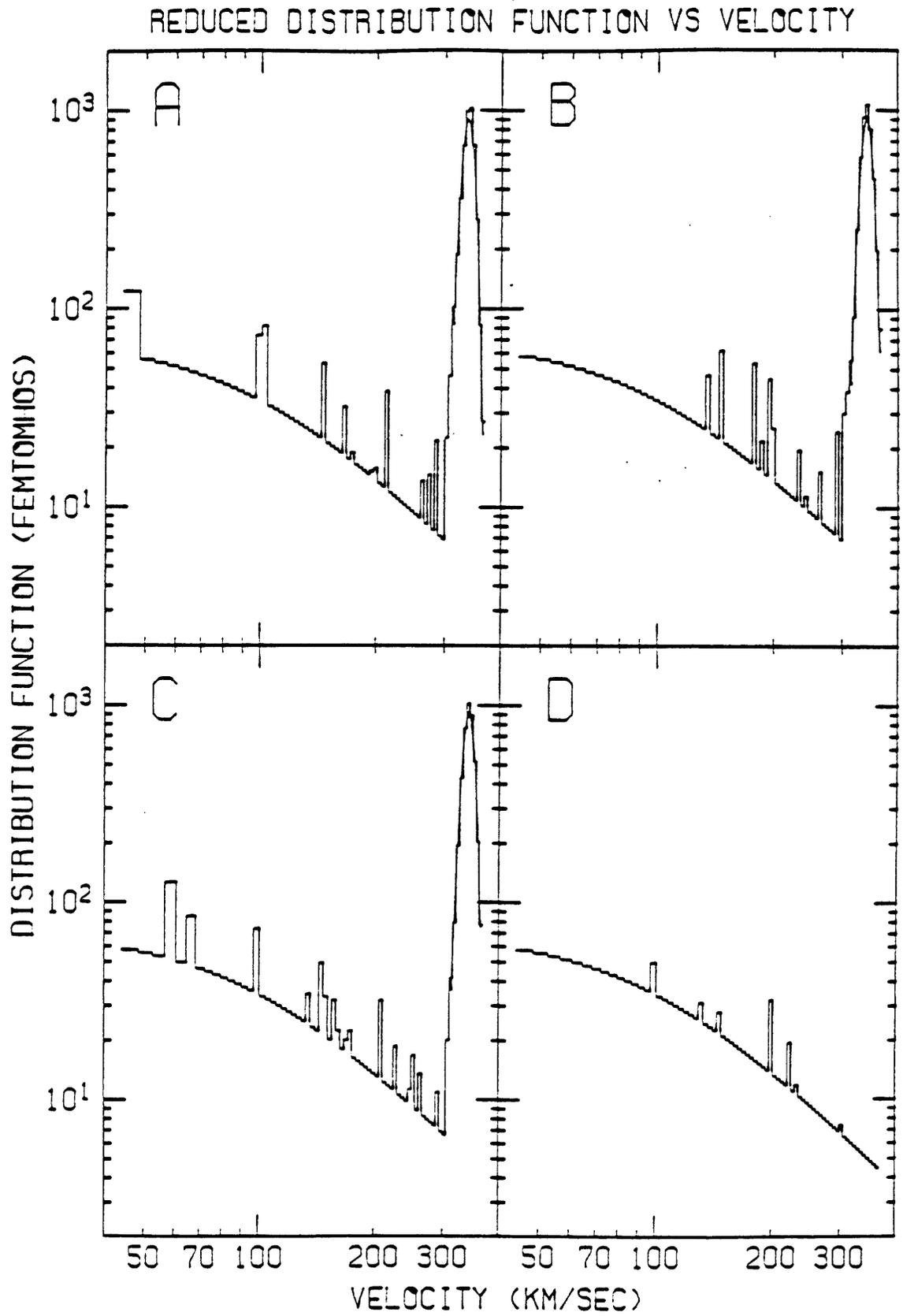


Figure 3.4

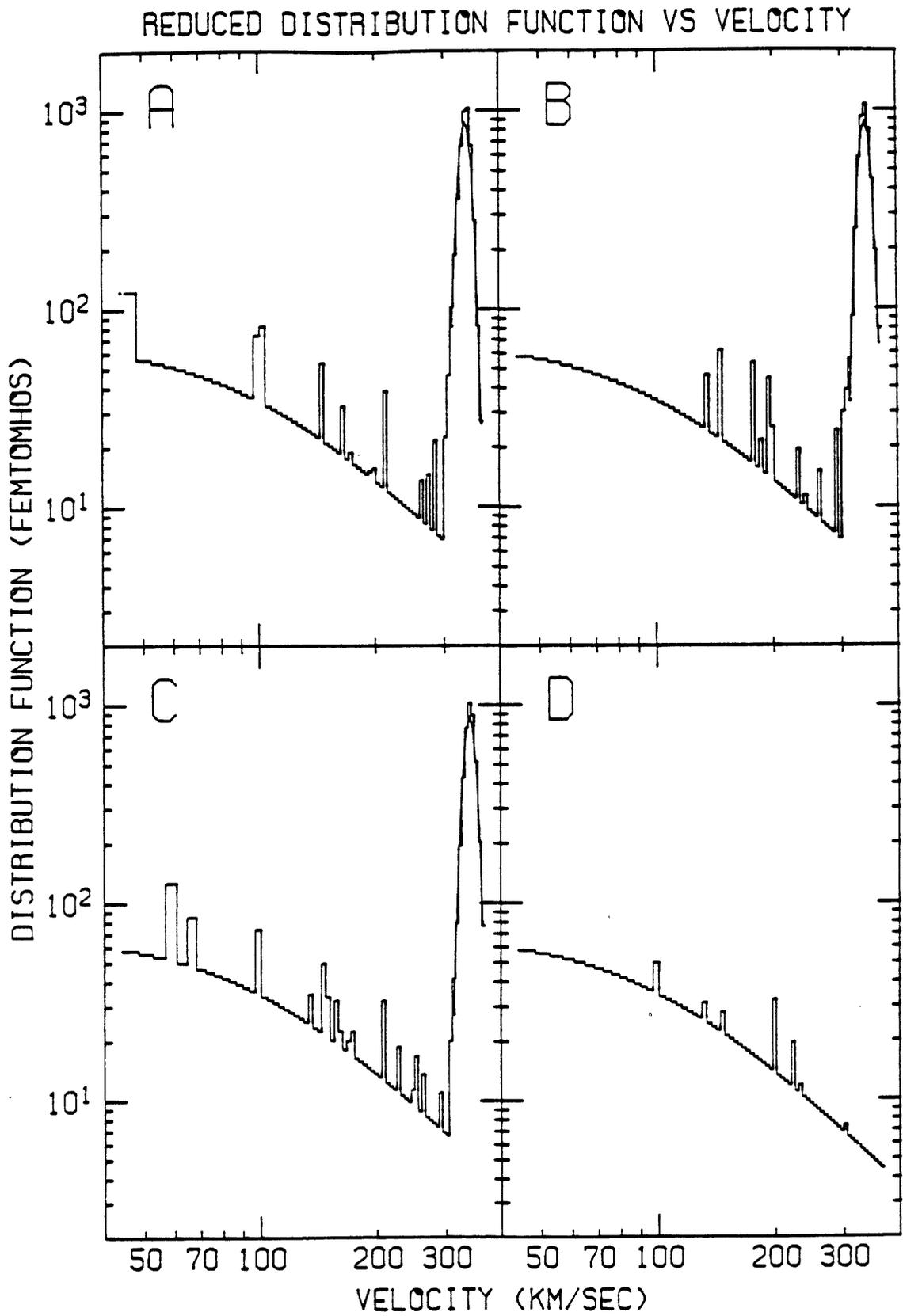


Figure 3.4a

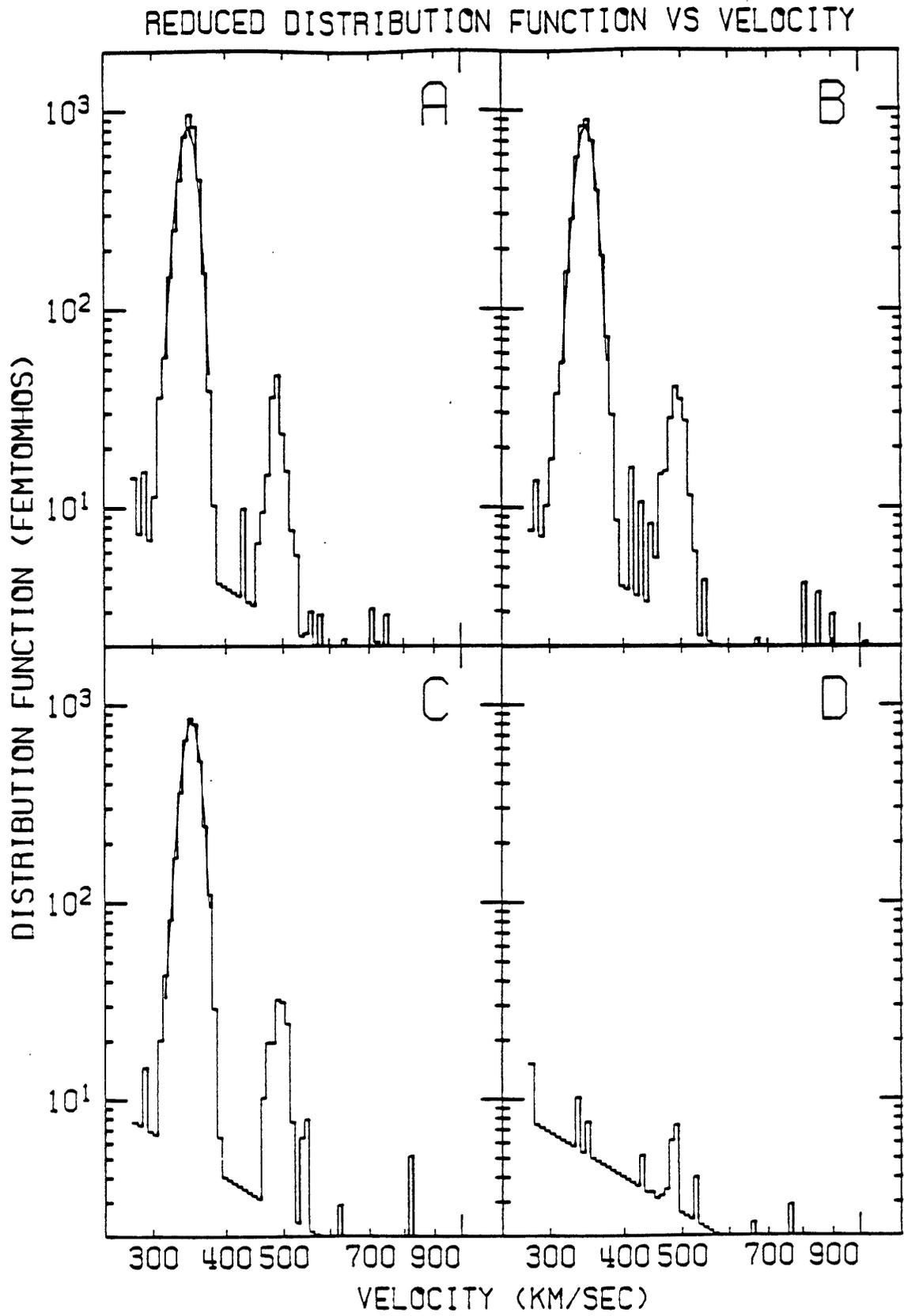


Figure 3.5

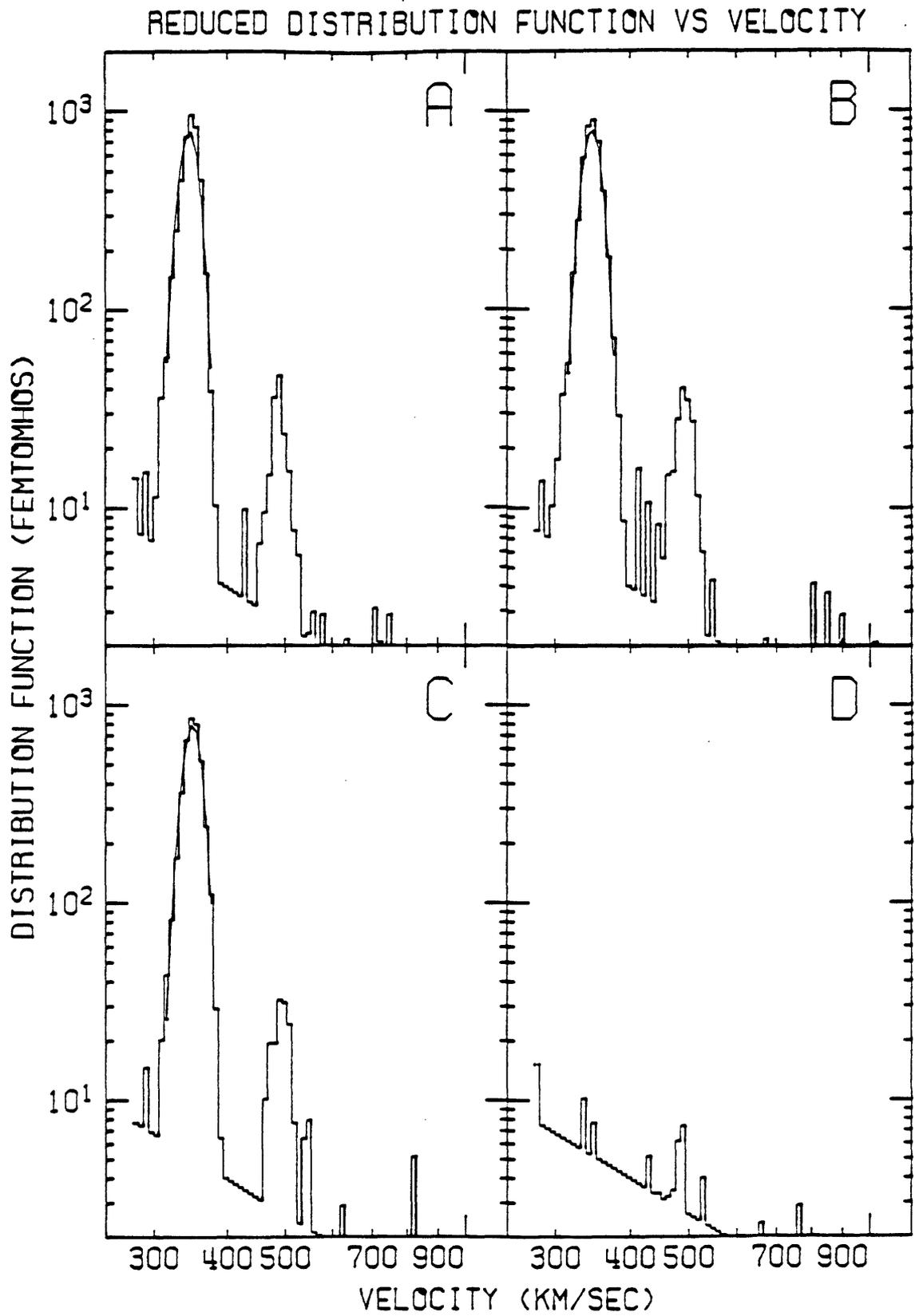


Figure 3.5a

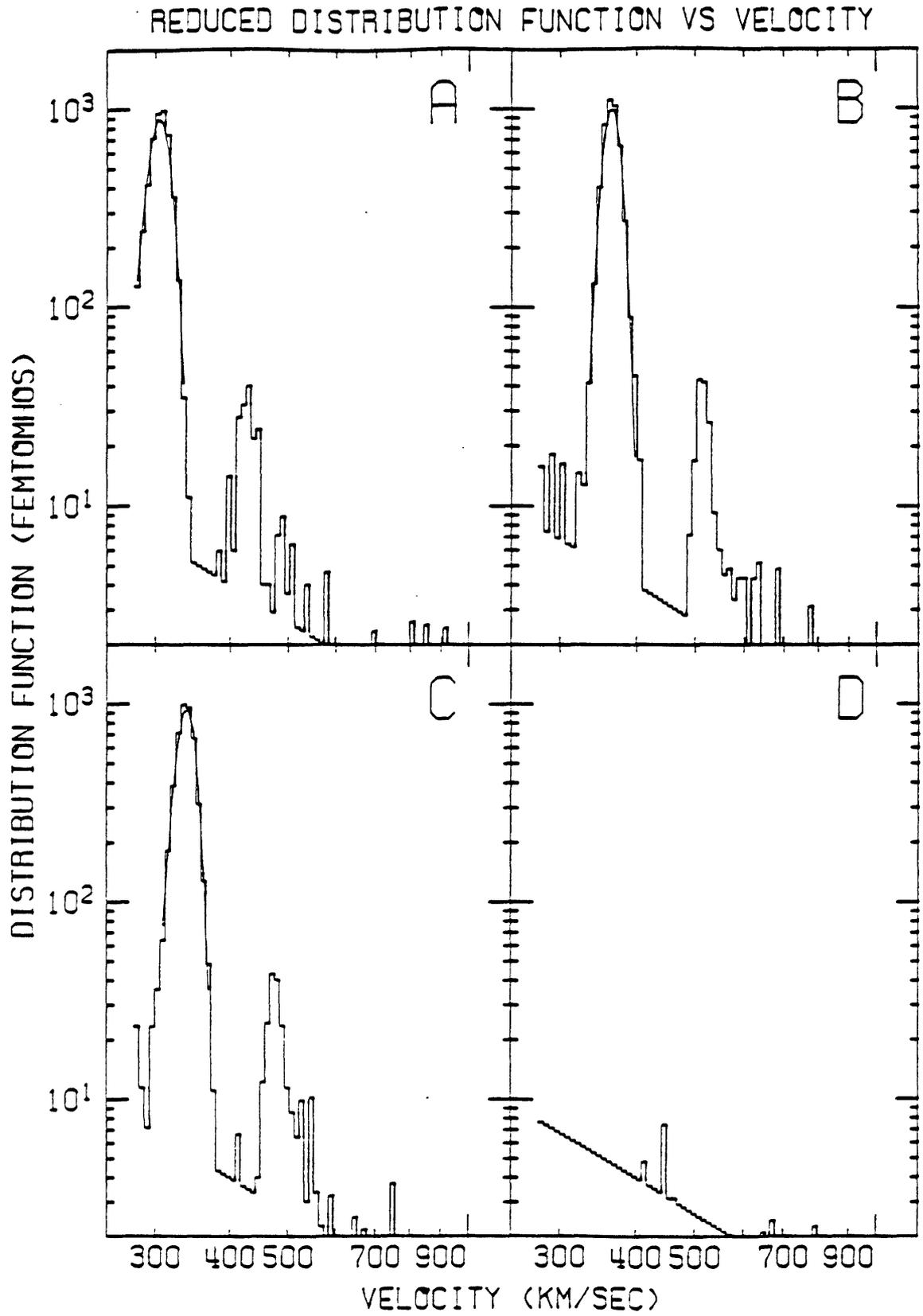


Figure 3.6

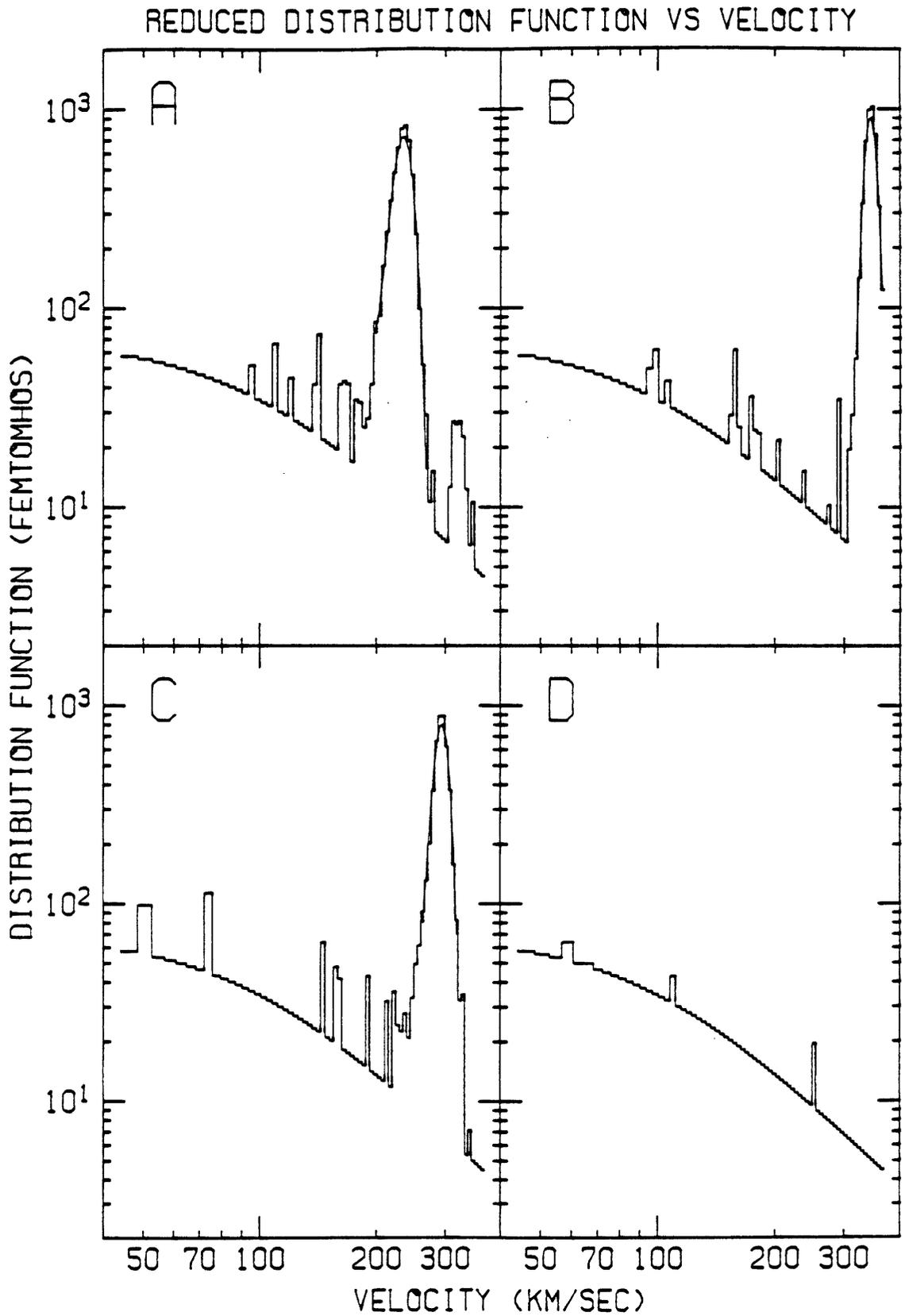


Figure 3.7

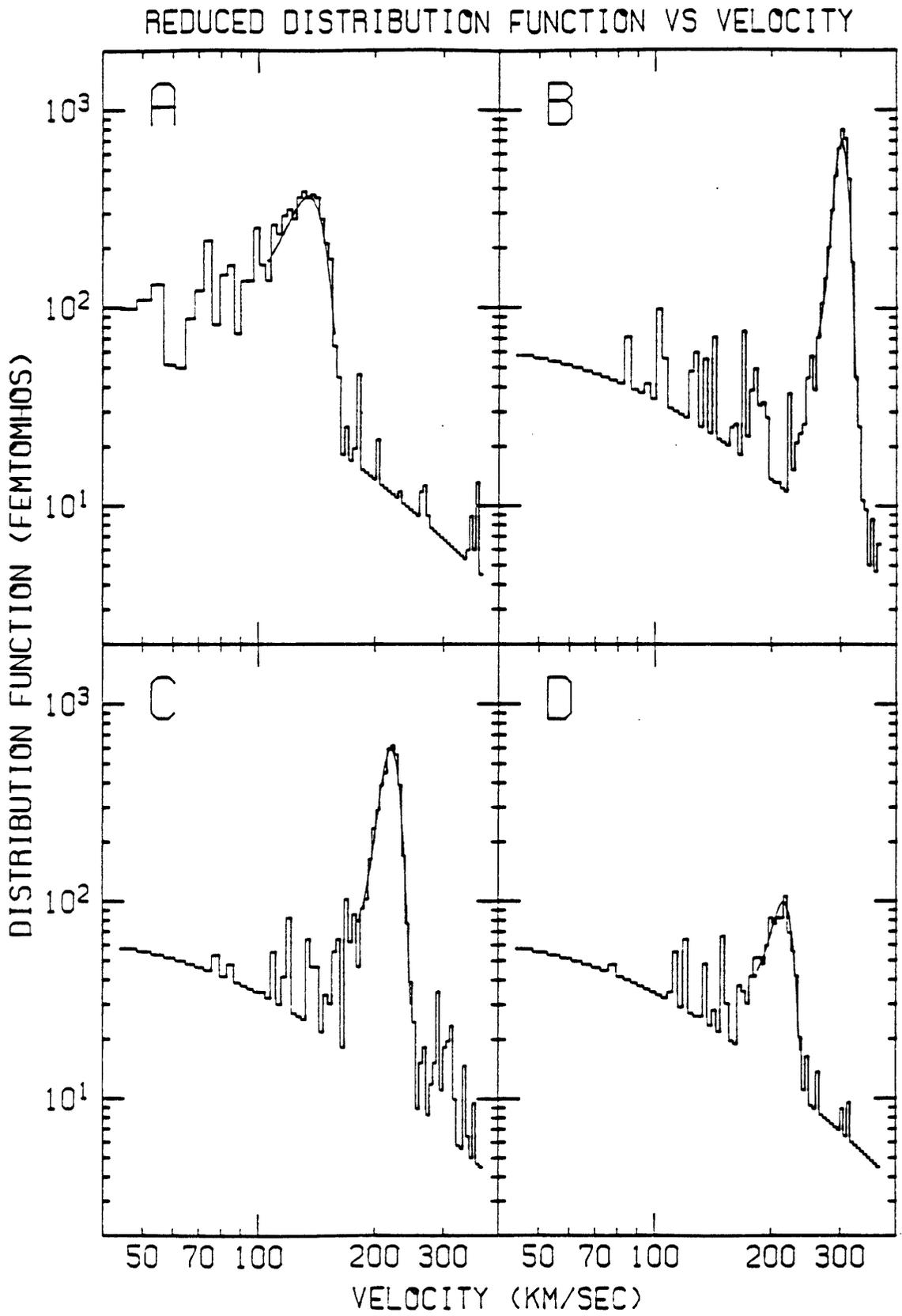


Figure 3.8

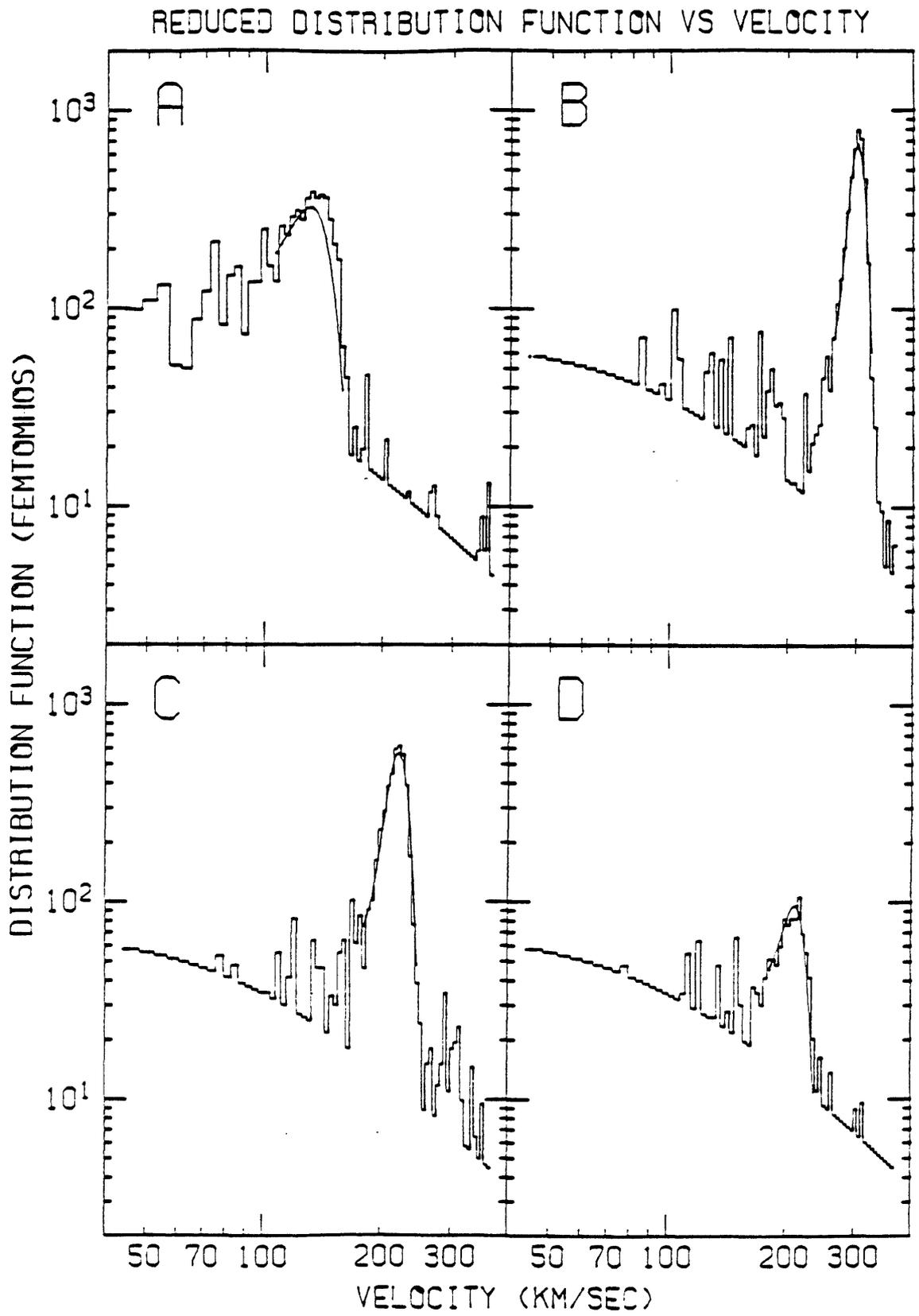


Figure 3.8a

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

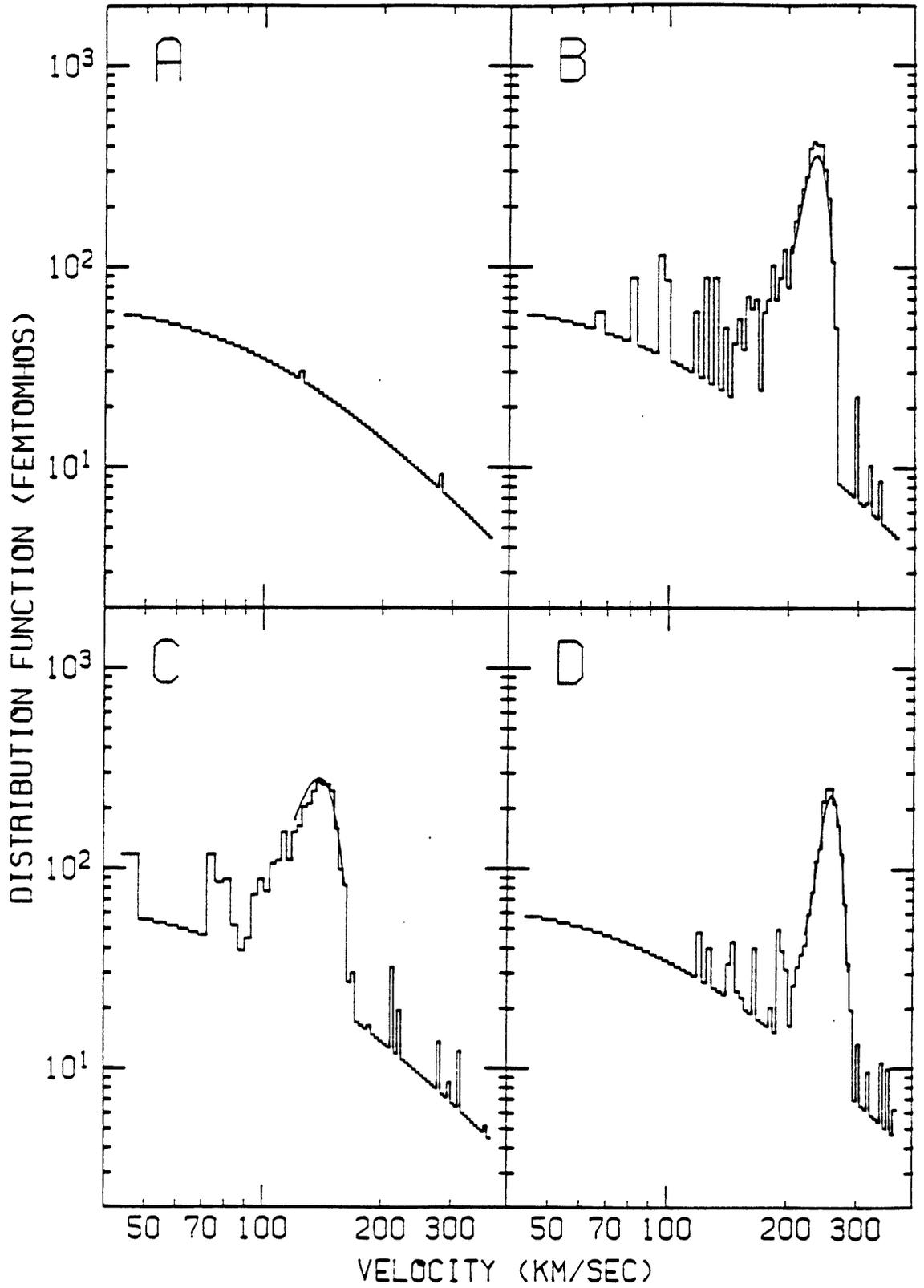


Figure 3.9

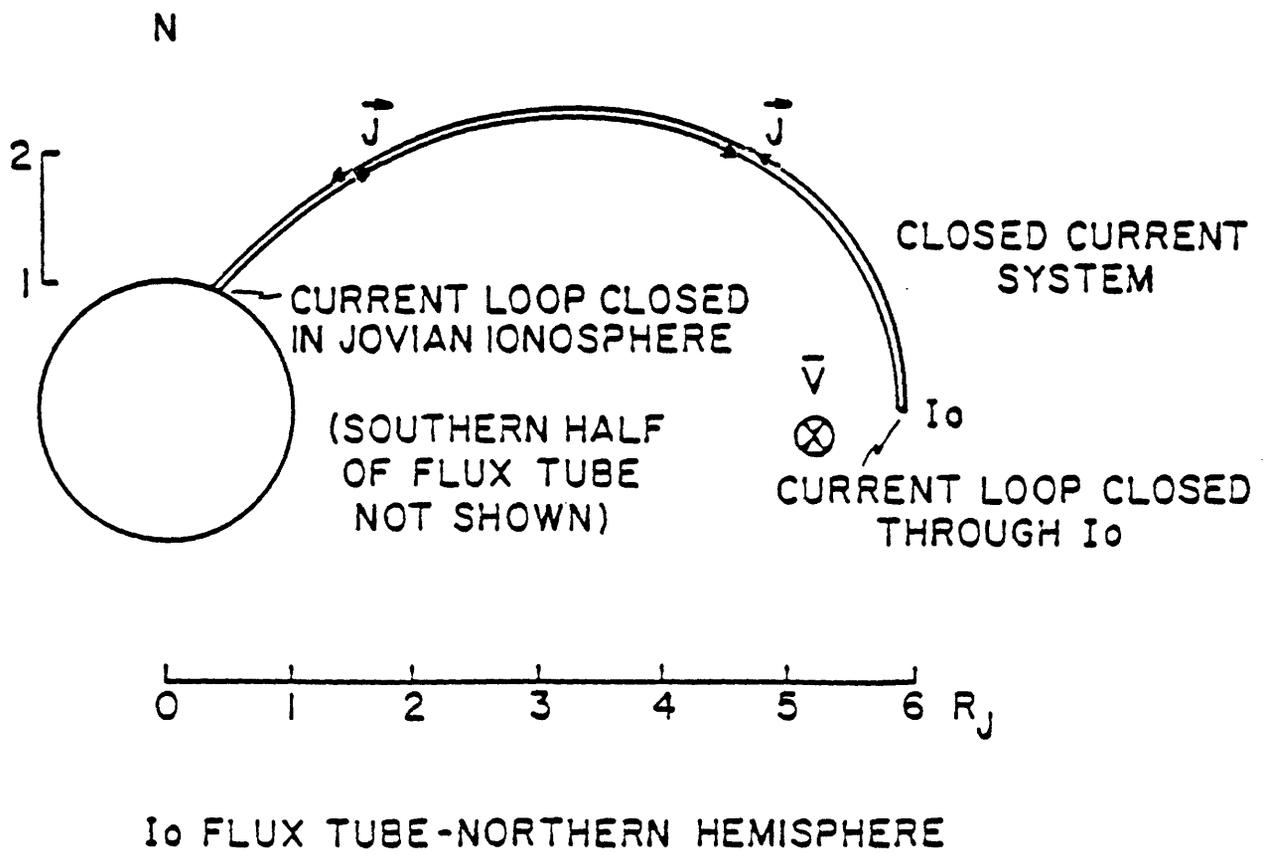


Figure 4.1

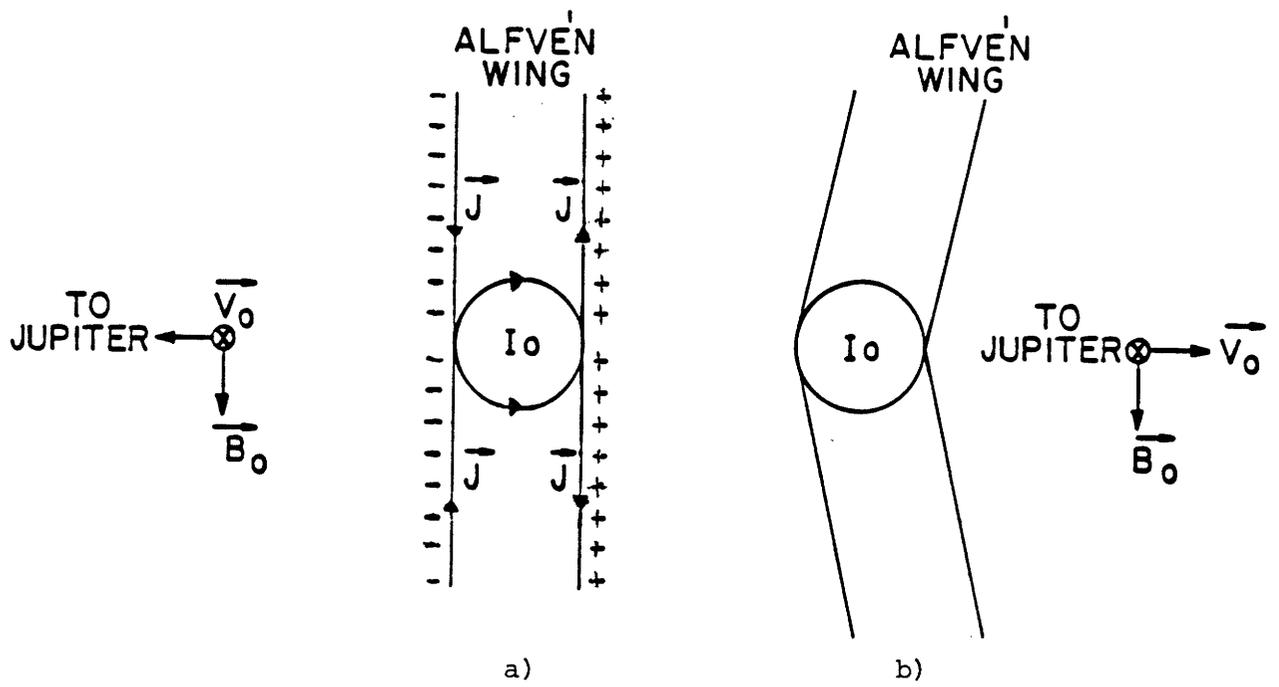


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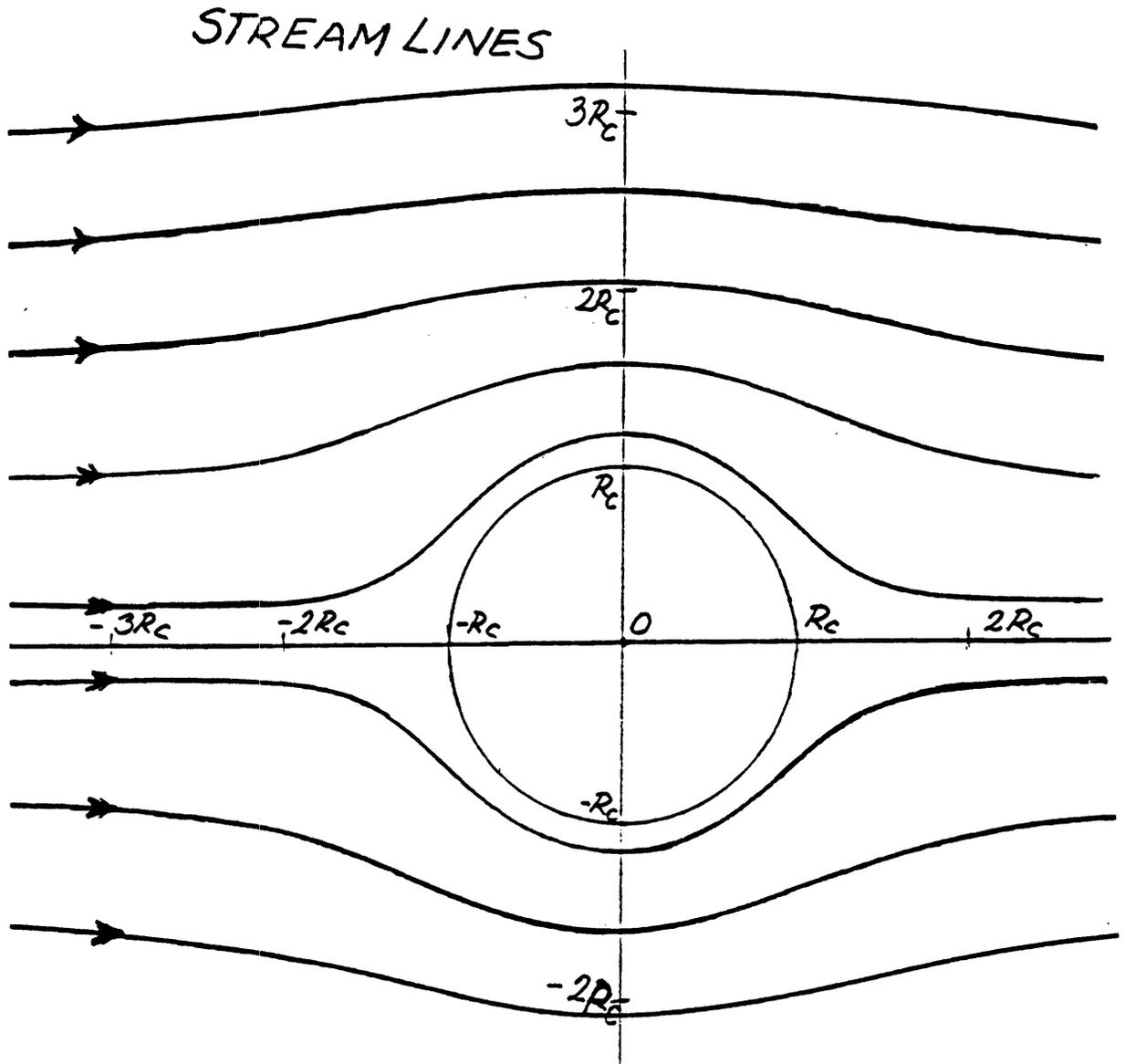


Figure 4.3

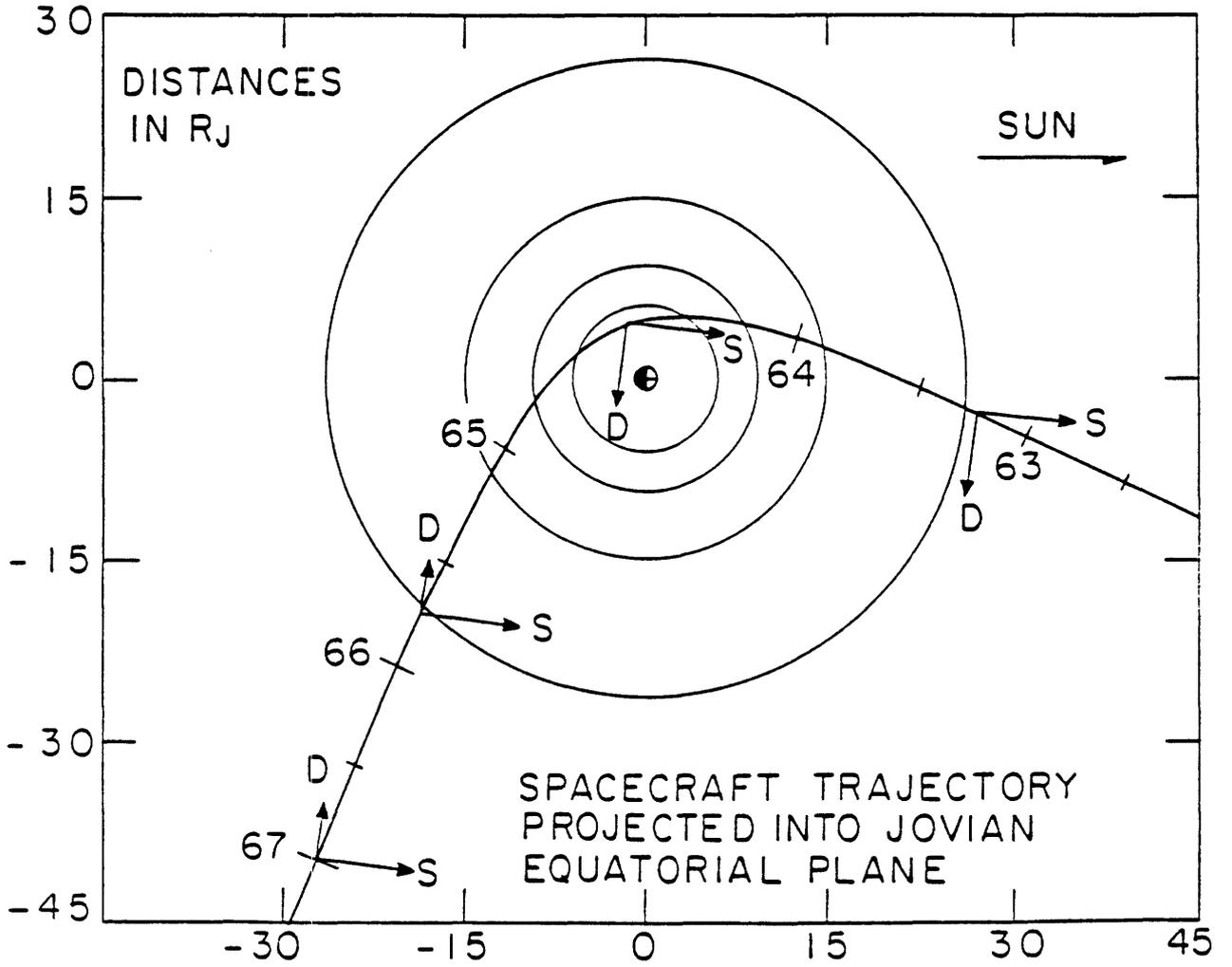


Figure 5.1

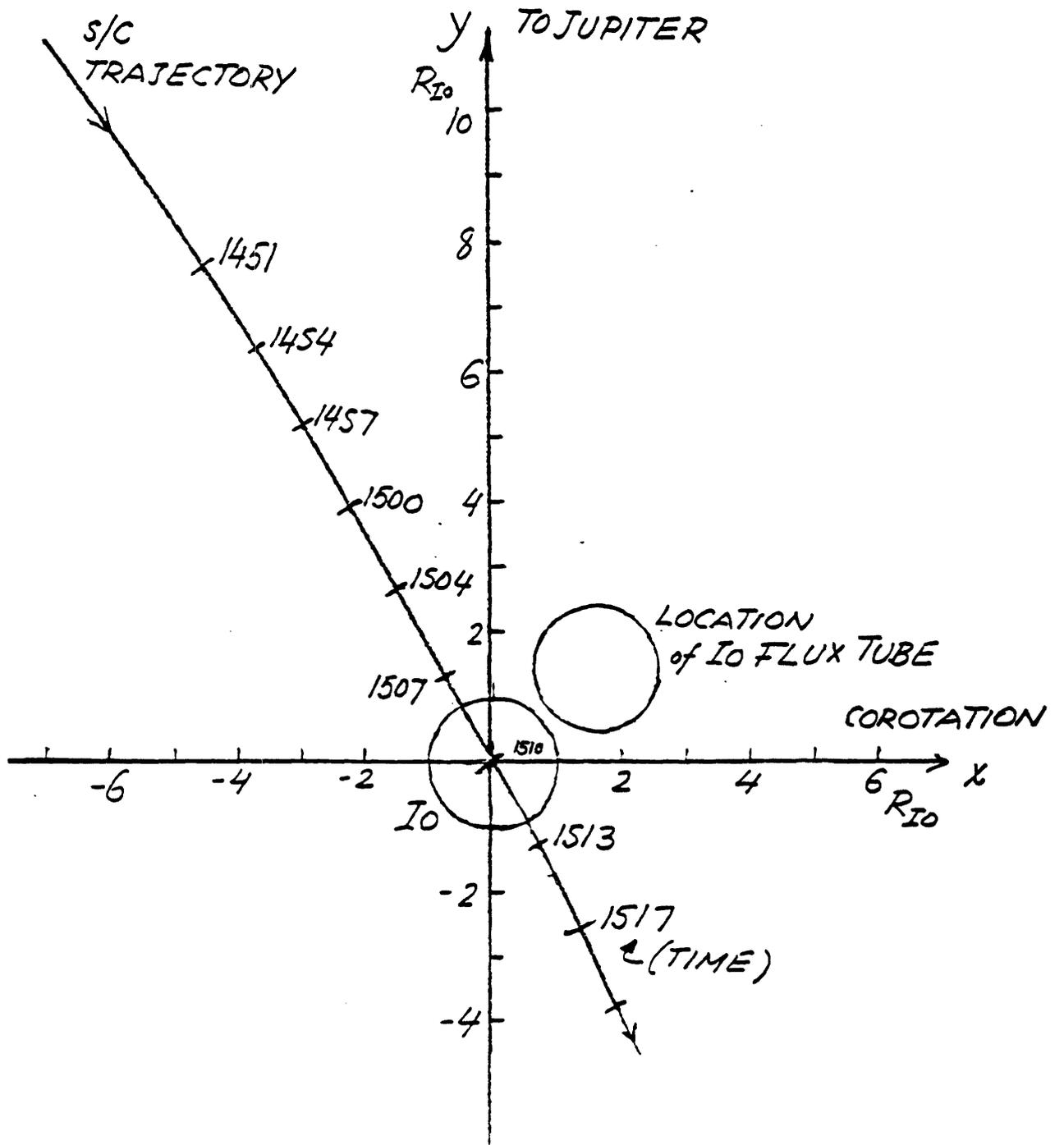


Figure 5.2

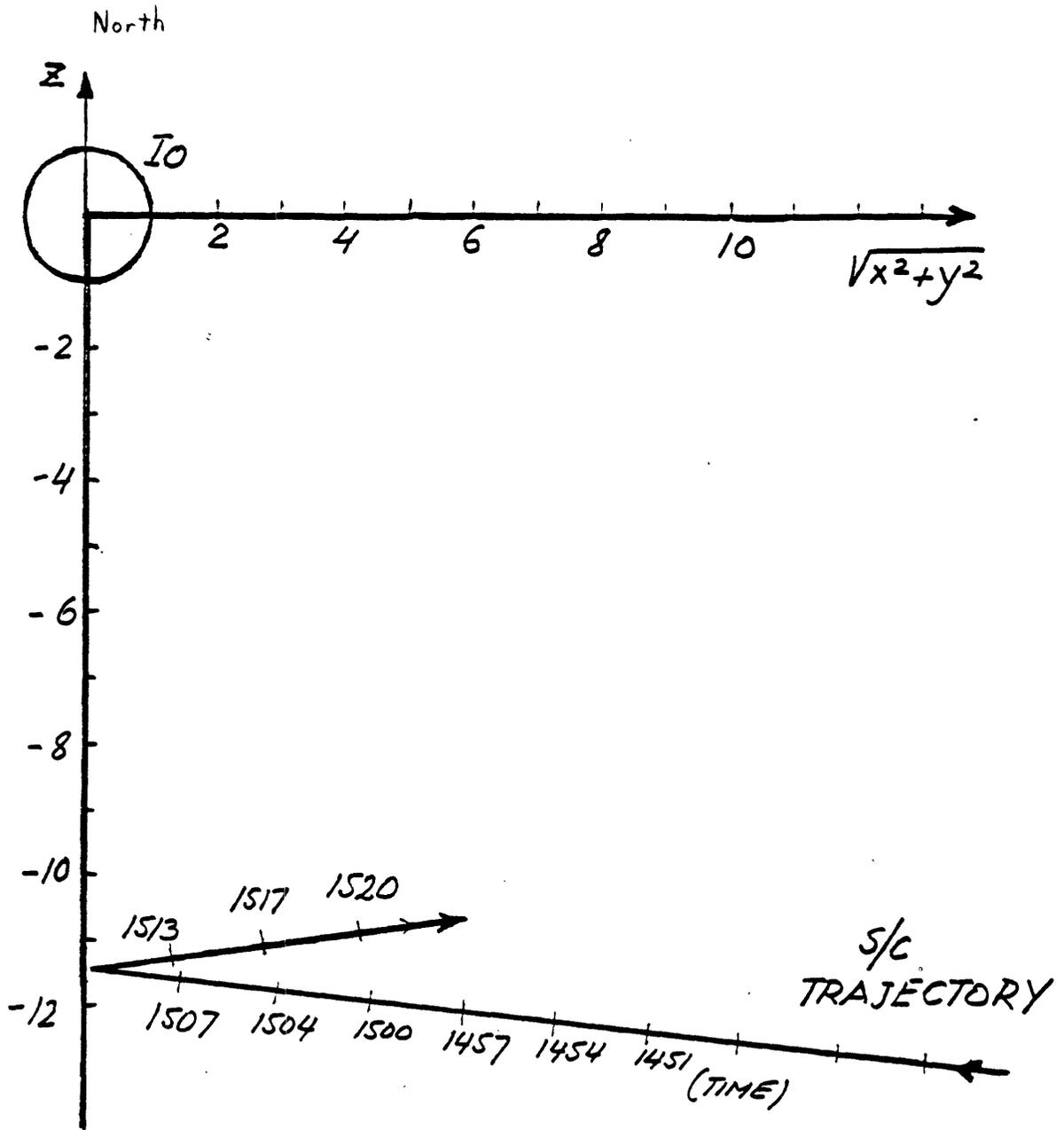


Figure 5.3

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

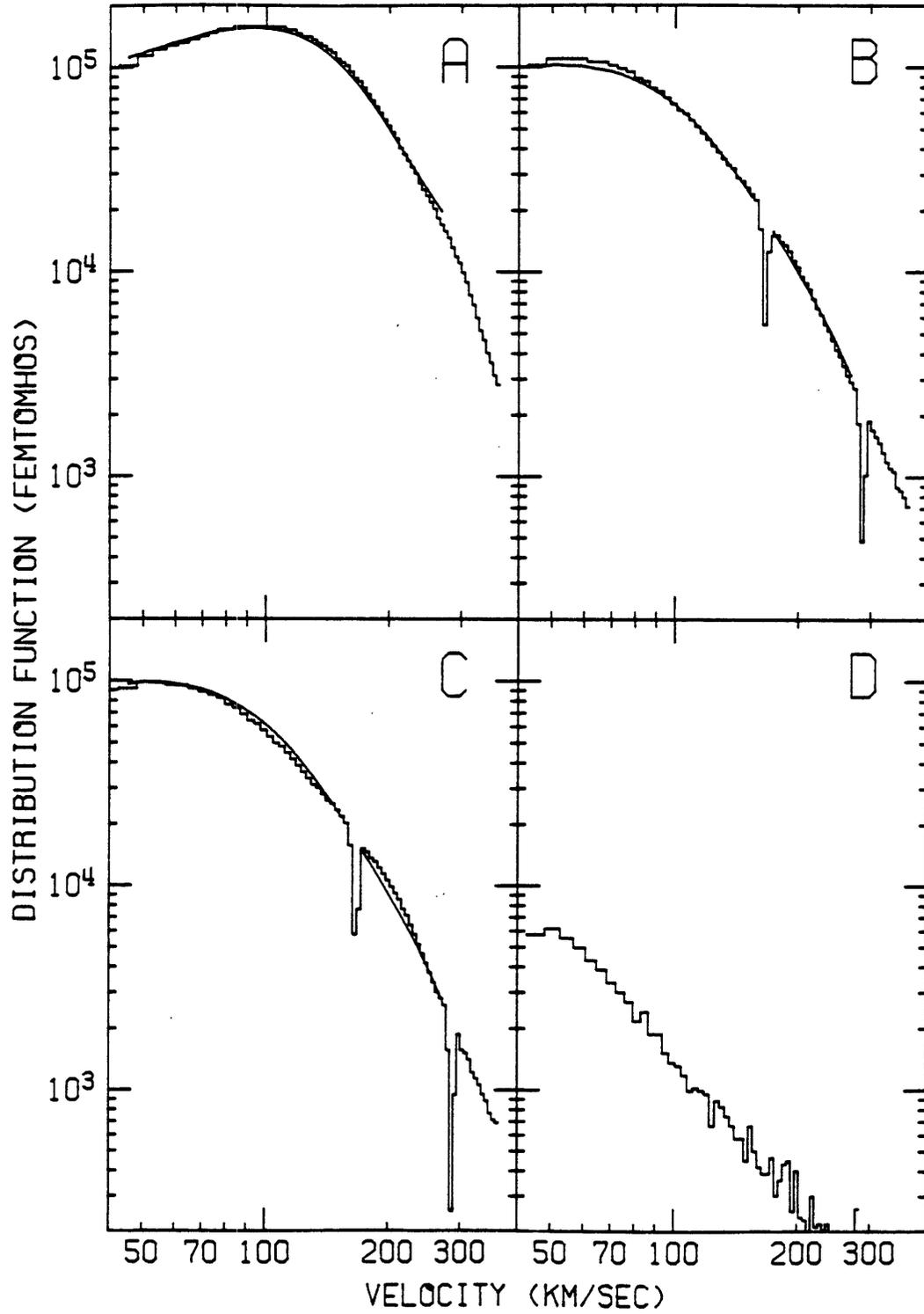


Figure 5.4

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

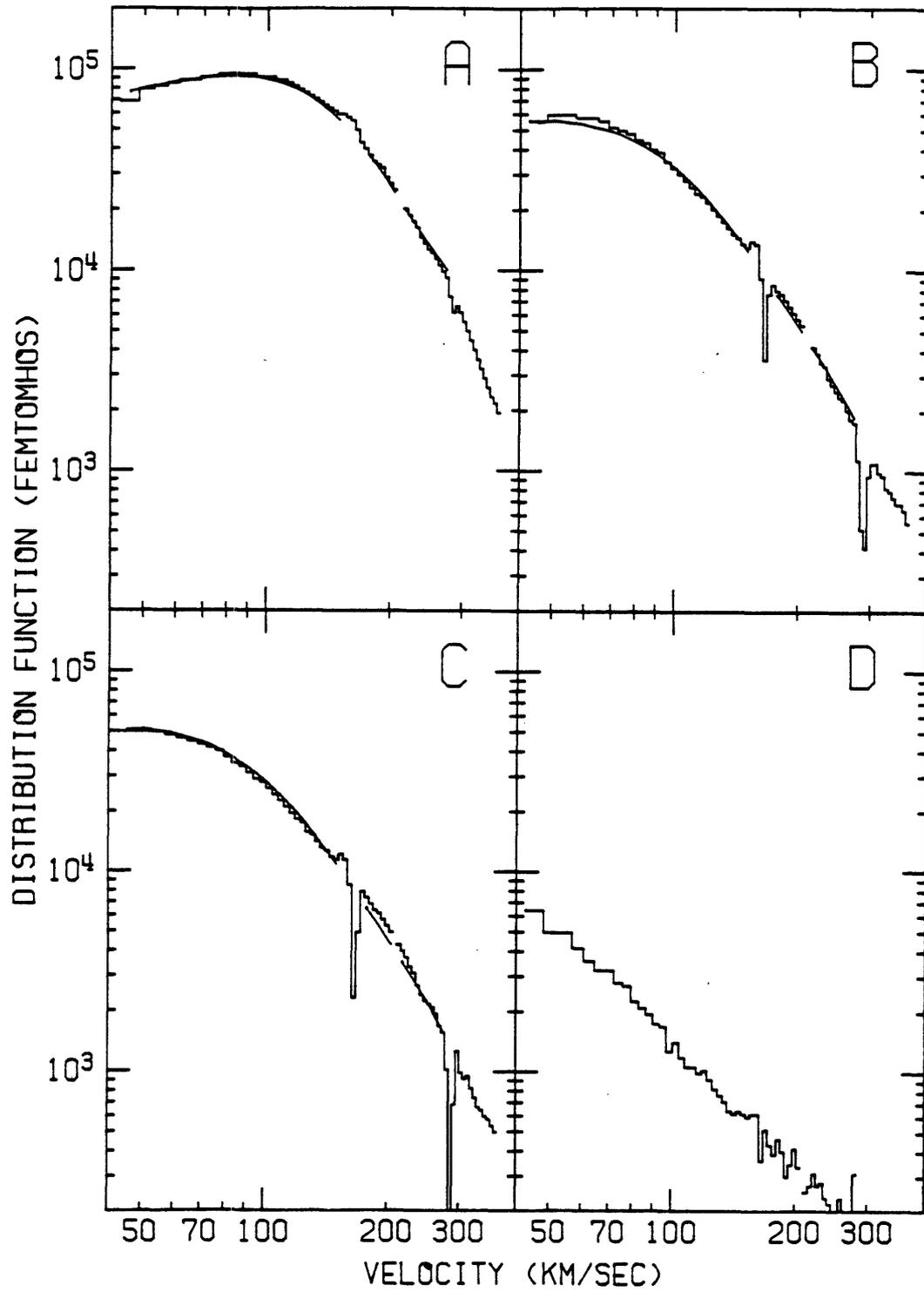


Figure 5.5

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

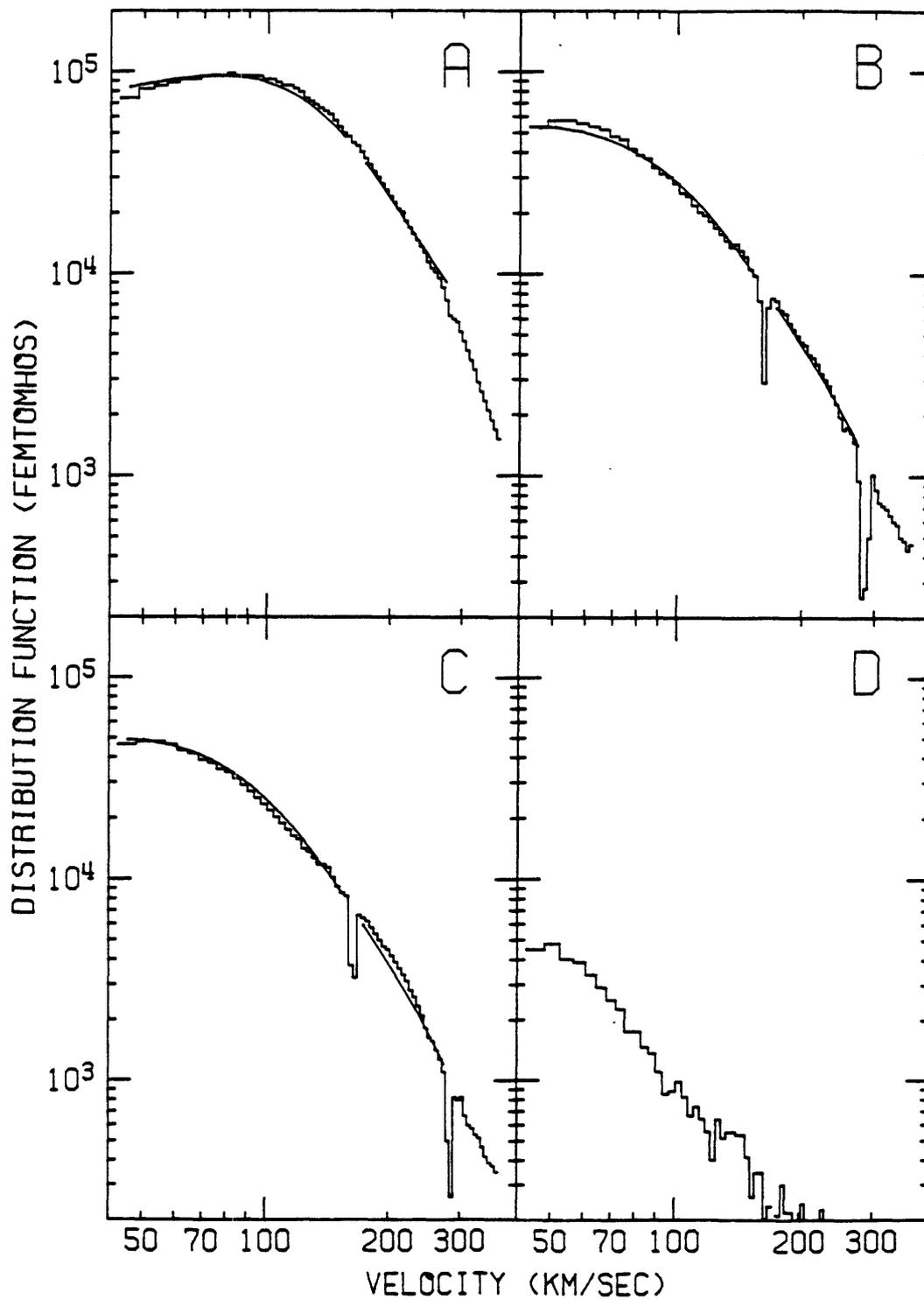


Figure 5.6

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

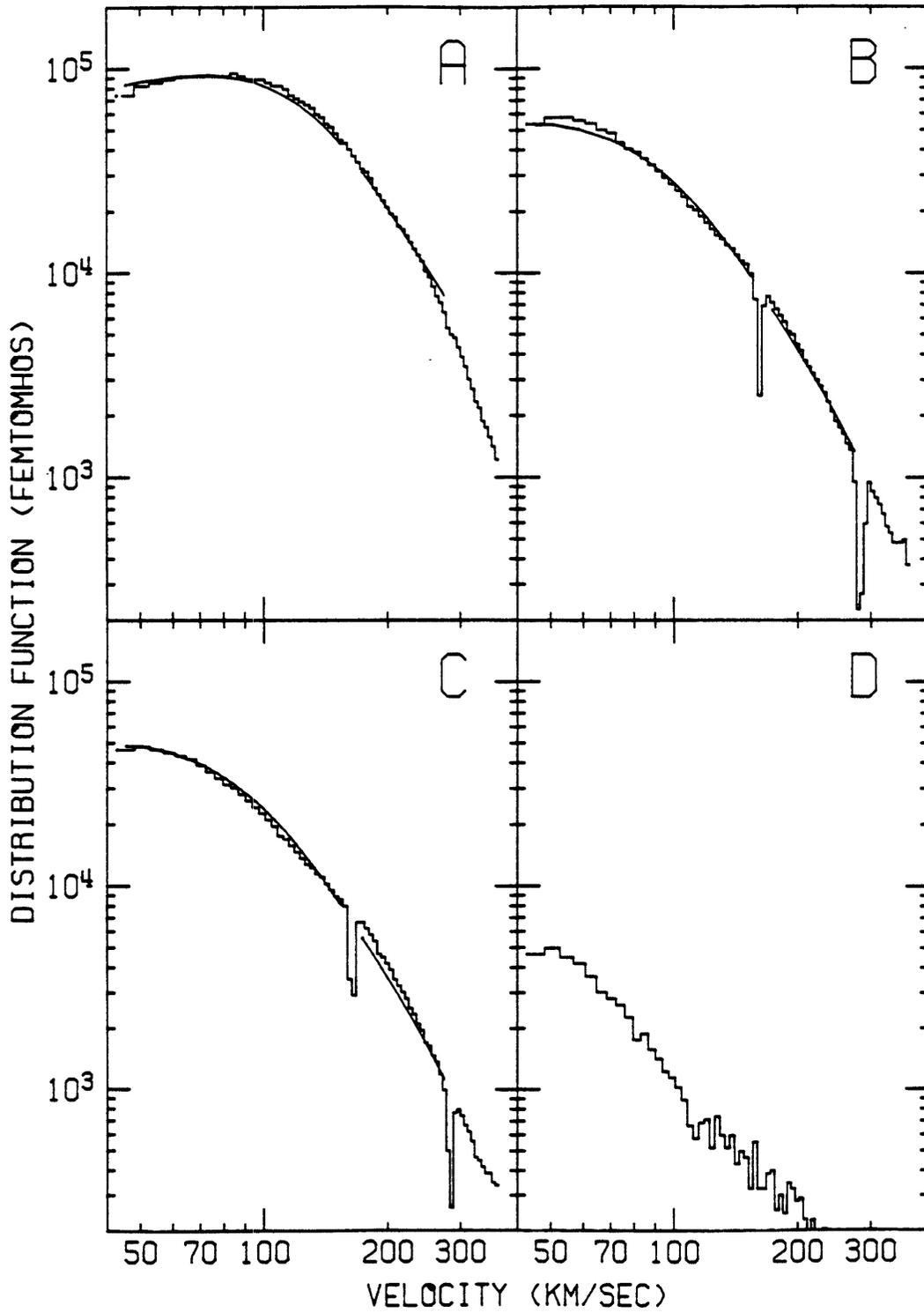


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REDUCED DISTRIBUTION FUNCTION VS VELOCITY

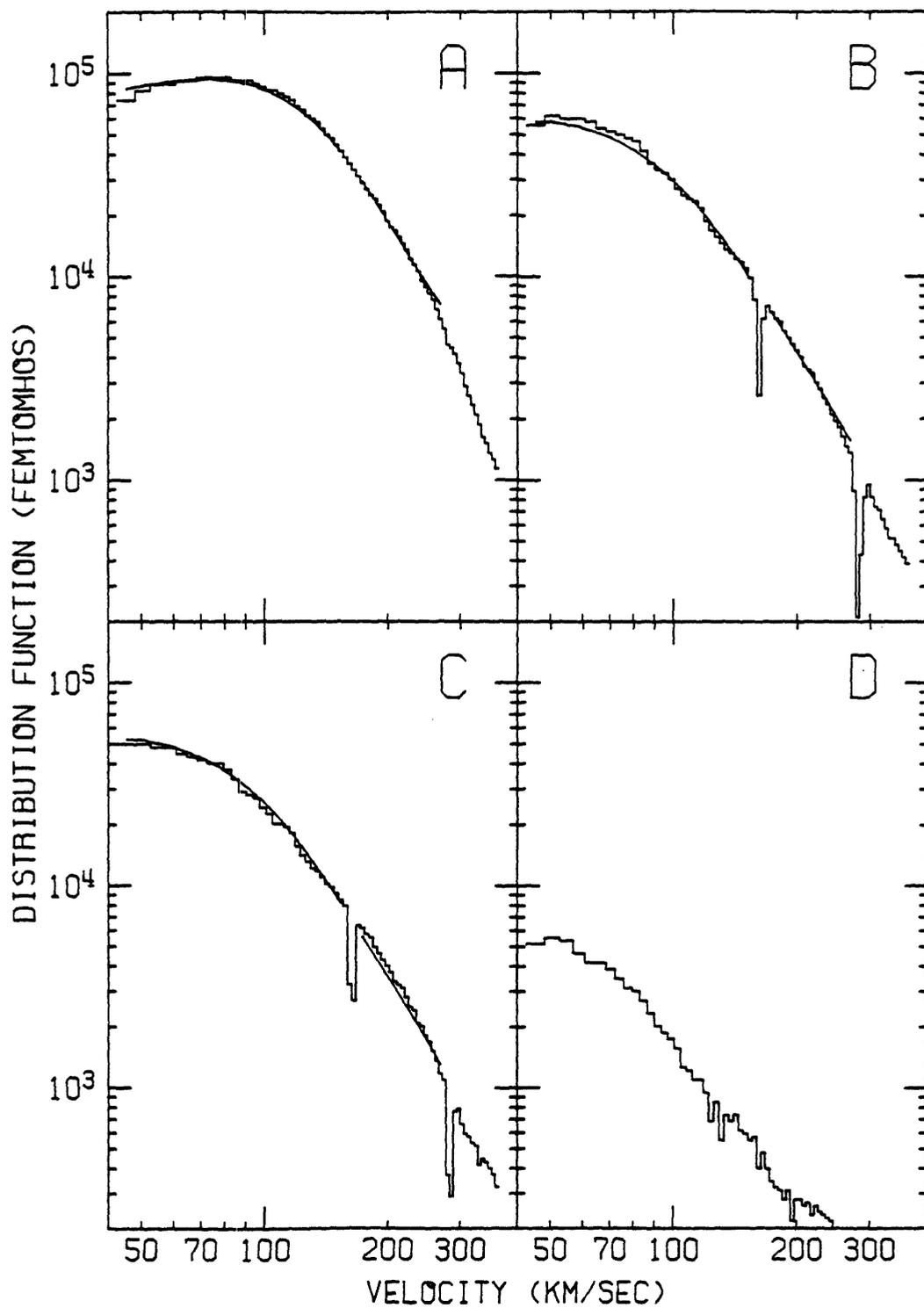


Figure 5.8

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

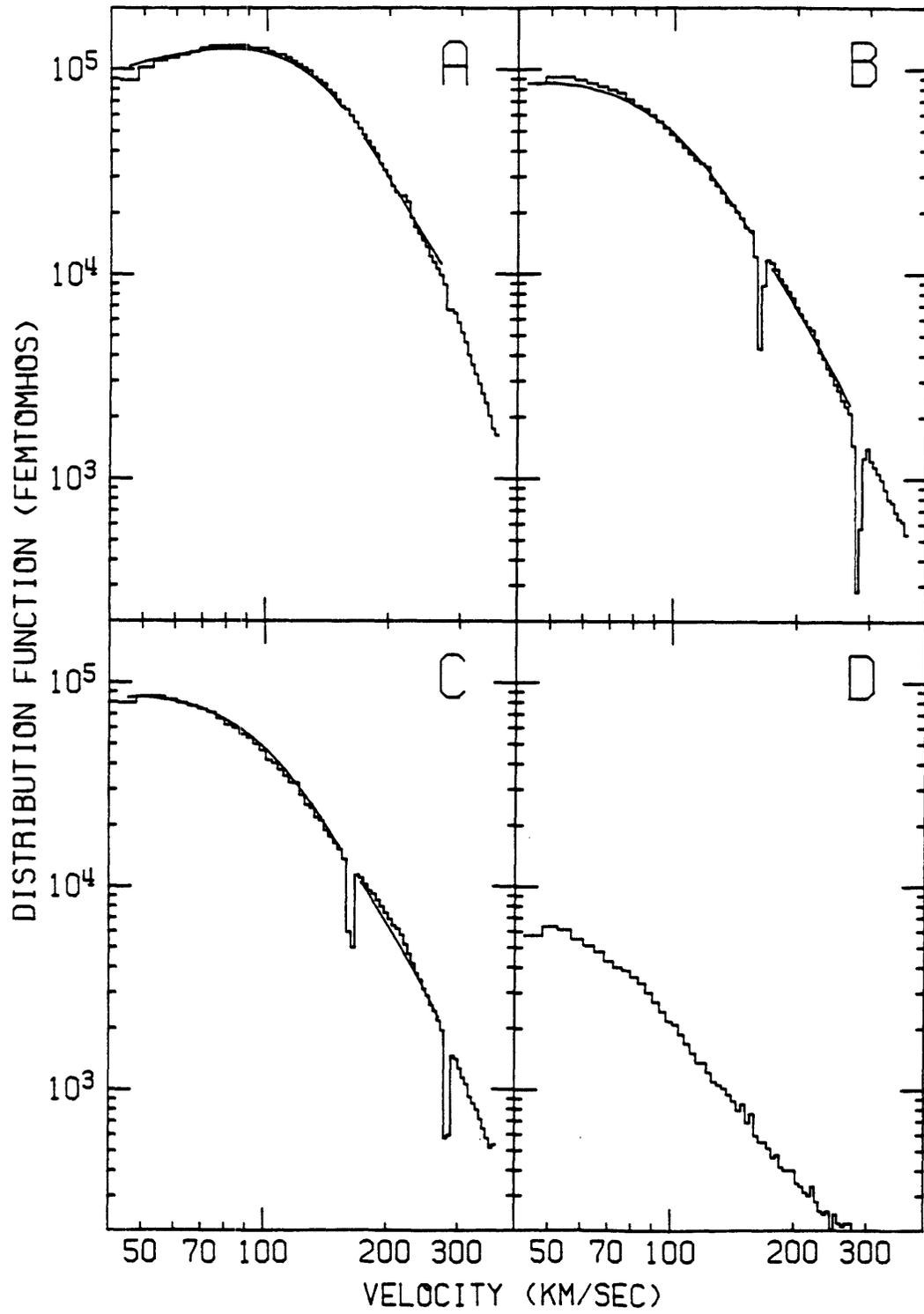


Figure 5.9

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

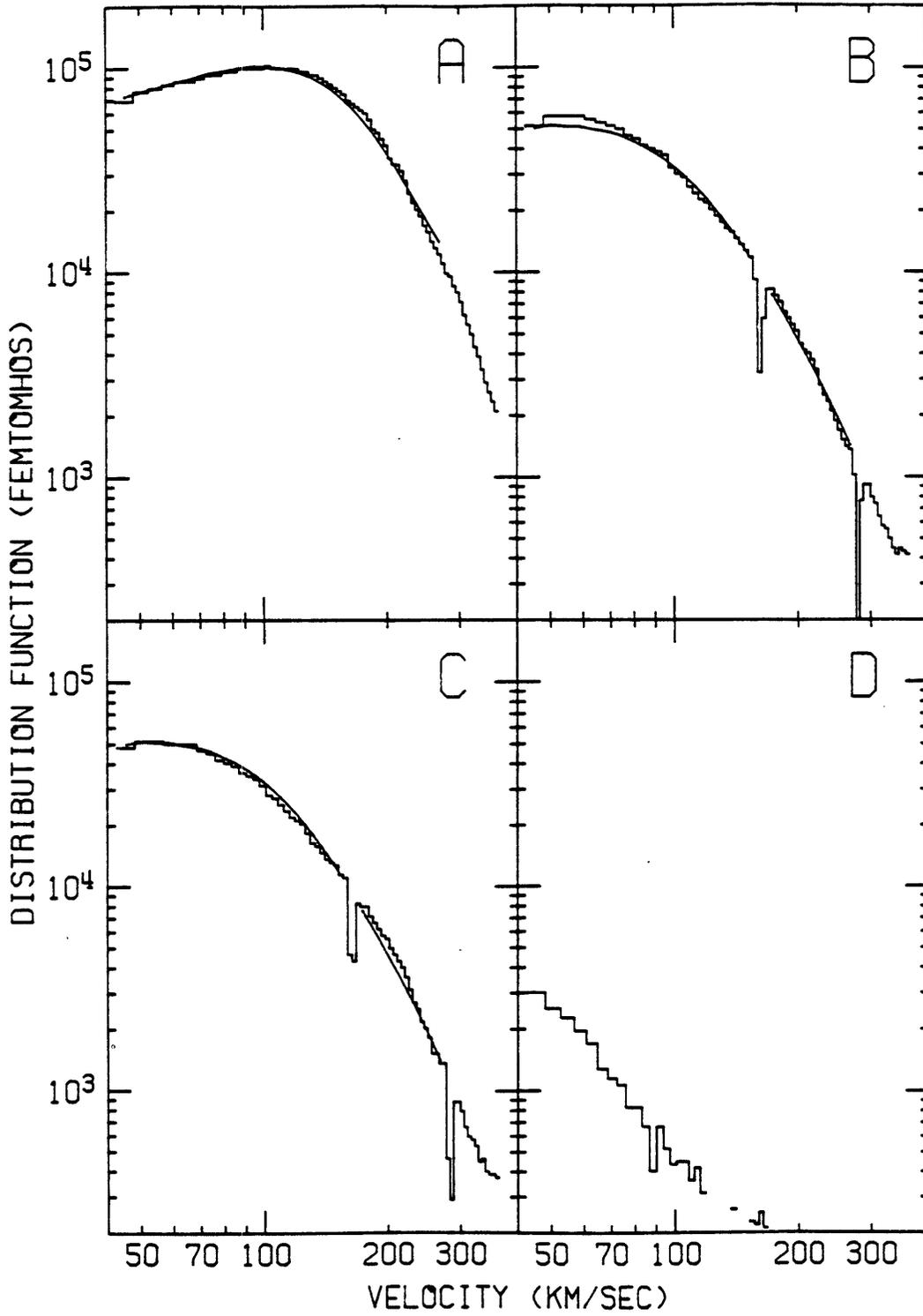


Figure 5.10

REDUCED DISTRIBUTION FUNCTION VS VELOCITY

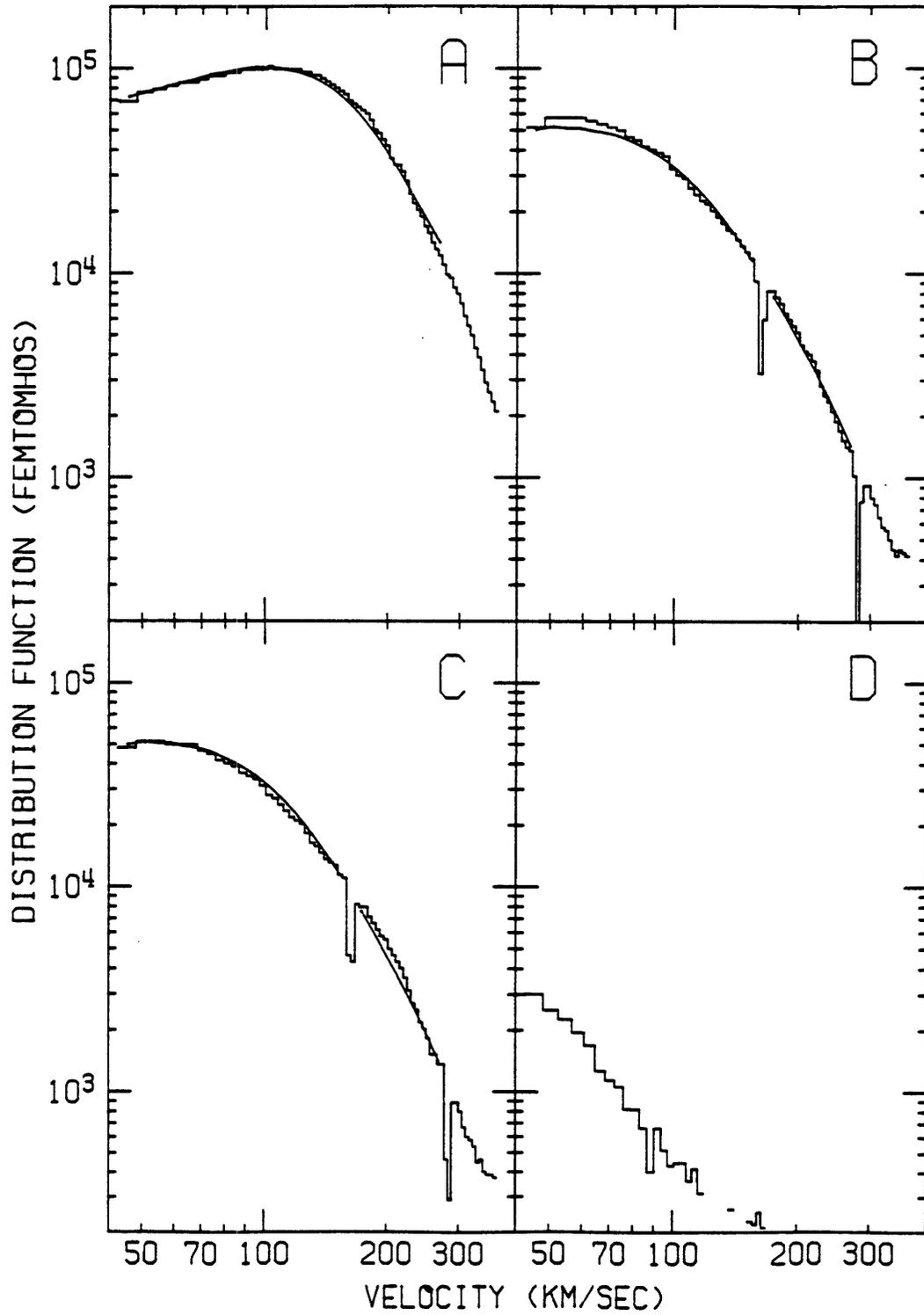


Figure 5.11

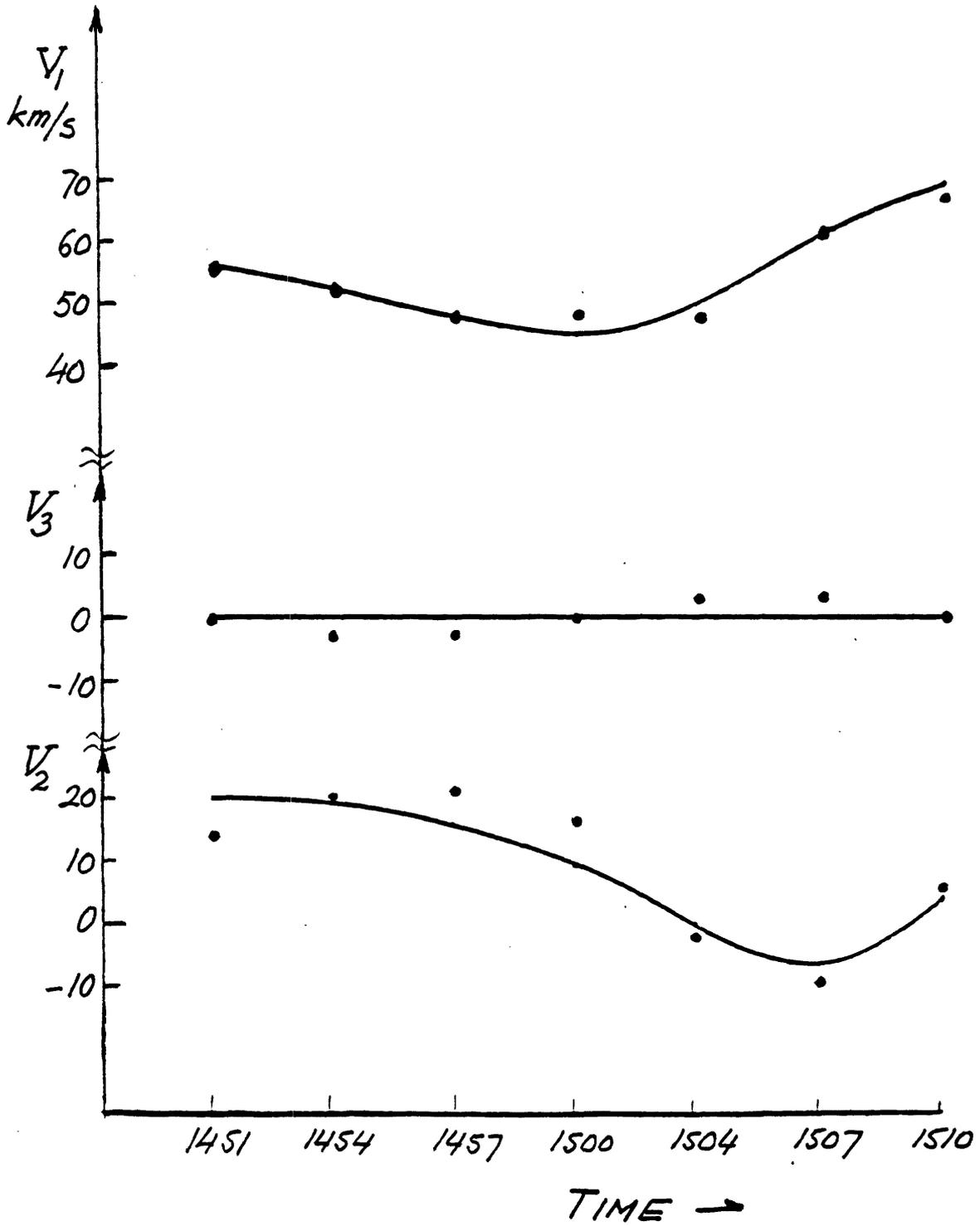


Figure 5.12

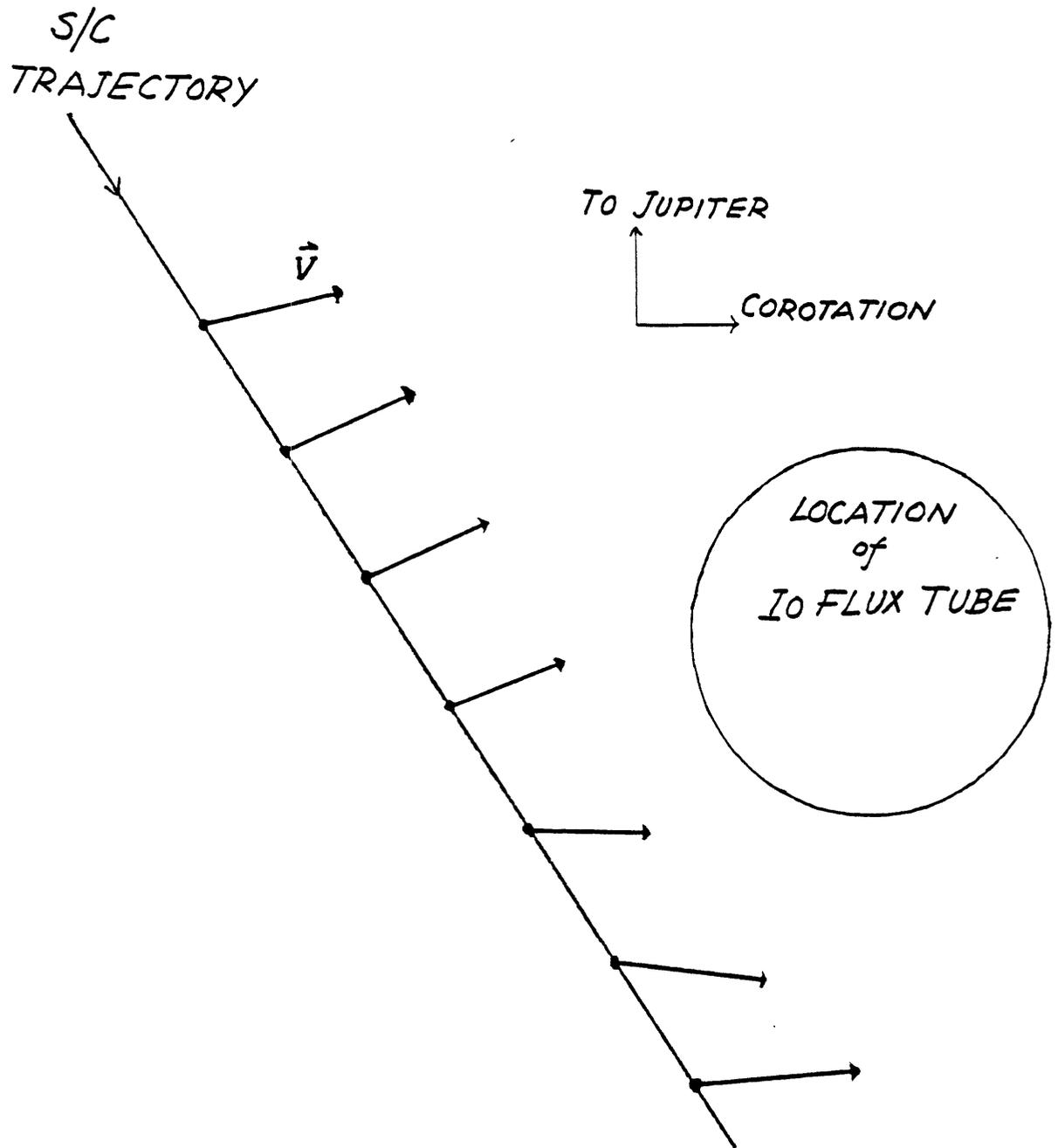


Figure 5.13

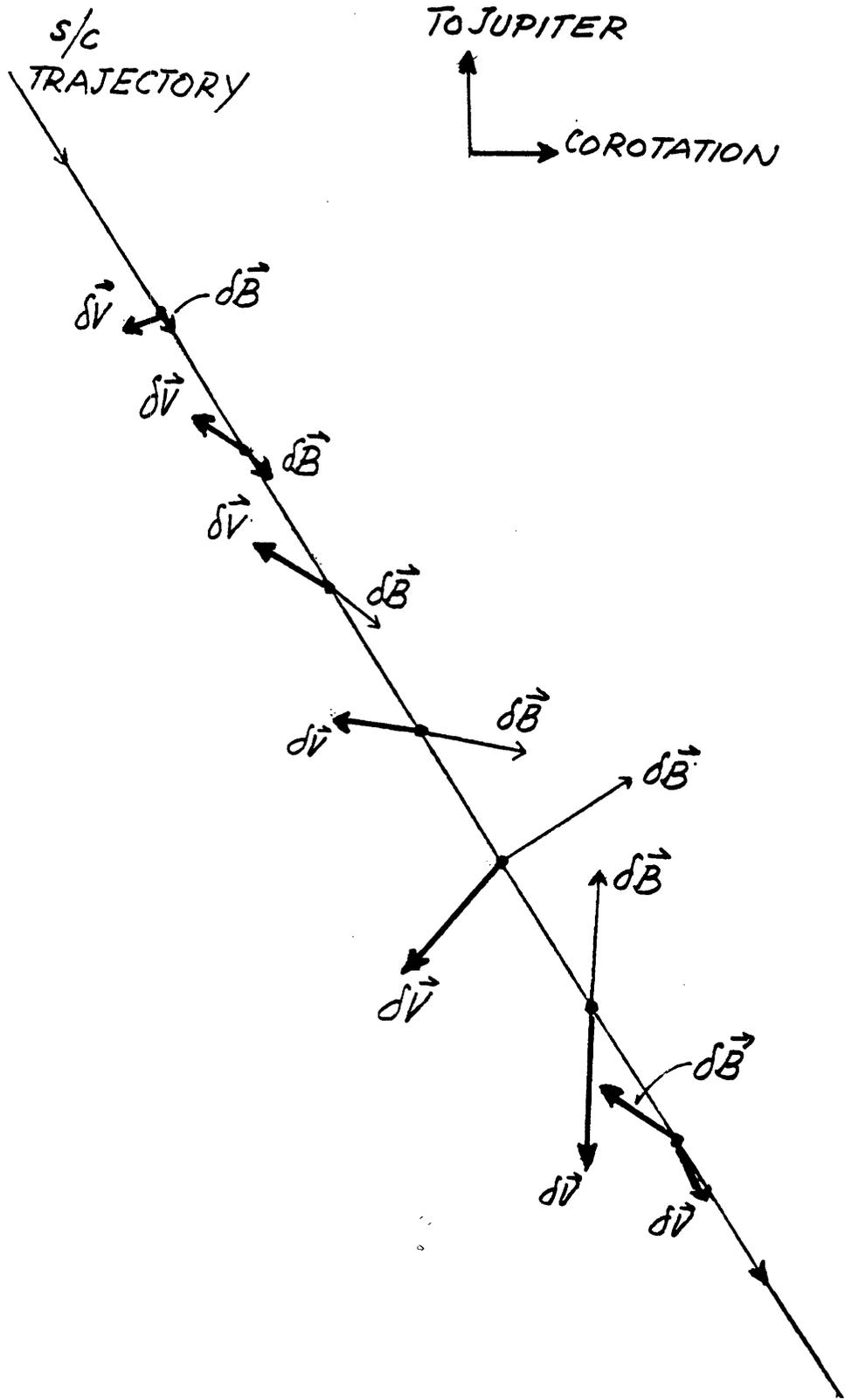


Figure 5.14

Appendix A

Table of nominal constants for MJS.

The delta values are the full width.

M.mode. $K = 1.036633$

| ACCELERATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|---------------------|----------------------|-----------------|--------------------|------------------|
| 1 | 10.0000 | 12.1980 | 11.0990 | 2.19795 |
| 2 | 12.1980 | 14.4764 | 13.3372 | 2.27846 |
| 3 | 14.4764 | 16.8383 | 15.6574 | 2.36192 |
| 4 | 16.8383 | 19.2868 | 18.0626 | 2.44846 |
| 5 | 19.2868 | 21.8249 | 20.5539 | 2.53815 |
| 6 | 21.8249 | 24.4561 | 23.1405 | 2.63112 |
| 7 | 24.4561 | 27.1836 | 25.8193 | 2.72751 |
| 8 | 27.1836 | 30.0110 | 28.5973 | 2.82742 |
| 9 | 30.0110 | 32.9420 | 31.4765 | 2.93100 |
| 10 | 32.9420 | 35.9803 | 34.4612 | 3.03836 |
| 11 | 35.9803 | 39.1300 | 37.5552 | 3.14967 |
| 12 | 39.1300 | 42.3951 | 40.7625 | 3.26505 |
| 13 | 42.3951 | 45.7797 | 44.0874 | 3.38466 |
| 14 | 45.7797 | 49.2884 | 47.5340 | 3.50865 |
| 15 | 49.2884 | 52.9256 | 51.1070 | 3.63718 |
| 16 | 52.9256 | 56.6960 | 54.8108 | 3.77042 |

| ACCELERATOR STEP | PROTONS | |
|---------------------|------------------|----------------|
| | AVERAGE SPEED | DELTA SPEED |
| 1 | 46.0847 | 4.57437 |
| 2 | 50.5341 | 4.32440 |
| 3 | 54.7646 | 4.13652 |
| 4 | 58.8298 | 3.99184 |
| 5 | 62.7640 | 3.87860 |
| 6 | 66.5970 | 3.78922 |
| 7 | 70.3517 | 3.71844 |
| 8 | 74.0422 | 3.66253 |
| 9 | 77.6828 | 3.61877 |
| 10 | 81.2848 | 3.58509 |
| 11 | 84.8573 | 3.55996 |
| 12 | 88.4083 | 3.54213 |
| 13 | 91.9447 | 3.53067 |
| 14 | 95.4725 | 3.52478 |
| 15 | 98.9968 | 3.52381 |
| 16 | 102.522 | 3.52729 |

Table of nominal constants for MJS.

page 2 of 11

The delta values are the full width.

M mode. $K = 1.036633$

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 17 | 56.5960 | 60.6045 | 58.6502 | 3.90854 |
| 18 | 60.6045 | 64.6562 | 62.6304 | 4.05171 |
| 19 | 64.6562 | 68.8564 | 66.7563 | 4.20013 |
| 20 | 68.8564 | 73.2104 | 71.0334 | 4.35400 |
| 21 | 73.2104 | 77.7239 | 75.4671 | 4.51350 |
| 22 | 77.7239 | 82.4027 | 80.0633 | 4.67885 |
| 23 | 82.4027 | 87.2529 | 84.8278 | 4.85023 |
| 24 | 87.2529 | 92.2809 | 89.7669 | 5.02792 |
| 25 | 92.2809 | 97.4930 | 94.8869 | 5.21210 |
| 26 | 97.4930 | 102.896 | 100.194 | 5.40303 |
| 27 | 102.896 | 108.497 | 105.696 | 5.60097 |
| 28 | 108.497 | 114.303 | 111.400 | 5.80614 |
| 29 | 114.303 | 120.322 | 117.313 | 6.01883 |
| 30 | 120.322 | 126.561 | 123.442 | 6.23932 |
| 31 | 126.561 | 133.029 | 129.795 | 6.46788 |
| 32 | 133.029 | 139.734 | 136.381 | 6.70432 |

| MODULATOR STEP | PROTONS | |
|-------------------|------------------|----------------|
| | AVERAGE SPEED | DELTA SPEED |
| 17 | 106.053 | 3.53475 |
| 18 | 109.594 | 3.54587 |
| 19 | 113.147 | 3.56033 |
| 20 | 116.716 | 3.57790 |
| 21 | 120.304 | 3.59835 |
| 22 | 123.914 | 3.62150 |
| 23 | 127.548 | 3.64718 |
| 24 | 131.209 | 3.67531 |
| 25 | 134.900 | 3.70569 |
| 26 | 138.622 | 3.73830 |
| 27 | 142.378 | 3.77303 |
| 28 | 146.169 | 3.80980 |
| 29 | 149.998 | 3.84853 |
| 30 | 153.867 | 3.88921 |
| 31 | 157.778 | 3.93175 |
| 32 | 161.731 | 3.97613 |

Table of nominal constants for MJS.

page 3 of 11

The delta values are the full width.

M mode. $K = 1.036633$

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 33 | 139.734 | 146.684 | 143.209 | 6.95042 |
| 34 | 146.684 | 153.889 | 150.287 | 7.20505 |
| 35 | 153.889 | 161.358 | 157.624 | 7.46898 |
| 36 | 161.358 | 169.101 | 165.230 | 7.74258 |
| 37 | 169.101 | 177.127 | 173.114 | 8.02621 |
| 38 | 177.127 | 185.447 | 181.287 | 8.32024 |
| 39 | 185.447 | 194.072 | 189.760 | 8.62503 |
| 40 | 194.072 | 203.013 | 198.543 | 8.94098 |
| 41 | 203.013 | 212.282 | 207.648 | 9.26830 |
| 42 | 212.282 | 221.890 | 217.086 | 9.60791 |
| 43 | 221.890 | 231.850 | 226.870 | 9.95996 |
| 44 | 231.850 | 242.174 | 237.012 | 10.3247 |
| 45 | 242.174 | 252.877 | 247.526 | 10.7029 |
| 46 | 252.877 | 263.972 | 258.425 | 11.0950 |
| 47 | 263.972 | 275.474 | 269.723 | 11.5015 |
| 48 | 275.474 | 287.396 | 281.435 | 11.9229 |

| MODULATOR STEP | AVERAGE SPEED | PROTONS DELTA SPEED |
|-------------------|------------------|---------------------------|
| 33 | 165.731 | 4.02234 |
| 34 | 169.777 | 4.07030 |
| 35 | 173.872 | 4.12004 |
| 36 | 178.018 | 4.17149 |
| 37 | 182.216 | 4.22468 |
| 38 | 186.468 | 4.27957 |
| 39 | 190.776 | 4.33617 |
| 40 | 195.141 | 4.39446 |
| 41 | 199.566 | 4.45434 |
| 42 | 204.051 | 4.51605 |
| 43 | 208.599 | 4.57947 |
| 44 | 213.211 | 4.64449 |
| 45 | 217.889 | 4.71125 |
| 46 | 222.634 | 4.77977 |
| 47 | 227.449 | 4.84991 |
| 48 | 232.335 | 4.92197 |

Table of nominal constants for MJS.

page 4 of 11

The delta values are the full width.

M mode. $K = 1.036633$

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 49 | 287.396 | 299.756 | 293.576 | 12.3596 |
| 50 | 299.756 | 312.568 | 306.162 | 12.8123 |
| 51 | 312.568 | 325.850 | 319.209 | 13.2817 |
| 52 | 325.850 | 339.618 | 332.734 | 13.7683 |
| 53 | 339.618 | 353.891 | 346.755 | 14.2727 |
| 54 | 353.891 | 368.687 | 361.289 | 14.7954 |
| 55 | 368.687 | 384.024 | 376.355 | 15.3374 |
| 56 | 384.024 | 399.923 | 391.973 | 15.8992 |
| 57 | 399.923 | 416.405 | 408.164 | 16.4817 |
| 58 | 416.405 | 433.490 | 424.948 | 17.0854 |
| 59 | 433.490 | 451.202 | 442.346 | 17.7114 |
| 60 | 451.202 | 469.562 | 460.382 | 18.3601 |
| 61 | 469.562 | 488.594 | 479.078 | 19.0327 |
| 62 | 488.594 | 508.324 | 498.459 | 19.7300 |
| 63 | 508.324 | 528.777 | 518.551 | 20.4526 |
| 64 | 528.777 | 549.979 | 539.378 | 21.2019 |

| MODULATOR STEP | PROTONS | |
|-------------------|------------------|----------------|
| | AVERAGE SPEED | DELTA SPEED |
| 49 | 237.294 | 4.99552 |
| 50 | 242.327 | 5.07096 |
| 51 | 247.437 | 5.14831 |
| 52 | 252.624 | 5.22735 |
| 53 | 257.892 | 5.30808 |
| 54 | 263.241 | 5.39071 |
| 55 | 268.674 | 5.47504 |
| 56 | 274.193 | 5.56147 |
| 57 | 279.798 | 5.64981 |
| 58 | 285.493 | 5.73984 |
| 59 | 291.279 | 5.83177 |
| 60 | 297.158 | 5.92602 |
| 61 | 303.132 | 6.02197 |
| 62 | 309.203 | 6.12003 |
| 63 | 315.373 | 6.21999 |
| 64 | 321.644 | 6.32227 |

Table of nominal constants for MJS.

page 6 of 11

The delta values are the full width.

M mode. K = 1.036633

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 81 | 1016.92 | 1056.00 | 1036.46 | 39.0840 |
| 82 | 1056.00 | 1096.52 | 1076.26 | 40.5156 |
| 83 | 1096.52 | 1138.52 | 1117.52 | 41.9999 |
| 84 | 1138.52 | 1182.06 | 1160.29 | 43.5383 |
| 85 | 1182.06 | 1227.19 | 1204.62 | 45.1333 |
| 86 | 1227.19 | 1273.98 | 1250.58 | 46.7866 |
| 87 | 1273.98 | 1322.48 | 1298.23 | 48.5005 |
| 88 | 1322.48 | 1372.76 | 1347.62 | 50.2773 |
| 89 | 1372.76 | 1424.88 | 1398.82 | 52.1191 |
| 90 | 1424.88 | 1478.90 | 1451.89 | 54.0293 |
| 91 | 1478.90 | 1534.91 | 1506.91 | 56.0076 |
| 92 | 1534.91 | 1592.97 | 1563.94 | 58.0591 |
| 93 | 1592.97 | 1653.16 | 1623.06 | 60.1860 |
| 94 | 1653.16 | 1715.55 | 1684.35 | 62.3909 |
| 95 | 1715.55 | 1780.22 | 1747.89 | 64.6763 |
| 96 | 1780.22 | 1847.27 | 1813.75 | 67.0437 |

| MODULATOR STEP | AVERAGE SPEED | PROTONS DELTA SPEED |
|-------------------|------------------|---------------------------|
| 81 | 445.969 | 8.40751 |
| 82 | 454.349 | 8.55270 |
| 83 | 462.976 | 8.70063 |
| 84 | 471.752 | 8.85173 |
| 85 | 480.681 | 9.00559 |
| 86 | 489.764 | 9.16213 |
| 87 | 499.007 | 9.32216 |
| 88 | 508.410 | 9.48468 |
| 89 | 517.978 | 9.65058 |
| 90 | 527.713 | 9.81964 |
| 91 | 537.619 | 9.99188 |
| 92 | 547.698 | 10.1671 |
| 93 | 557.955 | 10.3459 |
| 94 | 568.391 | 10.5278 |
| 95 | 579.012 | 10.7134 |
| 96 | 589.820 | 10.9025 |

Table of nominal constants for MJS.

page 5 of 11

The delta values are the full widths.

M mode. $K = 1.036633$

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 65 | 549.979 | 571.958 | 560.968 | 21.9785 |
| 66 | 571.958 | 594.741 | 583.349 | 22.7837 |
| 67 | 594.741 | 618.360 | 606.550 | 23.6184 |
| 68 | 618.360 | 642.843 | 630.601 | 24.4836 |
| 69 | 642.843 | 668.224 | 655.533 | 25.3804 |
| 70 | 668.224 | 694.534 | 681.379 | 26.3103 |
| 71 | 694.534 | 721.808 | 708.171 | 27.2739 |
| 72 | 721.808 | 750.081 | 735.944 | 28.2732 |
| 73 | 750.081 | 779.390 | 764.735 | 29.3088 |
| 74 | 779.390 | 809.772 | 794.581 | 30.3826 |
| 75 | 809.772 | 841.268 | 825.520 | 31.4956 |
| 76 | 841.268 | 873.917 | 857.593 | 32.6492 |
| 77 | 873.917 | 907.762 | 890.840 | 33.8452 |
| 78 | 907.762 | 942.848 | 925.305 | 35.0852 |
| 79 | 942.848 | 979.218 | 961.033 | 36.3704 |
| 80 | 979.218 | 1016.92 | 998.069 | 37.7026 |

| MODULATOR STEP | AVERAGE SPEED | PROTONS DELTA SPEED |
|-------------------|------------------|---------------------------|
| 65 | 328.018 | 6.42646 |
| 66 | 334.498 | 6.53276 |
| 67 | 341.085 | 6.64139 |
| 68 | 347.782 | 6.75213 |
| 69 | 354.590 | 6.86498 |
| 70 | 361.513 | 6.98037 |
| 71 | 368.552 | 7.09766 |
| 72 | 375.709 | 7.21749 |
| 73 | 382.988 | 7.33985 |
| 74 | 390.390 | 7.46432 |
| 75 | 397.918 | 7.59155 |
| 76 | 405.574 | 7.72088 |
| 77 | 413.361 | 7.85297 |
| 78 | 421.282 | 7.98780 |
| 79 | 429.338 | 8.12496 |
| 80 | 437.533 | 8.26465 |

Table of nominal constants for MJS.

page 7 of 11

The delta values are the full widths.

M mode. $K = 1.036633$

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 97 | 1847.27 | 1916.77 | 1882.02 | 69.5017 |
| 98 | 1916.77 | 1988.82 | 1952.79 | 72.0476 |
| 99 | 1988.82 | 2063.51 | 2026.16 | 74.6870 |
| 100 | 2063.51 | 2140.93 | 2102.22 | 77.4229 |
| 101 | 2140.93 | 2221.19 | 2181.06 | 80.2590 |
| 102 | 2221.19 | 2304.39 | 2262.79 | 83.1992 |
| 103 | 2304.39 | 2390.63 | 2347.51 | 86.2471 |
| 104 | 2390.63 | 2480.04 | 2435.34 | 89.4065 |
| 105 | 2480.04 | 2572.72 | 2526.38 | 92.6816 |
| 106 | 2572.72 | 2668.80 | 2620.76 | 96.0769 |
| 107 | 2668.80 | 2768.40 | 2718.60 | 99.5964 |
| 108 | 2768.40 | 2871.64 | 2820.02 | 103.245 |
| 109 | 2871.64 | 2978.67 | 2925.15 | 107.027 |
| 110 | 2978.67 | 3089.61 | 3034.14 | 110.948 |
| 111 | 3089.61 | 3204.63 | 3147.12 | 115.012 |
| 112 | 3204.63 | 3323.85 | 3264.24 | 119.225 |

| MODULATOR STEP | PROTONS | |
|-------------------|------------------|----------------|
| | AVERAGE SPEED | DELTA SPEED |
| 97 | 600.819 | 11.0948 |
| 98 | 612.011 | 11.2910 |
| 99 | 623.402 | 11.4907 |
| 100 | 634.995 | 11.6942 |
| 101 | 646.792 | 11.9013 |
| 102 | 659.799 | 12.1126 |
| 103 | 671.020 | 12.3275 |
| 104 | 683.457 | 12.5467 |
| 105 | 696.115 | 12.7697 |
| 106 | 708.998 | 12.9971 |
| 107 | 722.111 | 13.2285 |
| 108 | 735.457 | 13.4641 |
| 109 | 749.042 | 13.7044 |
| 110 | 762.868 | 13.9489 |
| 111 | 776.941 | 14.1979 |
| 112 | 791.266 | 14.4515 |

Table of nominal constants for MJS.

page 8 of 11

The delta values are the full widths.

M mode. $K = 1.036633$

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 113 | 3323.85 | 3447.44 | 3385.65 | 123.593 |
| 114 | 3447.44 | 3575.56 | 3511.50 | 128.120 |
| 115 | 3575.56 | 3708.38 | 3641.97 | 132.813 |
| 116 | 3708.38 | 3846.06 | 3777.22 | 137.679 |
| 117 | 3846.06 | 3988.78 | 3917.42 | 142.722 |
| 118 | 3988.78 | 4136.73 | 4062.75 | 147.948 |
| 119 | 4136.73 | 4290.09 | 4213.41 | 153.367 |
| 120 | 4290.09 | 4449.08 | 4369.59 | 158.988 |
| 121 | 4449.08 | 4613.89 | 4531.49 | 164.812 |
| 122 | 4613.89 | 4784.74 | 4699.32 | 170.848 |
| 123 | 4784.74 | 4961.85 | 4873.29 | 177.105 |
| 124 | 4961.85 | 5145.44 | 5053.64 | 183.594 |
| 125 | 5145.44 | 5335.76 | 5240.60 | 190.320 |
| 126 | 5335.76 | 5533.05 | 5434.41 | 197.293 |
| 127 | 5533.05 | 5737.57 | 5635.31 | 204.520 |
| 128 | 5737.57 | 5949.59 | 5843.58 | 212.012 |

| MODULATOR STEP | PROTONS | |
|-------------------|------------------|----------------|
| | AVERAGE SPEED | DELTA SPEED |
| 113 | 805.847 | 14.7099 |
| 114 | 820.688 | 14.9730 |
| 115 | 835.795 | 15.2410 |
| 116 | 851.173 | 15.5133 |
| 117 | 866.825 | 15.7917 |
| 118 | 882.759 | 16.0743 |
| 119 | 898.977 | 16.3628 |
| 120 | 915.487 | 16.6565 |
| 121 | 932.293 | 16.9553 |
| 122 | 949.400 | 17.2594 |
| 123 | 966.814 | 17.5695 |
| 124 | 984.542 | 17.8852 |
| 125 | 1002.59 | 18.2066 |
| 126 | 1020.96 | 18.5342 |
| 127 | 1039.66 | 18.8675 |
| 128 | 1058.70 | 19.2069 |

Table of nominal constants for MJS.

page 9 of 11

The delta values are the full width.

E_1 mode. $K = 1.074608$

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 1 | 10.0000 | 14.4765 | 12.2382 | 4.47647 |
| 2 | 14.4765 | 19.2869 | 16.8817 | 4.81044 |
| 3 | 19.2869 | 24.4563 | 21.8716 | 5.16934 |
| 4 | 24.4563 | 30.0113 | 27.2337 | 5.55501 |
| 5 | 30.0113 | 35.9807 | 32.9960 | 5.96947 |
| 6 | 35.9807 | 42.3956 | 39.1881 | 6.41483 |
| 7 | 42.3956 | 49.2890 | 45.8423 | 6.89343 |
| 8 | 49.2890 | 56.6967 | 52.9928 | 7.40773 |
| 9 | 56.6967 | 64.6571 | 60.6769 | 7.96040 |
| 10 | 64.6571 | 73.2114 | 68.9343 | 8.55431 |
| 11 | 73.2114 | 82.4040 | 77.8077 | 9.19254 |
| 12 | 82.4040 | 92.2823 | 87.3431 | 9.87837 |
| 13 | 92.2823 | 102.898 | 97.5900 | 10.6154 |
| 14 | 102.898 | 114.305 | 108.601 | 11.4074 |
| 15 | 114.305 | 126.564 | 120.434 | 12.2584 |
| 16 | 126.564 | 139.737 | 133.150 | 13.1730 |

| MODULATOR STEP | ELECTRONS | |
|-------------------|------------------|----------------|
| | AVERAGE SPEED | DELTA SPEED |
| 1 | 2067.48 | 381.335 |
| 2 | 2432.31 | 348.319 |
| 3 | 2770.76 | 328.585 |
| 4 | 3093.20 | 316.292 |
| 5 | 3405.69 | 308.703 |
| 6 | 3712.22 | 304.342 |
| 7 | 4015.56 | 302.344 |
| 8 | 4317.81 | 302.157 |
| 9 | 4620.60 | 303.422 |
| 10 | 4925.25 | 305.891 |
| 11 | 5232.89 | 309.388 |
| 12 | 5544.48 | 313.787 |
| 13 | 5860.87 | 318.995 |
| 14 | 6182.84 | 324.944 |
| 15 | 6511.10 | 331.582 |
| 16 | 6846.33 | 338.874 |

Table of nominal constants for MJS.

page 10 of 11

The delta values are the full widths.

L Mode.

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 1 | 10.0000 | 30.0113 | 20.0056 | 20.0113 |
| 2 | 30.0113 | 56.6968 | 43.3541 | 26.6855 |
| 3 | 56.6968 | 92.2825 | 74.4897 | 35.5857 |
| 4 | 92.2825 | 139.737 | 116.010 | 47.4543 |
| 5 | 139.737 | 203.018 | 171.377 | 63.2814 |
| 6 | 203.018 | 287.405 | 245.212 | 84.3870 |
| 7 | 287.405 | 399.937 | 343.671 | 112.532 |
| 8 | 399.937 | 550.001 | 474.969 | 150.064 |
| 9 | 550.001 | 750.114 | 650.058 | 200.113 |
| 10 | 750.114 | 1016.97 | 883.542 | 266.855 |
| 11 | 1016.97 | 1372.83 | 1194.90 | 355.858 |
| 12 | 1372.83 | 1847.37 | 1610.10 | 474.544 |
| 13 | 1847.37 | 2480.19 | 2163.78 | 632.815 |
| 14 | 2480.19 | 3324.06 | 2902.12 | 843.872 |
| 15 | 3324.06 | 4449.38 | 3886.72 | 1125.32 |
| 16 | 4449.38 | 5950.02 | 5199.70 | 1500.64 |

| MODULATOR STEP | PROTON AVERAGE SPEED | DELTA SPEED |
|-------------------|----------------------------|----------------|
| 1 | 59.8357 | 32.0763 |
| 2 | 90.0803 | 28.4129 |
| 3 | 118.668 | 28.7616 |
| 4 | 148.385 | 30.6730 |
| 5 | 180.531 | 33.6197 |
| 6 | 216.070 | 37.4585 |
| 7 | 255.889 | 42.1787 |
| 8 | 300.895 | 47.8334 |
| 9 | 352.069 | 54.5151 |
| 10 | 410.501 | 62.3493 |
| 11 | 477.421 | 71.4897 |
| 12 | 554.227 | 82.1219 |
| 13 | 642.519 | 94.4626 |
| 14 | 744.134 | 108.767 |
| 15 | 861.181 | 125.329 |
| 16 | 996.092 | 144.493 |

Table of nominal constants for MJS.

The delta values are the full widths.

E_2 Mode. $K = 1.333522$

| MODULATOR STEP | THRESHOLD VOLTAGE | PEAK VOLTAGE | AVERAGE VOLTAGE | DELTA VOLTAGE |
|-------------------|----------------------|-----------------|--------------------|------------------|
| 1 | 10.0000 | 30.0113 | 20.0056 | 20.0113 |
| 2 | 30.0118 | 56.6968 | 43.3541 | 26.6855 |
| 3 | 56.6968 | 92.2825 | 74.4897 | 35.5857 |
| 4 | 92.2825 | 139.737 | 116.010 | 47.4543 |
| 5 | 139.737 | 203.018 | 171.377 | 63.2814 |
| 6 | 203.018 | 287.405 | 245.212 | 84.3870 |
| 7 | 287.405 | 399.937 | 343.671 | 112.532 |
| 8 | 399.937 | 550.001 | 474.969 | 150.064 |
| 9 | 550.001 | 750.114 | 650.058 | 200.113 |
| 10 | 750.114 | 1016.97 | 883.542 | 266.855 |
| 11 | 1016.97 | 1372.83 | 1194.90 | 355.858 |
| 12 | 1372.83 | 1847.37 | 1610.10 | 474.544 |
| 13 | 1847.37 | 2480.19 | 2163.78 | 632.815 |
| 14 | 2480.19 | 3324.06 | 2902.12 | 843.872 |
| 15 | 3324.06 | 4449.38 | 3886.72 | 1125.32 |
| 16 | 4449.38 | 5950.02 | 5199.70 | 1500.64 |

| MODULATOR STEP | ELECTRONS | |
|-------------------|------------------|----------------|
| | AVERAGE SPEED | DELTA SPEED |
| 1 | 2564.08 | 1374.53 |
| 2 | 3860.12 | 1217.55 |
| 3 | 5085.13 | 1232.49 |
| 4 | 6358.58 | 1314.40 |
| 5 | 7736.11 | 1440.67 |
| 6 | 9259.03 | 1605.17 |
| 7 | 10965.3 | 1807.44 |
| 8 | 12893.9 | 2049.76 |
| 9 | 15086.9 | 2336.08 |
| 10 | 17590.8 | 2671.79 |
| 11 | 20458.4 | 3063.48 |
| 12 | 23749.7 | 3519.09 |
| 13 | 27533.2 | 4047.91 |
| 14 | 31887.6 | 4660.86 |
| 15 | 36903.3 | 5370.59 |
| 16 | 42684.5 | 6191.81 |

Appendix B

```

SUBROUTINE CUPINT(DN,U,W,NSTEP,CUR) DD 00010
IMPLICIT REAL*8(A-H,O-Z) DD 00020
REAL*8 U(3),VZSUP/134.9/,RO/100./,ECHRGE/1.6E-19/ DD 00030
REAL*8 PI/3.14152654/ DD 00040
REAL*8 AMVZ(128)/ DD 00050
+ 46.06, 50.50, 54.73, 58.79, 62.72, 66.56, 70.31, 74.00, DD 00060
+ 77.63, 81.23, 84.80, 88.35, 91.89, 95.41, 98.93, 102.46, DD 00070
+ 105.99, 109.52, 113.07, 116.64, 120.23, 123.83, 127.47, 131.12, DD 00080
+ 134.81, 138.53, 142.28, 146.07, 149.90, 153.77, 157.67, 161.62, DD 00090
+ 165.62, 169.66, 173.76, 177.90, 182.09, 186.34, 190.65, 195.01, DD 00100
+ 199.43, 203.91, 208.46, 213.07, 217.74, 222.48, 227.29, 232.18, DD 00110
+ 237.13, 242.16, 247.27, 252.45, 257.71, 263.06, 268.49, 274.00, DD 00120
+ 279.60, 285.29, 291.08, 296.95, 302.92, 308.99, 315.15, 321.42, DD 00130
+ 327.79, 334.26, 340.84, 347.54, 354.34, 361.26, 368.29, 375.44, DD 00140
+ 382.71, 390.11, 397.63, 405.28, 413.06, 420.98, 429.03, 437.22, DD 00150
+ 445.54, 454.02, 462.64, 471.41, 480.33, 489.41, 498.64, 508.04, DD 00160
+ 517.60, 527.32, 537.22, 547.29, 557.54, 567.97, 578.58, 589.38, DD 00170
+ 600.37, 611.55, 622.93, 634.52, 646.30, 658.30, 670.51, 682.94, DD 00180
+ 695.58, 708.46, 721.56, 734.89, 748.47, 762.28, 776.34, 790.65, DD 00190
+ 805.22, 820.05, 835.14, 850.51, 866.15, 882.07, 898.27, 914.77, DD 00200
+ 931.56, 948.65, 966.05, 983.76, 1001.80, 1020.10, 1038.80, 1057.90/ DD 00210
REAL*8 AMDVZ(128)/ DD 00220
+ 4.57, 4.32, 4.13, 3.99, 3.88, 3.79, 3.72, 3.66, DD 00230
+ 3.62, 3.58, 3.56, 3.54, 3.53, 3.52, 3.52, 3.53, DD 00240
+ 3.53, 3.54, 3.56, 3.58, 3.59, 3.62, 3.65, 3.67, DD 00250
+ 3.70, 3.74, 3.77, 3.81, 3.84, 3.89, 3.93, 3.97, DD 00260
+ 4.02, 4.07, 4.11, 4.17, 4.22, 4.28, 4.33, 4.40, DD 00270
+ 4.45, 4.51, 4.58, 4.64, 4.70, 4.78, 4.85, 4.92, DD 00280
+ 4.99, 5.06, 5.15, 5.22, 5.31, 5.38, 5.47, 5.56, DD 00290
+ 5.65, 5.73, 5.83, 5.92, 6.02, 6.11, 6.22, 6.32, DD 00300
+ 6.42, 6.53, 6.63, 6.75, 6.86, 6.97, 7.10, 7.21, DD 00310
+ 7.33, 7.46, 7.59, 7.71, 7.85, 7.98, 8.12, 8.25, DD 00320
+ 8.41, 8.54, 8.70, 8.84, 9.00, 9.15, 9.32, 9.47, DD 00330
+ 9.65, 9.81, 9.98, 10.16, 10.34, 10.52, 10.70, 10.89, DD 00340
+ 11.09, 11.28, 11.48, 11.69, 11.89, 12.10, 12.32, 12.53, DD 00350
+ 12.76, 12.99, 13.22, 13.45, 13.69, 13.94, 14.18, 14.44, DD 00360
+ 14.70, 14.96, 15.23, 15.50, 15.78, 16.06, 16.35, 16.64, DD 00370
+ 16.94, 17.24, 17.56, 17.87, 18.20, 18.50, 18.90, 19.10/ DD 00380
CUR=0.00 DD 00390
VZT=AMVZ(NSTEP)-AMDVZ(NSTEP)/2.00 DD 00400
VZ=VZT DD 00410
X1=(VZ-U(3))/W DD 00420
IF (X1 .GT. 3.00) RETURN DD 00430
DVZ=AMDVZ(NSTEP)/10.00 DD 00440
N=0 DD 00450
10 CONTINUE DD 00460
N=N+1 DD 00470
NN=MOD(N,2) DD 00480
VZ=VZ+DVZ DD 00490
X1=(VZ-U(3))/W DD 00500
IF (X1 .LT. -5.00) GO TO 10 DD 00510
IF (X1 .GT. 3.00) GO TO 100 DD 00520
P1=(VZT/VZ)**2 DD 00530
P2=(VZSUP/VZ)**2 DD 00540
X=DABS(U(1)/VZ) DD 00550

```

| | | |
|-----|--|----------|
| | Y=U(2)/VZ | DD 00560 |
| | CALL TRANSP(X,Y,P1,P2,T) | DD 00570 |
| | CALL AREAOV(X,Y,P1,P2,R) | DD 00580 |
| | IF(NN .EQ. 1)CUR=CUR+R*T*DEXP(-X1**2)*VZ*2.DO | DD 00590 |
| | IF(NN .EQ. 0)CUR=CUR+R*T*DEXP(-X1**2)*VZ | DD 00600 |
| | GO TO 10 | DD 00610 |
| 100 | CONTINUE | DD 00620 |
| | CUR=CUR*RO*ECHRGE*DN/(W*DSQRT(PI))*1.E20*DVZ*2.DO/3.DO | DD 00630 |
| C | CUR=CUR+R*T*DEXP(-X1**2)*VZ | DD 00640 |
| C | CUR=CUR*RO*ECHRGE*DN/(W*DSQRT(PI))*1.E20*DVZ | DD 00650 |
| | RETURN | DD 00660 |
| | END | DD 00670 |
| | SUBROUTINE TRANSP(X,Y,P1,P2,T) | DD 00680 |
| | IMPLICIT REAL*8(A-H,O-Z) | DD 00690 |
| | C=1/42.DO | DD 00700 |
| | X2=X**2 | DD 00710 |
| | Y2=Y**2 | DD 00720 |
| | T=((1.DO-C*DSQRT(1.DO+X2))*(1.DO-C*DSQRT(1.DO+Y2)))**5 | DD 00730 |
| | T=T*((1.DO-C*DSQRT(1.DO+X2/(1.DO-P1)))**3 | DD 00740 |
| | * (1.DO-C*DSQRT(1.DO+Y2/(1.DO-P1)))**3 | DD 00750 |
| | T=T*(1.DO-C*DSQRT(1.DO+X2/(1.DO+P2)))**3 | DD 00760 |
| | * (1.DO-C*DSQRT(1.DO+Y2/(1.DO+P2)))**3 | DD 00770 |
| | RETURN | DD 00780 |
| | END | DD 00790 |
| | SUBROUTINE AREAOV(X,Y,P1,P2,R) | DD 00800 |
| | IMPLICIT REAL*8(A-H,O-Z) | DD 00810 |
| | REAL*8 A/.33800/,B/.19700/,C/.09300/,D/.37200/ | DD 00820 |
| | SHIFT=A+B*2*(DSQRT(1.DO+P2)-1.DO)/P2+C/DSQRT(1.DO-P1)+ | DD 00830 |
| | + D*2*(1.DO-DSQRT(1.DO-P1))/P1 | DD 00840 |
| | X1=SHIFT*X | DD 00850 |
| | Y1=SHIFT*Y | DD 00860 |
| | IF(X1 .GT. 4.9400) GO TO 100 | DD 00870 |
| | R=(4.9400-X1)/3.8400 | DD 00880 |
| | IF(X1 .LT. 1.100) R=1.DO | DD 00890 |
| | IF(Y1 .LT. -2.0200) GO TO 10 | DD 00900 |
| | Y3=YU(X1) | DD 00910 |
| | Y4=YUP(X1) | DD 00920 |
| | IF(Y1-Y3) 1,1,2 | DD 00930 |
| 1 | RETURN | DD 00940 |
| 2 | IF(Y1 .GT. Y4) GO TO 100 | DD 00950 |
| | R=R*(Y4-Y1)/(Y4-Y3) | DD 00960 |
| | RETURN | DD 00970 |
| 10 | IF(Y1 .LT. -3.6300) GO TO 100 | DD 00980 |
| | R=R*(3.6300+Y1)/1.6100 | DD 00990 |
| | RETURN | DD 01000 |
| 100 | R=0.DO | DD 01010 |
| | RETURN | DD 01020 |
| | END | DD 01030 |
| | FUNCTION YU(X) | DD 01040 |
| | IMPLICIT REAL*8(A-H,O-Z) | DD 01050 |
| | A=.76200 | DD 01060 |
| | B=1.01800 | DD 01070 |
| | C=.24700 | DD 01080 |
| | D=.2500 | DD 01090 |
| | YU=A*DCOS(B*X+C)/(1.DO+D*X) | DD 01100 |

| | |
|---------------------------|----------|
| RETURN | DD 01110 |
| END | DD 01120 |
| FUNCTION YUP(X) | DD 01130 |
| IMPLICIT REAL*8 (A-H,O-Z) | DD 01140 |
| A=2.5D0 | DD 01150 |
| B=0.125D0 | DD 01160 |
| YUP=A-B*(X-1.D0)**2 | DD 01170 |
| RETURN | DD 01180 |
| END | DD 01190 |

Appendix C

```

SUBROUTINE DCPINT(DN,U,W,NSTEP,CUR) DD 00010
IMPLICIT REAL*8 (A-H,O-Z) DD 00020
REAL*8 U(3),VZSUP/134.9/,ECHRG/1.6E-19/,PI/3.141592654/,RC/6.35/, DD 00030
, RG/5.1308/,RA/5.6438/, DD 00040
, RCG2/13.99739136/,RCA2/8.47002156/,RAG2/5.5273698/,RO/82.7/ DD 00050
REAL*8 AMVZ(128)/ DD 00060
+ 46.06, 50.50, 54.73, 58.79, 62.72, 66.56, 70.31, 74.00, DD 00070
+ 77.63, 81.23, 84.80, 88.35, 91.89, 95.41, 98.93, 102.46, DD 00080
+ 105.99, 109.52, 113.07, 116.64, 120.23, 123.83, 127.47, 131.12, DD 00090
+ 134.81, 138.53, 142.28, 146.07, 149.90, 153.77, 157.67, 161.62, DD 00100
+ 165.62, 169.66, 173.76, 177.90, 182.09, 186.34, 190.65, 195.01, DD 00110
+ 199.43, 203.91, 208.46, 213.07, 217.74, 222.48, 227.29, 232.18, DD 00120
+ 237.13, 242.16, 247.27, 252.45, 257.71, 263.06, 268.49, 274.00, DD 00130
+ 279.60, 285.29, 291.08, 296.95, 302.92, 308.99, 315.15, 321.42, DD 00140
+ 327.79, 334.26, 340.84, 347.54, 354.34, 361.26, 368.29, 375.44, DD 00150
+ 382.71, 390.11, 397.63, 405.28, 413.06, 420.98, 429.03, 437.22, DD 00160
+ 445.54, 454.02, 462.64, 471.41, 480.33, 489.41, 498.64, 508.04, DD 00170
+ 517.60, 527.32, 537.22, 547.29, 557.54, 567.97, 578.58, 589.38, DD 00180
+ 600.37, 611.55, 622.93, 634.52, 646.30, 658.30, 670.51, 682.94, DD 00190
+ 695.58, 708.46, 721.56, 734.89, 748.47, 762.28, 776.34, 790.65, DD 00200
+ 805.22, 820.05, 835.14, 850.51, 866.15, 882.07, 898.27, 914.77, DD 00210
+ 931.56, 948.65, 966.05, 983.76, 1001.80, 1020.10, 1038.80, 1057.90, DD 00220
REAL*8 AMDVZ(128)/ DD 00230
+ 4.57, 4.32, 4.13, 3.99, 3.88, 3.79, 3.72, 3.66, DD 00240
+ 3.62, 3.58, 3.56, 3.54, 3.53, 3.52, 3.52, 3.53, DD 00250
+ 3.53, 3.54, 3.56, 3.58, 3.59, 3.62, 3.65, 3.67, DD 00260
+ 3.70, 3.74, 3.77, 3.81, 3.84, 3.89, 3.93, 3.97, DD 00270
+ 4.02, 4.07, 4.11, 4.17, 4.22, 4.28, 4.33, 4.40, DD 00280
+ 4.45, 4.51, 4.58, 4.64, 4.70, 4.78, 4.85, 4.92, DD 00290
+ 4.99, 5.06, 5.15, 5.22, 5.31, 5.38, 5.47, 5.56, DD 00300
+ 5.65, 5.73, 5.83, 5.92, 6.02, 6.11, 6.22, 6.32, DD 00310
+ 6.42, 6.53, 6.63, 6.75, 6.86, 6.97, 7.10, 7.21, DD 00320
+ 7.33, 7.46, 7.59, 7.71, 7.85, 7.98, 8.12, 8.25, DD 00330
+ 8.41, 8.54, 8.70, 8.84, 9.00, 9.15, 9.32, 9.47, DD 00340
+ 9.65, 9.81, 9.98, 10.16, 10.34, 10.52, 10.70, 10.89, DD 00350
+ 11.09, 11.28, 11.48, 11.69, 11.89, 12.10, 12.32, 12.53, DD 00360
+ 12.76, 12.99, 13.22, 13.45, 13.69, 13.94, 14.18, 14.44, DD 00370
+ 14.70, 14.96, 15.23, 15.50, 15.78, 16.06, 16.35, 16.64, DD 00380
+ 16.94, 17.24, 17.56, 17.87, 18.20, 18.50, 18.90, 19.10, DD 00390
CUR=0.00 DD 00400
IF(U(1).EQ.0.00.AND.U(2).EQ.0.00) GO TO 1 DD 00410
PHI=DATAN2(U(2),U(1)) DD 00420
GO TO 2 DD 00430
1 PHI=0.00 DD 00440
2 VT2=U(1)**2+U(2)**2 DD 00450
VZT=AMVZ(NSTEP)-AMDVZ(NSTEP)/2.00 DD 00460
VZ=VZT DD 00470
X1=(VZ-U(3))/W DD 00480
IF(X1.GT.3.00) RETURN DD 00490
DVZ=AMDVZ(NSTEP)/10.00 DD 00500
N=0 DD 00510
10 CONTINUE DD 00520
N=N+1 DD 00530
NN=MOD(N,2) DD 00540
VZ=VZ+DVZ DD 00550

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```

X1=(VZ-U(3))/W
IF (X1 .LT. -5.00) GO TO 10
IF (X1 .GT. 3.00)GO TO 500
P1=(VZT/VZ)**2
P2=(VZSUP/VZ)**2
PSI=VT2/VZ**2
CALL TRNSPD(PSI,PHI,P1,P2,T)
CALL SHIFT(PSI,P1,P2,DCA,DCG,DAG)
IF (DCA .EQ. 0.00) GO TO 150
XCG=(RCG2+DCG**2)/(2.00*DCG)
XCA=(RCA2+DCA**2)/(2.00*DCA)
IF (XCA-XCG) 100,200,200
100 CALL CLAROV(RCG2,RC,RG,DCG,A1)
CALL CLAROV(RAG2,RA,RG,DAG,A2)
R=A1+A2-R0
GO TO 250
150 R=R0
GO TO 250
200 CALL CLAROV(RCA2,RC,RA,DCA,R)
250 CONTINUE
IF(NN .EQ. 1)CUR=CUR+R*T*DEXP(-X1**2)*VZ*2.DO
IF(NN .EQ. 0)CUR=CUR+R*T*DEXP(-X1**2)*VZ
GO TO 10
500 CONTINUE
CUR=CUR*ECHRGE*DN/(W*DSQRT(PI))*1.0D20*DVZ*2.DO/3.DO
RETURN
END
SUBROUTINE TRNSPD(PSI,PHI,P1,P2,T)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 C/.02381D0/,A1/0.00/,A2/1.0821D0/,A3/.8727D0/,
A4/.9599D0/,A5/.6981D0/,A6/.0698D0/,AM/1.1868D0/,AS/0.00/
T=(1.00-C*DSQRT(1.00+PSI*DCOS(PHI-A1)**2))*
* (1.00-C*DSQRT(1.00+PSI*DSIN(PHI-A1)**2))*
* (1.00-C*DSQRT(1.00+PSI*DCOS(PHI-A2)**2))*
* (1.00-C*DSQRT(1.00+PSI*DSIN(PHI-A2)**2))*
* (1.00-C*DSQRT(1.00+PSI*DCOS(PHI-A3)**2))*
* (1.00-C*DSQRT(1.00+PSI*DSIN(PHI-A3)**2))*
* (1.00-C*DSQRT(1.00+PSI*DCOS(PHI-A4)**2))*
* (1.00-C*DSQRT(1.00+PSI*DSIN(PHI-A4)**2))*
* (1.00-C*DSQRT(1.00+PSI*DCOS(PHI-A5)**2))*
* (1.00-C*DSQRT(1.00+PSI*DSIN(PHI-A5)**2))*
* (1.00-C*DSQRT(1.00+PSI*DCOS(PHI-A6)**2))*
* (1.00-C*DSQRT(1.00+PSI*DSIN(PHI-A6)**2))*
* (1.00-C*DSQRT(1.00+PSI*DCOS(PHI-AM)**2/(1.00-P1)))*
* (1.00-C*DSQRT(1.00+PSI*DSIN(PHI-AM)**2/(1.00-P1)))*
* (1.00-C*DSQRT(1.00+PSI*DCOS(PHI-AS)**2/(1.00+P2)))*
* (1.00-C*DSQRT(1.00+PSI*DSIN(PHI-AS)**2/(1.00+P2)))
RETURN
END
SUBROUTINE CLAROV(R2,RL,RS,D,A)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PI/3.141592654D0/
IF (D-RL+RS) 1,1,2
1 A=PI*RS**2
RETURN

```

```

DD 00560
DD 00570
DD 00580
DD 00590
DD 00600
DD 00610
DD 00620
DD 00630
DD 00640
DD 00650
DD 00660
DD 00670
DD 00680
DD 00690
DD 00700
DD 00710
DD 00720
DD 00730
DD 00740
DD 00750
DD 00760
DD 00770
DD 00780
DD 00790
DD 00800
DD 00810
DD 00820
DD 00830
DD 00840
DD 00850
DD 00860
DD 00870
DD 00880
DD 00890
DD 00900
DD 00910
DD 00920
DD 00930
DD 00940
DD 00950
DD 00960
DD 00970
DD 00980
DD 00990
DD 01000
DD 01010
DD 01020
DD 01030
DD 01040
DD 01050
DD 01060
DD 01070
DD 01080
DD 01090
DD 01100

```

| | |
|--|----------|
| 2 IF(D-RL-RS) 3,4,4 | DD 01110 |
| 3 QS=(R2-D**2)/(2.DO*D*RS) | DD 01120 |
| QL=(R2+D**2)/(2.DO*D*RL) | DD 01130 |
| A=RS**2*(PI/2.DO+DARSIN(QS)+QS*DSQRT(1.DO-QS**2))+ | DD 01140 |
| + RL**2*(PI/2.DO-DARSIN(QL)-QL*DSQRT(1.DO-QL**2)) | DD 01150 |
| RETURN | DD 01160 |
| 4 A=0.DO | DD 01170 |
| RETURN | DD 01180 |
| END | DD 01190 |
| SUBROUTINE SHIFT(PSI,P1,P2,DCA,DCG,DAG) | DD 01200 |
| IMPLICIT REAL*8(A-H,O-Z) | DD 01210 |
| S=.495DO+.720DO*(1.DO-DSQRT(1.DO-P1))/P1+ | DD 01220 |
| +.290DO*(DSQRT(1.DO+P2)-1.DO)/P2 | DD 01230 |
| B=S/(.166DO+.139DO*(DSQRT(1.DO+P2)-1.DO)/P2) | DD 01240 |
| DCA=DSQRT(PSI)*6.DO*S | DD 01250 |
| DCG=DCA/B | DD 01260 |
| DAG=DCA-DCG | DD 01270 |
| RETURN | DD 01280 |
| END | DD 01290 |

Appendix D

```

SUBROUTINE ABCCUR(DN,W,U1,A,Z,NCHN,DISTF)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 CM(100),VZSC2,SCPOT/-12.5D0/
COMMON/CM/CM,N13
INTEGER*4 NCHN(2),NSTEP(129)/22,16,10,6,41*4,84*2/,NFIRST
INTEGER*4 NTAIL(2)
INTEGER MFIRST(128)/3,6,10,13,17,20,23,26,29,
, 32,35,38,39,41,42,44,45,46,48,49,50,52,53,54,55,56,58,59,60,61,
, 63,64,65,66,67,68,70,71,72,73,74,75,76,77,78,80,81,82,83,84,85,
, 86,87,88,89,90,92,93,94,95,96,97,98,99,100,101,102,103,104,105,
, 106,107,108,109,110,111,112,113,115,49*0/
REAL*4 AMA1,AMA2,AMC1,AMSFT,ABCS
REAL*4 AAVZ,AAVZ,AVZ,AAADVZ,AADVZ,ADVZ,DVOLT
REAL*8 U(3),ZX(4,2),ZY(4,2,2),U1(3),AA(2),SS(2),CC(2)
REAL*8 A1(2),XBAR(2),A2(2),B(2,2),D(2,2),AC(2),CRR(129)/129*0.D0/
REAL*8 X(4)/-4.94D0,-1.10D0,1.10D0,4.94D0/,
, Y(4,2)/-3.63D0,-2.02D0,2.02D0,3.63D0,
, -3.63D0,-2.02D0,2.02D0,3.63D0/
REAL*8 R/100./,T/.65/,ECHRGE/1.6D-19/,SQRTPI/1.772453851/
REAL*8 PI/3.141592654/,PRESET/Z7FFFFFFF/,DISTF(128),CUR1(241,5,2)
COMMON/TRNPAR/AMA1(100,129),AMA2(100,129),AMC1(100,129),
, AMSFT(100,129)
COMMON/ABCSET/ABCS(241,4,2)
COMMON/VZ/AAVZ(22,5),AAVZ(4,40),AVZ(2,84),AAADVZ(22,5),
, AADVZ(4,40),ADVZ(2,84),DVOLT(128),NFIRST(129)
DO 1 I=1,128
IF(I .GE. 80)MFIRST(I)=I+36
1 DISTF(I)=0.D0
C THE FOLLOWING LINE IS FOR NON-ZERO SPACECRAFT POTENTIAL
C VZSC2=2.D0*Z*ECHRGE*SCPOT/(A*1.67D-27*1.D6*W**2)
CONST=R*T*Z*ECHRGE*1.0D20/SQRTPI*DN
NCHAN1=NCHN(1)
NCHAN2=NCHN(2)
NCHAN3=NCHN(2)+1
NTAIL(1)=NFIRST(NCHAN1)
NTAIL(2)=MFIRST(NCHAN2)-1
CALL SETCUR(W,U1,A,Z,NTAIL,1,CUR1)
NTAIL(1)=MFIRST(NCHAN1)
NTAIL(2)=99+NFIRST(NCHAN2)-NSTEP(NCHAN2)
CALL SETCUR(W,U1,A,Z,NTAIL,2,CUR1)
C
C NORMALIZE VELOCITY
C
DO 3 I=1,3
3 U(I)=U1(I)/W
C
C INTEGRATE ACROSS CURRENT CHANNEL
C
DO 2700 NCHAN=NCHAN1,NCHAN3
CURENT=0.D0
NS=NSTEP(NCHAN)
DO 2600 JJ=1,NS
G=0.D0
C
DIS00010
DIS00020
DIS00030
DIS00040
DIS00050
DIS00060
DIS00070
DIS00080
DIS00090
DIS00100
DIS00110
DIS00120
DIS00130
DIS00140
DIS00150
DIS00160
DIS00170
DIS00180
DIS00190
DIS00200
DIS00210
DIS00220
DIS00230
DIS00240
DIS00250
DIS00260
DIS00270
DIS00280
DIS00290
DIS00300
DIS00310
DIS00320
DIS00330
DIS00340
DIS00350
DIS00360
DIS00370
DIS00380
DIS00390
DIS00400
DIS00410
DIS00420
DIS00430
DIS00440
DIS00450
DIS00460
DIS00470
DIS00480
DIS00490
DIS00500
DIS00510
DIS00520
DIS00530
DIS00540
DIS00550

```

```

C   SELECT VALUE OF VZ,DVZ
C
  IF(NCHAN .GE. 6 .AND. NCHAN .LE. 45)GO TO 10
  IF(NCHAN .GE. 46)GO TO 20
  VZ=DBLE(AAAVZ(JJ,NCHAN))*DSQRT(Z/A)/W
  DVZ=DBLE(AAADVZ(JJ,NCHAN))*DSQRT(Z/A)
  GO TO 100
10  VZ=DBLE(AAVZ(JJ,NCHAN-5))*DSQRT(Z/A)/W
  DVZ=DBLE(AADVZ(JJ,NCHAN-5))*DSQRT(Z/A)
  GO TO 100
20  VZ=DBLE(AVZ(JJ,NCHAN-45))*DSQRT(Z/A)/W
  DVZ=DBLE(ADVZ(JJ,NCHAN-45))*DSQRT(Z/A)
100 CONTINUE
C   THE FOLLOWING LINE CORRECTS FOR NON-ZERO SPACECRAFT POTENTIAL
  VZ2=VZ**2+VZSC2
  IF(VZ2 .LT. 0.00)GO TO 2600
  X1=(DSQRT(VZ2)-U(3))**2
C
C   X1=(VZ-U(3))**2
  IF (X1 .GT. 16.00) GO TO 2600
C
C   ASSIGN VALUES OF RESPONSE PARAMETERS
C
  S=DBLE(AMSFT(JJ,NCHAN))
  CC(1)=DBLE(AMC1(JJ,NCHAN))
  CC(2)=1.000-CC(1)
  AC(1)=DBLE(AMA1(JJ,NCHAN))
  AC(2)=DBLE(AMA2(JJ,NCHAN))
  AA(1)=(AC(1)+VZ**2)/S**2
  AA(2)=(AC(2)+VZ**2)/S**2
  SS(1)=S*VZ*U(1)/(VZ*VZ+AC(1))
  SS(2)=S*VZ*U(1)/(VZ*VZ+AC(2))
C
C   COMPUTE CURRENTS
C
  DO 2020 L=1,4
  ZX(L,1)=DSQRT(AA(1))*(X(L)-SS(1))
2020 ZX(L,2)=DSQRT(AA(2))*(X(L)-SS(2))
  XBAR(1)=(PSI(ZX(2,1),SS(1),AA(1))+PSI(ZX(4,1),SS(1),AA(1))-
-   PSI(ZX(1,1),SS(1),AA(1))-PSI(ZX(3,1),SS(1),AA(1))+
+   (ZX(4,1)-ZX(3,1))*FUN(SS(1),AA(1)))/
/   (DSQRT(PI*AA(1))*
*   (PHI(ZX(4,1))+PHI(ZX(1,1))-PHI(ZX(3,1))-PHI(ZX(2,1))))
  XBAR(2)=(PSI(ZX(2,2),SS(2),AA(2))+PSI(ZX(4,2),SS(2),AA(2))-
-   PSI(ZX(1,2),SS(2),AA(2))-PSI(ZX(3,2),SS(2),AA(2))+
+   (ZX(4,2)-ZX(3,2))*FUN(SS(2),AA(2)))/
/   (DSQRT(PI*AA(2))*
*   (PHI(ZX(4,2))+PHI(ZX(1,2))-PHI(ZX(3,2))-PHI(ZX(2,2))))
  IF (DABS(XBAR(1))-1.100) 2021,2021,2022
2021 A2(1)=1.000
  GO TO 2023
2022 A2(1)=1.25700-0.06300*DABS(XBAR(1))-
-   0.12600*DSQRT(XBAR(1)**2-5.1000*DABS(XBAR(1))+6.61200)
2023 CONTINUE
  IF (DABS(XBAR(2))-1.100) 2026,2026,2027
DIS00560
DIS00570
DIS00580
DIS00590
DIS00600
DIS00610
DIS00620
DIS00630
DIS00640
DIS00650
DIS00660
DIS00670
DIS00680
DIS00690
DIS00700
DIS00710
DIS00720
DIS00730
DIS00740
DIS00750
DIS00760
DIS00770
DIS00780
DIS00790
DIS00800
DIS00810
DIS00820
DIS00830
DIS00840
DIS00850
DIS00860
DIS00870
DIS00880
DIS00890
DIS00900
DIS00910
DIS00920
DIS00930
DIS00940
DIS00950
DIS00960
DIS00970
DIS00980
DIS00990
DIS01000
DIS01010
DIS01020
DIS01030
DIS01040
DIS01050
DIS01060
DIS01070
DIS01080
DIS01090
DIS01100

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```

2026 A2(2)=1.000
GO TO 2028
2027 A2(2)=1.25700-0.06300*DABS(XBAR(2))-
- 0.12600*DSQRT(XBAR(2)**2-5.1000*DABS(XBAR(2))+6.61200)
2028 CONTINUE
Y(3,1)=.76200*DCOS(1.01800*XBAR(1)+.24700)/(1.000+.2500*XBAR(1))
Y(3,2)=.76200*DCOS(1.01800*XBAR(2)+.24700)/(1.000+.2500*XBAR(2))
Y(4,1)=2.500-0.12500*(XBAR(1)-1.000)**2
Y(4,2)=2.500-0.12500*(XBAR(2)-1.000)**2
C WRITE(6,*)XBAR(1),Y(3,1),Y(4,1)
SS(1)=S*VZ*U(2)/(VZ*VZ+AC(1))
SS(2)=S*VZ*U(2)/(VZ*VZ+AC(2))
DO 2030 L=1,4
ZY(L,1,1)=DSQRT(AA(1))*(Y(L,1)-SS(1))
ZY(L,2,1)=DSQRT(AA(1))*(Y(L,2)-SS(1))
ZY(L,1,2)=DSQRT(AA(2))*(Y(L,1)-SS(2))
2030 ZY(L,2,2)=DSQRT(AA(2))*(Y(L,2)-SS(2))
A1(1)=(PHI(ZX(1,1))+PHI(ZX(4,1))-PHI(ZX(3,1))-PHI(ZX(2,1)))/
/ (ZX(4,1)-ZX(3,1))
A1(2)=(PHI(ZX(1,2))+PHI(ZX(4,2))-PHI(ZX(3,2))-PHI(ZX(2,2)))/
/ (ZX(4,2)-ZX(3,2))
2049 CONTINUE
DO 2500 I=1,2
DO 2450 J=1,2
B(I,J)=(PHI(ZY(4,I,J))-PHI(ZY(3,I,J)))/(ZY(4,I,J)-ZY(3,I,J))-
- (PHI(ZY(1,I,J))-PHI(ZY(2,I,J)))/(ZY(1,I,J)-ZY(2,I,J))
D(I,J)=-U(1)**2*AC(I)/(VZ**2+AC(I))-U(2)**2*AC(J)/(VZ**2+AC(J))
IF (D(I,J) .LT. -50.000) GO TO 2050
D(I,J)=CC(I)*CC(J)*DEXP(D(I,J))/DSQRT(AA(I)*AA(J))
GO TO 2051
2050 D(I,J)=0.
2051 CONTINUE
G=(VZ/S)**2*D(I,J)*A1(I)*A2(I)*B(I,J)/4+G
2450 CONTINUE
2500 CONTINUE
CUR=VZ*DEXP(-X1)*G*DVZ
CURENT=CURENT+CUR
C WRITE(6,2) JJ,CUR,CURENT,VZ
2 FORMAT(' JJ,CUR,CURENT,VZ=',I4,,3D16.7)
2600 CONTINUE
CRR(NCHAN)=CURENT
2700 CONTINUE
C WRITE(6,9000)(CRR(I),I=NCHAN1,NCHAN3)
9000 FORMAT(5D16.6)
C
C CALCULATE CONTRIBUTION FROM "TAIL"
C
DO 3000 NCHAN=NCHAN1,NCHAN2
DISTF(NCHAN)=CRR(NCHAN)-CRR(NCHAN+1)
IF(NCHAN .NE. N13)GO TO 17
CM(1)=CRR(NCHAN)
IF(NCHAN .LE. 4)CM(1)=DISTF(NCHAN)
17 NF=NFIRST(NCHAN)
IF(NCHAN .LE. 4)GO TO 2775
NF1=NFIRST(NCHAN+1)-1

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DIS01110
DIS01120
DIS01130
DIS01140
DIS01150
DIS01160
DIS01170
DIS01180
DIS01190
DIS01200
DIS01210
DIS01220
DIS01230
DIS01240
DIS01250
DIS01260
DIS01270
DIS01280
DIS01290
DIS01300
DIS01310
DIS01320
DIS01330
DIS01340
DIS01350
DIS01360
DIS01370
DIS01380
DIS01390
DIS01400
DIS01410
DIS01420
DIS01430
DIS01440
DIS01450
DIS01460
DIS01470
DIS01480
DIS01490
DIS01500
DIS01510
DIS01520
DIS01530
DIS01540
DIS01550
DIS01560
DIS01570
DIS01580
DIS01590
DIS01600
DIS01610
DIS01620
DIS01630
DIS01640
DIS01650

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```

N=NSTEP(NCHAN)+1
DO 2725 I=NF,NF1
IF(CUR1(I,1,1) .EQ. PRESET) GO TO 3000
IF(CUR1(I,1,1) .EQ. 0.DO) GO TO 2725
DISTF(NCHAN)=DISTF(NCHAN)+CUR1(I,1,1)+
+ CUR1(I,2,1)*DBLE(AMSFT(N,NCHAN)-ABCS(I,1,1))+
+ CUR1(I,3,1)*DBLE(AMA1(N,NCHAN)-ABCS(I,2,1))+
+ CUR1(I,4,1)*DBLE(AMA2(N,NCHAN)-ABCS(I,3,1))+
+ CUR1(I,5,1)*DBLE(AMC1(N,NCHAN)-ABCS(I,4,1))
C WRITE(6,9001)I,NCHAN,DISTF(NCHAN)
9001 FORMAT(' I,NCHAN,DISTF(NCHAN)=' I4,2D16.7)
N=N+1
2725 CONTINUE
N1=N
N2=NSTEP(NCHAN+1)+1
IF(NCHAN .EQ. N13)CM(2)=DISTF(NCHAN)
GO TO 2750
2775 CONTINUE
I=1
N1=NSTEP(NCHAN)+1
N2=NSTEP(NCHAN+1)+1
2750 DO 2900 LL=N1,100
M=1
IF(I .GE. MFIRST(NCHAN))M=2
IF(CUR1(I,1,M) .EQ. 0.DO)GO TO 2900
IF(CUR1(I,1,M) .EQ. PRESET)GO TO 3000
DISTF(NCHAN)=DISTF(NCHAN)+
+ CUR1(I,2,M)*DBLE(AMSFT(LL,NCHAN)-AMSFT(N2,NCHAN+1))+
+ CUR1(I,3,M)*DBLE(AMA1(LL,NCHAN)-AMA1(N2,NCHAN+1))+
+ CUR1(I,4,M)*DBLE(AMA2(LL,NCHAN)-AMA2(N2,NCHAN+1))+
+ CUR1(I,5,M)*DBLE(AMC1(LL,NCHAN)-AMC1(N2,NCHAN+1))
IF(NCHAN .EQ. N13)CM(I-NF1+2)=DISTF(NCHAN)
IF(NCHAN .EQ. N13 .AND. NCHAN .LE. 4)CM(I+1)=DISTF(NCHAN)
I=I+1
N2=N2+1
2900 CONTINUE
3000 CONTINUE
5000 CONTINUE
DO 5001 NCHAN=1,128
5001 DISTF(NCHAN)=DISTF(NCHAN)*CONST/DBLE(DVOLT(NCHAN))
RETURN
END
FUNCTION FUN(SIGMA,ALPHA)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 SQRTPI/1.772453851/
X=SIGMA*DSQRT(ALPHA)
FUN1=2.0DO*SQRTPI*X*DERF(X)
IF (DABS(X) .GT. 10.0DO) GO TO 1
FUN1=FUN1+2.0DO*DEXP(-X*X)
1 FUN=FUN1
RETURN
END
FUNCTION PHI(Z)
IMPLICIT REAL*8 (A-H,O-Z)
SQRTPI=1.772453851

```

DIS01660
DIS01670
DIS01680
DIS01690
DIS01700
DIS01710
DIS01720
DIS01730
DIS01740
DIS01750
DIS01760
DIS01770
DIS01780
DIS01790
DIS01800
DIS01810
DIS01820
DIS01830
DIS01840
DIS01850
DIS01860
DIS01870
DIS01880
DIS01890
DIS01900
DIS01910
DIS01920
DIS01930
DIS01940
DIS01950
DIS01960
DIS01970
DIS01980
DIS01990
DIS02000
DIS02010
DIS02020
DIS02030
DIS02040
DIS02050
DIS02060
DIS02070
DIS02080
DIS02090
DIS02100
DIS02110
DIS02120
DIS02130
DIS02140
DIS02150
DIS02160
DIS02170
DIS02180
DIS02190
DIS02200

```
IF (DABS(Z) .GT. 10.D0) GO TO 1
PHI=Z*DERF(Z)+DEXP(-Z*Z)/SQRTPI
RETURN
1 PHI=Z*DERF(Z)
RETURN
END
FUNCTION PSI(Z,SIGMA,ALPHA)
IMPLICIT REAL*8 (A-H,O-Z)
SQRTPI=1.772453851
PSI=SQRTPI*DERF(Z)*(-.5D0+SIGMA*DSQRT(ALPHA)*Z)
IF (DABS(Z) .GT. 10.D0) RETURN
PSI=PSI+SIGMA*DSQRT(ALPHA)*DEXP(-Z*Z)
RETURN
END
```

DIS02210
DIS02220
DIS02230
DIS02240
DIS02250
DIS02260
DIS02270
DIS02280
DIS02290
DIS02300
DIS02310
DIS02320
DIS02330
DIS02340

```

SUBROUTINE SETCUR(W,U1,A,Z,NTAIL,M,CUR1)
C
C THIS PROGRAM COMPUTES THE CURRENTS IN THE TAIL AND THE DERIVATIVES
C OF THOSE CURRENTS WITH RESPECT TO THE PARAMETERS
C WHICH DESCRIBE THE RESPONSE FUNCTION
C
IMPLICIT REAL*8 (A-H,O-Z)
INTEGER NTAIL(2),M
REAL*8 U(3),ZX(4,2),ZY(4,2,2),U1(3),AA(2),SS(2),CC(2)
REAL*8 SCPOT/-12.5D0/,ECHRGE/1.6D-19/
REAL*8 A1(2),XBAR(2),A2(2),B(2,2),D(2,2),AC(2),PRESET/Z7FFFFFFF/
REAL*8 X(4)/-4.94D0,-1.10D0,1.10D0,4.94D0/,CUR1(241,5,2)
REAL*8 Y(4,2)/-3.63D0,-2.02D0,2.02D0,3.63D0/,
, -3.63D0,-2.02D0,2.02D0,3.63D0/
REAL*8 PI/3.141592654D0/,DCUR(4)/0.D0,.05D0,.001D0,.01D0/
REAL*4 ABCS,TVZ,TDVZ
COMMON /ABCSET/ABCS(241,4,2)
COMMON /TAIL/TVZ(241),TDVZ(241)
C THE FOLLOWING LINE IS FOR NON-ZERO SPACECRAFT POTENTIAL
VZSC2=2.D0*Z*ECHRGE*SCPOT/(A*1.67D-27*1.D6*W**2)
C
C
C NORMALIZE THE VELOCITY
C
DO 1 I=1,3
1 U(I)=U1(I)/W
2000 CONTINUE
NT1=NTAIL(1)
NT2=NTAIL(2)
DO 3700 JJ=NT1,NT2
C
C CHOOSE VZ AND DVZ
C
VZ=DBLE(TVZ(JJ))*DSQRT(Z/A)/W
VZ2=VZ**2+VZSC2
IF(VZ2 .LT. 0.D0) GO TO 3690
DVZ=DBLE(TDVZ(JJ))*DSQRT(Z/A)
C THE FOLLOWING LINE IS FOR NON-ZERO SPACECRAFT CHARGE
X1=(DSQRT(VZ2)-U(3))
C
C X1=(VZ-U(3))
IF (X1 .LT. -4.D0) GO TO 3690
IF (X1 .GT. 4.D0) GO TO 3800
X1=X1**2
C
C THIS LOOP COMPUTES THE DERIVATIVES
C
DO 3600 KK=1,4
G=0.D0
S=DBLE(ABCS(JJ,1,M))
CC(1)=DBLE(ABCS(JJ,4,M))
AC(1)=DBLE(ABCS(JJ,2,M))
AC(2)=DBLE(ABCS(JJ,3,M))
IF(KK .EQ. 2)S=S+DCUR(2)
IF(KK .EQ. 3)AC(1)=AC(1)+DCUR(3)

```

CUR00010
CUR00020
CUR00030
CUR00040
CUR00050
CUR00060
CUR00070
CUR00080
CUR00090
CUR00100
CUR00110
CUR00120
CUR00130
CUR00140
CUR00150
CUR00160
CUR00170
CUR00180
CUR00190
CUR00200
CUR00210
CUR00220
CUR00230
CUR00240
CUR00250
CUR00260
CUR00270
CUR00280
CUR00290
CUR00300
CUR00310
CUR00320
CUR00330
CUR00340
CUR00350
CUR00360
CUR00370
CUR00380
CUR00390
CUR00400
CUR00410
CUR00420
CUR00430
CUR00440
CUR00450
CUR00460
CUR00470
CUR00480
CUR00490
CUR00500
CUR00510
CUR00520
CUR00530
CUR00540
CUR00550

```

IF(KK .EQ. 4)AC(2)=AC(2)+DCUR(4)
CC(2)=1.000-CC(1)
AA(1)=(AC(1)+VZ**2)/S**2
AA(2)=(AC(2)+VZ**2)/S**2
SS(1)=S*VZ*U(1)/(VZ*VZ+AC(1))
SS(2)=S*VZ*U(1)/(VZ*VZ+AC(2))
DO 3020 L=1,4
3020 ZX(L,1)=DSQRT(AA(1))*(X(L)-SS(1))
ZX(L,2)=DSQRT(AA(2))*(X(L)-SS(2))
XBAR(1)=(PSI(ZX(2,1),SS(1),AA(1))+PSI(ZX(4,1),SS(1),AA(1))-
- PSI(ZX(1,1),SS(1),AA(1))-PSI(ZX(3,1),SS(1),AA(1))+
+ (ZX(4,1)-ZX(3,1))*FUN(SS(1),AA(1)))/
/ (DSQRT(PI*AA(1))*
* (PHI(ZX(4,1))+PHI(ZX(1,1))-PHI(ZX(3,1))-PHI(ZX(2,1))))
XBAR(2)=(PSI(ZX(2,2),SS(2),AA(2))+PSI(ZX(4,2),SS(2),AA(2))-
- PSI(ZX(1,2),SS(2),AA(2))-PSI(ZX(3,2),SS(2),AA(2))+
+ (ZX(4,2)-ZX(3,2))*FUN(SS(2),AA(2)))/
/ (DSQRT(PI*AA(2))*
* (PHI(ZX(4,2))+PHI(ZX(1,2))-PHI(ZX(3,2))-PHI(ZX(2,2))))
IF (DABS(XBAR(1))-1.100) 3021,3021,3022
3021 A2(1)=1.000
GO TO 3023
3022 A2(1)=1.25700-0.06300*DABS(XBAR(1))-
- 0.12600*DSQRT(XBAR(1)**2-5.1000*DABS(XBAR(1))+6.61200)
3023 CONTINUE
IF (DABS(XBAR(2))-1.100) 3026,3026,3027
3026 A2(2)=1.000
GO TO 3028
3027 A2(2)=1.25700-0.06300*DABS(XBAR(2))-
- 0.12600*DSQRT(XBAR(2)**2-5.1000*DABS(XBAR(2))+6.61200)
3028 CONTINUE
Y(3,1)=.76200*DCOS(1.01800*XBAR(1)+.24700)/(1.000+.2500*XBAR(1))
Y(3,2)=.76200*DCOS(1.01800*XBAR(2)+.24700)/(1.000+.2500*XBAR(2))
Y(4,1)=2.500-0.12500*(XBAR(1)-1.000)**2
Y(4,2)=2.500-0.12500*(XBAR(2)-1.000)**2
SS(1)=S*VZ*U(2)/(VZ*VZ+AC(1))
SS(2)=S*VZ*U(2)/(VZ*VZ+AC(2))
DO 3030 L=1,4
ZY(L,1,1)=DSQRT(AA(1))*(Y(L,1)-SS(1))
ZY(L,2,1)=DSQRT(AA(1))*(Y(L,2)-SS(1))
ZY(L,1,2)=DSQRT(AA(2))*(Y(L,1)-SS(2))
3030 ZY(L,2,2)=DSQRT(AA(2))*(Y(L,2)-SS(2))
999 FORMAT(8D10.2)
A1(1)=(PHI(ZX(1,1))+PHI(ZX(4,1))-PHI(ZX(3,1))-PHI(ZX(2,1)))/
/ (ZX(4,1)-ZX(3,1))
A1(2)=(PHI(ZX(1,2))+PHI(ZX(4,2))-PHI(ZX(3,2))-PHI(ZX(2,2)))/
/ (ZX(4,2)-ZX(3,2))
DO 3500 I=1,2
DO 3450 J=1,2
B(I,J)=(PHI(ZY(4,I,J))-PHI(ZY(3,I,J)))/(ZY(4,I,J)-ZY(3,I,J))-
- (PHI(ZY(1,I,J))-PHI(ZY(2,I,J)))/(ZY(1,I,J)-ZY(2,I,J))
D(I,J)=-U(1)**2*AC(I)/(VZ**2+AC(I))-U(2)**2*AC(J)/(VZ**2+AC(J))
IF (D(I,J) .LT. -50.00) GO TO 3050
D(I,J)=CC(I)*CC(J)*DEXP(D(I,J))/DSQRT(AA(I)*AA(J))
GO TO 3051
CUR00560
CUR00570
CUR00580
CUR00590
CUR00600
CUR00610
CUR00620
CUR00630
CUR00640
CUR00650
CUR00660
CUR00670
CUR00680
CUR00690
CUR00700
CUR00710
CUR00720
CUR00730
CUR00740
CUR00750
CUR00760
CUR00770
CUR00780
CUR00790
CUR00800
CUR00810
CUR00820
CUR00830
CUR00840
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CUR00860
CUR00870
CUR00880
CUR00890
CUR00900
CUR00910
CUR00920
CUR00930
CUR00940
CUR00950
CUR00960
CUR00970
CUR00980
CUR00990
CUR01000
CUR01010
CUR01020
CUR01030
CUR01040
CUR01050
CUR01060
CUR01070
CUR01080
CUR01090
CUR01100

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| | | |
|------|---|----------|
| 3050 | D(I,J)=0.DO | CUR01110 |
| 3051 | CONTINUE | CUR01120 |
| | G1=(VZ/S)**2*D(I,J)*A1(I)*A2(I)*B(I,J)/4 | CUR01130 |
| | G=G+G1 | CUR01140 |
| C | | CUR01150 |
| C | COMPUTE DERIVATIVE WITH RESPECT TO C ANALYTICALLY | CUR01160 |
| C | | CUR01170 |
| | IF (KK .NE. 1) GO TO 3440 | CUR01180 |
| | IF (I+J .EQ. 2)GD=G1*2.000/CC(1) | CUR01190 |
| | IF (I+J .EQ. 3)GD=G1*(CC(2)-CC(1))/(CC(2)*CC(1))+GD | CUR01200 |
| | IF (I+J .EQ. 4)GD=GD-G1*2.000/CC(2) | CUR01210 |
| 3440 | CONTINUE | CUR01220 |
| 3450 | CONTINUE | CUR01230 |
| 3500 | CONTINUE | CUR01240 |
| C | | CUR01250 |
| C | COMPUTE DERIVATIVES NUMERICALLY | CUR01260 |
| C | | CUR01270 |
| | CUR1(JJ, KK, M)=VZ*DEXP(-X1)*G*DVZ | CUR01280 |
| | IF(KK .NE. 1)CUR1(JJ, KK, M)= | CUR01290 |
| | = ((CUR1(JJ, KK, M)-CUR1(JJ, 1, M))/DCUR(KK)) | CUR01300 |
| 3550 | CONTINUE | CUR01310 |
| | CUR1(JJ, 5, M)=VZ*DEXP(-X1)*GD*DVZ | CUR01320 |
| 3600 | CONTINUE | CUR01330 |
| | GO TO 3700 | CUR01340 |
| 3690 | CONTINUE | CUR01350 |
| | CUR1(JJ, 1, M)=0.DO | CUR01360 |
| 3700 | CONTINUE | CUR01370 |
| | RETURN | CUR01380 |
| 3800 | CONTINUE | CUR01390 |
| | DO 3850 L=JJ, 200 | CUR01400 |
| | CUR1(L, 1, M)=PRESET | CUR01410 |
| 3850 | CONTINUE | CUR01420 |
| | RETURN | CUR01430 |
| | END | CUR01440 |

```

SUBROUTINE PREP
INTEGER NSTP(5)/20,14,8,4,2/
REAL*4 VOLT(228),CW(227),RATIO/1.036633/
REAL*4 DVZ(2,129),VZT(228)
REAL*4 AAVZ,AAVZ,AVZ,TVZ,AAADVZ,AAADVZ,ADVZ,TDVZ,DVOLT
COMMON /VZ/AAVZ(22,5),AAVZ(4,40),AVZ(2,84),AAADVZ(22,5),
, AADVZ(4,40),ADVZ(2,84),DVOLT(128),NFIRST(129)
COMMON /TAIL/TVZ(241),TDVZ(241)
C
C
C
GENERATE TABLE OF THRESHOLD SPEEDS AND CHANNEL WIDTHS
DO 1 K=1,228
1 VOLT(K)=RATIO**(K-1)*60.-50.
DO 2 K=1,227
IF(K .LE. 128)DVOLT(K)=VOLT(K+1)-VOLT(K)
2 CW(K)=ALOG(VOLT(K+1)/VOLT(K))* .5
VZT(1)=43.76871
DO 3 K=1,227
3 VZT(K+1)=VZT(K)*EXP(CW(K))
C
C
C
GENERATE TABLE OF SPEEDS FOR INTEGRATION OF FIRST 5 CHANNELS
DO 4 NCHAN=1,5
NFIRST(NCHAN)=1
AAVZ(2,NCHAN)=1.02*VZT(NCHAN)
AAVZ(1,NCHAN)=(VZT(NCHAN)+AAVZ(2,NCHAN))/2.
DVZ(1,NCHAN)=AAVZ(2,NCHAN)-AAVZ(1,NCHAN)
NSTEP=NSTP(NCHAN)
DVZ(2,NCHAN)=(VZT(6)-AAVZ(2,NCHAN))/FLOAT(NSTEP)
AAADVZ(1,NCHAN)=DVZ(1,NCHAN)*4./3.
AAADVZ(2,NCHAN)=(DVZ(1,NCHAN)+DVZ(2,NCHAN))/3.
DO 4 N=1,NSTEP
AAADVZ(N+2,NCHAN)=DVZ(2,NCHAN)*2./3.
IF(MOD(N,2) .EQ. 1)AAADVZ(N+2,NCHAN)=DVZ(2,NCHAN)*4./3.
IF(N .EQ. NSTEP)AAADVZ(N+2,NCHAN)=DVZ(2,NCHAN)/3.
4 AAVZ(N+2,NCHAN)=AAVZ(N+1,NCHAN)+DVZ(2,NCHAN)
C
C
C
GENERATE TABLE OF INTEGRATION VELOCITIES FOR CHANNELS 6-45
DO 5 NCHAN=6,45
DVZ(1,NCHAN)=(VZT(NCHAN+1)-VZT(NCHAN))/4
AADVZ(1,NCHAN-5)=DVZ(1,NCHAN)*4./3.
AADVZ(2,NCHAN-5)=DVZ(1,NCHAN)*2./3.
AADVZ(3,NCHAN-5)=DVZ(1,NCHAN)*4./3.
AADVZ(4,NCHAN-5)=DVZ(1,NCHAN)/3.
DO 5 NSTEP=1,4
5 AAVZ(NSTEP,NCHAN-5)=VZT(NCHAN)+FLOAT(NSTEP)*DVZ(1,NCHAN)
C
C
C
GENERATE TABLE OF INTEGRATION VELOCITIES FOR CHANNELS 46-128
DO 6 NCHAN=46,129
DVZ(1,NCHAN)=(VZT(NCHAN+1)-VZT(NCHAN))/2.
ADVZ(1,NCHAN-45)=DVZ(1,NCHAN)*4./3.
ADVZ(2,NCHAN-45)=DVZ(1,NCHAN)/3.
DO 6 NSTEP=1,2

```

```

PRE00010
PRE00020
PRE00030
PRE00040
PRE00050
PRE00060
PRE00070
PRE00080
PRE00090
PRE00100
PRE00110
PRE00120
PRE00130
PRE00140
PRE00150
PRE00160
PRE00170
PRE00180
PRE00190
PRE00200
PRE00210
PRE00220
PRE00230
PRE00240
PRE00250
PRE00260
PRE00270
PRE00280
PRE00290
PRE00300
PRE00310
PRE00320
PRE00330
PRE00340
PRE00350
PRE00360
PRE00370
PRE00380
PRE00390
PRE00400
PRE00410
PRE00420
PRE00430
PRE00440
PRE00450
PRE00460
PRE00470
PRE00480
PRE00490
PRE00500
PRE00510
PRE00520
PRE00530
PRE00540
PRE00550

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```

2 IF(JTLMOD-ITLMOD) 1,3,1
3 CONTINUE
WRITE(35)(CURRNT(I),I=1,NET)
N=0
DO 20 NCP=1,4
DO 10 NCHAN=1,128
IF(.NOT. LCHAN(NCHAN,NCP)) GO TO 10
N=N+1
IF (ITLMOD .EQ. 1) N1=NCHAN+16*(NCP-1)
IF (ITLMOD .EQ. 2) N1=NCHAN+128*(NCP-1)
DRAY(N)=CURRNT(N1)
WRAY(N)=DSQRT((4.D-2*DRAY(N))**2+(DBLE(CN)/DBLE(DVOLT(NCHAN)))**2)
10 CONTINUE
20 CONTINUE
READ(1,*)ABCS
DO 30 I=1,128
READ(15){SHIFT(J),J=1,60}
READ(25){SHIFT(J),J=61,100}
READ(16){A1(J),J=1,60}
READ(26){A1(J),J=61,100}
READ(17){A2(J),J=1,60}
READ(27){A2(J),J=61,100}
READ(18){C1(J),J=1,60}
READ(28){C1(J),J=61,100}
DO 29 J=1,100
AMSFT(J,I)=SHIFT(J)
AMA1(J,I)=A1(J)
AMA2(J,I)=A2(J)
AMC1(J,I)=C1(J)
29 CONTINUE
30 CONTINUE
READ(30,9100)NINT
READ(30,9101)AS
READ(30,9101)BS
9100 FORMAT(20I4)
9101 FORMAT(5D16.7)
LARRY=.TRUE.
RETURN
END
SUBROUTINE SIMCOM(PARM,LARRY,ARRY,ARRY1)
LOGICAL*1 LCHAN,LARRY
INTEGER*4 NCHN(2),NPTS,NPARAM,NP,NINT
C THE FOLLOWING STATEMENT IS A MODIFICATION
C REAL*4 VO(3)/41.2,28.95,3.5/,DB(3)/-25.8,33.0,37.2/
C
REAL*4 AS,BS,C,CN,CC
REAL*4 AAVZ,AAVZ,AVZ,TVZ,AAADVZ,AAADVZ,ADVZ,TDVZ,DVOLT
REAL*8 DN/1.DO/,A/64.DO/,Z/1.DO/,DIST(128),PARAM(10)
REAL*8 F(71,4),WSP,VSC(3),VCP(3),ARRY1(512,6),ARRY(512)
COMMON/LCHAN/LCHAN(128,4),CN,CC
COMMON/VZ/AAVZ(22,5),AAVZ(4,40),AVZ(2,84),AAADVZ(22,5),
AAADVZ(4,40),ADVZ(2,84),DVOLT(128),NFIRST(129)
COMMON/AS/AS(72,4),BS(72,4),C(4),NINT(72,4)
WSP=PARAM(4)
VSC(1)=PARAM(1)
FXR00560
FXR00570
FXR00580
FXR00590
FXR00600
FXR00610
FXR00620
FXR00630
FXR00640
FXR00650
FXR00660
FXR00670
FXR00680
FXR00690
FXR00700
FXR00710
FXR00720
FXR00730
FXR00740
FXR00750
FXR00760
FXR00770
FXR00780
FXR00790
FXR00800
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FXR00930
FXR00940
FXR00950
FXR00960
FXR00970
FXR00980
FXR00990
FXR01000
FXR01010
FXR01020
FXR01030
FXR01040
FXR01050
FXR01060
FXR01070
FXR01080
FXR01090
FXR01100

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```

VSC(2)=PARAM(2)
VSC(3)=PARAM(3)
C THE FOLLOWING TWO STATEMENTS ARE A MODIFICATION
C DO 9991 MP=1,3
C9991 VSC(MP)=DBLE(VO(MP))+PARAM(3)*DBLE(DB(MP))
C
NP=0
DO 300 NCP=1,3
CALL VSCCP(VSC,NCP,VCP)
NCHN(1)=1
NCHN(2)=127
C WRITE(6,9000)DN,WSP,VCP,A,Z
9000 FORMAT(5D16.7)
CALL ABCCUR(DN,WSP,VCP,A,Z,NCHN,DIST)
C WRITE(6,9000)(DIST(NN),NN=1,71)
DO 100 J=1,4
DO 100 NCHAN=1,71
N1=NINT(NCHAN,J)
N2=N1+1
F(NCHAN,J)=(DBLE(AS(NCHAN,J))*DIST(N1)+
+ DBLE(BS(NCHAN,J))*DIST(N2))*DBLE(C(J))
100 CONTINUE
DO 150 NCHAN=1,71
IF(.NOT. LCHAN(NCHAN,NCP))GO TO 150
NP=NP+1
C ARRAY(NP)=PARAM(5)**2*F(NCHAN,1)+PARAM(6)**2*F(NCHAN,2)+
C + PARAM(7)**2*F(NCHAN,3)+PARAM(8)**2*F(NCHAN,4)+
C + DBLE(CC)/DVOLT(NCHAN)
ARRAY(NP)=PARAM(5)*F(NCHAN,1)+PARAM(6)*F(NCHAN,2)+
+ PARAM(7)*F(NCHAN,3)+PARAM(8)*F(NCHAN,4)+DBLE(CC)/DVOLT(NCHAN)
IF(.NOT. LARRY)GO TO 150
DO 130 NSP=1,4
C 130 ARRAY1(NP,NSP)=F(NCHAN,NSP)*2.DO*PARAM(NSP+4)
130 ARRAY1(NP,NSP)=F(NCHAN,NSP)
150 CONTINUE
300 CONTINUE
RETURN
END
SUBROUTINE VSCCP(VSC,NCP,VCP)
REAL*8 TM(9,4),VJE(3),VSC(3),TJESC(9),VCP(3),T(9),V(3)
DATA TM/0.5,C.8137977,0.2961981, -0.8660254,0.4698463,0.1710101,
1 0.,-0.3420201,0.9396926,
2 0.5,-0.8137977,-0.2961981, 0.8660254,0.4698463,0.1710101,
*0.,-0.3420201,0.9396926,
3 -1.,0.,0., 0.,-0.9396926,-0.3420201, 0.,-0.3420201,0.9396926,
4 -0.6819984,-0.02552388,-0.7309082,
* 0.7313537,-0.02380140,-0.6815829, 0.,-0.9993908,0.03489950/
CALL DVTRNS(VCP,TM(1,NCP),VSC)
RETURN
END
SUBROUTINE DVTRNS (VP,T,V)
C VP = TV+VO
REAL*8 VP(1),T(1),V(1)
VP(1) = T(1)*V(1)+T(4)*V(2)+T(7)*V(3)
VP(2) = T(2)*V(1)+T(5)*V(2)+T(8)*V(3)

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FXR01110
FXR01120
FXR01130
FXR01140
FXR01150
FXR01160
FXR01170
FXR01180
FXR01190
FXR01200
FXR01210
FXR01220
FXR01230
FXR01240
FXR01250
FXR01260
FXR01270
FXR01280
FXR01290
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FXR01540
FXR01550
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FXR01570
FXR01580
FXR01590
FXR01600
FXR01610
FXR01620
FXR01630
FXR01640
FXR01650

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```
VP(3) = T(3)*V(1)+T(6)*V(2)+T(9)*V(3)  
RETURN  
END
```

```
FXR01660  
FXR01670  
FXR01680
```



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