DYNAMICAL PROPERTIES OF THE MAGNETOSPHERE

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I want to start with a general review of the whole geomagnetic field and mention problems some of which I think have been pretty well solved, in the sense of being pretty well understood, and some of which are very definitely not understood, even though there are some people that think they are understood. If there is more than one person who feels that a problem is understood and these persons do not agree with each other, I usually assume that the problem is not understood. I have to admit that I fall into this category myself at times, where I think something is understood and someone else does not. (See PARKER and FERRARO (1968) for a general review of the magnetosphere.)

The earth and certain aspects of the geophysical environment are represented in Figure 1. The earth's radius is designated algebraically by $R_E (\approx 6400$ km) and is a handy unit when talking about the geomagnetic field. The solid earth is a conducting body and this means that the magnetic lines of force are *frozen in*, as will be discussed further below. The central half of the earth, i.e., to $\frac{1}{2} R_E$, is a liquid core with perhaps another solid core at the center. The liquid core consists apparently of molten iron and

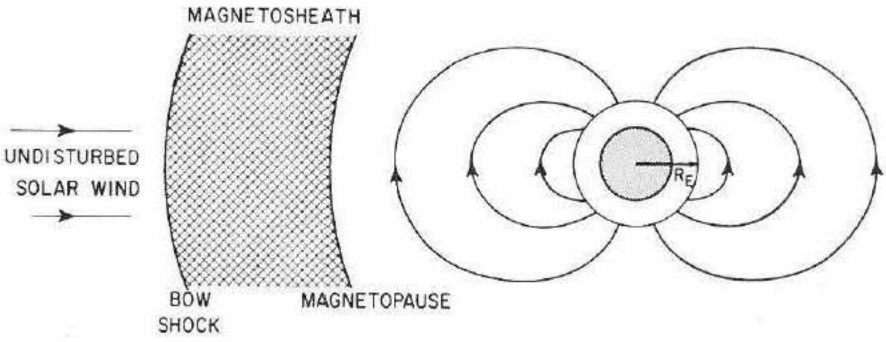

Fig. 1. Schematic representation of the geomagnetic field in meridian profile. R_E is the radius of the earth. The core of the earth is shown shaded. Typical field lines emanate from the surface of the earth and are contained within the magnetopause. Beyond the bow shock is the undisturbed solar wind, and the magnetosheath is the domain between the bow shock and the magnetopause.

nickel and is in a convective state. The dipole field of the earth is usually attributed to the motions in the core. At the surface of the earth the magnetic lines of force come in at the North and go out at the South with roughly the shape indicated in Figure 1. The geomagnetic field is very approximately a dipole field near the surface of the earth and is certainly generated inside the earth, apparently from liquid motions in the core. These motions are of the order of a few millimeters per second, and their effects can be seen at the surface of the earth in the general, slow churning of irregularities in the magnetic field. In fact this is one way of inferring that the core is liquid; the other way is that the core fails to transmit shear waves during seismic activity.

There are several schemes that have been described which might possibly account for the generation of the dipole field by liquid motions in the core. I pointed out in 1955 that the cyclonic motions, the rising and falling of columns of materials with the usual cyclonic motion as occurs in the atmosphere, will produce a dipole field. A little later Backus produced a somewhat more idealized model that could be treated rigorously mathematically and was able to push it through more completely. Then Rosenfeld introduced what is called a steady-state dynamo (which does not involve transient cyclonic motions) and showed that there is a configuration of conducting spheres which gives a dynamo that generates predominately a dipole field. There are also higher modes generated but they are lost in the distance between the core and the surface. Work on the theory of the origin of the dipole field of the earth has not been a very active one because, discouragingly, there is very little that one can do to prove anything. The mathematical calculations are too complex to push through adequately to ascertain whether there is a theory. Even the Mohole was not going deep enough to do any experiments on the subject. The field has been relatively inactive for quite a number of years now, with only a few new ideas kicking around, and will not be dealt with here.

The dipole field of the earth is not entirely static. The dipole axis is approximately 11° from the geographic axis and fluctuates in time. In fact, the polarity of the geomagnetic field reverses apparently almost catastrophically every million or so years. It is laughable that people should suggest that field reversal has the slightest effect upon evolution, but it does make good newspaper copy. The relaxation time of the field is of the order of 50000 years. That is to say, if the dynamo action down in the core were shut off, the field would decay with a characteristic time of the general order of 50000 years, depending on the conductivity assigned to the interior of the earth. Letting σ denote the conductivity of the core in *esu*, the relaxation time is

$$\tau_D = 4\pi\sigma L^2/c^2 , \tag{1}$$

where L is the spatial scale of the sources of the field and c is the speed of light. As a general rule of thumb, for any deformations of the field that take place over a few years the lines of force may be considered to be locked into the surface of the earth. This is the usual case – the lines of force are *frozen in* – for small periods of time. Sometimes in an idealized calculation one dispenses with the solid earth completely and replaces it with a simple point dipole. This is perfectly legitimate so long as the effect interested in does not follow directly from the *frozen in* character of the lines of force at the surface of the earth.

To complete this introduction some brief comments will be made about the following:

(i) the atmosphere,
(ii) the ionosphere,
(iii) the magnetosphere and magnetopause,
(iv) the magnetosheath,
(v) the bow shock, and
(vi) the solar wind.

Surrounding the conducting, solid earth is the non-conducting atmosphere, and this is a very important layer from the point of view of geomagnetic activity. Of course the atmosphere is important to us because we breathe it, but it is important to geomagnetic activity because its conductivity is essentially zero. This permits convection of the geomagnetic field, on which more will be said later. The atmosphere is very dense compared to the next layer above it, the ionosphere. The atmosphere is roughly 60–80 km thick, depending on its precise definition, and the ionosphere extends upwards to about 1000 km.

The principal characteristic of the ionosphere is its large conductivity. It is a gas sufficiently dense to be collision dominated and this allows electric currents to flow along lines of force and across lines of force.

Outside the ionosphere there is what is sometimes called the *magnetosphere*. It is the region between the ionosphere and the *magnetopause*. The magnetopause is very crudely represented in Figure 1. The outer boundary of the magnetosphere is the magnetopause. I use these terms even though I do not understand them: The magnetosphere is not a sphere and the magnetopause does not have feet. The magnetosphere contains the magnetic dipole field of the earth but the total field is compressed into a form that departs rather markedly from a dipole field far out. The dipole symmetry is completely distorted at large radial distances. The magnetosphere is filled with both a neutral gas, mainly hydrogen, and an ionized gas. The collision rate is very low so that for most but not all purposes the region is collisionless. Each particle, even a thermal particle, moves essentially independently of its neighbors. In fact, this is the way that I am defining the boundary between the ionosphere and the magnetosphere: namely, at a high enough altitude that the collision rate between atoms, ions, and electrons is relatively low. Strangely enough, the density of the neutral hydrogen and the ionized gas is roughly comparable. I do not think there is any fundamental reason why this is so. It is more or less just a coincidence; given the temperature of the sun, and other things, it just happens to come out that way. Because the ionized gas is collisionless it has a high electrical conductivity along the magnetic field and a low electrical conductivity across the field. Electric currents tend to flow principally along the field, but we must be careful. As will be elaborated upon later, a small transverse conductivity does not mean that currents cannot flow across the field. It merely means that you cannot make them flow across the field by introducing an electric field across the magnetic field. But there are other ways to make currents flow besides introducing electric fields.

The magnetopause is the boundary between the magnetosphere and the wind. A rough characteristic thickness is 100 km; this is extremely small and in many cases it is sufficient to treat the magnetopause as infinitely sharp. There are some exceptions to this which I shall come back to later.

The *magnetosheath* is gas in the solar wind which is in a turbulent or disordered state as a consequence of having passed through a shock transition. There is some speculation as to the degree and the nature of the disorder which stems from both a lack of observation and an inability to write down a complete shock theory.

The magnetosheath extends to the bow shock. This is a collisionless shock, i.e., the particles do not collide significantly because their mean free path is long. The shock is not produced, therefore, by ordinary collisions – in the classical, laboratory way – but is produced by electrostatic and magnetic forces. There are several ideas as to how the collisionless shock works, and my own view is that, in this one case at least, the collisionless shock seems to be sufficiently complicated that probably more than one of these ideas actually contributes to the real thing. If you vary the parameters for the shock, you find a great variety of the effects coming in.

Upstream from the bow shock is the undisturbed wind, the solar wind, which is completely ionized and mainly hydrogen. There is always helium present, probably 10 or 15%, but this adds nothing essential and complicates the arithmetic. The wind is mainly ionized hydrogen, but sometimes the helium in it makes a difference. Typical quiet-day velocities are observed to be (350 ± 100) km/sec. A typical quiet-day density is $5/\mathrm{cm}^3$ at the orbit of the earth, but this value can also be anywhere from 1 to 10 per cubic centimeter. The solar wind has a magnetic field in it which consists of lines of force pulled out from the sun. The mean value of the field magnitude is about 7γ (10^{-5} gauss $= 1\gamma$) and the average direction is about 45° with the radial direction. The temperature of this gas as it streams past the earth is typically 5×10^4 °K, but depending upon conditions it can vary anywhere from 10^4 to 10^6 °K. The temperature tends to be highest when the sun is active and lowest when the sun is relatively quiet. During active times, the wind velocity, the density, and the degree of turbulence in the wind tend to increase. But most of the complete observations are from sunspot minimum and not enough is known yet to say what conditions prevail when there is a really big blast on the sun, as occasionally occurs during sunspot maximum. Judging from the degree to which the wind compresses the field of the earth, the evidence is that when there is a large blast from the sun the velocity goes as high as 1000–2000 km/sec and a little later the density perhaps as high as $50/\mathrm{cm}^3$; this is only a very rough estimate. There are some pseudo-observational estimates which purport to give much higher densities – numbers like $(1\text{–}3) \times 10^5/\mathrm{cm}^3$ – but I think there is no justification for them. One simply does not find so much pressure being exerted on the geomagnetic field. It is a very interesting and very important question, and we all can look forward in the next 5 or 10 years to seeing some really reliable numbers. There was some evidence from Pioneer 5 that at the time of a big blast the geomagnetic field may be compressed by as much as $(30\text{–}40)\gamma$. This is a perfectly believable number, but of course one would like to check this with more information when the sun becomes more active again.

Much of the subject of solar-terrestrial relations, and certainly the whole subject of geomagnetic activity, deals solely with the interaction of the interplanetary gas, namely, the solar wind, blowing against the geomagnetic field, and I think one should understand that this is a problem much like the weather. It is extremely complicated and you have to ask yourself how far you really want to go into this subject because there is endless detail. At present, one is more or less scouting around trying to understand just the more basic and obvious effects; and the obvious is not always the basic. The aurora which is particularly obvious and spectacular is certainly not one of the basic effects. Indeed, it is not an easy effect to understand, and I think only in the last few years have there begun to be ideas which make sense so far as the aurora is concerned. A great variety of geomagnetic activity goes on. In the simplest case, the solar wind would be perfectly steady, and a steady wind blowing over a steady field would produce no time variations. It turns out that the wind varies over days and it has turbulence in it which varies over seconds. It also turns out that in the field there are some strange things going on, not entirely understood, which cause fluctuations and agitations. From these we derive a lot of effects. Externally there is cosmic-ray modulation, a subject not to be discussed here. There is also the aurora and an enormous range of magnetic activity. The latter includes micropulsations, short-period fluctuations, and very large things as the world-wide magnetic storm. And then there are the Van Allen radiation belts which have at least two different sources. The low-energy particles seem to come mainly from the same source as the electrons. Not all of the trapped particles are frustrated auroral particles, but certainly the lower-energy particles seem to be. In addition, there is a great host of ionospheric effects, many of which will be discussed elsewhere in this monograph. We are in the process of trying to understand, at least in a crude way, where each of these effects comes from. And one has to continually ask how far to push a question. My own taste is to push it to where the gross, and hopefully the novel, features are understood, and then if one wants to go farther, presumably he has some specific reason.

1. The Equations for Plasmas and Fields

A. THE HYDRODYNAMIC EQUATIONS

The fundamental equations for plasmas and fields provide the basis from which the physics of the magnetosphere may be understood. This is a subject that has been horribly mutilated in the past and is even mutilated to this day. The problem is how to treat the dynamical behavior of gases and fields. It is a dynamical problem since fast particles, wiggling fields, etc., are involved. How does one describe the motions of the gas? If the gas is collision dominated, and it is in the atmosphere and in the ionosphere, then its motion can be described by the hydrodynamic equation

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}, \tag{2}$$

where ρ is the mass density and $\mathbf{v}$ the velocity of the gas, p is the pressure, and the last term is the force exerted on the material due to the magnetic field $\mathbf{B}$. If the gas is conducting (which excludes the atmosphere) then the magnetic field is *frozen* into the gas and is carried along at the velocity $\mathbf{v}$:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{3}$$

The term *magnetic lines of force* should be made clear at this point. There is a superstition in some fields of physics, I think not in space science, but in other fields of physics, that the magnetic lines of force are only abstractions, and of course this is correct. But the superstition goes on to suggest that therefore we should not talk about lines of force. I think the best reply to this is that wave functions are also an abstraction and therefore I choose to ignore the speaker who is himself only a wave function. To be very clear that we understand what we are talking about now, the magnetic lines of force are the solutions of the equations

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}. \tag{4}$$

The fields may be functions of time, and generally they are. The solutions of (4) comprise a two parameter family that fills space, and at any instant of time it is perfectly obvious what is meant by a line of force. At each point P in space there is one and only one solution to (4), and this is the line of force through point P. This line of force goes wherever taken by Equation (4). Now the question is: what happens if the field is time varying and we wish to identify a line of force a little later in time? Formally there is no restriction on how to define this. It is simply natural to say that at some initial time two points P and P' connected by a solution of Equation (4) lie on the same line of force. Equation (3) points out that if points P and P' are moved with the material then at all subsequent times the two points remain on a solution of Equation (4), and for convenience this line of force is identified with the original one. In this way, the solutions of (4) are thought of as moving with the fluid. This is a perfectly arbitrary definition, but it is extremely convenient.

B. THE COLLISIONLESS PLASMA

If the gas is collision dominated then Equations (2) and (3) apply without hesitation. In the magnetosphere, however, the gas is not collision dominated and in fact is essentially collisionless. Very fortunately, for most purposes, where only the gross motion of the collisionless material is of interest, there are no significant modifications of the hydrodynamic equations. It is true that some effects will be missed but almost always the right answer is obtained by just writing down the simple hydrodynamic equations and not worrying about it. The justification to the statement that collisionless gases ordinarily obey the hydrodynamic equations presented here will be deliberately heuristic. To begin, the possibility must be admitted for a collisionless gas that the pressure tensor is not isotropic and the hydrodynamic Equation (5) becomes, in component notation,

$$\rho \frac{dv_i}{dt} = -\frac{\partial p_{ij}}{\partial x_i} + \frac{\partial M_{ij}}{\partial x_j}, \tag{5}$$

where summation ($j = 1, 2, 3$) is implied over the repeated index. Of course the existence of pressure does not in any way depend upon collisions since it is merely defined from the mean square particle velocity and the number density. For example, in the absence of collisions,

$$p_{xx} = NM\langle v_x^2 \rangle,$$

where the angular brackets imply an ensemble average over the available particles. The quantity M_{ij} in (5) is the Maxwell stress tensor due to the magnetic field. It provides an isotropic pressure of $B^2/8\pi$ and an additional tension $B^2/4\pi$ in the direction of the field. This is equivalent to saying there is a pressure of $B^2/8\pi$ in the two directions perpendicular to the lines of force and a net tension of $B^2/8\pi$ along the lines of force. Explicitly,

$$M_{ij} = -\delta_{ij}\frac{B^2}{8\pi} + \frac{B_i B_j}{4\pi}, \tag{6}$$

the first term giving the isotropic pressure and the second the tension along the field line. Gravitational terms may also be included in (5).

The equation of motion will now be constructed for a collisionless gas. In a collisionless gas where the magnetic field is steady or slowly varying, the electric field if non-zero is very weak along the lines of force:

$$\mathbf{E} \cdot \mathbf{B} = 0. \tag{7}$$

In general the potential differences along the lines of force will not exceed the thermal energy of the lowest-energy plasma present, which is never more than a couple of volts. If there is an electric field present, all the particles drift with the electric drift velocity $\mathbf{v}$;

$$\mathbf{v} = c\,\frac{\mathbf{E} \times \mathbf{B}}{B^2}. \tag{8}$$

From (7) and (8), the electric field is

$$\mathbf{E} = -(\mathbf{v}/c) \times \mathbf{B} \tag{9}$$

and this is simply another way of saying that the electric field in the frame of reference moving with the particles is zero, neglecting relativistic terms. The particles always set themselves in motion such that this is the case, no matter what external field is artifically applied. The point is that in a collisionless plasma electric fields and plasma velocities are one and the same; they are exactly equivalent. If a motion is made by pushing on the gas, then there is automatically an electric field. If an electric field is artifically introduced, as can be done in a laboratory, then motion in the gas results in such a way that there is no electric field in the particle frame of reference. This shows at once the futility in attempting to appeal to electric fields to accelerate particles unless extremely special conditions prevail. Switching on an electric field only sets the particles into a relatively slow motion so that they do not see any electric field. There is the old statement that nature abhors a vacuum, and one might also say that collisionless plasmas abhor electric fields. They always move in such a way that they are able to avoid them.

Using the Maxwell equation

$$\frac{\partial \mathbf{B}}{\partial t} = - c\nabla \times \mathbf{E}, \tag{10}$$

with $\mathbf{E}$ given by (9), it follows that Equation (3) is reproduced, and this says that the magnetic lines of force move with the velocity $\mathbf{v}$. Actually the velocity $\mathbf{v}$ is the principal motion of the gas, but gradient drifts have been left out. This is satisfactory providing

$$(R/L) \ll 1, \tag{11}$$

where R is the radius of gyration or cyclotron radius of the particle and L is the scale of the magnetic field. There are some additional motions to the gas besides that of the electric drift, but these are very small if (11) is satisfied and to a first approximation can be ignored.

With this discussion, the equations obtained are similar in form with the original hydrodynamic equations, with only small differences because of the tensor forces. In a kinetic-theory approach we would write down either a Boltzmann or a Vlasov equation, and (5) would be equivalent to the first velocity moment of it. The second velocity moment would determine p_{ij}, etc. But as is well known, this is not a particularly fruitful approach. Every case becomes a special one and it is difficult to proceed.

The next task will be to write (5) in a more useful form employing mostly heuristic arguments. In a magnetic field the principal function of a particle is to circle about the field, although there may also be a velocity component along the field. These two components of the thermal velocity will be denoted $w_\perp$ and $w_\parallel$ and provide definitions for the pressure perpendicular and parallel to the lines of force:

$$p_\perp = \tfrac{1}{2} NMw_\perp^2 \tag{12}$$

$$p_\parallel = NMw_\parallel^2. \tag{13}$$

Of course $w_\parallel$ and $w_\perp$ need not be equal. The factor of $\frac{1}{2}$ occurs in (12) because $w_\perp^2$ is the mean squared velocity over two dimensions perpendicular to the field, whereas $w_\parallel^2$ is the mean squared velocity in only one dimension. One might be tempted to break the equations of motion (5) into perpendicular and parallel components by writing

$$\rho \frac{\mathrm{d}v_\perp}{\mathrm{d}t} = - \nabla_\perp \left(p_\perp + \frac{B^2}{8\pi} \right) + \left[\frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B} \right]_\perp \tag{14}$$

$$\rho \frac{\mathrm{d}v_\parallel}{\mathrm{d}t} = - \nabla_\parallel p_\parallel. \tag{15}$$

No magnetic term appears in (15) because (curl $\mathbf{B}$) $\times$ $\mathbf{B}$ has no component parallel to the magnetic field. As they stand, Equations (14) and (15) are incomplete. Additional effects enter from the anisotropy of the pressure and the non-uniformity of the magnetic field. But the extra terms are relatively simple and can be calculated as follows.

Consider the easiest case first, for motions with perpendicular accelerations. If the pressure is isotropic, there is no problem and Equation (14) is complete. But if the pressure is anisotropic and in particular if $p_\parallel > p_\perp$, so that the particles are mainly whizzing back and forth along the lines of force, then there is a centrifugal force given by $NMw_\parallel^2 K = p_\parallel K$. K is the curvature of the line of force and is determined by the rate of change of the perpendicular component of the field in the direction of the field:

$$K = \frac{1}{B^2} [(\mathbf{B} \cdot \nabla) \mathbf{B}]_\perp.$$

The net centrifugal force tending to accelerate the flow across the lines of force depends upon the anisotropic part of the pressure, i.e., upon the difference of $(p_\parallel - p_\perp)$ from unity, and (14) becomes

$$\rho \frac{\mathrm{d}v_\perp}{\mathrm{d}t} = - \nabla_\perp \left(p_\perp + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} [(\mathbf{B} \cdot \nabla) B]_\perp \left(1 - \frac{p_\parallel - p_\perp}{B^2/4\pi} \right). \tag{16}$$

The correction term in (16) is due to the centrifugal force. If the anisotropy is so large that

$$p_\parallel - p_\perp > \frac{B^2}{4\pi}$$

is satisfied, then the sign of the last term in (16) changes so that what was a tension becomes a compression. The field lines react like wet spaghetti under compression and buckle; the dominant centrifugal force hauls off the field.

In order to obtain the modifications in (15), a brief preliminary discussion is necessary. Consider a non-uniform magnetic field, either static or varying extremely slowly in time. The magnetic moment invariant of the individual particles moving in this field may be expressed as

$$\frac{\sin^2 \theta}{B} \cong \text{constant}, \tag{17}$$

where θ is the pitch angle. Also, if the field is constant in time, so is the particle speed. To a first approximation a particle moves along a line of force; let s measure the position along the line. The density function ψ for particles with pitch angle θ within $\mathrm{d}\theta$ at time t is defined by

$$\text{particles/cm}^3\text{-steradian} = \psi(s, \theta, t) \tfrac{1}{2} \sin\theta \mathrm{d}\theta.$$

Monoenergetic particles will be treated and in the end a sum over energy will be performed. To understand how the particle density varies along the line of force remember that Liouville said that in a conservative system – presumably he did not mean politically conservative – the particle density in phase space – both coordinate space and velocity space – is preserved along the particle trajectory. That is to say, the total derivative of the density function ψ is zero in moving with the particle along the line of force:

$$\frac{d\psi}{dt} = 0 = \frac{\partial\psi}{\partial t} + \frac{\partial\psi}{\partial s}\frac{ds}{dt} + \frac{\partial\psi}{\partial\theta}\frac{d\theta}{dt}. \tag{18}$$

Now $ds/dt = w\cos\theta$ is just the particle velocity along the field, and from (17) it follows that

$$\frac{d\theta}{ds} = \frac{\tan\theta}{2B}\frac{dB}{ds} \tag{19}$$

and (18) becomes

$$0 = \frac{\partial\psi}{\partial t} + w\cos\theta\frac{\partial\psi}{\partial s} + \frac{w\sin\theta}{2B}\frac{dB}{ds}\frac{\partial\psi}{\partial\theta}. \tag{20}$$

Suppose now that the particle distribution has splashed around sufficiently for time variations to be unimportant. It then follows from (20) by replacing s by B as the independent variable that the general solution for the time-independent density is of the form

$$\psi(s, \theta) = f\left(\frac{\sin^2\theta}{B}\right), \tag{21}$$

where f is an arbitrary function.

A convenient form is to use

$$\psi = A\left(\frac{\sin^2\theta}{B}\right)^\alpha, \tag{22}$$

where different values of the constants A and α correspond to different physical situations. For example, with an isotropic distribution of particles the density must be independent of θ and $\alpha = 0$ is required by (22). But then the density is also independent of the field and so with an isotropic distribution the density of particles is constant everywhere along the field line. Physically one might expect that if the field is converging then some particles are always mirroring and turning back and therefore the density should be declining. As the field converges, however, the particles are being compressed together, and the two effects exactly compensate for each other to keep the density constant. A density distribution with $\alpha < 0$ is peaked toward small pitch angles and here the density increases with field strength. Lots of other cases can be constructed.

In considering now the equation of motion in the direction parallel to the field more care must be taken because particle density gradients along the field may occur even in a static solution. It is necessary to subtract off the portion of the density gradient due to pitch angle effects since only a pressure gradient in excess of this static pressure gradient accelerates the particles. Now

$$p_\parallel = \tfrac{1}{2}\int_0^\pi d\theta \sin\theta\psi\, Mw^2\cos^2\theta$$

and using (19)

$$\frac{\partial p_\parallel}{\partial s} = \frac{Mw^2}{4B}\frac{dB}{ds}\int_0^\pi d\theta \sin^2\theta\cos\theta\frac{\partial\psi}{\partial\theta}.$$

This last integral is conveniently transformed by integration-by-parts to the result

$$\frac{\partial p_\parallel}{\partial s} = \frac{1}{B}\frac{dB}{ds}(p_\parallel - p_\perp). \tag{23}$$

Equation (23) shows that the pressure gradient along the line of force occurs whenever the pressure distribution is not isotropic; this is the equilibrium pressure gradient and will not accelerate the gas since it reacts upon, but is balanced by, the field. Only the difference between $\nabla_\parallel p_\parallel$ and the right side of (23) accelerates the gas:

$$\rho\frac{dv_\parallel}{dt} = -\nabla_\parallel p_\parallel + \frac{[(\mathbf{B}\cdot\nabla)B]_\parallel}{4\pi}\frac{p_\parallel - p_\perp}{(B^2/4\pi)}. \tag{24}$$

The derivative (dB/ds) is written as $[(\mathbf{B}\cdot\nabla)\mathbf{B}]_\parallel$ in (24) to be symmetrical in form to (16), and these are the two equations of motion for a collisionless gas. Modifications from the hydrodynamic equations are especially pronounced when the gas anisotropies in pressure are large compared to the magnetic pressure. Fortunately, in the magnetosphere deviations from isotropy usually are not large. Furthermore, the pressure terms are small compared to the magnetic terms so there are only rather small corrections. Without going into any further detail here, it may be said that (16) and (24) are the equations of motion, that these equations very closely resemble the ordinary hydrodynamic equations which actually suffice to a first approximation unless some very special effects are being sought.

Equations (16) and (24) are essentially the Chew-Goldberger-Low equations. There are two more equations to go with them having to do with the compression of the magnetic lines of force and the adiabatic compression of the field. In order to use (16) and (24) a method is needed for computing $p_\parallel$ and $p_\perp$. This is done with the equation of continuity expressing mass conservation:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \tag{25}$$

and the equations of state:

$$\frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = 0, \quad \frac{d}{dt}\left(\frac{B^2 p_\parallel}{\rho^3}\right) = 0. \tag{26}$$

These formulas look formidable but become clear after a little thought. Consider as a special case how to calculate p under a transverse compression. During the compression the magnetic moment, $w_\perp^2/B$, is invariant and so is N/B since the gas is compressed right along with the field. Other constants can very easily be constructed from these, one of which is $p_\perp/N^2$. If the system were subject to longitudinal compression, $w_\perp$

would not be affected at all and the ratio $p_\perp/\rho$ would be a constant; B also would be constant. The first relation in (26) applies to $p_\perp$ in both of these cases, and similar arguments may be used for $p_\parallel$ and the second relation in (26). The Chew-Goldberger-Low approximation is not the only one, it has many gaps in it, but it is all that will be discussed here.

C. HYDROMAGNETIC WAVE PROPAGATION

This subject will be discussed in more detail elsewhere in this monograph. Under most conditions in the geomagnetic field the pressure of the gas is rather less than the pressure of the magnetic field: $p \ll B^2/8\pi$. This condition is not true down low in the atmosphere and is not true probably out at some distance when in the main phase of a magnetic storm. But generally it is a good approximation and pressure terms can be dropped in treating the bulk motions of the gas, and a description is given by

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} \tag{27}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{28}$$

Suppose now that the field becomes $(\mathbf{B}+\mathbf{b})$, where $\mathbf{B}$ is the original dipole field and $\mathbf{b}$ is a small perturbation, so that (27) and (28) may be linearized, i.e., only terms linear in $\mathbf{v}$ and $\mathbf{b}$ are kept. For example, $\mathrm{d}/\mathrm{d}t$ is replaced by $\partial/\partial t$ in (27). Then

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \frac{1}{4\pi}(\nabla \times \mathbf{b}) \times \mathbf{B} \tag{29}$$

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{30}$$

The decoupled equation for the velocity $\mathbf{v}$ is essentially a wave equation second order in space and time derivatives:

$$\frac{\partial^2 \mathbf{v}}{\partial t^2} = \frac{\mathbf{B}}{4\pi\rho} \times \operatorname{curl}\operatorname{curl}(\mathbf{B} \times \mathbf{v}). \tag{31}$$

The characteristic velocity is the Alfvén velocity:

$$v_A^2 = \frac{B^2}{4\pi\rho}.$$

The dispersion relation for a uniform medium is simply

$$\omega^2 - k^2 v_A^2 = 0,$$

where ω is the circular frequency and k is the wave number of the wave. This case is very simple because the sound speed has been taken to be zero and all waves propagate with the Alfvén speed. The displacement must be perpendicular to the magnetic lines of force because the force is perpendicular to them. A good rule of thumb is that in the magnetosphere $v_A \approx 1000$ km/sec. The Alfvén speed reaches a maximum at an altitude of a few thousand kilometers and declines outward from there to a few hundred kilometers per second, but for a nice round number the figure given is a fairly good estimate.

Treating the hydromagnetic waves with the hydrodynamic equations (29) and (30) makes a mistake which for some purposes is not so serious and for other purposes is absolutely fatal. Equation (31) has no damping; there are no dissipation terms in it. Recently BARNES (1966, 1967) has solved the equations for a collisionless plasma treating waves where the wave frequency is small compared to the ion cyclotron frequency (i.e., hydromagnetic waves). If correct, the calculations indicate the important result that these waves are subject to Landau damping, i.e., there is some fraction of the particles where the thermal velocity carries them along with the wave speed and the particles gain energy at the expense of the wave energy. He finds that no matter which way the wave propagates, so long as it is not within a degree of the field or a degree of being perpendicular to the field, it dies by the factor e over short distances, of anywhere from about 2 to 10 wavelengths. The damping is specified in terms of a given number of wavelengths and so applies to all waves unless of course the wavelength is so long that it passes through the magnetosphere before damping occurs. This is a somewhat surprising result. I think it is an extremely important result that should be kept in mind. If Barnes is correct, then hydromagnetic waves in the magnetosphere can be damped very quickly without building up into shocks or any of the other standard means of wave dissipation. Barnes has recalculated his results by an independent method, using some special cases to check the first calculations. The calculations are complicated, and some questions still remain, but we must seriously consider that hydromagnetic waves are subject to heavy Landau damping. Sometimes this is important and sometimes it is unimportant.

D. SOME MAGNETOSPHERIC PROBLEMS WELL UNDERSTOOD

The methods for treating theoretically the magnetopause, the bow shock, and the solar wind, shown in Figure 1, are relatively simple and well understood. The solar wind is a supersonic, hydrodynamic flow which runs into a blunt obstacle. This is an old problem in aerodynamics, and people have studied it both theoretically and in wind tunnels for a long time. The Mach number in the wind is typically anywhere from 3 to 10 so that the problem is quite standard. An upstream shock wave is expected, and people have applied the theory for the formation of these standoff shocks to blunt objects. Spreiter, Kellogg, and others have worked on the problem with varying degrees of improved approximation, and I think it is fair to say that given the shape of the magnetopause as an object in the flow the bow shock configuration is fairly well understood. There is no theory for the details of the shock transition itself since in hydrodynamics the theory of the transition does not enter into the hydrodynamic flow. The shock is merely a discontinuity in the flow.

The shape of the magnetopause on the sunward side and even perhaps a little toward the rear of the earth is also fairly well understood. It is a matter of pressure balance:

$$p_2 = \frac{B^2}{8\pi} + p_1, \tag{32}$$

where p_2 is the normal pressure on the geomagnetic field due to the solar wind and p_1 is the gas pressure inside the magnetosphere (which is usually negligible). A mathematical problem is involved here which is not easy. Equation (32) is a boundary condition, and the problem is to find the boundary. Normally boundary value problems involve a *given* boundary for which a solution is found, but here the boundary is part of the game.

There are three mathematical methods for calculating the shape of the magnetopause, all different, that have been worked out by Beard, Davis, and Slutz. They all give approximately the same results and each has some difficulty with the neutral point. But Blum has worked out an expansion that covers that. It is fair to say that one not only understands the gross physics of the pressure balance (32) between the compressed field and the impact of the wind, but also that methods exist for calculating this boundary which are at least as accurate as the experimental determinations. Of course the experimental determinations are plagued by the fact that only point measurements are being made and the whole boundary is never seen at once. It would be fun to go into this problem but it will have to suffice merely to mention it here. There might be some question about the force law exerted on the magnetopause surface, but this is another story and requires a tale about the tail of the magnetic field. This will be taken up later.

One can study how the field within the magnetosphere is increased by the compression, and in fact this is part of the calculation involved in determining the shape of the magnetosphere. On quiet days when the subsolar distance to the magnetopause is 10 R_E, the compression of the field around the vicinity of the earth involves an increase of the Northward component, i.e., the component parallel to the axis of the earth, of about 15–20γ. This is not a very large effect but it is interesting to see how it follows from the calculations. This compression can be nicely illustrated by taking simpler boundaries than the actual magnetopause. For example, a long time ago Chapman and Ferraro considered the effects of pushing a plane up and found

$$\left(\frac{\Delta B}{B_0}\right)_{\text{plane}} = \frac{1}{8}\left(\frac{R_E}{L}\right)^3.$$

Here ΔB is the change in field produced by the compression, B_0 is the value of the field at the surface of the earth on the equator, and L is the distance to the plane. If the compression is done with a sphere of radius L, by enclosing the field from all sides, then the coefficient is 16 times larger:

$$\left(\frac{\Delta B}{B_0}\right)_{\text{sphere}} = 2\left(\frac{R_E}{L}\right)^3.$$

The actual case lies somewhere in between these results, the coefficient being slightly less than unity.

E. THE MAGNETOSPHERE AS AN ELASTIC MEDIUM

In the discussions above, emphasis has been given to the fact that for all of the seeming complexities of the behavior of particles moving without collisions in a magnetic field, nonetheless in a large fraction of the cases encountered the dynamics can be understood on the basis of stresses in the magnetic field and hydrodynamic motions. To rephrase it very slightly, a medium which consists of a gas and a magnetic field behaves much like an elastic medium. It has the stresses in it due to the pressure of the gas (which is not necessarily isotropic) and due to the magnetic field. The stress variables are:

$$p_\perp, p_\parallel, M_{ij},$$

as defined in (6), (12), and (13). When the magnetic field is in equilibrium under its own stresses and not exerting any stresses on the material, this means that $\partial M_{ij}/\partial x_j = 0$, or equivalently,

$$\mathbf{B} \times \operatorname{curl} \mathbf{B} = 0. \tag{33}$$

Normally it would be sufficient to satisfy (33) by taking curl **B** parallel to **B**. But it must be emphasized that, generally, within the geomagnetic field there is no component of curl **B** parallel to the magnetic field (because of the non-conducting atmosphere) and the required solution of (33) is

$$\operatorname{curl} \mathbf{B} = 0. \tag{34}$$

This means that the magnetic field within the magnetosphere is derivable from a scalar potential:

$$\mathbf{B} = -\nabla\psi. \tag{35}$$

The magnetic field here is a compressed dipole field – compressed by external forces and there are no internal forces. If curl $\mathbf{B} \neq 0$, then a Lorentz force would be exerted on the trapped gases in the field, but except during the main phase of a magnetic storm these gases will not support forces. For there to be no force exerted by the field on the gas, curl **B** must be parallel to **B** and this would imply that there is a torsion in the lines of force, i.e., a twisting of the flux tubes. But being in an elastic medium these flux tubes will unwind – if they can. And they can because there is a non-conducting atmosphere at the foot of every flux tube down at the surface of the earth. The atmosphere does not support torsion – at least not very much – and permits field lines to unwind. So, as a general rule under steady conditions – not under transient conditions – curl **B** must be zero and the field is approximated fairly well by a scalar potential.

Now when one begins to inquire about distorting the field away from its equilibrium confinement inside the magnetopause he is talking about distortions of the magnetic field which involve a non-vanishing curl, and then **B** is no longer expressible in the form (35). The easiest way to think about this situation is again in terms of an elastic medium. If a curl is produced which distorts the field then forces are exerted on the medium and some reaction of the gas must be introduced. The simplest way to express this, if the situation is more or less steady, is from (2):

$$0 = -\nabla p + \frac{1}{4\pi}(\mathbf{V} \times \mathbf{B}) \times \mathbf{B}. \tag{36}$$

The theory for the distortions of the geomagnetic field, both the distortions that might occur during storms inside the magnetopause and for the shape of the magnetopause itself, is a problem in elasticity. This is an important point that is fundamental for understanding what is happening. Prior to about 10 years ago the search for the cause of magnetic storms centered more around electromotive forces and currents, and I think that is one reason why considerable difficulty was encountered. The prime mover is the *force*. The force of course gives rise to e.m.f.'s and currents, but the current is the secondary quantity which more or less is taken care of quite automatically. Nowhere in the hydromagnetic equations does a current occur. Of course, sometimes it is easiest to make a *calculation* using the currents. This is another matter and in fact some examples below employ currents explicitly. But basically the problem is one in elasticity.

As far as I know, the first mention or direct application of hydromagnetic theory to the geomagnetic field was made by Dungey about 1955 when he pointed out, and discussed, the fact that hydromagnetic waves should propagate in the magnetic field of the earth. About 1956, I pointed out that the more or less steady deformations associated with the magnetic storm, in particular with the main phase, are also hydromagnetic deformations. Shortly thereafter Dessler pointed out that some of the rise time of the sudden commencement of a magnetic storm has to do with the propagation of the impulse from the magnetopause where it is first applied inward to an observer at the earth. The fact is that the gross features of magnetospheric deformations generally can be understood on the basis of simple stresses and hydrodynamics.

F. THE GEOMAGNETIC TAIL

As mentioned briefly above, methods of calculating the pressure balance and determining the shape of the magnetopause have been worked out and seem to apply fairly well. Calculations for the position of the bow shock also seem to check fairly well with the observations, as nearly as anyone can tell. Continuing to take the hydrodynamic picture quite literally, as Johnson did several years ago, one would conclude that the magnetopause would close off behind the earth with something like the Mach angle. The geomagnetic field becomes so weak that it exerts little pressure and the solar wind closes in as rapidly as its thermal motions allow, and this would be at an angle of 10–20° depending on the Mach number of the wind. It seems to be a fact, however, that this does not happen. Instead, the observations indicate that the earth has a very long and broad tail. There seems to be some question as to exactly how wide it is at large distances, but it goes back an extremely long distance having been observed at about 80 R_E by the unanchored anchored IMP and perhaps at 1000 R_E in a more recent measurement. The point is the tail is an extremely long structure, and I want to emphasize that on the basis of simple hydrodynamic theory there should not be a long

tail; i.e., if the wind slid smoothly along the field surface without agitating it the tail would quickly close. A schematic representation of the observed tail is given in Figure 2. Here there is also a region of relatively weak field shown, sometimes called a *neutral sheet* although it is not strictly neutral, across which the field changes direction; this aspect of the tail needs to be explored much further.

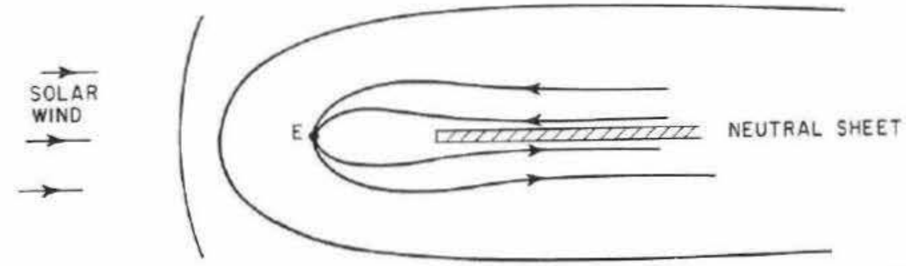

Fig. 2. Schematic representation emphasizing the nature of the geomagnetic tail. The neutral sheet is indicated along with certain field lines in the tail in relationship to the earth, *E*.

By some means there is a tension being exerted on the tail to hold it out since here the lines of force extend radially and there is a tension $B^2/4\pi$ along the lines of force from which the component of the isotropic pressure in that direction is subtracted. There are a number of possibilities as to how the tail might be stretched out because an active force is required. One suggestion is that in the tail there is a pressure p_1, a gas pressure or perhaps a turbulent pressure of some kind, which of course tends to push out and just equals the inward tension on the tail:

$$p_1 = \frac{B^2}{8\pi}.$$

In addition, in order for there to be some kind of transverse equilibrium, the total pressure outside the tail p_2, must balance the internal pressure:

$$p_2 = p_1 + \frac{B^2}{8\pi} = \frac{B^2}{4\pi}.$$

Using a typical value of 15 γ for the tail field

$$p_2 \approx 1.8 \times 10^{-9}\,\text{dynes/cm}^2.$$

This is a fairly high value implying that the total tension to hold out the tail requires a pressure which is about $\frac{1}{10}$ of the solar wind impact pressure on the front side of the magnetopause. There is nothing implausible with this requirement but I suspect that there are other contributing factors. For instance, it has also been suggested that there is some enhanced mixing of the wind plasma as it rushes by the tail, either due to surface instabilities of the tail, as discussed by Dungey, or any number of things, and the wind itself may eventually penetrate into the tail at large distances. The inertia of the wind acts directly then as a tension on the lines of force. This, too, is plausible since only about $\frac{1}{10}$ of the total wind impact momentum is required to provide the necessary tension.

Even though there are a lot of ideas on the extension of the tail, I think that it is very definitely an unanswered question. There may very well be a number of effects all of which contribute in varying proportions depending upon conditions in the wind. But it should be kept in mind that there must be a rather large tension pulling on the tail; otherwise it would not be there. Also, at least in some small scale sense, the tail is a somewhat non-hydrodynamic phenomenon and involves surface effects or internal heating to achieve high gas pressures. Next I will discuss a somewhat related problem because it too involves tensions.

2. Convection

The convection of field lines depends on the fact that the atmosphere is non-conducting. The basic idea was pointed out first by Gold and then developed in an explicit way by Axford and Hines. The earth is blanketed by the atmosphere which is very good insulator. Gold pointed out that while magnetic lines of force may be frozen in at the interior of the earth, and while they may be frozen into the gas in the ionosphere and above, they were *not* frozen in in the atmosphere, and as a result two lines of force can be interchanged in a continuous manner. Referring to Figure 3, if the two field line end points P_1 and P_2 are interchanged in the atmosphere then the whole line of force connected to each of these points would turn over along its length and the gas that is tied to each line of force would interchange as well. The lines of force have identities given to them by the material that lies along them, and the exchange of material that takes place is why it is sensible to talk about interchanging lines of force.

Axford and Hines used this idea in considering the polar D_s currents in the ionosphere. The D_s current systems are deduced from the distortions of the geomagnetic field that are observed at high latitudes. These currents flow through the ionosphere and are shown schematically in Figure 4. Since the ionosphere is a resistive medium, there must be an electric field driving these currents. Because of the Hall conductivity the electric field and the current are not strictly parallel, and in fact make a fairly large angle with each other. But this is merely an arithmetic complication that implies that the electric field pattern is not quite the same as Figure 4. Since there are fairly reasonable estimates of the conductivity of the polar ionosphere, potential differences that are involved can be deduced. Typical values range from 10^4 to 10^5 V for the total potential differences involved in the flow of D_s currents. During active times, the potential difference tends to be 10^5 V, and during quiet times it is more likely to be 10^4 V. Now the magnetic lines of force passing through the ionosphere are bathed in a plasma which is an excellent conductor, and a potential difference between any two points in the polar ionosphere must exist everywhere along the two lines of force containing these points. This is because the lines of force themselves are approximately equipotentials. Just as a piece of copper is an equipotential under steady conditions, the plasma lying along any line of force is an equipotential and therefore a potential difference between two points in the ionosphere must be maintained all along the lines of force. This means that there is an electric field between these two lines of force and the plasma there must drift with the velocity

$$\mathbf{v} = c\frac{\mathbf{E} \times \mathbf{B}}{B^2} \tag{37}$$

in order that there be no electric field in the rest frame of the plasma. During active times, this drift velocity gets to be as high as 1 km/sec. So, electric fields can be

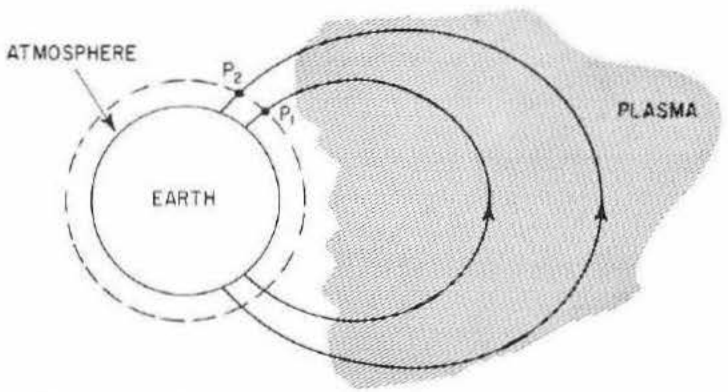

Fig. 3. Schematic representation of the geomagnetic field lines passing through the neutral atmosphere and the plasma which extends upward from the ionosphere. Points P_1 and P_2 lie at the top of the atmosphere.

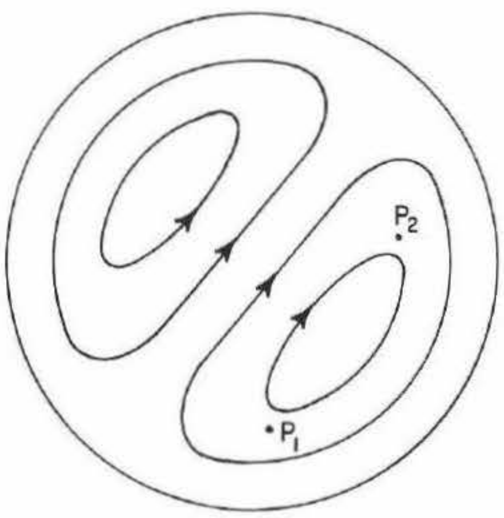

Fig. 4. Schematic representation of the D_s current system. The contour lines show the direction of current flow in the ionosphere required to generate an observed high-latitude magnetic disturbance at the surface of the earth. The points P_1 and P_2 are the same as those in Figure 3.

calculated from the observations of the polar D_s current systems, and on this basis it is concluded from (37) that the lines of force are convecting. The convection is manifested at the ends of the field lines by the D_s current system and is a straightforward deduction from an observation. Having made this first step, the next thing to ask is whether anything interesting follows from it or whether it is just another natural phenomenon with no connection with anything else. A few things can be said without going through an extremely long analysis, and I will begin by investigating just a little further the physics of the slippage down in the non-conducting atmosphere.

A. SLIPPAGE IN THE NON-CONDUCTING ATMOSPHERE

Consider the following problem. Imagine that a stationary, perfectly conducting plane is located at $z=0$ and that a similar plane moving with velocity $v(t)$ is located at $z=h$, as shown in Figure 5. Between these two plates is a non-conducting region, taken to represent the atmosphere, which is occupied by a uniform, vertical magnetic field. The region $z>h$ is intended to represent the ionosphere and the magnetosphere, and $z<0$ represents the solid earth. I will deal with these domains as though they were just two sheets, but it would make no difference to treat them as semi-infinite blocks of perfectly conducting material with frozen in fields. It will be instructive to treat the stationary and transient situations separately.

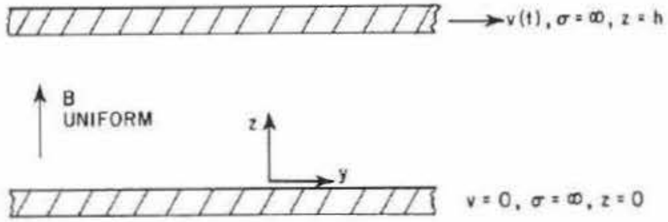

Fig. 5. Simple model of field line convection. Perfectly conducting plane surfaces are located at $z=0$ and $z=h$ and the non-conducting region in between is permeated by a uniform, vertical magnetic field.

Suppose that the velocity v of the top sheet at $z=h$ is constant so that it is sliding along uniformly in the y-direction. In the frame of reference of this sheet, which is a perfect conductor, there is no electric field; in the fixed frame of reference, however, there is an electric field which, according to Lorentz, is given by

$$\mathbf{E} = -\frac{\mathbf{v}}{c} \times \mathbf{B} = -\left(\frac{v}{c} B\right)\hat{\mathbf{x}}. \tag{38}$$

Or defining

$$E_0 = vB/c \tag{39}$$

the electric field at $z>h$ is

$$\mathbf{E} = -E_0 \hat{\mathbf{x}}.$$

Since the lower conducting sheet is stationary, the electric field at $z<0$ must vanish. These values determine the boundary conditions on the electric field. In between the plates, the electric field is derivable from a scalar potential which satisfies Laplace's equation:

$$\mathbf{E} = -\nabla\psi; \quad \nabla^2\psi = 0.$$

These relations express the fact that the field is electrostatic and that there is no free charge confined between the plates. The solution that satisfies the proper boundary conditions at $z=0$, h is:

$$\psi = E_0 \frac{xz}{h}; \quad E_x = -E_0 \frac{z}{h}; \quad E_z = -E_0 \frac{x}{h}. \tag{40}$$

The electric field lines are contained in the x-z plane and are sketched in Figure 6, where the z-axis has been chosen to lie along the center of the field. The electric field lines become more nearly vertical in going away from the z-axis so that if h is chosen to be the height of the ionosphere the field is usually vertical at $x \simeq 300$ km. The potential differences that are involved are of the order of 10^4–10^5 V as deduced directly from the D_s current systems or from the velocity that follows from it, and the electric field between the ionosphere and the earth results when the geomagnetic field is in a state of convection. The faster it convects the higher is the potential difference; the two go in direct proportion to each other.

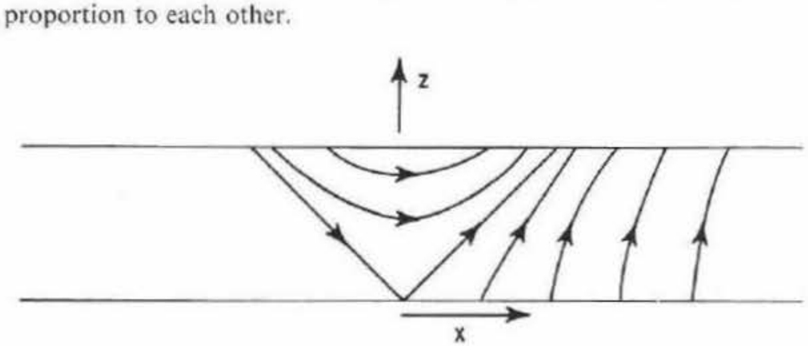

Fig. 6. Steady-state electric field lines in the neutral atmosphere.

In case someone should be tempted, the electric field (40) will not account for the aurora even though the potential differences are just the right order of magnitude. This electric field exists only so long as the medium refuses to conduct electricity; it is a high impedance e.m.f. in that sense. If even a very small amount of conductivity should be introduced into the atmosphere, the convection would simply stop because the lines of force would suddenly be nailed into the fluid. So it is not an electric field to which one can appeal for the production of the aurora. It exists only in the non-conducting part of the atmosphere, well below where much of the auroral activity is observed. Having steered around that bit of temptation, we should go on now to briefly treat the time-dependent problem.

Suppose that the velocity of the conducting sheet at $z=h$ is changing in time, as when it goes from zero up to some final steady-state value. Both v and E_0 in Equation (39) would now be time-dependent, and the magnetic field in the non-conducting gap would be affected. Using the Lorentz gauge,

$$\nabla\cdot\mathbf{A}+\frac{1}{c}\frac{\partial\psi}{\partial t}=0, \tag{41}$$

the equations in the gap are

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}=0 \tag{42}$$

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi=0. \tag{43}$$

Here $\mathbf{A}$ is the vector potential and ψ is the scalar potential. Equations (42) and (43) are homogeneous because there are no currents and no free charge in the non-conducting atmosphere. In any circumstance relevant to this discussion, the velocity of convection $v(t)$ changes rather slowly in times comparable to the light propagation time, and this means that all time derivatives in (41)–(43) will be small. Time derivatives will therefore be neglected (we could come back and calculate them as a kind of perturbation, but they turn out to be completely uninteresting) and both the vector and scalar potential are to satisfy Laplace's equation. As before, we take

$$\psi=E_0(t)\frac{xz}{h} \tag{44}$$

and require from (41) that

$$\frac{\partial A_x}{\partial x}+\frac{\partial A_z}{\partial z}=-\frac{1}{c}\frac{\mathrm{d}E_0}{\mathrm{d}t}\frac{xz}{h}. \tag{45}$$

With (44) it is suggestive that $E_y=0$ and therefore that $A_y=0$; this condition is imposed in (45). Both A_x and A_z in (45) must satisfy Laplace's equation and after a little thought it becomes apparent that the solutions must be of third order in the coordinates:

$$A_x=a_1(xz^2-\tfrac{1}{3}x^3)+a_2(zx^2-\tfrac{1}{3}z^3)$$

$$A_z=b_1(xz^2-\tfrac{1}{3}x^3)+b_2(zx^2-\tfrac{1}{3}z^3).$$

Here a_1, a_2, b_1, and b_2 are independent of the coordinates. The boundary requirements $E_x=0$ at $z=0$, and $E_x=-E_0(t)$ at $z=h$, are satisfied by taking $A_x=0$. The gauge requirement (45) then requires $b_2=0$ and

$$b_1=-\frac{1}{2ch}\frac{\mathrm{d}E_0}{\mathrm{d}t}.$$

Thus,

$$\mathbf{A}=\hat{\mathbf{z}}\left(\frac{-1}{2ch}\frac{\mathrm{d}E_0}{\mathrm{d}t}\right)(xz^2-\tfrac{1}{3}x^3), \tag{46}$$

which gives rise to a magnetic field along $\hat{\mathbf{y}}$:

$$B_y=-\frac{\partial A_z}{\partial x}=\frac{1}{2ch}\frac{\mathrm{d}E_0}{\mathrm{d}t}(z^2-x^2). \tag{47}$$

The electric field is essentially the same as before (Equation (40)) since the solution (46) contributes a term which is second order in the time derivative and hence small. This term must be small for consistency since second-order time derivatives have been systematically neglected. The small contributions in the electric and magnetic fields from time derivatives of $E_0(t)$ are present only while there is acceleration, i.e., while the rate of convection is changing. These transient contributions vanish upon reaching a steady state and are of relatively little interest. The magnetic field remains essentially undistorted by the convection and the electric field is essentially as shown in Figure 6. This is what takes place beneath the convecting magnetosphere. The electric fields obtained are interesting but, as I say, can not produce auroras.

B. CONVECTION CONTRIBUTING TO AURORAS

Axford and Hines have a rather extended discussion of some of the consequences of this convection, which is extremely interesting. I will mention briefly one point which is more interesting and impressive to me than the others. It was pointed out by Axford and Hines that the convection pattern, as well as the actual magnitude of its speed, can be deduced from the polar D_s current system. Very roughly, if you are looking downward at the earth from the North, the solar wind is streaming by and the convection pattern resembles that shown in Figure 7. The whole system shown in Figure 7 revolves around once a day because of the daily rotation of the earth and this tends to skew the convection pattern somewhat. The extremely interesting point that Axford and Hines make is to suppose that an electron (a thermal electron of 1 or 10eV, something of that order) is folded into the field somewhere back in the tail near point A in Figure 7. After all, there are lots of thermal particles zooming by and there is no

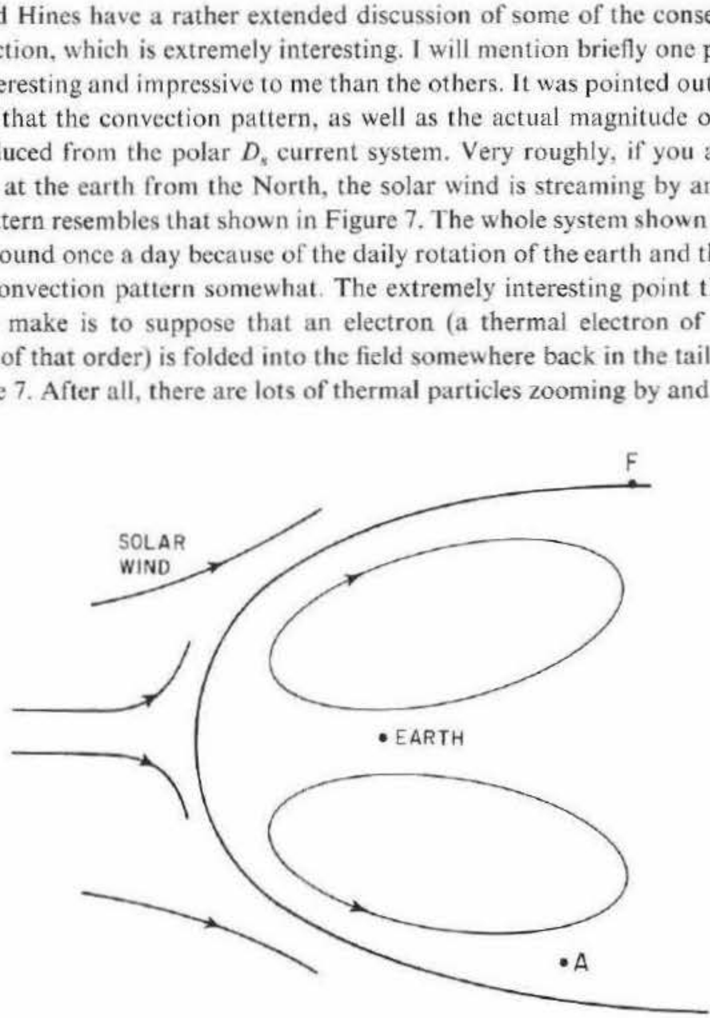

Fig. 7. Schematic view of the convection pattern traced by field lines.

reason why one should not be folded into the field. This electron rides with the convection starting at a region out in the tail where the field is 1.5×10^{-4} gauss and is carried close in toward the earth where the field strength is about $\frac{2}{3}$ gauss, or about 4000 times larger than at point A. Axford and Hines made some rough estimates based on the magnetic moment invariant and the longitudinal invariant and found that these convecting particles should come awfully close to the polar regions, and perhaps be precipitated there. Since the magnetic moment invariant requires that the perpendicular kinetic energy be proportional to the field magnitude, the kinetic energy of the particle starting from point A will increase by a factor of the order of a few thousand. So starting with, say, 2 eV in the tail the particle energy becomes something like 8 keV near the earth. Of course the solar wind has a lot hotter particles than 2 eV, and you might just as well start with 10 eV in the tail and end up with 40000 V. This process is very simple and straightforward, and probably unavoidable, so long as you have convection. Axford and Hines mention that these particles should come close to the earth at high latitudes because it is mainly the high-latitude lines of force that are convecting. The region down at low latitudes seems not to be convecting, judging from what current is seen in the ionosphere. Convection is mainly a high-latitude phenomenon, extending 40–45° from the poles depending upon the degree of activity. The convection region expands when things are very active.

Recently Taylor and Hones have actually gone through the detailed arithmetic in following the convecting particles. Using a computer and the two invariants, they drew maps of the points of precipitation of particles at earth. It is uncanny the way the individual particle energies are precipitated into narrow rings, but of course with a distribution of energies this actually does not occur. This work is presented in a very nice article but is marred by strange statements which are not true at all. The calculations as far as I know are entirely correct, but the authors claim their analysis has nothing whatever to do with the convection of Axford and Hines. I do not know what they think *they* are doing but they did a beautiful job of it in spite of their disclaimer.

The suggestion then is that convection is responsible for much of the aurora although certainly not for its detailed forms. There is a lot yet to be done on auroral forms but the basic idea is that it is convection which folds in the particles and accelerates them by a factor of a few thousand in energy, and this is probably the origin of much if not most of the fast auroral particles. Convection also is the source of the lower-energy trapped particles, which are just those particles that do not hit the earth and contribute to an aurora; particles left as a residue. Some of the higher-energy trapped particles certainly have a different origin, but I think that probably one of the most important consequences of the idea of convection, apart from the fact that it is interesting in itself, is the production of high-energy particles.

A number of ideas concerning the possibility of accelerating particles in the neutral sheet in the tail have also been proposed. These are very tempting ideas but they have to be explored observationally first, I think, to track them down. Some of the other speakers may discuss this point.

I do not want to leave the idea that the aurora is due only to convection, but it seems fairly straightforward to say that some portion of the aurora must be due to the convection. And I think that any further steps in demonstrating this must be made observationally. One of the most straightforward and obvious experiments to observe the convection is to fire rockets at high latitudes at appropriately active times, leaving behind a trail of, say, ionized barium such as Lüst's group at Munich has developed. The motion of the convection could then actually be plotted and checked out against inferences made from the polar D_s current system. This would be a much more direct approach than the present somewhat indirect one of measuring magnetic-field variations, deducing currents, measuring or deducing conductivities, and so forth. I think it would be important to try a direct observation of convection at least once during the next solar maximum, and it would not be a terribly expensive undertaking now that the technique is developed.

3. The Geomagnetic Storm

Among the various activities that go on in the geomagnetic field as a consequence of the solar wind blowing over it is the magnetic storm. The magnetic storm is best described in terms of the horizontal component of the magnetic field of the earth at low latitudes, not necessarily at the equator which sometimes is complicated with other effects. Let us say that we look at the records of a lot of low-latitude magnetic observing stations and concern ourselves with the horizontal component. It turns out that this is the component easiest to discuss and to understand. Now the first thing to do is to average over many stations around the world in order to remove all local fluctuations. When a storm begins due to an enhanced solar wind, the magnetic field of the earth reacts by flopping and jerking around. At any one station, particularly at an auroral latitude, both the horizontal and the vertical components of the field fluctuate up and down by large amounts. This aspect of the disturbance is not the heart of the problem, at least not directly in connection with the main compression. These variations are big effects, but by averaging out the fluctuations a world-wide view is obtained. What is then found for the horizontal component is shown schematically in Figure 8.

The quiet day value of the geomagnetic field is abruptly increased at the onset of the storm shown in Figure 8. This rise in field magnitude is called the *sudden commencement* and usually is abbreviated by SC. This rise is typically 10–30 γ, depending upon how violent the storm is, and the rise time is about 2–3 min. After SC the enhanced field value endures, with fluctuations or a net gradual increase or decrease, for anywhere from 1 to 10 hours. Then the magnitude drops and may fall below the normal or quiet-day value by 0 or 50 or 100 γ, depending upon the magnitude of the storm. In rare instances, the field may drop 200 γ or more below the quiet-day value. The recovery phase of the storm is next, first recovering very rapidly with a time scale of a few hours and then with a time scale of about a day; the recovery is largely completed in about a week, but there is really no unique time that can be assigned. Normally all changes in field during a storm are less than 1% of the quiet-day value. It should be emphasized that this characterization of a storm averages out many fluctuations that would be seen at a particular station. The period of enhanced field magnitude is called the *initial phase* of the storm and the subsequent period the *main phase*. For some storms one of the major phases, i.e., the sudden commencement, the initial phase, or the main phase, may actually be missing. All features of a *typical* storm are shown in Figure 8.

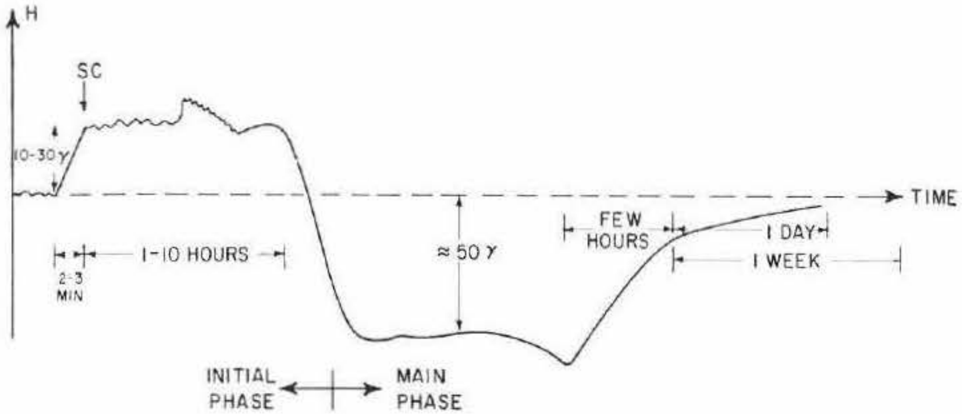

Fig. 8. Schematic representation of the development of a magnetic storm as seen on the surface of the earth. The ordinate is the horizontal component of the magnetic field as observed at low latitudes, averaged over many stations and storms. The abscissa is a non-linear scale of time.

A. THE INITIAL PHASE

What apparently happens at sudden commencement is a compression in the geomagnetic field as explained by Chapman and Ferraro. They did not use hydromagnetic language, and liberties will be taken in restating their ideas. They pointed out that the solar corpuscular radiation, as one called the solar wind in those days, suddenly began to press hard on the outer boundary of the geomagnetic field causing a compression. Now we would say that the sudden commencement is caused by the initial slap of the shock wave in the solar wind on the geomagnetic field. Dessler pointed out that a hydromagnetic wave then propagates throughout the magnetosphere which, along with the time it takes for the enhanced wind to envelop the magnetopause, accounts for the sudden commencement rise time of 2–3 min.

To understand the initial phase, stresses in the field must be considered. It was pointed out above that even on a quiet day the magnetic field of the earth is confined within a surface and compressed. Examples for plane and spherical boundaries were given, but the magnetosphere is neither. The computations of Beard, Mead, and others for the magnetopause give the result

$$\frac{\Delta B}{B_0} = 0.6\left(\frac{R_E}{L}\right)^3, \tag{48}$$

where B_0 is the field magnitude at the surface of the earth on the equator ($\approx \frac{1}{3}$ gauss) and ΔB is the increase in field due to the compression. L is the distance from the center of the earth sunward to the magnetopause. On quiet days, $L \approx 10\ R_E$ and $\Delta B \approx 15$–$20\ \gamma$. Suppose now that the horizontal component is increased by $20\ \gamma$ more, for a total increase of $40\ \gamma$. Then L decreases by $2^{1/3}$, from 10 to 8 R_E. This sets the proper framework of discussion. The quiet-day wind holds the magnetopause in at about 10 R_E. A blast wave, or at least a shock, from the sun comes along and suddenly pushes harder upon the magnetopause. The boundary is moved inward to about 8 R_E and produces an average world-wide increase in the horizontal field of the order of $20\ \gamma$. Of course these numbers will vary from storm to storm.

With some simple calculations, these considerations can be used to place limits upon the density of the solar wind during active times. For a quiet-day wind, taking a density $N = 5/\text{cm}^3$ and a velocity $V = 316$ km/sec as representative, the impact pressure is

$$NMV^2 = 8 \times 10^{-9}\ \text{dynes/cm}^2 \tag{49}$$

and this just balances the pressure of the field at the stagnation or subsolar point of the magnetopause. The quiet-day balance is achieved at about 10 R_E. Now take an extreme case and imagine that the compression of the geomagnetic field increases from its quiet-day value to about $100\ \gamma$. This would be for a very big storm, larger than would be normally seen although larger ones have occurred. From (48) this would give $L = 5.6\ R_E$, and the pressure of the wind p_S during the storm has increased by about a factor of 30 over its quiet-time value (49). This increase is caused by changes in the solar wind density and/or velocity. For example, if the quiet-day velocity triples then $N \approx 17/\text{cm}^3$; if the velocity doubles then $N \approx 40/\text{cm}^3$. One knows, however, that a fairly high storm velocity applies, near 1000 km/sec, since this is the time it takes for the wind to get here from the sun. Roughly speaking the observed sudden commencements and initial phases of storms suggest that the density of the solar wind may *sometimes* go as high as $30/\text{cm}^3$, and only on rare occasions will this value be exceeded. With the large velocity that one associates with the solar-wind density, a larger density would produce a compression of the field, which we recognize as the initial phase of the storm, that is vastly greater than actually observed.

This is just a very crude kind of estimate but it is an important one because there are some observational statements in the literature, the most modest of which is $N = 300/\text{cm}^3$, and they go up from there to about 10^5. I think that one can place a much lower limit on the density for at least most of the large storms.

B. THE MAIN PHASE

Whereas the initial phase of a magnetic storm is caused by compression, the main phase is caused by the inflation of the magnetic field of the earth by gas pressure. For a moderate (or slightly above moderate) sized storm, the main phase decrease is about $100\ \gamma$, or about $\frac{1}{3}$ of 1% of the main field. This is not a large change in the total field, and I am amazed to read in books how the magnetic storm was discovered way back in the days when people had magnetized needles suspended by threads. Someone with real patience and good eyesight apparently sat there and looked at that needle and caught it in the act of moving a fraction of 1%. Of course there are excursions during the storm which are bigger than the average world-wide depression and I suppose that is what they were actually seeing. But I must say, my hat is off to them. They probably applied for a grant from NASA for 25 cents to buy the needle and from then on it was all blood, sweat, and tears.

First some of the mathematical techniques in elasticity for grinding out distortions will be discussed, and then some specific mechanisms that produce the distortions of the field will be mentioned. The main phase is an expansion of the field, i.e., the field decreases and so necessarily expands. There are other ways of expressing this. Chapman and Ferraro said the main phase was due to a Westward ring current, which is an equally tautological statement since Maxwell says if you have a curl of $\mathbf{B}$ you must have a current density. In my statement, if you have a rarefaction in the field then you must have an expansion. These are equally trivial statements, entirely equivalent. Now look at the stress problem. The net force on the medium is due to the pressure gradient plus the magnetic force (or the divergence of the Maxwell stress tensor):

$$-\nabla p+\frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}=0. \tag{50}$$

If the field is undistorted, its curl is zero and no force is exerted by the field on the gas; so at equilibrium, the pressure is uniform. (Gravity will be neglected in this discussion.) Now what one would like to do in principle is to prescribe the pressure in space and then find the resulting field. In other words, inflate the magnetic field of the earth with gas and then ask what distortions are produced. Then, by comparing the calculated field distortions with the observed distortions, one could eventually determine what pressure variations are needed. Unfortunately Equation (50) is non-linear in the field $\mathbf{B}$, and even though the non-linear case has been discussed at some length under conditions of axisymmetry by Chandrasekhar, Schluter, Lüst, and others, it has not been solved in most cases. I shall not deal with the non-linear problem.

One could play the game the other way around: specify the distortions of the field and then deduce what pressure might be required. It turns out there is a difficulty here too. For, the distortions assumed must really be expressible as the gradient of a scalar or the problem has no answer. And this is a difficult requirement except for very simple cases. So what one does is to rework Equation (50) a bit and put it in a better form for deducing the resulting field distortions from the pressure.

Suppose the magnetic field $\mathbf{B}(\mathbf{r})$ is localized in some region, as is the dipole field of the earth, and some distortion $\Delta\mathbf{B}$ is introduced. The field $\Delta\mathbf{B}$ would go away if the distortion were relieved. A theorem that can be proven, and will soon be recognized, states that $\Delta\mathbf{B}$ is related to $\nabla\times\mathbf{B}$ by

$$\Delta\mathbf{B}=\frac{1}{4\pi}\int(\mathrm{d}\mathbf{r}')\,\frac{[\nabla'\times\mathbf{B}(\mathbf{r}')]\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}. \tag{51}$$

Here ∇' acts on $\mathbf{r}'$. Using (51), $\Delta\mathbf{B}$ can be calculated from curl $\mathbf{B}$, which from Equation

(50) is given by

$$\nabla\times\mathbf{B}=-4\pi\,\frac{\nabla p\times\mathbf{B}}{B^2}, \tag{52}$$

where $\mathbf{B}\cdot\nabla\times\mathbf{B}$ has been taken equal to zero because of the non-conducting atmosphere at the base of the field lines. The component of the curl in the direction of the field represents a torsion on the field, and a parallel component of the curl means that the field is twisted like rope. Like an elastic, it wants to untwist and because of the non-conducting atmosphere it can untwist. So, generally speaking, one does not have a parallel component of the curl, or if you do it is small or transient. This is justified in the case of the main phase of the storm which lasts many hours or days. On the average, field-aligned currents are essentially negligible, i.e., you cannot have the lines of force strongly twisted like a rope for very long because they will simply unwind in a matter of a few minutes. Denoting $\mathbf{R}=\mathbf{r}-\mathbf{r}'$, and substituting (52) in (51),

$$\begin{aligned}\Delta\mathbf{B}&=-\int(\mathrm{d}\mathbf{r}')\,\frac{[\nabla'p(\mathbf{r}')\times\mathbf{B}(\mathbf{r}')]\times\mathbf{R}}{B^2(\mathbf{r}')\,R^3}\\&=-\int\frac{\mathrm{d}\mathbf{r}'}{B^2R^3}\,[\mathbf{B}\mathbf{R}\cdot\nabla'p-(\nabla'p)\,\mathbf{B}\cdot\mathbf{R}].\end{aligned} \tag{53}$$

In component form,

$$\Delta B_i=-\int\frac{\mathrm{d}\mathbf{r}'}{B^2r^3}\left[B_iR_j\,\frac{\partial p_{jk}}{\partial x_k}-\frac{\partial p_{ik}}{\partial x_k}\,B_jR_j\right]. \tag{54}$$

These equations are still non-linear. Equation (54) is the same one that results if an anisotropic pressure were used at the outset.

The way to use Equation (54) is to consider only small distortions of the field so that to a first approximation the field $\mathbf{B}$ can be replaced by the undistorted dipole field under the integral. With this replacement the pressure must be small in some sense, the equation is linearized, and the distortion $\Delta\mathbf{B}$ can be cranked out directly. The process can be iterated using the calculated $\Delta\mathbf{B}$ as a correction to the original dipole field, and again performing the integration. But it turns out that unless there are extraordinarily large distortions, this is hardly worth doing. Even fairly large pressures do not distort field magnitudes enough to make it worth doing the second-order calculation in most cases.

Several points should be made about Equation (54). This equation gives the *direct* relation between the distorting forces and the distorted field. $\Delta\mathbf{B}$ and not curl $\mathbf{B}$ is the measured quantity, so (54) is the direct relation. Sometimes in putting in various forms of the pressure it is algebraically easier to compute the current first, which is equivalent to the curl of the field, and to work directly with the equation

$$\Delta\mathbf{B}(\mathbf{r})=\int\frac{\mathrm{d}\mathbf{r}'}{cR^3}\,\mathbf{j}(\mathbf{r}')\times\mathbf{R}, \tag{55}$$

which is recognized as the Biot-Savart law. The connection between (51) and (55), which involves the stresses directly, is simply the Maxwell equation

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\,\mathbf{j}.$$

Now to examine the currents briefly. A particle moving in a non-uniform magnetic field will undergo drift motions. If there is a gradient perpendicular to the field direction, the particle will drift along a path of constant field density at the rate

$$u_G = \tfrac{1}{2} w_\perp \frac{R_c}{l}, \tag{56}$$

where $w_\perp$ is the particle speed transverse to the field direction, R_c is the particle cyclotron radius, and l defines the scale of the gradient. l is given by

$$\frac{1}{l} = \frac{\nabla_\perp B}{B} = \frac{3}{r},$$

where the last equality applies to a dipole field. If the lines of force along which the particle moves are curved, then the particle experiences a centrifugal force and undergoes an additional (curvature) drift. The Lorentz force on the particle must balance the centrifugal force:

$$q\left(\frac{u_c}{c}\right) B = M w_\parallel^2 K.$$

Here u_c is the curvature drift, $w_\parallel$ the longitudinal speed, and K is the curvature of the line of force. Thus

$$u_c = \frac{Mcw_\parallel^2 K}{qB}. \tag{57}$$

Since K is the reciprocal of the radius of curvature and $R_c = (w_\perp\, Mc/qB)$, the results (56) and (57) are quite similar in algebraic form. The vector expression for the combined gradient and curvature drifts is

$$\mathbf{u} = \frac{Mc}{qB^4}\mathbf{B} \times \left[\frac{w_\perp^2}{4}\nabla B^2 + w_\parallel^2 (\mathbf{B}\cdot\nabla)\,\mathbf{B}\right]. \tag{58}$$

The first term in this expression gives the gradient drift across the field lines, and the second term provides the curvature drift due to the change of the field along its direction.

These drifts produce currents due to the motion of the guiding center and additional currents are provided by the particle cyclotron motion. Conceptually this circular motion of the particle is very simple, but to add up the total current involved account must be taken of the fact that as the line of force curves the planes of the cyclotron motion tilt and are closer together on one side than the other. The easiest way to treat the current due to the cyclotron motion is to remember that the particle has a diamagnetic moment of $\frac{1}{2}Mw_\perp^2/B$ and to simply add the fields of dipoles. So the current is

broken up into two parts: the drift part to which the Biot-Savart law is applied, and the cyclotron part which may be treated in the same way or as a sum of dipole moments. Some examples are treated next.

1. *The Simple Ring Current*

This first example is so simple that the above formulas need not be used. Imagine, as shown in Figure 9, that a dipole is at the origin. The dipole is drawn right side up; it is only an archaic custom that puts South at the bottom of all maps. Now place a thin ring of gas with radius r in the equatorial plane. The calculations are independent of the shape of the cross-section of the ring, so suppose it is circular. Suppose that the pressure in this ring acts so as to increase its major diameter. The total force in the

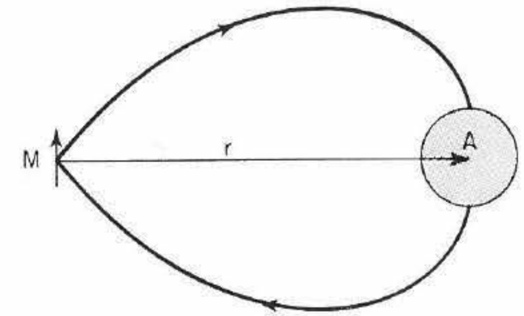

Fig. 9. Simple ring-current model in meridian profile. A dipole is at origin and a symmetrical ring current is located at an average distance r in the equatorial plane. The ring has cross-section A and particle pressure p.

ring is pA and pA/r is the force per unit circumference pushing outward. But this ring of plasma is imbedded in a magnetic field, and a current will be induced to resist an outward movement of the ring. If the ring is trapped in the field, the current is induced to the point where the Lorentz force just balances the tendency of the ring to expand:

$$\frac{pA}{r} = \frac{IB}{c} \quad \text{or} \quad I = \frac{pAc}{Br}.$$

Now outside the ring current the magnetic field is expressible as the gradient of a scalar, since, apart from the dipole, there are no other currents in the system. The magnetic field satisfies Laplace's equation outside the ring and may be calculated as if in a vacuum. The field at the center of a ring with current I is

$$\Delta B = \frac{2\pi I}{cr} = \frac{2\pi pA}{r^2 B}. \tag{59}$$

This gives the expansion of the field in the vicinity of the origin due to the outward stresses exerted by the dipole. The volume of the ring is $(2\pi rA)$ and the pressure, which of course is due to internal particle motions, is $p = NMw^2$, where N is the particle number density and w is the mean square velocity in the tangential direction. The pressure is therefore twice the particle kinetic-energy density. Equation (59) may now be written as

$$\Delta B = \frac{2E}{r^3 B}, \tag{60}$$

where E is the total kinetic energy of the particles in the ring. Further, the dipole field at distance r in the equatorial plane is $B = B_0 (R_E/r)^3$ where B_0 is the field at the earth's surface. So Equation (60) becomes

$$\frac{\Delta B}{B_0} = \frac{2E}{B_0^2 R_E^3}. \tag{61}$$

Since ΔB is a uniform field in the vicinity of origin, the left side of (61) may be regarded as the fractional change in field on earth at any point on the equator. The denominator on the right in (61) is an energy characteristic of the dipole field. Denoting by $E_M(R_E)$ the energy in the dipole field above the surface of the earth, we have

$$E_M(R_E) = \iiint_{r \geqslant R_E} \frac{B^2}{8\pi} \, dV = \tfrac{1}{3} B_0^2 R_E^3 \approx 8 \times 10^{24} \text{ ergs} \tag{62}$$

and (61) becomes

$$\frac{\Delta B}{B_0} = \frac{2E}{3E_M(R_E)}. \tag{63}$$

The fact that the dipole field should cut off somewhere near $10 R_E$ does not change the result (62) much.

2. *Pancake Distribution*

A somewhat more artificial problem is to take a distribution of particles confined to the equatorial plane. The particles drift around the earth in the equatorial plane with 90° pitch angles. The easiest way to calculate the inflation of the field is to obtain the current in the distribution by multiplying the velocity (58) by qN. In this model there is no term $w_\parallel$. The inflation due to the drift motions is then found to be $(\Delta B)_D / B_0 = E/E_M$. In addition, the diamagnetic moments of the particles contribute minus $\frac{1}{3}$ this result, so that the total inflation is

$$\frac{\Delta B}{B_0} = (1 - \tfrac{1}{3}) \frac{E}{E_M(R_E)},$$

i.e., the same as (63).

This result may also be calculated in a different way which will seem to be a bit trickier at first. Let $\hat{\mathbf{z}}$ denote the direction of the dipole and let ϖ denote the radial distance from the polar axis. Since there is no motion along $\hat{\mathbf{z}}$, the component of the pressure tensor p_{zz} must vanish. Also,

$$p_{\varpi\varpi} = p_{\phi\phi} = \tfrac{1}{2} N M w_\perp^2 .$$

Suppose that the particles form a thin ring at the particular radius ϖ. The pressure is then a delta function in z times a fairly localized function in ϖ. Using (54),

$$\frac{\Delta B(0)}{B_0} = - \frac{2\pi}{B_0^2 R_E^3} \int dz \int d\varpi \varpi^2 \frac{\partial p}{\partial \varpi}.$$

The integration over z is trivial because of the delta function, and integrating by parts gives

$$\frac{\Delta B}{B_0} = - \frac{2\pi}{B_0^2 R_E^3} \int_{-\infty}^{\infty} dz \int_0^{\infty} d\varpi \left[\frac{\partial}{\partial \varpi} (\varpi^2 p) - 2\varpi p \right].$$

The first term integrates directly and vanishes at both limits. The remaining integral is proportional to the volume integral of the pressure, and again we find

$$\frac{\Delta B}{B_0} = \frac{2}{3} \frac{E}{E_M(R_E)}.$$

3. *Field-Aligned Ring Current*

As a final example, consider all of the particles to have zero pitch angle, moving on the same magnetic shell as though having been ejected from the North pole and returned through the South pole. Calculating the change ΔB at origin for this configuration gives three times the result (63)!

C. ARBITRARY DISTRIBUTION

Other theorems may also be proven directly. For example, for a given pitch-angle distribution the result $\Delta B/B_0$ is independent of the scale of the ring current. In other words, particles of a given pitch-angle distribution and total energy can be located anywhere – 2 R_E, 5 R_E, or 8 R_E – and the same inflation ΔB results. And very recently Sckopke has found that in fact the inflation does not even depend on the pitch angle; in all cases of particles mirroring in the field, Equation (63) applies with the same factor $\frac{2}{3}$. Sckopke's result is a great simplification since it says that the average inflation ΔB in the vicinity of the earth, i.e., the origin, depends only on the total energy of the particles without regard to their pitch-angle distribution.

It should be mentioned that in all the examples given here including Sckopke's result, the particle distribution is assumed to be axisymmetric. But of course any particle distribution will quickly become axisymmetric after drifting around the source and becoming mixed up.

For a particle distribution in a quadrupole field, say a linear quadrupole so it has axisymmetry, then the inflation ΔB depends on the total particle kinetic energy and also upon the location of the particles through a factor like r/R_E. Hence the further out the particles are located from the source, the more effective they become in decreasing the field. In an octupole field the spatial dependence goes like $(r/R_E)^2$. In the case of the dipole there is no dependence on the spatial location of the particles, which is convenient in a way, but inconvenient, too, as will become apparent.

D. PROPERTIES OF THE MAIN PHASE

The above mathematical results may now be applied to the magnetic storm. Modifying an earlier suggestion made by Alfvén, Singer suggested, about 1956, that the ring current responsible for the main phase of the magnetic storm was the result of particles moving inside the dipole field of the earth. The ring-current particles were not skimming along the magnetopause as had been suggested earlier, and were not moving in and out of the dipole field as suggested by Alfvén, but were drifting around the earth and constituted a Westward current source. In 1959 Dessler and I became interested in this question, and we adopted this idea, formulated it and worked out a lot of examples, some of which are presented above (DESSLER and PARKER, 1959). We missed seeing Sckopke's theorem, but at least it is a simplification we can be grateful for once it is discovered. The question now is: where are the particles that are producing the storm, what kind of particles are they, what puts them there, and what takes them away when the storm is over? This last point is an important one and will affect the discussion.

There are several little things to mention before conjecturing on what makes the magnetic storm. The magnetic dipole energy outside the surface of the earth is given in (62), and the magnetic energy beyond a distance r is

$$E_M(r) = E_M(R_E)\left(\frac{R_E}{r}\right)^3. \tag{64}$$

The r^{-3} dependence in this expression is expected since the field-energy density goes as r^{-6} and the volume goes as r^3. In inflating the magnetic field of the earth, laboratory experiments suggest that it will hold plasma only up to a certain point. If inflated beyond a certain amount, the gas simply runs away with the field. In fact, if the pressure of the gas exceeds the pressure of the field, one expects that the field would be enormously distorted and that the gas would escape; at least this is one's laboratory experience. This suggests, therefore, that in adding particle kinetic energy beyond some distance r the total particle energy beyond that distance would probably be less than the magnetic energy beyond the same distance. For instance, a storm with a 100 γ main phase requires 3×10^{22} ergs of gas energy, based on Equation (63). That this should equal the field energy (64) prevents the particles from being too far out in the field. A field-particle energy balance would not be possible at 9–10 R_E.

Let the amount of particle energy beyond a distance r be a fraction α times the dipole magnetic energy beyond the same distance:

$$E(r) = \alpha E_M(r). \tag{65}$$

α is a number less than unity which presumably can be obtained from observation. A value $\alpha\approx\frac{1}{2}$ is probably indicative. The relation (65) restricts the location of the particle energy. It requires, to make a big storm, that particles move down to small values of r to inflate the field. To state some numbers, for $\alpha=\frac{1}{2}$ and $\Delta B=100\ \gamma$ the gas must extend in at least as far as $r=5.5\ R_E$. For $\alpha=\frac{1}{2}$ and $\Delta B=300\ \gamma$, which is really a big storm, the particles extend inward to at least $r=3.7\ R_E$. These numbers are calculated from (64) and are insensitive to the value of α since its cube root is taken. It follows that the size of the storm determines how close to the earth the ring current must be. The original expression (63) is independent of the location of the particles.

The next question to deal with is what makes the average world-wide magnetic storm effect. Only the symmetric part of the storm will be discussed here; asymmetric effects will be discussed briefly below. Storms are frequently large enough that the gas that inflates the field must often extend in to 5 R_E or even less. Several questions must be asked at this point in order to proceed. The first is: what gases are heated up and placed in the ring-current zone? In the early days, people, including me, spoke glibly of injecting gas directly from the solar wind, and this is a beautiful idea until examined closely. I think now that the idea is untenable. The field is very stable and there is no evidence that particles are ever injected directly. The next idea is that maybe a lot of shock waves simply heat up the gas already present in the field. The particles do not have to be a solar gas. There is no shortage of gas in the earth's field. In a magnetic storm things are extremely active and hydromagnetic waves are in abundance; there is no reason that the ambient background gas – ionized gas – is not simply heated.

AXFORD and HINES (1961) have made an alternative suggestion involving a very rapid convection system. As discussed above, gas is convected in and out of the field. The gas is heated normally – increasing the kinetic energy – when convected deep into the field. If the convection is very rapid, gas being taken from the solar wind or somewhere will be adiabatically compressed and heated enormously.

I suspect that both of these effects occur and contribute to heating the gas. I do not know which one will be the dominant effect and at the moment do not know that one can make much of a distinction. Ideas are plausible but the particles still can not be identified. We know the energy required of the particles – about 3×10^{22} ergs for a 100 γ main-phase decrease – and that the particles can be anywhere in the field so long as they are at about 5.5 R_E or closer to the earth. The simplicity of Equation (63) is unfortunate in this respect; if it were more complicated it might have given a clue as to where the gas resides.

The next question, once the storm has been turned on, is that it can be turned off in a day. The explanation of this, given in 1959, is perhaps the strongest argument for the general nature and location of the particles that inflate the field. People have suggested that some of the high-energy particles in the radiation belt – electrons, protons, etc. – may be responsible for the storm. In considering how the storm can be turned off, i.e., how the particles can be cooled again, both convection and collisions might possibly slow the particles down.

One should recognize with collisions that it does not help to have one particle slam into another and transfer its energy. According to (63) only the total kinetic energy matters in inflating the field and a redistribution of particle energy will not change the resultant ΔB. So in the decay of a storm one must be concerned only with those collisions which actually remove energy from the ionized gas. Radiative collisions are one obvious possibility. Here an electron collides with an ion or atom, loses its energy, and the energy absorbed by the ion or atom is quickly lost by radiation. The cross-section for this process is too small, however, giving a relaxation time of the order of months. The only significant process that we could find was the charge exchange interaction which fortunately had just recently been measured fairly accurately in the laboratory. In this reaction a fast proton interacts with a neutral hydrogen atom. The bound electron finds itself in the eternal triangle and occasionally gives up its stationary partner to take a ride with the freewheeling proton going by. The result is the formation of a fast neutral hydrogen atom and a proton bereft of its electron. The process is:

$$p_f + \mathrm{H} \rightarrow \mathrm{H}_f + p,$$

where p stands for proton, H for hydrogen atom, and the subscript f labels the fast particle. The fast neutral hydrogen atom is no longer coupled to the magnetic field and is free to carry away the energy of the incident, fast proton. In its place is left the abandoned proton with an energy of the order of 1 eV – depending on the value of the thermal energy.

The formula for the lifetime of the charge exchange process is fairly simple: a proton moving through a neutral hydrogen background of N atoms per cubic centimeter has a life expectancy of

$$\tau \approx (10^7/N)\,\mathrm{sec}. \tag{66}$$

The same expression applies for a neutral atom passing through a background of protons – the process is reversible. Equation (66) is a crude approximation but gives representative values for proton energies extending from about 400 eV to 40 keV. Over this range the coefficient in (66) is the same to within a factor of 2, which is quite amazing. (The charge exchange cross-section varies inversely with the velocity so that the lifetime is independent of the particle velocity over a factor of about 10, and independent of the particle energy over a factor of about 100.)

In order for the charge exchange mechanism to cool off the gas in a day, say $\tau = 10^5$ sec, it requires a background density of neutral hydrogen of about $10^2/\mathrm{cm}^3$. Although it varies with the sunspot cycle, the book value for the density of neutral hydrogen above the earth gives this value at about 4 R_E, which is compatible with the restriction (65), mentioned earlier. We suggested then that the particles producing the main phase of the storm were protons in the general vicinity of 4 R_E (DESSLER and PARKER, 1959). Their density would be about $10^2/\mathrm{cm}^3$, assuming the average proton energy to be of the order of 1 keV to get the necessary total energy. (We could not specify the energy from the decay time (66) since it is independent of the energy. So again the formula is so simple that we could not really go further.)

Dessler pointed out that one direct test for the occurrence of the charge exchange process would be to detect the flux of neutral hydrogen atoms being produced by the incident protons. He calculated the flux and found it to be about 10^5 hydrogen atoms/cm^2-sec striking the top of the atmosphere as a result of charge exchange. Although 1 keV neutral hydrogen atoms would be difficult to detect, I get the impression from

talking to people that it can be done. It would not be an easy experiment because it must be performed during the decay of a large or fairly large storm.

The other possibility for relaxing the storm is convection. Convection slows down after the active phase of the storm, and although proceeding slowly I am sure it contributes to the decay. But I think it does not contribute a very large portion. Convection may be important way out in the tail in the late stage of the decay.

Other predictions follow. For example, the gas inflating the field during a large storm must penetrate deeper into the field (see (64)). Consequently, the large storm should decay more rapidly at first than a small storm; and this seems to be the case. Johnson has pointed out that during sunspot minimum the upper atmosphere is cooler and the neutral hydrogen density builds up to 2 or even 3 times its value at sunspot maximum. One predicts, therefore, that for two storms of the same size, one during sunspot minimum and one at maximum, the storm at sunspot minimum should decay 2 or 3 times more rapidly because of the larger hydrogen background. Matsushita examined the data and to our delight found that this seems to be the way it actually goes – storms of a given size decay more rapidly at sunspot minimum than at sunspot maximum. There was also a lot of joy in Mudville when Van Allen found that kilovolt particles were in great abundance during a magnetic storm and with about the right density. The observed particle energies extend from about 200 eV to 50 keV – the same range covered by the charge exchange formula (66) – and probably are widely distributed through this range. FRANK (1967) finds that protons contribute about $\frac{3}{4}$ of the total energy which is the right order of magnitude to give the observed main phase decrease. We have been quite happy about these results confirming our old predictions. But now that we are entering a time of solar activity there will be a lot more data which will give a better idea as to what is happening.

Van Allen's data displaying the radial profile of the proton density show the tendency for the particle-energy density to be more or less comparable to the magnetic-energy density and then drop off rather suddenly on the inward side. I will hazard a speculation that this inner boundary of the low-energy protons (which are the source of the magnetic storm main phase) also constitutes the inner boundary of the Axford-Hines convection pattern at that particular time. If this suggestion is correct, it implies that a large part of the heating is due to the compression of particles being convected.

E. ASYMMETRIC INFLATION

It is an observed fact that the inflation of the geomagnetic field during a storm is asymmetric. The development of a storm differs at various parts of the globe from the world-wide average. There is no reason, of course, that the storm should not be asymmetric. A brief and qualitative description of asymmetric inflation can be made from Figure 10.

Suppose that instead of having gas spread uniformly around the earth there is a blob of plasma in the equatorial plane. It is perfectly clear that the pressure of this plasma can be put into Equation (54) to obtain a ΔB. This is correct as far as it goes, but there are additional effects that must be included arising from the fact that under quasi-steady conditions the current has zero divergence. As shown in Figure 10, the plasma blob in the equatorial plane has a current, or curl, flowing across the magnetic-field lines. The current does not vanish at the end of the plasma blob and – by its conservation – must go somewhere. The magnetic field is filled by a very tenuous plasma, which would not be able to sustain the force should the current continue across the lines of force. The only place the current, or curl, can flow is along the lines of force. The equatorial current therefore splits and, assuming symmetry, a current $I/2$ runs along the lines of force, as in Figure 10. Without the ionosphere, the current along the lines of force would flow into the origin which masks a lot of stresses which do not appear in the equations above.

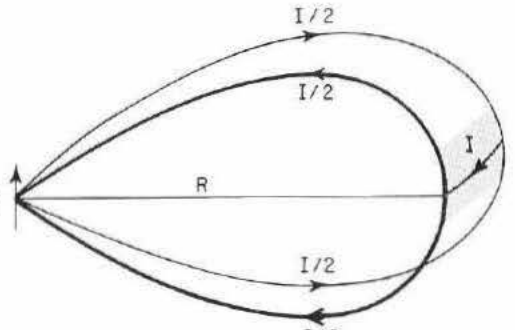

Fig. 10. Schematic current distribution for a plasma blob in the equatorial plane of a dipole field. The plasma blob carries a current I at a distance R in the equatorial plane. This current splits symmetrically at the ends of the blob and flows along the dipole field lines into the origin.

With this asymmetric current distribution flowing into the origin, the inflation of the field is again given by Equation (63). The resultant field is asymmetric about the origin, but the energy of the blob is related to the change ΔB at the origin by the same relation (63). The ionosphere changes this result. Very briefly, the effect of the ionosphere and the non-conducting atmosphere is to prevent the currents coming down the field from continuing toward the origin. Instead, the current coming down the field line is diverted through the ionosphere across the field and back out into the magnetosphere to complete the circuit; this is shown in Figure 11. The current cross-over in the ionosphere exerts stresses, but the ionosphere is dense enough to support these stresses. The easiest way to calculate the total field at the origin for this current configuration is to use the Biot-Savart law (55) directly instead of (54). Ignoring the effects of the solid earth, one finds a radically new result (PARKER, 1966):

$$\frac{\Delta B}{B} = \frac{1}{3} \frac{E}{E_M(R_E)}, \tag{67}$$

and here the field at origin is *increased*, whereas before it always decreased. In obtaining this result, one finds (a) that the currents along the field lines have canceling

effects at the origin and (b) that the field due to the currents crossing over in the ionosphere exactly cancel the current I borne by the asymmetric blob in the equator. All that is left then is the diamagnetic effect of the particles in the plasma blob, and this gives the result (67).

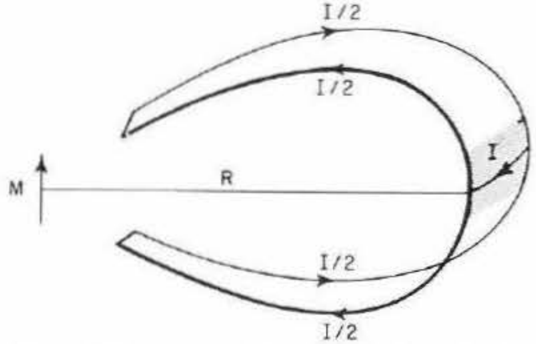

Fig. 11. Plasma-blob current configuration with ionospheric cross-over.

Actually the current does not go directly across an ionospheric segment, as shown in Figure 11. The ionospheric current spreads out and flows all the way around the world in an appropriate pattern. There is now a tremendous degree of flexibility depending upon the details of ionospheric conditions; but one point is clear. The asymmetric part of a storm decays extremely rapidly because it has to drive currents through the ionosphere which has a high resistivity. Whereas the symmetric part of the storm decays in about 10^5 sec, the asymmetric part dies away in about 1 or 2 hours. It should die away rapidly and is observed to do so. When the quiet time of the storm is reached, the ring current is very symmetrical and remnants of it may be seen in some storms after a week or two.

4. Structure of the Magnetopause

The next topic will be to discuss the equilibrium, or lack of it, at the magnetopause. This is an interesting problem that so far is unsolved and poses somewhat of a dilemma.

A. STATIONARY STATE MAGNETOPAUSE

Upon encountering the geomagnetic field, the solar wind plasma passes the bow shock, and is deflected and disordered to some extent, and then streams past the magnetopause in a manner depicted in Figure 12. At the magnetopause there is a balance between the total internal and external pressure:

$$\frac{B_i^2}{8\pi} + p_g = \left(\frac{B^2}{8\pi} + p\right)_w. \tag{68}$$

Here the subscript w refers to the wind variables outside the magnetopause, and on the left the gas pressure p_g is small compared to the geomagnetic field pressure. Equation (68) has been used to compute the magnetopause and the shock positions fairly successfully with regard to the gross geometry. The main concern here will be to discuss the detailed structure of the magnetopause, i.e., the 100–200 km transition region between the geomagnetic field and the plasma in the wind. This should be recognized to be a general problem in plasma physics. The magnetopause is perhaps the most striking illustration of the balance between plasma and field at an interface, but nonetheless it is a general problem that occurs in laboratory plasmas as well as in astrophysical circumstances, as with the geomagnetic field.

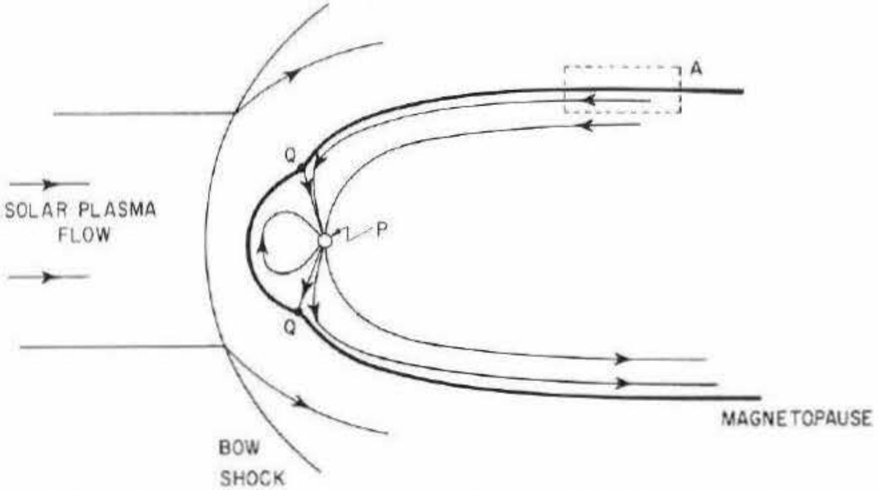

Fig. 12. Schematic representation of the magnetosphere. The incident solar plasma flow establishes the shock surface and the magnetopause. A few field lines are drawn within the magnetopause including the one connected to the neutral points labeled Q in diagram. The point P on the earth is labeled because of a question, as described in the Appendix.

In the fluid approximation, there is no real interface, and the situation is quite simple. With no particles to worry about, if you start out with a perfectly sharp boundary between field and gas – taken to be an infinitely conducting fluid – then the boundary would remain perfectly sharp for all subsequent times. If the fluid possesses some resistivity or diffusivity then the gas will slowly penetrate into the field. In c.g.s. units the diffusion coefficient η is related to the conductivity σ by

$$\eta = \frac{c^2}{4\pi\sigma}$$

and the rate at which the gas diffuses into the field may easily be expressed with the error function:

$$B(z, t) = B_0\left[1 - erf\,\frac{z}{(\eta t)^{\frac{1}{2}}}\right].$$

Here z is the vertical coordinate across the boundary as shown in Figure 13, and t denotes the time that the fluid sliding along the boundary has been in contact with the field:

$$t = \int \frac{\mathrm{d}s}{v}.$$

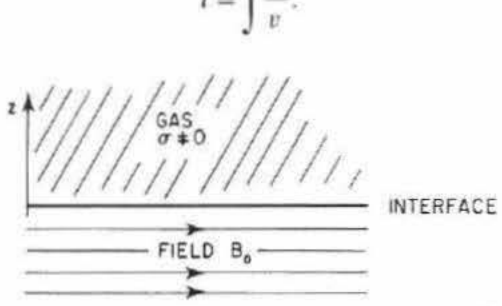

Fig. 13. Simple representation of a plane interface between a conducting fluid and a magnetic field.

The fluid approximation is normally used for the gross features of the problem but it avoids the detailed behavior of the particles in the plasma which will now be considered.

B. NORMALLY INCIDENT PARTICLES

Consider first the confinement of a portion of the geomagnetic field out in the tail region, as domain A of Figure 12, where the curvature of the surface is not essential, and suppose that the confinement is accomplished by the impact of particles normally incident on the surface (cf. Figure 14). As the particles swing around in the field, they generate a current. The Lorentz force acting on this current must be balanced by the pressure gradient of the particles. Thus

$$\frac{\mathrm{d}p}{\mathrm{d}z} = \frac{jB}{c} \quad \text{and} \quad \frac{\mathrm{d}B}{\mathrm{d}z} = -\frac{4\pi}{c}j,$$

where the second relation is the Maxwell equation. Combining these relations,

$$\frac{\mathrm{d}p}{\mathrm{d}z} + \frac{B}{4\pi}\frac{\mathrm{d}B}{\mathrm{d}z} = 0$$

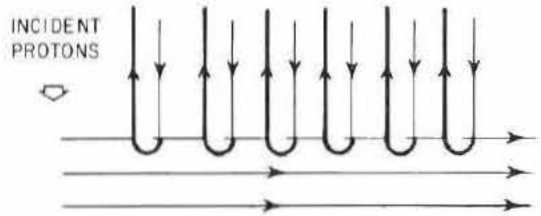

Fig. 14. Confinement of the magnetic field **B** by particles incident normal to the interface. The bold portion of the trajectory is to be viewed so that a proton would execute a diamagnetic orbit while in the field.

and this integrates directly to give the familiar relation

$$p + \frac{B^2}{8\pi} = \text{constant}.$$

The particle motions determine the current density j which automatically satisfies Maxwell's equation with the result that the equilibrium solution requires a constant total pressure. There would be no net motion in this circumstance.

The case of normal incidence of a plasma on a field has been solved in considerable detail – by Chapman and Ferraro, later by Ferraro, and more recently by Gross, Grad, and Rosenbluth, who worked out detailed models – and will only be discussed briefly for purposes of illustration. Referring to Figure 15, suppose that electrons and protons are normally incident on the magnetic field. The particle-field boundary is not sharp. It is assumed that complete equilibrium exists and that any electric field has been shorted out, as will be discussed in more detail below. The protons will penetrate deeply into the field and the electrons will hardly penetrate at all before bouncing back. If there are N atoms/cm^3 in the incident beam of particles, each with mass M (the sum of the electron plus proton mass) and speed w, then the impact pressure $2NMw^2$ must balance the magnetic-field pressure at equilibrium:

$$2NMw^2 = \frac{B^2}{8\pi}.$$

The variation of the magnetic field as a function of z, where $z = 0$ represents the ion turning point, is shown in Figure 16. The field falls off slowly from its uniform value, tending asymptotically to zero with a tail expressible with elementary functions. There is one complication that occurs. When the particles confining the field are first switched on, the picture would not be as in Figure 16. The protons would like to penetrate a full gyroradius – about 100 km – into the field at the magnetopause, but are prevented from doing so by electrostatic forces. There is a sufficient particle number density that

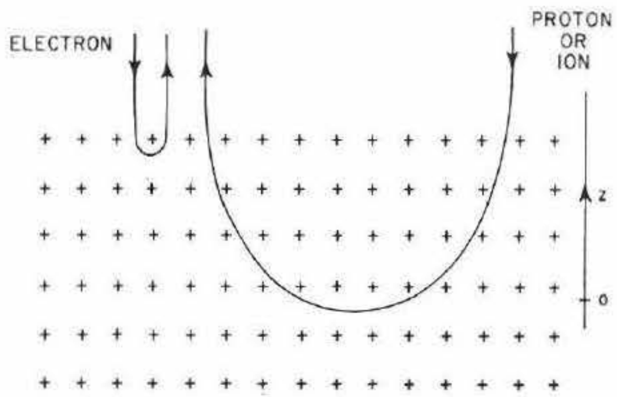

Fig. 15. Schematic representation of the boundary region with electrons and protons normally incident on a uniform magnetic field. The plus signs denote magnetic-field lines pointing upward out of the page.

over distances of more than a few meters the electrons and ions cannot be separated. So when the electrons are stopped by the magnetic field, electrostatic forces prevent the ions from penetrating very much deeper than the electrons; this was pointed out by Chapman and Ferraro about 35 years ago.

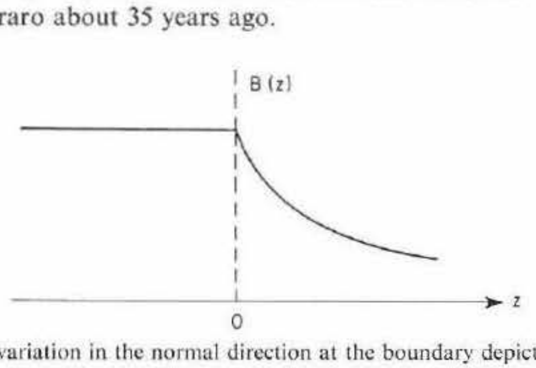

Fig. 16. Field variation in the normal direction at the boundary depicted in Figure 15.

In the magnetic field of the earth, however, the lines of force near the magnetopause all connect down to the ionosphere which acts as a short circuit. Whereas a potential difference between lines of force far out in the magnetosphere will not cause a current to flow across the lines of force, electrical current will be produced in the ionosphere. As the electrons incident at the magnetopause are turned back and prevent the protons from penetrating much further, the electric field between lines of force near the surface will rapidly propagate down to the ionosphere. There a current flows which probably sends electrons up along the lines of force to neutralize the ion space charge. After a brief period the ion trajectory relaxes into a final equilibrium configuration in which there is no potential difference across the lines of force. In the terminology of plasma physics one would say there are conducting end plates and the lines of force are all shorted together. A round number for the length of time for this shorting out process to occur is one minute. The shorting goes part of the way very rapidly – in the first second – and then it depends upon whether the ambient plasma density is more or less than the incident beam density. Hence it may take about a minute for complete relaxation to occur, although some of the neutralization takes place in the first second. The subsequent remarks here relate to the final equilibrium state where there are no longer any electric fields.

C. OBLIQUELY INCIDENT PARTICLES

The material in the above section relates to the kinds of discussions that have been going on the last 30 years or so. Next I would like to point out a difficulty in this rather simple picture (Parker, 1967; Lerche, 1967). So far the treatment has been confined to normal incidence of the particles; this is a mathematically ideal case and does not represent the actual circumstances in space.

There is plasma both inside and outside of the magnetopause. It is convenient to choose a frame of reference in which the plasma inside the magnetopause – or a piece of it – is at rest. The bulk velocity of the streaming plasma outside will then be several hundred kilometers per second, and the value 400 km/sec will be taken as representative. The thermal velocity of the gas outside is rather less than the wind velocity except at the stagnation point of the flow at the subsolar point. Individual particles are incident upon the magnetopause at an angle as shown in Figure 17. Due to the normal component of their velocity, the particles will turn back out of the field as before (cf. Figure 14). The additional component of the particle motion in the tangential direction will constitute a current. This is an ion current. The incident electrons hardly penetrate the surface at all and will be dealt with below.

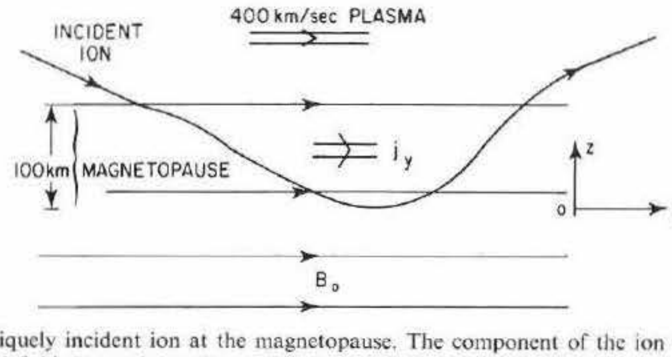

Fig. 17. Obliquely incident ion at the magnetopause. The component of the ion velocity parallel to the internal field B_0 produces the current j_y. The coordinate axes are chosen as indicated: the z-axis is normal to the interface and $z=0$ is defined by the ion turning point; the y-axis is in the direction of the field B_0 that is to be confined and is essentially parallel to the incident solar plasma-flow direction.

The ion current in the direction of the magnetic field creates a problem that is easiest to explain by adopting the artifice that the solar wind is switched on suddenly at some initial time. This makes the exposition time-dependent, but some aspects of the steady-state situation will be discussed later, as far as possible. When the wind is first switched on, the ions do not penetrate any farther than the electrons for the first second or so. It is some time later that ions are able to deeply penetrate the field and generate the current j_y shown in Figure 17. As this ion current is switched on, it will induce an equal and opposite current in the background plasma. This latter current must flow down through the ionosphere, and a rough estimate, as will be shown shortly, suggests that this induced current dies out in a minute or two. After that, the raw solar wind ion current j_y remains, passing through the lower part of the 100 km magnetopause.

This current j_y is not the current that terminates the magnetic field of the earth; the termination is effected by currents across the field lines as in Figure 14. The current j_y generates a new field not considered before. In attempting an analysis in which pressures are balanced at the magnetopause, one finds there is no way to balance the field generated by the current j_y of the penetrating ions. The currents j_y generate a field across the initial field direction and there is no particle pressure left with which to compress this additional component. Perhaps the electrons could be heated to

compress this component, but the degree of heating is critical and the problem is not entirely solved. With no way of balancing this extra field component, there would be no equilibrium solution for the boundary.

It should be kept in mind that saying there is no equilibrium is not the same as saying there is an instability. If there is no static equilibrium then I do not know how to proceed, since all of our techniques for handling such problems involve perturbing equilibria. In the end I will suggest that the magnetopause is a bit unsteady in time. The argument will be: since I can find no steady solutions and since there are solutions, as determined by observation, the solutions must be unsteady in time. I am at a total loss to describe the unsteady solutions in detail.

In order to demonstrate the equilibrium dilemma mathematically, the simplest possible case of a monoenergetic or cold wind will be treated (PARKER, 1967). The analysis has been done in more complexity (LERCHE, 1967), but the following will suffice. The particles interpenetrate as in the original model of Chapman and Ferraro. The incident particles each have the velocity $\mathbf{w}$ and the coordinate system is chosen so that w_y is essentially the wind velocity, and w_x and w_z represent the thermal velocities in the wind in so far as the analogy applies in this simple case. The magnetic-field component taken to represent the geomagnetic field is $B_y(z)$ so that all variation is in the z-coordinate; this is the field that is to be confined by the beam of ions. As already suggested, whether we like it or not, a field component $B_x(z)$ will also be generated. A particle incident on this magnetic field will penetrate the field, being deflected in a forward loop, and then emerge back into the wind. An observer sitting out in the wind will see the particle hit the field and bounce, undergoing the usual specular reflection. The equations describing the particle trajectories are

$$\ddot{x} = -\frac{q}{Mc}\dot{z}B_y(z), \quad \ddot{y} = \frac{q}{Mc}\dot{z}B_x(z)$$

$$\ddot{z} = \frac{q}{Mc}\left[\dot{x}B_y(z) - \dot{y}B_x(z)\right].$$

These equations may easily be integrated in terms of new functions F and G introduced to represent the magnetic field in the form

$$\frac{\mathrm{d}F}{\mathrm{d}z} = -\frac{qB_x(z)}{Mcw} \quad \frac{\mathrm{d}G}{\mathrm{d}z} = -\frac{qB_y(z)}{Mcw}. \tag{69}$$

With these definitions the field strength is expressed in units of the particle radius of gyration in the field. The equations of motion are now

$$\ddot{x} = w\frac{\mathrm{d}G}{\mathrm{d}z}\dot{z} = w\frac{\mathrm{d}G}{\mathrm{d}t}$$

$$\ddot{y} = -w\frac{\mathrm{d}F}{\mathrm{d}z}\dot{z} = -w\frac{\mathrm{d}F}{\mathrm{d}t}$$

$$\ddot{z} = w\left(\dot{y}\frac{\mathrm{d}F}{\mathrm{d}z} - \dot{x}\frac{\mathrm{d}G}{\mathrm{d}z}\right), \tag{70}$$

where $(\mathrm{d}F/\mathrm{d}t)$ and $(\mathrm{d}G/\mathrm{d}t)$ are the time derivatives of F and G observed by a person moving with the particle. The first two equations in this last set integrate directly:

$$\dot{x} = wG \qquad \dot{y} = -wF, \tag{71}$$

where the constants of integration have been absorbed into F and G. With these solutions and multiplying (70) by $\dot{z}$,

$$\dot{z}\ddot{z} = -w^2\left(F\frac{\mathrm{d}F}{\mathrm{d}t} + G\frac{\mathrm{d}G}{\mathrm{d}t}\right)$$

and this integrates directly to give

$$\dot{z}^2 = w^2(1 - F^2 - G^2). \tag{72}$$

The particle is moving in a constant magnetic field so its speed remains constant, and the constant of integration is specified in (72) by the requirement that when $F=G=0$ then $\dot{z}=w$.

With the particle equations of motion solved, the remaining half of the problem is to satisfy Maxwell's equations. The current density $\mathbf{j}$ satisfies the relations

$$4\pi j_x = -c\frac{\mathrm{d}B_y}{\mathrm{d}z} \qquad 4\pi j_y = c\frac{\mathrm{d}B_x}{\mathrm{d}z} \tag{73}$$

and is related to the particle motion by

$$j_x = 2N(z)q\dot{x} \qquad j_y = 2N(z)q\dot{y}. \tag{74}$$

Here $N(z)$ is the particle number density at a given level, and in order to conserve particles

$$N(z)\dot{z} = N_0 w_z, \tag{75}$$

where N_0 is the initial beam density and w_z the initial velocity along z. The factor of 2 in (74) accounts for the fact that there are inward and outward going particles in equal amounts. Combining the requirements of (74) and (75) with the solutions (71) and (72) for the particle motion, the Maxwell relations in (73) become

$$\frac{\mathrm{d}^2G}{\mathrm{d}z^2} = \frac{8\pi N_0 q^2 w_z}{Mc^2 w}\frac{G}{(1 - F^2 - G^2)^{\frac{1}{2}}} \tag{76}$$

$$\frac{\mathrm{d}^2F}{\mathrm{d}z^2} = \frac{8\pi N_0 q^2 w_z}{Mc^2 w}\frac{F}{(1 - F^2 - G^2)^{\frac{1}{2}}}. \tag{77}$$

The coefficients in these last two equations contain the plasma frequency of the incident beam.

In order to solve these differential equations the boundary conditions must be specified. This is done in terms of a pressure balance:

$$\frac{B_0^2}{8\pi} = 2N_0 M w_z^2 + \frac{B^2(h)}{8\pi}. \tag{78}$$

The pressure on the left is exerted by the geomagnetic field being compressed, and this is balanced by the pressure of the particles incident normally on the surface plus the

external field pressure. The latter is represented in terms of the field $B(h)$ where h is some arbitrarily large height above the boundary. It would be nice to be able to set $B(h)\equiv 0$, so that there is no field at infinity, but it turns out that this is not possible. Introducing the particle cyclotron frequencies in the fields B_0 and $B(h)$,

$$\Omega_0 = \frac{qB_0}{Mc} \qquad \Omega(h) = \frac{qB(h)}{Mc} \tag{79}$$

Equations (76) and (77) may be written in the form

$$\frac{\mathrm{d}^2G}{\mathrm{d}z^2} = \frac{\Omega_0^2 - \Omega^2(h)}{2ww_z}\frac{G}{(1 - F^2 - G^2)^{\frac{1}{2}}} \tag{80}$$

$$\frac{\mathrm{d}^2F}{\mathrm{d}z^2} = \frac{\Omega_0^2 - \Omega^2(h)}{2ww_z}\frac{F}{(1 - F^2 - G^2)^{\frac{1}{2}}}. \tag{81}$$

These equations are non-linear but possess a certain symmetry that enhances the possibility of direct integration. For (80) and (81) can be written in the form

$$\frac{\mathrm{d}^2G}{\mathrm{d}z^2} = -\frac{\partial V}{\partial G}, \quad \frac{\mathrm{d}^2F}{\mathrm{d}z^2} = -\frac{\partial V}{\partial F}, \tag{82}$$

where

$$V = C(1 - F^2 - G^2)^{\frac{1}{2}} \tag{83}$$

$$C = \frac{\Omega_0^2 - \Omega^2(h)}{2ww_z}. \tag{84}$$

In this form, the problem is analogous to one in particle mechanics where z represents the time, F and G represent the coordinates, and V represents the potential. The potential V has a relatively simple form, being dome-shaped as indicated in Figure 18. The surface defined by V is an oblate or prolate spheroid, depending on the value of the constant C. This analogy is worth developing in order that the solutions to (82) can be understood without doing much mathematics.

The functions F and G are the coordinates of the fictitious particle moving in the potential V, and the 'velocity' components $(\mathrm{d}G/\mathrm{d}z)$ and $(\mathrm{d}F/\mathrm{d}z)$ represent the field strength as a function of z in accordance with the definition (69). Let $z=0$ be defined by the turning-point in the boundary layer field of the incident beam of particles. Then at 'initial time', $\dot{z}=0$, and from (72) this requires the fictitious particle to be located on the unit circle in the FG-plane where $V=0$. Approaching $z=0$ from negative values, the field is B_0 in the y-direction; B_x is zero since the particles penetrate only down to $z=0$. Hence, from (69), $\mathrm{d}F/\mathrm{d}z=0$ and $\mathrm{d}G/\mathrm{d}z\neq 0$ at $z=0$, so that at $z=0$ the trajectory of the fictitious particle is parallel to the G-axis, as shown in Figure 19. The particle trajectory ends in the incident or reflected beam of particles. Suppose that the incident particles form a stream with velocity U tangent to the boundary plus a non-vanishing component w_z perpendicular to the boundary, so that for the total beam velocity $w^2 = w_x^2 + U^2 + w_z^2$. Then at the end point, labeled by the subscript e,

$$\dot{x} = wG_e \qquad \dot{y} = U = -wF_e,$$

where U is the stream velocity. If the latter is large, then $F_e \approx -1$ and $G_e \approx 0$. $\dot{x}$ is unchanged by the specular reflection and the case $\dot{x}=0$, where $F_e=-1$ and $G_e=0$, is shown in Figure 19. In any case, the game to play to determine the trajectory of the fictitious particle is to find the curve that starts on the unit circle and connects to the desired end point.

The case of normal beam incidence is the simplest to describe since the trajectory goes vertically from the unit circle to the origin as shown in Figure 19. (This solution can be carried out in terms of elementary functions and is the familiar one that can be found in the literature.) The fictitious particle moves exactly on the G-axis, coasting rapidly at first and slowing down to come to rest right at the origin. For any other end point, the trajectory is more complicated, and, because the fictitious

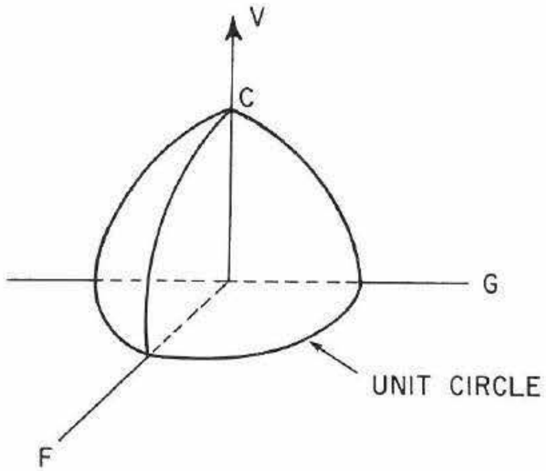

Fig. 18. The spheroid defined by the fictitious potential V. The intersection with the FG-plane, in which the fictitious particle moves, is a unit circle.

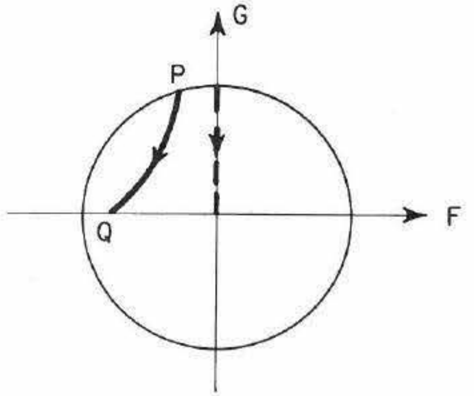

Fig. 19. Sample trajectories for the fictitious particle moving in the FG-plane. The particle starts at point P and moves toward point Q. For normal incidence the particle moves vertically down the G-axis.

particle is sliding off the dome-shaped potential, the end-point velocity is not zero (as at point Q in Figure 19). Since the velocity of the fictitious particle represents the magnetic field, the end-point magnetic field $B(h)$ is not zero. This is the dilemma: we set out to terminate a magnetic field with a plasma and find that generally there are no equilibrium solutions with vanishing fields at infinity. If the plasma is incident normal to the field, confinement is possible, but not when the plasma is also sliding along the field.

D. DIFFICULTIES IN PRODUCING CONFINEMENT

The next question to consider is what can be done to avoid this non-equilibrium result. The difficulty arises because the ions swoop deeply into the field producing the current j_y which generates the field B_x. The latter represents the end-point velocity, as at point Q in Figure 19. When the particles enter the field B_0, they are deflected downward and back out into the wind and exert a net *outward* force on B_x. The tendency is to push B_x out into the wind and not to confine it. One possibility to produce an equilibrium is to have enough hot electrons exerting a sufficient pressure to balance the tendency for the field B_x to move outward. The electron pressure required for this can be determined by calculating the field $B(h)$ in the above model.

From (82), the energy equation for the fictitious particle motion is simply

$$\left(\frac{dF}{dz}\right)^2 + \left(\frac{dG}{dz}\right)^2 = 2(E - V) = 2C\left[1 - (1 - F^2 - G^2)^{\frac{1}{2}}\right], \tag{85}$$

where $E=C$ is the conserved energy. From (69) and (72), asymptotic or end point values are given by

$$\left(\frac{dF}{dz}\right)^2 \sim \frac{B^2(h)}{w^2}, \quad \left(\frac{dG}{dz}\right)^2 \sim 0, \quad \frac{w_z}{w} = 1 - F^2 - G^2$$

since $B_x(z) \sim B(h)$ and $B_y(z) \sim 0$. Inserting these values into (85) and using (84), the field pressure at $z=h$ is simply

$$\frac{B^2(h)}{8\pi} = \frac{B_0^2}{8\pi}\left(1 - \frac{w_z}{w}\right). \tag{86}$$

The field $B(h)$ is in the x-direction and so is rotated 90° from the field $\mathbf{B}_0$. When the incident beam is normal to $\mathbf{B}_0$, then $w_z = w$ and the field vanishes at the top: $B(h)=0$. But when the direction of incidence becomes more and more oblique, the fraction (w_z/w) decreases toward zero and can be neglected at small angles. In this limit the field has not diminished at all and our best efforts have merely rotated the field by 90° without diminishing its magnitude.

At this point I would like to introduce electrons to push down on the field to produce the confinement, and physically it seems to me that this should work. Unfortunately I mentioned this analysis to Lerche one day and he set out to solve the problem using a distribution function in place of the monoenergetic beam used here. The mathematics is much more complicated, but he has worked it out for a general distribution of both ions and electrons. He has proven that for any analytically well-behaved distribution function, with an arbitrary ratio of the ion to electron temperature, there is no equilibrium (static) solution. So although I would expect that the electrons should help out, something is wrong with the idea. I think the electrons do bring things a little closer to equilibrium, but still there is no rigorous equilibrium.

To ask what this all means turns out to be a hard question. The analysis shows that there is no simple equilibrium in the way we had always imagined. On a small scale there seems not to be an equilibrium. Let me hazard a guess – call it a prediction if you like – as to what happens at the magnetopause. Once electrostatic neutrality is achieved, so that the ions penetrate more deeply than the electrons, and after waiting one more minute while the induced electron currents die away, then you find that a field is generated which cannot be confined. That field springs upward into the wind, probably in a non-uniform way, and my guess is that it is simply convected away in the wind. Of course this would expose the next layer of field underneath, and the whole process repeats itself in some characteristic way every couple of minutes. In the actual case, it is probably not a nice periodic peeling of layers 100 km thick off the magnetosphere; it is probably a more continuous process. I think the net effect will be to enhance the mixing of plasma and field at the magnetopause boundary. I am not predicting that you will see a simple 90° rotation in magnetic field across the magnetopause as obtained in the above perfectly static model. The system had to be held down in an artifical way to get that result. I think that in the actual case the lines of force spring away from the magnetopause, probably being rotated some in being carried away by the wind. I do not know how to calculate meaningfully how rapidly the process occurs, but taking a characteristic time of a couple of minutes you can peel off enough lines of force in a few hours, or a day at least, to account for the total flux in the geomagnetic tail. But this is based upon an extremely rough calculation and I do not know whether it should be taken seriously. The one conclusion I can make is that the magnetopause structure is more complicated then we had imagined. There are no static solutions – it is a time-dependent thing – and the problem certainly deserves a lot more scrutiny.

The above calculations are on a small scale – 10^2 km – and the findings in no way invalidate the calculations that have been done on the magnetopause shape. Beard, Davis, and Slutz were interested in the large-scale effect – 10^5 km – and the little local tempest in the 10^2 km boundary thickness in no way upsets the overall momentum balance that the large-scale calculations are based upon. The small-scale effects have only to do with the rate at which the field mixes into the plasma. As you know, this is already a sticky question involving rates of merging of oppositely directed fields, etc.

Appendix

A. PREFACING COMMENT

Editor's Note: Professor Parker, noticing that all the tutorial lecturers were in attendance, initiated his first lecture with the comment: "Most of the tutorial lecturers are curious to find out what the others are talking about so we won't duplicate each other. I suspect that in a few days there won't be so many speakers present. We all have had the benefit of a long quiet airplane ride to prepare our first lecture, and once we have shot that off then we will have to sit down and go to work. I just didn't want there to be any disappointment if some of you notice that we weren't showing up. It isn't that we're not friends or not enemies anymore, it's just that we have other things that we have to do."

B. ON THE IRROTATIONAL NATURE OF THE GEOMAGNETIC FIELD

Carovillano: In your discussion of Equation (35), you would regard the currents, say, due to gradient drift as a secondary effect in violating curl $\mathbf{B}=0$.

Parker: No. Let's talk about the steady situation since this is the basic question. If you put in particles, you make a pressure which distorts the field. And incidentally there are currents flowing because Maxwell insists that whatever we do we must satisfy curl $\mathbf{B}=(4\pi/c)\,\mathbf{j}$. If you ask why do the currents flow, you find that when you introduce the distortion you introduce an e.m.f. which induces them and causes them to flow. That's probably the simplest way to say it.

Carovillano: But this actual total $\mathbf{B}$ would not be derivable from a potential.

Parker: Not from a scalar potential, not once you have distorted the field. Only when you keep all particles out of it can you describe it by a scalar potential. That's right.

C. ON THE ROTATION OF THE TAIL MAGNETIC FIELD

Kahalas: Is the magnetic field in the tail rotating with the earth?

Parker: The tail always points in the anti-solar direction, but as far as one knows the lines of force remain connected to the earth. So as the earth revolves, the fields in the tail revolve. I don't know how one would detect the rotation exactly because there is only a rather weak electric field and some plasma motions of rather low velocity, but I'm fairly sure it rotates.

D. DISCUSSION ON FIELD-LINE CONVECTION

Baggerly: What is the geographic scale of the phenomena [for the model portrayed in Figures 5 and 6]?

Parker: They would extend over the polar regions to some 45° either side of the pole. This isn't a precise figure but it is roughly the scale of these things. The scale is very large and that is why such large potential differences occur. The potential is only about 10^{-4} V/cm but extended over 10000 km and this provides a large potential difference.

Question: I don't clearly understand what you mean by convection, and secondly what is special about $x=0$ in Figure 6?

Parker: You mean why is $x=0$ the center of the field pattern? The center of the field pattern is determined by wherever the edges are, and $x=0$ is just midway between. This defines the middle of the pattern.

Question: In other words, the plates [at $z=0, h$] are not infinite in extent?

Parker: No, I was just too lazy to put in the boundaries; it messes up the algebra.

Question: And by convection you mean the horizontal motion of the top plate?

Parker: Yes. The top plate represents the ionosphere which is at the feet of the lines of force. The ionosphere and everything above, along these lines of force, turn around in the same way.

Dessler: By convection you don't mean driven by **E**, like ordinary convection; you mean a circulation pattern.

Parker: I mean a circulation pattern, yes. A circulation driven by forces as yet unidentified.

Dessler: May be identified.

Parker: Well, since the two antagonists are sitting right here in front of me, I refuse to get involved. How many of you have seen the movie, 'A Man for All Seasons'? I'm Sir Thomas More at this point, and I ain't going to sign any oaths, I tell you! Yes, there are a lot of ideas as to what drives the convection, and one or more of them may be correct.

Question: What actually is moving at 1 km/sec?

Parker: The feet of the lines of force, during active periods.

Question: Not the electrons, or protons, or...?

Parker: Oh sure, all the material moves along the lines of force. This can be illustrated from Figure 3. Consider two flux tubes arbitrarily chosen from the entire field and let these two flux tubes be centered about the lines of force that pass through the points P_1 and P_2 in Figure 3. Now I will perform an operation. I will paint all of the charged particles in the flux tube labeled by P_1 red, and all in the tube labeled by P_2 green. Electrons, protons, anything that has charge on it and is tied to the magnetic field is colored. The convection that I am talking about is simply a continuous deformation, an interchange, of these two flux tubes. The intervening field simply gets out of the way and lets them through in the interchange. When the interchange is completed, the green and red particles have changed places. What was green originally is now red, and what was red is green. Everything convects. The material convects too, and that is the only way that I can identify a line of force. Remember, if the material were not tied to the lines of force, there would be no way of really talking in a meaningful way about a given line of force. We must identify the convection with the material on the lines of force. The material actually overturns too. And this overturn takes place in a day or less, particularly during active periods when the convection rate is as much as 1 km per second. The velocity 1 km per second is for the motions in the ionosphere at the feet of the lines of force. These lines extend outward into the magnetosphere and there the velocity is rather higher, reaching about 20 km/sec during active periods.

Carovillano: Why don't the colored particles intermix?

Parker: Because their drift velocity is relative to the lines of force and for these low-energy particles the drift is very slow compared to the convection velocities. If I were dealing with the high-energy particles among the trapped particles, these go drifting through so quickly that they hardly partake of the convection motion. But if I'm dealing with thermal particles, which is what I have in mind, they drift slowly. The drift is proportional to the square of the velocity and it is extremely slow. So to a first approximation each particle just sits on its line of force and goes around in a small cyclotron circle and goes along with the ride wherever the line of force is headed.

Question: What energies would you be talking about? These are thermal particles of what energy, sir? Would the energy be a fraction of an electron volt?

Parker: Yes.

Question: Would this apply then to temperatures in the *F*-region?

Parker: It applies in order of magnitude roughly from the *F*-region and above.

Question: But how far can one extend this outward in the polar regions?

Parker: You can extend it out up to the magnetopause at about 10 R_E – as long as you have particles around to identify the lines of force, and as I remember the estimate for the density way out there is a particle or a fraction of a particle per cubic centimeter. I think Professor Helliwell will have more details on this kind of thing.

Question: Will you compare the number density of high-energy trapped particles with thermal trapped particles?

Parker: The number density of the high-energy trapped particles is extremely low compared to the densities of the thermal particles. It depends on how far down the spectrum you want to go, but 10^{-5} or 10^{-8} per cubic centimeter [is a representive density for high-energy particles].

Van Allen: Well, if you go down to a kilovolt, Gene, then you get comparable number densities beyond an L value of about 3 or 4, namely, of the order unity.

Parker: Well, at $L=3$ or 4 the thermal background is 100 or more. I was going clear out to the magnetopause, and there of course both densities are down some. At a distance of about 4 R_E the thermal plasma has a density of a couple hundred particles per cubic centimeter and Van and Lou Frank say we get about 1 particle per cubic centimeter at a kilovolt. Is that fair, Van? And farther out I know the thermal density drops off and I am sure the high-energy particle density drops off too.

E. ON THE PRODUCTION OF AURORAL PARTICLES BY CONVECTION

Carovillano: Would the change in energy in the acceleration of auroral particles be an electromagnetic effect, i.e., due to electromagnetic fields?

Parker: Well, yes and no. It's simply adiabatic compression of a particle, just as when you put a gas in a cylinder and squash it down. It's adiabatic compression. If you actually follow the detailed motions of the particles, then yes, electric fields are involved. In fact some people would call this the betatron effect. But one doesn't have to ask that kind of a question as long as he stays with thermodynamics. Mainly you compress the particles enormously, and you heat them up by a factor of 1000 or more.

Carovillano: Normally, the magnetic moment is conserved.

Parker: It is just adiabatic compression, yes. You see this is two-dimensional compression. Effectively, $\gamma=2$ [for magnetic moment conservation] where

$$\gamma = 1 + 2/f$$

and f denotes the number of degrees of freedom. The temperature T is related to the density N by

$$T \propto N^{\gamma-1}$$

and all [the convection argument says] is that the temperature is proportional to the density of the material which is frozen into the field. Both the first and the second invariants are nothing more than statements of adiabatic compression. The longitudinal invariant says that the velocity along the field times the length between the bounce points is a constant. And you'll find that's just adiabatic compression in one dimension, with $\gamma = 3$. To my own taste at least, in treating the acceleration of particles I think it is much more profitable to think in terms of simple adiabatic compression than to get all tangled up in the details of what fields are directly involved. Always do thermodynamics if you can, and if you can't, then you have to go into details. But the present case is sufficiently simple. That's why I've sort of dropped betatron acceleration from my vocabulary. I don't know what the word means anymore.

Question: Is this a night-time phenomenon which you are describing here? Is that correct? Does that eliminate the possibility of day-time aurora which has been specifically observed?

Parker: No. If you look at the detailed curves that Taylor and Hones grind out – I'm trying to remember now – you get more precipitation on the night side than on the day side. It just happens to be the way the longitudinal and transverse invariants bring the particles down to the surface of the earth. I don't remember the details well enough to tell you what it implies for daytime aurora, but as I remember, it brings most of the particles on the night side, which is where you want them.

Question: Would this predict at what time of the night [auroras occur] – evening, midnight, or morning?

Parker: It does, but again I couldn't recite in great detail. I would emphasize though that whatever it predicts, whether or not it agrees in detail with this or that observation, the calculations definitely do not give the individual auroral forms. There is more to it. [Convection provides] a general supply of particles and there's something else that shapes the curtains and the rays and all the rest of it. So I look upon this as a kind of a gross overall feature with a lot left to be done if one wants to understand the detailed auroral forms.

Question: If the particles are convected in, would they not be convected out again?

Parker: If they don't hit the earth, they are convected back out. If they hit the earth, they are stopped and do not get convected back out.

Question: So if you regard the low-energy trapped particles, then they wouldn't be trapped since they would convect back out.

Parker: No. This convective pattern is not so steady. A particle which comes in from one place will go out in another. Orbital patterns are highly irregular. It is just like the current in a river; it kind of sloshes about a bit. A particle may go around a hundred times before it happens to find its way out of the magnetosphere again.

Question: You used conducting plates extending to infinity in your discussion [cf. Figure 5]. What happens when there are tubes of ionization in the space between these plates?

Parker: The dense non-conducting atmosphere is between the two plates and tubes of ionization are not present here. Above the upper plate, above the ionosphere, there would be tubes of ionization and these would be convected around with the field.

Question: Tubes of ionization are produced between the plates by particle precipitation?

Parker: So you mean what happens if something punches through the upper plate. Well, then you would simply lower the ionosphere a little bit. Lower the plate. Remember the plate is merely an idealization for the bottom of the ionosphere. If you poke a hole in it somewhere, then you simply lower the level of the plate a little bit. The point is that an aurora never comes down to the ground level. It's always 70–80 km up.

Question: What are convection circulation times?

Parker: During active times, as I remember, it goes around in about 6–10 hours. During quiet times it goes around much more slowly. Roughly it is the characteristic time for auroral storms, and so forth, which build up.

Question: What is the ultimate energy source of the convection?

Parker: That is the next point to discuss. Something is needed to drive the convection. The convection seems to be there. Now, what makes the convection go? There have been a number of suggestions but I don't think the question is at all settled. The suggestion that Axford and Hines made – but it was only a suggestion and the validity [of the convection theory] in no way depends upon it – is simply that you have friction in the solar wind due to the viscosity in the gas. This would tend to push the outer boundary in the direction of the flow which tends to push some of the inner material along with it. Then as you begin to pile up too many lines of force, they squeeze the others out and force the return flow. I think this is probably the most plausible of all the suggestions, but I don't know if it is the right one. The point is that convection is there and one needs to look for a mechanism which will drive it.

It has also been suggested that the convection can be driven by particles trapped in the field. But to my way of thinking that's the tail wagging the dog. I think the convection produces particles in the geomagnetic field, and not vice versa. Instabilities in the boundary, non-equilibrium, anything that will let the wind get a grip on the lines of force in the tail would contribute to driving the convection. This problem is not unrelated to the formation of the geomagnetic tail which also requires a kind of grip to stretch it outward. There may be some connection between the two mechanisms. Dungey and some of the others may talk about this a little later. I think that one just doesn't know for sure exactly where all the forces come from that drive the convection.

F. THE DIRECTION OF THE TAIL

Question: You mention that the orientation of the tail is away from the sun. Is the direction of the tail to be understood as radial, or could it also be 45° or so from this?

Parker: It is radial, in the direction of the wind, in other words.

Question: It must be in the direction of the radial wind?

Parker: Yes. I say it must be, not understanding what makes the tail. But if in some way I assume it is the pressure of the wind, this would certainly orient the tail around so it would be blowing down stream like a wind sock.

Question: But what about the magnetic field in the solar wind?

Parker: That doesn't have much effect. It just rides along and there's no reason why the tail would line up with the field. There are some effects due to variations in Mach number with field changes, but these are smaller than what we are talking about here.

G. ON THE INFLATION OF AN AMBIENT FIELD

Carovillano: $\Delta\mathbf{B}$ is some part of $\mathbf{B}$. How do you pick $\Delta\mathbf{B}$ out of $\mathbf{B}$? [See Equation (51), ff.]

Parker: $\Delta\mathbf{B}$ is the part of the field associated with the distortion of the field, through curl $\mathbf{B}$.

Carovillano: So the total field has a part with zero curl, which is an external field to the region, plus the intrinsic field to the region which is $\Delta\mathbf{B}$.

Parker: I would say that $\Delta\mathbf{B}$ is just a distortion of the intrinsic field; I don't think of it as having external sources.

Carovillano: But the distortion $\Delta\mathbf{B}$ takes place outside the sources of the intrinsic field.

Parker: The total curl present in the field is contained in $\Delta\mathbf{B}$ and the rest of the field is curl free, so that the total field can be written as

$$\mathbf{B} = -\nabla\psi + \Delta\mathbf{B}$$

and the gradient has zero curl.

Carovillano: So you want to inflate the original field outside the domain of its own sources; otherwise you could not maintain the form of this equation.

Parker: Yes, that's right. In application there is a dipole of zero size at the origin so there is no problem. You could include source forces if you wanted to, but I don't use it that way.

Carovillano: It is clear in using the original expression (51) for $\Delta\mathbf{B}$ that if the field $\mathbf{B}$ were approximated to zero order where it is derivable from a scalar potential, then $\Delta\mathbf{B}=0$ results. You seem to go from (51) to (53) or (54) without changing anything, but you do not get zero there in making the same zero-order approximation.

Parker: That's because I have fed in some information, namely, the relation (52) between the curl and the pressure gradient. With the curl removed, if I approximate the field a bit I won't get zero.

Carovillano: But there is an imbalance; with the same approximation the left side of (52) is zero and the right side is not.

Parker: Let me not make any approximations so that (52) applies (with or without the term involving the field aligned currents). I am now at liberty to approximate the field a bit, and if it is not too badly distorted I can replace the field by the undistorted dipole. I am no longer taking the curl of the field. I agree that the curl of the undistorted dipole field is zero, by definition. Your intuition is bothering you, but this time I'll stand by the mathematics. [*To the audience:*] You see, usually it's the other way around. He complains that my mathematics is sloppy and I tell him the result is obviously right.

H. ON THE ENERGY CONTENT IN AN INFLATED FIELD

Carovillano: Would it really be disturbing if, say, the value $\alpha=2$ were used in Equation (65)?

Parker: That value would be hard to define; my definition depends upon the field not being distorted too much. I don't quite know what you mean – in terms of the distorted or undistorted field?

Carovillano: In terms of the field that you're going to work from in defining the particle energy.

Parker: That would be the field before I put in the gas. Yes, I would be surprised. I have some asymptotic solutions for which, as I recall, the value $\alpha=2$ is too large. But only the cube root of α enters the discussion here so its actual value is not too critical.

J. ON THE ASYMMETRIC INFLATION OF A DIPOLE FIELD

Dessler: You are refering to the result at the origin [cf. Equation (67) and Figure 10].

Parker: Yes. If I move away from the origin the field increases in some regions and decreases in others, and life gets extremely complicated. Cummings has worked with models, although he didn't include an ionospheric cross-over segment. That would be the next degree of complexity that should be done.

Dessler: No, he included the cross-over in an analogous situation.

Parker: I see. He was trying to convince me that the cross-over was not important, and that's why I got the impression he had not put it in.

Dessler: He wiggled the current around to see its effects.

Parker: You can see what enormous world-wide variations you are going to get. In the symmetrical case the field is nicely uniform across the earth. But as soon as you have cross-over segments right overhead, a big effect comes mainly from them. It gets extremely complicated and somewhat less interesting, even though it may still be important.

Carovillano: There would still be an average increase over the world.

Parker: That's right, and that increase is correctly given by ΔB at the origin. The Laplacian of the disturbance field is zero so the field at origin is an average of the field over any small sphere drawn about the origin. So it is still significant to compute the field at origin. But although it gives the world-wide average, the field actually varies an awful lot over the world. The next step is to go into the complexities of the ionosphere. Here I sort of tell myself: Well, I've got the principles straight and I'll quit while I'm ahead.

K. ON THE GEOMAGNETIC CONFINEMENT BY OBLIQUELY INCIDENT PARTICLES

Maguire: I know you covered this, but would you say again why in this particular situation you won't get the same shorting-out of the electric field between the electrons and protons that you do in the normal incidence case?

Parker: I do get the same shorting-out. If I wait a few seconds the electric fields are communicated to the ionosphere and shorted out so that there should be no electric fields in the picture as I have drawn it.

Dessler: Why don't you draw the whole magnetosphere to show where you are?

Parker: If I take you literally and the magnetosphere is represented in Figure 12, then I'm right about at point P. But that isn't what you want.

I have in mind a piece of the magnetosphere somewhere like region A of Figure 12, say back in the tail. The plasma is rushing by, and if I had a suitable microscope I would see the individual particles exerting a normal pressure which confines the geomagnetic field. The transverse current needed to terminate the field is provided by the Lorentz force that deflects the normal component of particle velocity. In addition, I get something that I didn't want – or didn't bargain for at any rate – namely, a current parallel to the lines of force. And currents in their own perverse way tend to generate lines of force perpendicular to themselves, which puts a layer of rotated field – by 90° – at the magnetopause.

Maguire: That's the point I don't understand. In order to get that current you have to have a charge separation between the protons and electrons.

Parker: That's right.

Maguire: And why doesn't the electric field pull the two together like it does out in the nose?

Parker: Well, as I explained, there isn't an electric field here because electrons are drawn up from the ionosphere to neutralize the space charge of the ions. I pointed out that in the first minute you will, in fact, induce an equal and opposite current in the electrons. But that also has to go through the ionosphere and, as nearly as I can estimate, dies out in a fraction of a minute. Then I'm left with a net current j_y and no electric field.

Questioner A: After you turn the solar wind on at a given time with an infinitely long region, in order to keep charge neutrality wouldn't you have to drive more and more electrons out so that all the background electrons would go along with the protons?

Parker: I'm not sure what you mean. You say with an infinitely long medium; of course then it would take infinitely many electrons. Is that what you are referring to?

Questioner A: Yes, it would take more and more electrons.

Parker: Well, let's say that I have a region of some long finite length so that a finite number of electrons do all this neutralizing for me.

Frankenthal: But you are asking for them to be stationary and not to have the same drift velocity that the ions would have.

Parker: I would very much like to have them moving with the same velocity as the ions for it would get me out of my difficulty. But the electron current would have to flow through the ionosphere, and I think that within a minute it will stop flowing. You see these electrons would like to move along with the ions, but now there is a resistive part of the circuit through which they must flow and there is only an initial inductive field to get them moving. Thus the electron current would jump and decay with whatever characteristic time this resistor provides them.

Questioner B: If you had a finite length then the protons can't get out either.

Parker: Why not?

Questioner B: It depends on some sort of an end cut, shaven off in space, that creates a magnetic field.

Parker: Well, I can put the ends far enough away, as long as it is finite distance. I don't think the end effects are essential here. The magnetopause is only 100 km thick and as long as the ends are a couple of hundred thousand kilometers away I don't think we have to worry about them. We can just say that they are neutralized suitably.

Dessler: With a cylinder you don't have end effects to worry about, i.e., so long as you have cylindrical symmetry.

Parker: No, the length involved is out along the tail. I don't want to become involved in any argument as to how long the tail is, but it's long anyway – a million kilometers probably.

Ness: The tail field that you are using for the discussion of the relationship of the plasma and the boundary is actually the final or net observed field. So the secondary field that you are adding has to be subtracted from the observed field in order to see what the primary field has to be.

Parker: The field I am investigating is the geomagnetic field – the one that a person would observe with a magnetometer – and....

Ness: The field is distorted and what is observed is the result of all the currents present.

Parker: Well, I don't know whether anybody has observed this field yet or not. The point is I'm posing a problem which I will argue has some analogy to the magnetosphere: namely, I have a magnetic field which I believe to be confined by a plasma. At the moment, at least, let's not try to interpret the results until I work them out. Then I'm saying that I cannot help but generate another field in the perpendicular direction. Now whether you observe that or not is another question. Let's divorce ourselves from the actual case in Nature and just consider, in a perfectly general way, the confinement of a magnetic field by a plasma. Perhaps that would be simpler. In the end I'm going to ask the observationalist what does he see as he makes that crossing, particularly in the tail region.

Ness: My point was that what one measures is, of course, the net sum of all....

Parker: Right. The magnetometer passing out through the magnetopause would see the sum of these fields, and presumably it would see the tail field and then a rotated field upon coming out.

Frankenthal: It would seem that you would always have some electric field, and I don't know whether that's reasonable or not. Just because you bring some electrons out of the ionosphere you will have an excess of protons there and...

Parker: Oh, I'm sorry. I also have some extra solar-wind ions because the ions have been slightly longer in the field. I've forgotten how this inventory goes. I have an excess of ions in the ionosphere because I've pulled up some extra electrons. The ionospheric electrons want to congregate just inside the region where the electrons in the wind are bouncing. The charge balance is preserved, there is no violation.

Frankenthal: There is no extra positive charge immediately in the vicinity of the boundary.

Parker: Right. Because if there were an extra positive charge it would be shorted out by an ionospheric current.

Questioner C: There must be an extra positive charge left in the ionosphere.

Parker: No, I have no net charge out at the boundary and therefore I have no net charge at the ionosphere.

Frankenthal: Within which region are we making charge balance? Are we doing it within the boundary surface or ...

Parker: I have charge balance at the magnetopause and therefore I must have charge balance in the ionosphere because I have taken no net charge from the ionosphere. I have taken both electrons and ions from the ionosphere. The ions want to sit just inside the boundary where the electrons are hitting. I start with electrical neutrality and require that it be maintained at every point in space. I do not violate conservation of charge. There are extra ions from the ionosphere that tend to appear out at the magnetopause where the electrons from the solar wind hit. There are extra electrons in the solar wind because the ions from the wind remain inside the field a little longer, but the whole thing adds up to no net charge. I'm simply saying that I'll wait 2 min, or whatever it takes, to let things calm down and become distributed the way they want to go. There is no sleight of hand here with regard to the electrostatic charges. The sleight of hand comes later when I try to interpret all these things I'm getting you into.

L. ON THE POSSIBILITY OF AN EQUILIBRIUM

Vasyliunas: What about the possibility of the plasma moving at right angles to the original field? This should also give an equilibrium solution. [Cf. Figure 19 and the related discussion in the text.]

Parker: If you do it from the right side, yes. It is possible to arrive at the end point with zero [fictitious particle] velocity. There is a set of equilibrium solutions where you are normal to the field but not normal to the surface. But if conditions are changed slightly, I'm in trouble again because the end point velocity will not be zero.

Question: How about equal momenta for electrons and protons? [Cf. the discussion following Equation (86).]

Parker: That is what this would require and it doesn't work. At least part of the answer for why it doesn't work is that electrons with so much momentum would also dig a hole in the field, generate currents, and create the same problems.

Frankenthal: This has gotten you away from the problem that any electrostatic fields will develop in the first place due to the role of the ionosphere.

Parker: That's right. But that means you may not have to wait a couple of minutes as for this solution [Equation (89)]. Still you are back in the same boat solving the same equations. The electrons dig into the field and apparently they too generate twisted fields.

Frankenthal: But the longitudinal motion of the electrons and protons is the same and therefore there is no current j_y.

Parker: No, for then the electrons are moving much faster. You see, you may not have the electrons in bulk moving faster than the ions or you have net currents in space. So I guess that solution would be prohibited. Although I didn't mention it in the beginning, I must not have a wind that has a net current, where the electrons are moving faster than the protons. The electrons can have a higher thermal velocity but not a higher bulk velocity. That means I can never match the velocities exactly and achieve this neutralization that you are talking about.

Vasyliunas: There's one difficulty in connection with this problem. When you short out the field you bring up a stationary plasma from the ionosphere to keep charge neutrality. This is a rather cold plasma, say about 1 eV for the ionosphere, and a beam of protons is to flow through it at several hundred kilometers per second. Then would this not produce a two-stream instability?

Parker: It would, and it would warm up the plasma until the temperature is high enough that the two-stream instability vanishes.

Vasyliunas: Isn't it also possible that this would set up a fluctuation, say plasma oscillations of some kind, that would have to be taken into account in all the pressure-balance requirements?

Parker: This is why I have never attempted to actually work out what really happens. That is correct, yes. There are lots of other questions too. I've been glossing over them, but since you are worrying about that one, let me point out another. Suppose you take an ideal magnetopause where there is no boundary friction between the plasma and the field, in other words where there is equilibrium and no instability complications exist so that there would be no friction. The lines of force that cover the surface of the magnetopause all map down to the surface of the earth in the vicinity of the neutral lines of force (cf. Figure 12). The magnetopause layer roughly 100 km thick would map into a column of flux in the vicinity of the neutral line of force, and the radius of this column would be about 80 km at the ionosphere. In discussing the electrostatic neutralization that has to take place, I suggested that when the ions first penetrate they may induce an equal and opposite electron current. That electron current is pulled up from the polar ionosphere in this small 80 km patch of magnetic flux and the ionospheric current density required to neutralize the ion current would be prohibitive. The change in the field at the ionosphere would be about 1 gauss and of course there would be all kinds of violent instabilities, etc., in the region. One can simply go on and find endless catastrophes occurring in the system. This is why I am very hesitant to make any real predictions about the rate at which the lines of force may be pushed into the wind and then convected away. This may be a way of carrying lines of force off the front of the magnetopause but I'm very hesitant about giving any numbers. I just don't know how fast it will go because of all these complications.

Baggerly: If you're going to turn the magnetic-field lines around at the magnetopause and then blow them off, there may be a considerable torque resulting.

Parker: It turns out that the torque balances out for reasons of symmetry. But I'm not sure what....

Baggerly: The upper one and lower one would tend to turn the tail.

Parker: No, the magnetopause is only a very thin layer about 100 km thick so the torques are small compared to the total stresses. I haven't been able to find any interesting effects; maybe there are some, but I haven't been able to find any.

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