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A Quarter Century of Collisionless Shock Research

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This review highlights conceptual issues that have both governed and reflected the direction of collisionless shock research in the past quarter century. These include MHD waves and their steepening, the MHD Rankine-Hugoniot relations, the supercritical shock transition, nonlinear oscillatory wave trains, ion sound anomalous resistivity and the resistive-dispersive transition for subcritical shocks, ion reflection and the structure of supercritical quasi-perpendicular shocks, the earth’s foreshock, quasi-parallel shocks, and, finally, shock acceleration processes.

1. Introduction

Twenty-five years ago it was hotly debated whether collisionless shock waves even existed. Some argued that the rarity of collisions in a high-temperature plasma precluded the existence of shocks, while others maintained that collective microturbulence would replace particle collisions to create a shock with a thickness much less than a collision mean free path. The solar wind proved, upon its discovery in 1960, to have an enormous mean free path—comparable to the distance from the earth to the sun—yet the rapid rise times of the sudden commencements initiating magnetic storms suggested that solar flare plasma injection did create a thin collisionless shock (T. Gold (1955) [Bonett and Abrams, 1979]). Since it had been difficult to make collision-free plasmas in the laboratory, some foresaw that the first truly collisionless shock would be discovered in space. And so it was, standing in the solar wind ahead of the earth’s magnetosphere (Sonett and Abrams, 1983; Noses et al., 1984).

The following decade (1964–1974) was a golden age of collisionless shock research. The study of nonlinear collective plasma processes was in its infancy as the golden age opened, and collisionless shocks were the simplest example that illuminated the self-regulating interrelationship between macroscopic flows and microscopic collective processes that is central to most plasma configurations. High-altitude nuclear weapons studies and magnetic pinch fusion research motivated major laboratory investigations of collisionless shocks in the United States, Europe, and the Soviet Union. The discovery of the earth’s bow shock ensured that space observations would play a major role in collisionless shock research. Some of the first numerical simulations were of collisionless shocks. There was a marvelous collaboration between laboratory and space experimentalists, theorists, and specialists in numerical simulation.

The marvelous collaboration ended suddenly in 1974, largely because financial support for laboratory experiments disappeared when interest in magnetic pinch fusion waned. The space community was left to its own devices. Actually, because space plasmas are collision free and boundary free, and because the quality and variety of space plasma data were increasing rapidly, the space community was beginning to assert its dominance in collisionless shock research even before laboratory activity ceased.

The major achievement of the bridge years between the first and second golden ages, 1974–1979, was a phenomenological classification of the dependence of the earth’s bow shock structure on upstream solar wind parameters which revealed a richness of shock structure that did not
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1. How does collisionless shock structure depend on plasma parameters, which even today is not widely ap­

2. Section 5 deals with dispersive shocks, in which a non­

3. Possible to measure scale lengths, easy to do in the lab­

4. The ISEE spacecraft program initiated a second golden age of shock research, which began in 1979--1980 when

5. The first six sections discuss fluid theories of shock structure, a natural starting point, and the main success of the first golden age. Sagdeev [1979] has reviewed the theory of collisionless shocks in unmagnetized plasma; we will consider only the magnetized case. Section 1 begins by defining the three small-amplitude magnetohydrodynamic waves which determine the characteristics along which information about boundary conditions is propagated in magnetohydrodynamic flows. Viewing them as shocks of infinitesimal amplitude illustrates also the properties propagated along which characteristics show how fast and slow compressional waves steepen to form shocks, and illuminates the properties of the finite amplitude shocks. Considerations of shock evolution or steepening, which follow naturally from the use of fluid theory, eliminate certain extraneous solutions, namely standing waves and mirror solutions (section 3), explain the formation of dissipative subshocks (section 4) and dissipative wave trains (section 5), and define the transition between dissipative and dispersive structure in shocks with ion sound anomalous resistance (section 7).

4. The space community also consolidated numerous individ­

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Fig. 1. MHD Friedrichs diagrams. The Friedrichs diagram displays the dependence of the 1 MHD plane waves speeds (radial coordinate) on their angle of propagation to the magnetic field (angular coordinate) in a polar plot whose vertical axis is parallel to the magnetic field. Four cases are shown, with $C_0^2/C^2 = 1$ and 4 in the left-hand column and $C_0^2/C^2 = 5$ and 2 in the right. Cases 1 and 2 illustrate fast and slow waves speeds, respectively. The fast and slow waves, which are indicated by solid lines, while the intermediate wave, which does not steepen, is indicated by a dashed line. The magnetostatic wave propagates perpendicular to the magnetic field with the speed $C_s^2/C^2 = 1$. For parallel propagation, the fast and intermediate speeds equal the Alfven speed when $C_0^2/C^2 < 1$, while the slow and intermediate speeds equal the Alfven speed when $C_0^2/C^2 > 1$. $C_0^2/C^2 = 1$, all three speeds are equal.

3. MHD Rankine-Hugoniot Relations

3.1. Introduction

A nonlinear pressure pulse steepens until there is sufficient dissipation to form a steady shock. The structure of the dissipation layer is not describable by ideal MHD. However, these states asymptotically up stream and downstream of the shock are spatially uniform and therefore free of dissipation. As a result, ideal MHD does not describe the change in flow parameters between the two stationary states provided that the specific entropy is allowed to increase. These jump conditions, the MHD Rankine-Hugoniot (RH) relations, are obtained by integrating the MHD conservation laws (with an equation for conservation of energy replacing that for entropy) across the shock, which is considered to be discontinuous.

In (5), $A = U/C$ is the speed of the second wave relative to the first, is the fluid velocity component parallel to the wave normal, and $C$ is the wave speed relative to the fluid (call this $C_{1}$). The expression (5) is always positive, so that compressional waves ($\delta p = 0$) steepen. By resolving a smooth pressure pulse into a number of small-amplitude step waves, we infer that the compressional parts of the pulse steepen, while its rarefactive portions separate. Intermediate waves do not steepen, because they do not alter the density, the normal component of the flow speed, or the Alfven and sound speeds.

The quantity $\delta U = C_{1}/C_{2}$ in (5) may also be interpreted as the ratio of the steepening rates, $\gamma_{A} = \lambda C_{1}/C_{2}$, to the frequency, $w = \lambda C_{1}/C_{2}$, of a periodic fast or slow wave of a given density amplitude, $\delta p$. The normalized fast (solid) and slow (dotted) mode steepening rates are plotted in a Friedrichs diagram format in the top row of Figure 2 for $C_{1}/C_{2} = 0.3$, 1, and 3, assuming $\delta p_{1} = 1$. The exact steepening rates may be obtained by dividing the quantities in Figure 2 by $\delta p_{1}$. For a given $\delta p_{1}$, the fast and slow steepening rates are roughly independent of propagation angle, are almost equal for all $C_{1}/C_{2}$, and are identical for $C_{1}/C_{2} = 1$.

Because parallel fast and slow waves do not change the density when $C_{0}/C_{2} < 1$ and $C_{1}/C_{2} > 1$, respectively, it is more illuminating to derive the expressions analogous to (6) for a given perturbed fluid speed, $\delta U$, or, better yet, for a given perturbed energy density. The second and third rows of Figure 2, which show the steepening rates for $\delta U = (C_{1}/C_{2})^{1/2}$ and $\delta (\text{energy density} - \delta p)$, respectively, indicate that parallel fast and slow waves do not steepen when $C_{1}/C_{2} < 1$ and that parallel slow mode do not steepen when $C_{1}/C_{2} > 1$.

In order that nonlinear waves actually steepen, the above fluid steepening rates must exceed the wave damping rates calculated from kinetic theory. For the solar wind conditions prevailing at 1 AU, fast waves will nearly always steepen, whereas slow waves of comparable energy density will steepen only if the ion $\beta$ is very low.

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Fig. 2. Normalized MHD wave steepening rates. The normalized steepening rate defined in equation (6) is displayed in a polar plot for $C_{1}/C_{2} = 0.3, 1$, and 3, and $\gamma_{A} = 1$ in the top row. The vertical and horizontal axes are parallel and perpendicular to the magnetic field, respectively. The fast and slow mode steepening rates are indicated by solid and dashed lines, respectively. The steepening rates for step waves with $\delta p = 0$ are shown, those may be adjusted to any given density amplitude by dividing by $\delta p_{1}$. When $C_{1}/C_{2} = 1$, the fast mode steepens faster than the slow wave, and vice versa when $C_{1}/C_{2} > 1$. The two steepening rate are equal when $C_{1}/C_{2} = 1$. The parallel propagating fast and slow waves do not perturb the density when $C_{1}/C_{2} < 1$ and $C_{1}/C_{2} > 1$ respectively, and therefore do not steepen. It is more illuminating to plot the steepening rates for a velocity perturbation equal to the magnetosonic speed (middle row) or for a perturbed energy density equal to $\delta U/C_{2}$ (bottom row).
The solutions for shock normal angles (denoted in the top panels show a fast speed upstream to between the fast and intermediate speeds equal the slow and intermediate speeds upstream to below the slow speed downstream. A slow shock takes the flow speed from between the slow and intermediate speeds upstream to below the slow speed downstream. For parallel propagation, the fast and intermediate speeds equal the Alfvén speed and the shock "switches on" a tangential component of magnetic field and flow speed. Switch-on shocks occur when \( C_s^f < C_s^i \) upstream and the fast Mach number is less than or equal to 2. For the maximum strength slow shock, the flow speed equals the intermediate speed upstream. This shock "switches off" the tangential component of the magnetic field downstream.

Fig. 3 MHD shock evolutionary conditions. The top and bottom panels show a \( C_s^f/C_s^i = 2 \) Friedrichs diagram of the fast, intermediate, and slow wave speeds. A fast shock takes the normal component of the shock frame flow speed from above to below the fast and intermediate speeds downstream. A slow shock takes the flow speed from between the slow and intermediate speeds upstream to below the slow speed downstream. For parallel propagation, the fast and intermediate speeds equal the Alfvén speed when \( C_s^f/C_s^i < 1 \). If \( C_s^f < C_s^i \) downstream, the normal component of the flow speed must equal the Alfvén speed. In this case, the shock "switches on" a tangential component of magnetic field and flow speed. Switch-on shocks occur when \( C_s^f < C_s^i \) upstream and the fast Mach number is less than or equal to 2. For the maximum strength slow shock, the flow speed equals the intermediate speed upstream. This shock "switches off" the tangential component of the magnetic field downstream.

Fig. 4: Fast shock Rankine-Hugoniot solutions for \( C_s^f = 0 \) upstream. The density compression ratio (top), the magnetic field compression ratio (middle), and the ratio of the downstream internal energy density to the upstream flow energy density (bottom) are plotted as a function of the fast Mach number \( M \) assuming \( \gamma = 1.28 \). The solutions for shock normal angles (denoted in this figure by \( \beta_0 \)) in 18° intervals from 0° to 90° are shown. The discontinuous change at \( \beta_0 = 0 \) and \( M = 2 \) is due to the disappearance of the switch-on shock. Above \( M = 2, B_0/B_1 \) is unity, and the density compression ratio and normalized internal energy density equal their strong shock limits of 4 and 3, respectively. The more oblique the shock, the higher the Mach number at which it approaches the strong shock limit. The RH solutions depend weakly on shock normal angle for 45° < \( \beta \) ≤ 90°. Thus, quasi-parallel (\( \beta_0 < 45° \)) and quasi-perpendicular (\( \beta_0 > 45° \)) shocks differ considerably. This figure is from Kantrowitz and Petschek (1966).

For \( C_s^f < C_s^i \) and \( \beta_0 = 0 \) the fast and intermediate speeds are identical; if \( C_s^f < C_s^i \) downstream of a parallel fast shock, the evolutionary conditions demand that the normal component of the downstream flow velocity precisely equal the intermediate speed. To accomplish this, parallel fast shocks "switch-on" tangential components of flow velocity and magnetic field downstream when the fast Mach number is less than 2 and \( C_s^f < C_s^i \) upstream. Such shocks are called "switch-on" shocks.

3.2 Numerical Rankine-Hugoniot Solutions for Fast Shocks

The Rankine-Hugoniot relations calculate the dependence of the downstream flow state on the upstream flow speed or Mach number, the ratio of the sound to Alfvén speeds upstream (or, equivalently, the ratio of the thermal and magnetic pressures, \( \beta_0 \)), and the shock normal angle. Although it is sometimes convenient experimentally to specify the Alfvén or magnetic Mach numbers, the steepening argument indicates that the physically rigorous parameter is the fast Mach number, the ratio of the upstream flow speed to the upstream fast speed based upon the shock normal angle.

Figure 4, from Kantrowitz and Petschek (1966), shows the dependences on the fast Mach number and shock normal angle of the density compression ratio \( \rho_2/\rho_1 \), the magnetic field compression ratio \( B_2/B_1 \), and the flow internal energy (enthalpy) downstream of shocks that propagate into a cold plasma (\( \beta_0 = 0 \)) whose ratio of specific heats, \( \gamma \), is 1.5. As the fast Mach number approaches infini-
ty, the density and magnetic compression ratios approach a limit of 6 for all shock normal angles. In general, it may be shown that \( \rho_2/\rho_1 \) and \( B_2/B_1 \) approach 6/4 - 1 and the downstream internal energy density approaches \( 3/2 \) of the upstream flow energy density (when \( \gamma = 5/3 \)) in the strong shock limit.

The complex structure for fast Mach numbers less than 2 and quasi-parallel shocks is highlighted by the perpendicular and parallel limiting cases. For quasi-parallel shocks, the magnetic compression ratio and velocity contrast are virtually independent of both the shock normal angle and upstream velocity. Perpendicular shocks have the largest maximum compression ratio and produce the smallest \( \beta_1 \) for a given fast Mach number. However, by the time \( M_1 \) reaches 3.5, even a perpendicular shock propagating into a cold plasma creates \( \beta_1 = 1 \) downstream. Thus, if the fast Mach number exceeds 2, switch-on shocks no longer exist, and this structure disappears.

The three columns of Figure 6 contour the dependences on the fast Mach number, \( M_1 \), and the upstream shock normal angle of the magnetic compression ratio (left column), the velocity contrast (middle column), and the ratio \( \beta_1 \) of the downstream thermal to magnetic pressure for \( \gamma = 5/3 \). The top and bottom rows correspond to \( \beta_1 = 1 \) and 2, respectively. The velocity contrast is the ratio, \( U_2/U_1 \), of the normal components of the downstream and upstream flow velocities and is the inverse of the density compression ratio.

The properties of quasi-perpendicular (\( \theta_{NB} > 45^\circ \)) and quasi-parallel (\( \theta_{NB} < 45^\circ \)) shocks are highlighted by the perpendicular and parallel limiting cases. For quasi-perpendicular shocks, the magnetic compression ratio and velocity contrast are virtually independent of both the shock normal angle and upstream velocity. They depend primarily on the fast Mach number and approach their strong shock limits by the time \( M_1 \) reaches 5. The downstream \( \beta_1 \) does depend upon the upstream \( \beta_1 \), but is still relatively independent of shock normal angle. Perpendicular shocks have the largest maximum compression ratio and produce the smallest \( \beta_1 \) for a given fast Mach number. However, by the time \( M_1 \) reaches 3.5, even a perpendicular shock propagating into a cold plasma creates \( \beta_1 = 1 \) downstream. Thus, if the fast Mach number exceeds 2, switch-on shocks no longer exist. When \( \beta_1 = 0 \), all parallel fast shocks with Mach numbers less than 2 switch-on a tangential magnetic field component downstream, and the magnetic compression ratio exceeds unity. When the Mach number exceeds 2, parallel shocks leave the magnetic field unchanged in direction and magnitude. When \( C^2_s > C^2_p \) upstream, there can be no switch-on shock, as in the case \( \theta = 90^\circ \).

The velocity contrast depends significantly on the shock upstream normal angle for \( \beta_1 = 0 \), whereas it is virtually independent of \( \theta_{NB} \) when \( \beta_1 > 2 \). Since high \( \beta_1 \) shocks are decelerated, their downstream state should depend weakly on the upstream magnetic field magnitude and direction.

Because \( V \cdot B = 0 \), the normal component of the magnetic field is conserved across plane shocks. However, except for non-switch-on parallel shocks, the tangential component increases. Moreover, the magnetic tension induced by the increased tangential field refocuses the downstream flow velocity away from the shock normal. Figure 6 contours the dependences of the angles \( \theta_{NS}, \theta_{VR}, \) and \( \theta_{NS} \) upon fast Mach number and upstream shock normal angle. Figure 6 also shows the shock normal angle dependence of the magnetic field, and downstream velocity vectors, respectively. The top and bottom panels are for \( \beta_1 = 0 \) and 2, respectively. The shaded regions correspond to "subcritical" shocks, discussed in the next section.

The magnetic field does not change direction across perpendicular shocks or across non-switch-on parallel shocks. For all others, the downstream magnetic field is refracted away from the shock normal, as is the velocity. The magnetic field is always more strongly refracted than the flow velocity. When \( \beta = 0 \), the velocity refraction is essentially pronounced for low Mach number, and \( \beta_1 \) becomes less than 2 switch-on a tangential magnetic field. As the Mach number increases, magnetic stresses become proportionally less important, and the change in flow direction across the shock diminishes. When \( \beta_1 = 2 \), the shock is closer to the gas dynamic limit for which there is no velocity refraction, and the downstream flow velocity makes an angle of 10.8° or less to the shock normal.

In closing, we emphasize that the MHD Rankine-Hugoniot relations relate the uniform, dissipation-free states of local thermodynamic equilibrium asymptotically far upstream and downstream of the shock. The RH relations are valid only after all the dissipation processes in the spatially nonuniform shock transition have been accounted for. Several such processes are expected to occur and to have different characteristic scale lengths. Of these, the scale length over which the electron and ion temperatures equalize will typically be the longest. In principle, the RH relations apply only to states separated by a distance greater than the longest dissipative length.

4. Dissipative MHD Shocks

4.1. Introductory Remarks

A natural first approach to shock structure is to add scalar resistivity, viscosity, and thermal conductivity to the MHD equations and to solve the resulting nonlinear differential equation that describes the transition between the upstream and downstream stationary states. This is valid except where switch-on shocks occur, parallel shocks produce a large downstream \( \beta_1 \), because the magnetic field is not compressed. The magnetic field is nearly independent of shock normal angle. Each anisotropy is almost devoid of physical content, because the plasma processes that lead to dissipation are not specified. Nonetheless, its use has led to one general result—the identification of a critical Mach number above which resistivity cannot provide all the dissipation required by the Rankine-Hugoniot conditions.

Dissipative MHD has a basic scale length for each dissipation process: the length that makes the magnetic and ordinary Reynolds numbers unity, and a thermal conductivity scale length [Courant, 1970]. A nonlinear MHD pulse should steepen until it arrives at the largest scale length over which sufficient dissipation occurs to satisfy the Rankine-Hugoniot conditions. The question is, to which length will it steepen? Without a microscopic theory of the dissipation, one cannot go further. Moreover, plasma dissipation is not always a diffusive process, as the fluid description assumes. Nonetheless, the fluid equations do indicate that resistivity can initiate a fast shock, while viscosity alone cannot provide a complete fast shock transition, and thermal conduction alone is sufficient only for weak shocks [Courant, 1939].

The argument above made it natural to investigate when the entire fast shock transition can be accomplished by resistivity. It was always assumed that resistivity would provide enough dissipation for weak shocks. Such shocks would steepen until they arrive at the magnetic Reynolds length, and it was up to plasma physics to estimate the anomalous resistivity resulting from the saturation of current-driven instabilities in the shock front, in order to calculate the shock thickness.

The question whether resistivity provides enough dissipation for strong as well as weak shocks was first studied by Marshall [1965]. He found that a perpendicular shock propagating above a fast Mach number of 2.76 into a cold MHD fluid required more dissipation than the maximum possible from resistivity. At the critical Mach number, the normal component of the shock frame downstream flow speed, \( U_2 \), equaled the ordinary sound speed, \( C_s \). It was natural to assume that the additional dissipation was due to viscosity. Indeed within the fluid framework, viscosity was the only option, since finite thermal conductivity cannot provide for strong shocks.

Courant [1970] showed that the condition \( U_2 = C_s \) de-
number. We introduce the stationary point analysis in section 4.3, subsequently add dispersion to resistivity and apply the analysis to dispersive shocks in section 5.2, and finally use it to determine when subcritical shocks should be resistive and when they should be dispersive in section 6.12.

4.2 General Definition of Critical Mach Number

The fast magnetosonic speed in a resistive MHD fluid depends on wavelength, since causality requires that dissipation be accompanied by dispersion:

\[ c_s = \sqrt{\frac{B_0^2 \mu_0}{\rho}} \]

where \( \rho \) is the mass density, \( B_0 \) is the magnetic field, and \( \mu_0 \) is the permeability of free space. The fast magnetosonic speed is given by:

\[ c_s^2 = \frac{B_0^2}{\mu_0 \rho} \]

and \( R_m \) is the magnetic Reynolds length, which is defined as:

\[ R_m = \frac{\mu_0 L}{\nu} \]

where \( L \) is the characteristic length scale and \( \nu \) is the magnetic Reynolds number.

The fast magnetosonic speed in a resistive MHD fluid is sketched in the upper left-hand panel of Figure 7. The dotted and dashed lines correspond to the fluid velocity downstream of a subcritical and supercritical shock, respectively. The phase speed approaches the magnetosonic speed of ideal MHD in the long-wavelength limit. Finite resistivity progressively depletes the magnetic and fluid oscillations as the wavelength decreases, so that the phase speed ultimately approaches the sound speed, \( c_s \), when the wavelength is comparable to the magnetic Reynolds length. By adding the dispersion due to viscosity, we may set in a shorter Reynolds length, \( R_m \), when the wave length is comparable to the magnetic Reynolds length. The dotted and dashed lines correspond to the flow speeds downstream of subcritical and supercritical shocks respectively. The supercritical shock structures expected in the dispersive and dispersive cases are sketched in the lower left and lower right panels, respectively. The magnetic field increases on the magnetic Reynolds (left) or electron inertial (right) scale lengths, and a viscous (left) or ion sound subshock (right) is embedded within a broader magnetic field structure.

First Critical Mach Number

The supercritical shock structure expected from the arguments sketched in the lower left-hand panel of Figure 7. The magnetic field and the temperature of the electrons, which are resistively heated, should increase smoothly on the resistive scale length, whereas the temperature of the ions, which are heated by viscosity, should increase across a thin embedded subshock whose scale length is the Reynolds length. Part of the density increase and the associated decrease in fluid velocity should take place on the resistive scale length, and part in the viscous subshock.

The above argument can be extended from perpendicular to oblique fast shocks by noting that in resistive MHD, the speed \( c_s \) always replaces the Alfven speed in
the fast mode dispersion relation, so that when \(M_r > 1\), all fast modes speed up. Thus, equating the normal component of the downstream flow velocity to the downstream sound speed defines, for all shock normal and incident conditions, a critical Mach number above which resistivity is unable to provide all the required shock dissipation.

### 1.3. Stationary Point Analysis

To solve for the full shock structure, the dissipative MHD equations should be reduced to a nonlinear differential equation that describes the change in one of the fluid variables between the upstream and downstream stationary points. The fluid variables at the two stationary points satisfy the RH relations. Some of the effects of dissipation on shock structure can be obtained by studying the linear development of small perturbations at the stationary points [Coroniti, 1970]. For a proper shock transition, an upstream perturbation to increase approaching the shock, and all downstream perturbations to die away leaving the shock.

After equating the coefficients of viscosity and thermal conductivity to zero, and perturbing fluid variables about the upstream or downstream stationary points, we arrive at the following differential equation which describes the evolution in the shock frame of the tangential magnetic field [Coroniti, 1970]:

\[
R \frac{d^2 B_j}{d \phi^2} + D \frac{dB_j}{d \phi} = \frac{(U_j - C_j) \frac{dU_j}{d \phi}}{(U_j - C_j) \frac{dC_j}{d \phi}} B_j (39)
\]

where superscript prime denotes an axial derivative, \(R = c^2/4 \pi \mu_0 \) is the magnetic Reynolds length, and \( \sigma \) is the electrostatic conductivity. Note that the definition of \( R_i \) used in (9) differs slightly from (39). We choose a Cartesian coordinate system such that the shock normal is parallel to the negative y axis, and the magnetic field upstream and downstream is contained in the y-z plane.

Equation (9) applies to all shocks, and to both their upstream and downstream states. The Rankine-Hugoniot conditions must be used to determine the magnetic field direction, the flow speed, and the fast, slow, and sound speeds. Figure 5 suggests that for the argument to follow, we will need only the most general properties of the RH solutions. Since \( U_j > C_j > C_i > C_s \), upstream and downstream are in the y-z plane. 

### 5.1. Introductory Remarks

The two-fluid model of plasma contains three basic scale lengths, \( \lambda_{De} \) and \( \lambda_{ci} \), respectively, and the Debye length, \( \lambda_D \), which represents the jumps that define the characteristic length of the electron or ion Debye length. When \( U_j > C_j > C_i > C_s \), downstream and upstream are in the y-z plane. 

### 5.2. Dispersive Shocks

#### 5.2.1. Characteristics

The two-fluid model of plasma contains three basic scale lengths, \( \lambda_{De} \) and \( \lambda_{ci} \), respectively, and the Debye length, \( \lambda_D \), which represents the jumps that define the characteristic length of the electron or ion Debye length. When \( U_j > C_j > C_i > C_s \), downstream and upstream are in the y-z plane. 

### 5.3. Summary

1. Equating the normal component of the downstream shock frame speed to the sound speed defines a critical fast Mach number above which resistivity alone cannot provide all the dissipation required by the shock jump conditions.

2. The critical Mach number is a strong function of the upstream thermal velocity, \( \beta \), and shock normal angle. For typical solar wind parameters, it is less than 2.

3. Another dissipation mechanism in addition to resistivity must play a role in supercritical shocks. Although in MHD it is natural to assume that viscosity is the second dissipation mechanism, anything that converts flow momentum into heat will do.

In closing, we note that the critical Mach number is defined by a limiting argument that indicates when a second dissipation mechanism must exist. It need not suddenly turn on at the critical Mach number, and it could well be present in subcritical shocks.

#### 5.3.1. Dispersion

Dispersion can limit wave steepening. As a compressional wave steepens, flow nonlinearities populate the short-wavelength dispersion part of its Fourier decomposition spectrum. The short-wavelength energy is carried away by a nonlinear wave radiated by the steepening front. Dispersion, always necessary, ultimately damps the nonlinear wave, and a steady, spatially oscillatory shock is formed.

Many properties of nonlinear dispersive waves can be obtained from the RH solutions, but in the corresponding limit of small-amplitude waves. Whether the short-wavelength linear waves propagate faster or slower than the MHD fast speed determines whether the wave trains on or trails the main shock ramp. The nonlinear wave train must damp to a dissipative scale length far from the shock. Since the entire wave train is time stationary in the shock frame, the asymptotic small-amplitude wave must phase stand in the flow. Thus, the oscillatory scale length of the wave train may be estimated from the wave length of the corresponding small-amplitude wave that phase stands in the far upstream or far downstream flow.

Furthermore, since the dispersive wave must carry energy away from the steepening shock front, its phase velocity should be less than the downstream flow speed, if it stands upstream. Similarly, its group velocity should be less than the downstream flow speed, if it stands downstream.

We begin by considering dispersive supercritical shocks in section 5.2. Once again, the dispersion relation of small-amplitude waves suggests the existence of a critical shock transition when the downstream flow speed equals the sound speed. This suggestion is confirmed by a generalization of the stationary point analysis of section 4.2 to include both finite electron inertia dispersive waves and resistivity. A resistively damped wave train with a \( \alpha_d \) scale length stands downstream of subcritical shocks, whereas new forms of dispersion and dissipation are required for supercritical shocks.

Fine ion inertia plays no role in exactly perpendicu lar shocks, but it dominates the dispersive structure of even slightly oblique small-amplitude waves and shocks [Galeev and Karpman, 1963; Karpman, 1964]. In section 5.3, we infer the properties of oblique nonlinear wave trains and standing shocks for the corresponding small-amplitude waves. We will calculate the maximum Mach number for which a nonlinear dispersive shock can stand upstream, and comment upon the possible structures of supercritical oblique shocks.

#### 5.3.2. Supercritical Dispersive Shock

The two-fluid magnetosonic wave dispersion relation in the quasi-neutral approximation is formally similar to (15):
stream. Since the plasma currents cannot follow oscillations at scales less than the electron inertial length, the electrostatic oscillations will be decoupled from those of the magnetic field, and the nonlinear ion sound wave train will form a subshock that is embedded in a broader magnetic field structure. This wave train is expected to be similar to the one that forms in an unmagnetized plasma, except for the form of dissipation, besides resistivity, is required to damp the ion sound wave train and complete the supercritical shock transition. The variations in flow parameters across a supercritical dispersive shock are sketched in the bottom right-hand panel of Figure 7. The magnetic field forms a trailing wave train with an electron inertial scale length. The number of magnetic field oscillations depends upon the resistive damping rate. Embedded in the magnetic field structure near the local sonic point is a dispersive ion sound subshock, illustrated by the Debye length oscillations (not to scale) in the electrostatic potential $\phi$. Part of the reduction in flow speed required by the RH relations occurs across the electron inertia wave train, and part across the ion sound subshock.

At this point, we approach the limits of fluid theory. Nonlinear ion sound waves are damped, and rendered irreversible, by ion reflection, a nonfluid effect (Madsen and Sagdeev, 1963; Sagdeev, 1979). This indicates that the extra dissipation needed for supercritical shocks cannot be anything as simple as viscosity. Nonetheless, the two-fluid equations have served us well, for they too, do indicate the existence of the supercritical shock transition and, also, of the physics needed to describe supercritical shocks.

5.3. Oblique Dispersive Shocks

The left-hand panel of Plate 1 sketches the quasi-neutral two-fluid dispersion relation for $C_{T}/C_{E}$ small and large $\beta_{i}$ in comparison to $\beta_{e}$ (Formisano and Kennel, 1989). It has three branches whose phase speeds approach the three MHD speeds in the long-wavelength limit. At short-wavelengths, the fast wave becomes an electrostatic solitary wave as dispersion changes to a dispersive relativistic solitary wave, and then into an elliptically polarized whistler wave whose speed exceeds the fast MHD speed. Finite ion inertia dispersion progressively decouples the ion mass from the magnetic field oscillations as the wavelength of the fast mode decreases. Its phase speed therefore increases and approaches the maximum of about the electron Alfvén speed, in which only the electron mass inerentially loads the magnetic field oscillations. Finite electron inertia dispersion creates a leading whistler magnetic field and fluid oscillations, and its phase speed decreases. Eventually, it approaches the sound speed, at which point the fast mode becomes electrostatically relativistic. The intermediate speed decreases to the sound speed, and the slow speed tends to zero, as the wavelength decreases.

The right-hand panel of Plate 1 sketches the two-fluid quasi-neutral dispersion relation for oblique propagation and $C_{T}/C_{E}$ large. There are two important differences with respect to the previous $C_{T}/C_{E}$ small case. First, the wavelength at which the whistler phase speed first exceeds the fast speed and the ratio of the maximum whistler phase speed to the fast speed both decrease with increasing $C_{T}/C_{E}$. Second, when $C_{T} = C_{E}$, both $\omega_{pe} / \omega_{pT}$ and $\epsilon_{T}$ become the intermediate speed increases to the sound speed at the ion inertial wavelength.

The fact that the fast wave speed increases with decreasing wavelength implies that the nonlinear wave train will lead oblique shocks, in contrast to the 90° case discussed above. We may use an evolutionary argument to determine the shock normal angle at which the wave trains switch from trailing to leading. The upstream ion inertial scale length, $R_{i}$, is determined rigorously in Coroniti's (1971) derivation of the stationary point differential equation (22)-(34) in the next section:

$$R_{i} = \frac{C_{E}}{\omega_{pT} C_{T} / C_{E}}$$

Note that $R_{i}$ tends to zero as $\beta_{i}$ approaches $45^\circ$. The ion inertial scale length for small-amplitude waves may be obtained by setting the fast Mach number, $M_{f}$, equal to unity in (16). A nonlinear pulse will steepen until it encounters the first scale length at which a dispersive shock can form. Thus, if $R_{i} > R_{c}$, the wave train will lead the shock, and vice versa. Equating $R_{i}$ to $R_{c}$ defines the shock normal angle $\theta_{s}$ at which the dispersive structure changes:

$$\cos \theta_{s} = \frac{m_{i} C_{E}}{m_{e} C_{T} / C_{E}}$$

where $m_{i}$ and $m_{e}$ are the electron and ion mass, respectively.

For a low $\beta_{i}$ hydrogen plasma and $M_{f} = 1$, the angle $\theta_{s}$ is approximately $87^\circ$. Thus, we expect leading inertial wave trains for nearly all shock normal angles. The trailing 90° electron inertial wave train has never been definitively identified in space.

Let us now calculate the upper limit fast Mach number for which a whistler can phase-stand in the upstream flow. This dashed line in the right-hand panel of Plate 1 indicates that the upstream flow speed intersects the fast mode branch that provides it is the maximum whistler phase speed. $C_{T}/C_{E}$ is high, the fast mode becomes oblique subcritical shocks. The maximum whistler Mach number, $M_{f}$, corresponding to the maximum whistler phase speed. No wave train stands ahead of perpendicular shocks, and the "whistler critical Mach number," $M_{w}$, which is relatively small for very oblique shocks, increases rapidly with decreasing $\beta_{i}$ and approaches an upper limit of $\beta_{i} = 2$ for parallel shocks in zero hydrogen plasma. The whistler critical Mach number at all $\beta_{i}$ decreases rapidly once $2^\circ$, $C_{T}/C_{E}$ increases $\cos \theta_{s}$ decreases $R_{c}$, the shock will be initiated by a monotonic rarefaction wave on electron inertial scales when $R_{c} = R_{i}$. It is particularly important to note that for typical solar wind parameters an upstream whistler wave train need not form since the shock is sufficiently oblique and has a sufficiently high Mach number.

We next consider the waves that phase-stand downstream of shocks, focusing on the subcritical case first. The dashed lines in Plate 1 indicate a flow speed between the fast and intermediate speeds, the state downstream of fast shocks. The limit of the flow is the shock normal angle at which the wave trains switch from trailing to leading. The supercritical waves, we apply to the downstream state, correspond to a subcritical shock with $U_{s} < C_{T}$. In this case, a short-wavelength whistler standing in the downstream flow can carry energy away from the shock, thereby leading to a stable shock transition when the downstream wave train is damped by viscosity.

The right-hand panel, which, when it is applied to the downstream state, corresponds to a supercritical shock, suggests that a dispersive mode on the intermediate branch could stand downstream. Because its group velocity exceeds the downstream flow speed, it could carry energy towards the shock and thereby might cause the shock to steepen. However, since the long-wavelength MHD intermediate wave does not steepen, evolutionary arguments shed no light on how such a standing wave might develop. Debye length dispersion is also expected at short wavelengths. In short, although it has not been investigated in detail, the structure of oblique dispersive shocks at the critical Mach number is predicted by two-fluid theory.

5.4. Summary

We have outlined the physical picture of shock structure that emerges from two-fluid theory. The two-fluid approximation contains ion and electron inertial scale lengths in the quasi-neutral approximation, and, in addition, the Debye length, when quasi-neutrality is relaxed. Small-amplitude waves are dispersive at each of these basic scale lengths. When the ion $\beta$ is significant, low-frequency waves are also dispersive at cyclotron wavelengths (Fredericks and Kennel, 1968; Coroniti, 1971), an effect we neglected in order to focus on the classical wave train analyses in the literature. Ours discussion is therefore strictly valid for plasma in which the ratio of ion to electron temperatures is large, though we believe it illustrates shock behavior over a wider range of plasma parameters.

We tacitly assumed that all dissipative scale lengths are shorter than all pertinent dispersive scale lengths in order to emphasize the possible dispersive wave trains expected from two-fluid theory. We reached the following conclusions:

1. Finite electron inertia dispersion creates a trailing wave train with a $\epsilon_{T}$, magnetic field scale length downstream of perpendicular subcritical shocks.

2. Supercritical magnetosonic shocks steepen to form a trailing ion sound shock plus the whistler. The dissipation required to damp the ion sound wave train and thereby complete the shock transition cannot be resistivity.

3. Finite ion inertia dispersion creates a leading whistler wave train with the scale length $R_{c}$ (defined in (15)) upstream of oblique shocks. Leading ion inertial wave trains are expected for all shock normal angles.

4. Above the whistler critical Mach number defined in section 5.3, a small-amplitude whistler cannot stand in
the upstream flow, and the shock will be initiated by a monotonic ramp with a c_{De}^\perp amplitude.  

5. Inspection of the two-fluid dispersion relation suggests that supercritical oblique shocks potentially involve downstream oscillations on the intermediate branch. An electrostatic ion sound wave train may also form.  

This section primarily illustrated the content of classical theoretical structures obtained in the weak dissipation limit, and not necessarily the true structure of shocks. To approach greater realism, it is necessary to specify the shock dissipation mechanisms and to grapple with the limitations of the fluid approach.  

1. A microscopic theory of resistivity is needed to decide whether subcritical shocks will be resistive or dispersive (section 6).  

2. The weak dissipation limit probably does not apply to supercritical shocks, although it does define the critical shock transition. It is dangerous to proceed above the critical Mach number without including the new dissipation processes that must operate. For example, without explicitly considering the dissipation, one cannot say for sure whether nonlinear whistlers will stand upstream of supercritical shocks.  

3. The use of fluid theory requires that heat flow parallel to the magnetic field be neglected, an assumption which is suspect for quasi-perpendicular shocks.  

6. Subcritical Shocks With Ion Sound Anomalous Resistance  

6.1. Introductory Remarks  

Sections 4 and 5 summarized two very different theories of subcritical shock structure, one of which predicts a monotonic shock jump and the other an oscillatory shock. Both types have been found in studies of the earth's bow shock. According to the thickest shock hypothesis, nonlinear steepening is limited by dissipation or viscosity, whichever occurs first. Since resistivity is the only dissipation required for subcritical shocks we only need to compare the resistive and dispersive scale lengths to decide whether a subcritical shock will be resistive or dispersive. This can only be done by examining the theory of anomalous resistivity in collisionless plasma.  

6.2. Ion Sound Anomalous Resistance  

Since the earliest investigations of collisionless shocks [Sagdeev, 1965], it has been believed that the ion sound instability could provide the intrinsic resistivity necessary to complete the subcritical shock transition. The theoretical reasons for this belief have been good ones. The ion sound instability has a low current threshold when the electron temperature exceeds the ion temperature. It produces Debye length electrostatic fluctuations which are microscopic compared to the electron inertial length, so that fluid theory may be used even for perpendicularly damped shocks. When the electron plasma frequency exceeds the electron cyclotron frequency, as it does in the solar wind, ion sound waves are essentially unmagnetized and can interact with the bulk of the electron distribution, so that runaway can be prevented for most of the electrons. Finally, since resistivity implies the transfer of momentum from streaming electrons to ions, a good anomalous resistivity instability must involve both electrons and ions, as the ion sound instability does.  

The quasi-linear theory of the ion sound instability is well understood [Galeev, 1975]. When \( T_e > T_i \), the growth rate is given by  

\[
\gamma = \left( M_i/M_e \right)^{1/2} \left( \omega_c / \omega_e \right)^{1/2} |k \cdot \mathbf{v}_{ni} - \mathbf{a}| 
\]  

(17)  

where \( \omega_c = (T_i/M_i)^{1/2} \), the ion sound speed, \( \mathbf{a} \) and \( \mathbf{k} \) are the frequency and wave vector, respectively, and \( \mathbf{v}_{ni} \) is the electron drift velocity associated with the current. Since ion sound waves obey  

\[
u^2 = k^2 C_s^2 + \frac{\lambda_{De}^2}{1 + k^2 \lambda_{De}^2} 
\]  

(18)  

where \( \lambda_{De} \) is the electron Debye length, they will be unstable when \( \mathbf{v}_{ni} > \mathbf{a} \).  

In steady state anomalous resistance, the waves radiated by drifting electrons must be absorbed by ions, in order that electron momentum be transferred to ions. The ion distribution therefore develops a high-energy tail extending to speeds comparable to the ion sound speed. The electron distribution develops a flat top at low velocities.  

The above quasi-linear solution, the one most pertinent to typical solar wind conditions, is valid for relatively small driving currents and electric fields. For stronger sideband currents, \( \lambda_{De} \) increases. When \( \mathbf{v}_{ni} > \mathbf{a} \), the electron Debye length, they will be unstable when \( \mathbf{v}_{ni} > \mathbf{a} \).  

Assuming that \( \mathbf{v}_{ni} > \mathbf{a} \), we find, using (18),  

\[
\nu^2 = \left( \frac{M_i}{M_e} \right) C_s^2 \frac{1}{1 + k^2 \lambda_{De}^2} 
\]  

(19)  

where \( \lambda_{De} \) is the effective temperature and \( X \) the fractional depth of the hot tail. \( \nu^2 \) above, the speed at which the electron drift is limited by ion sound anomalous resistance.  

The above quasi-linear solution, the one most pertinent to typical solar wind conditions, is valid for relatively small driving currents and electric fields. For stronger sideband currents, \( \lambda_{De} \) increases. When \( \mathbf{v}_{ni} > \mathbf{a} \), the electron Debye length, they will be unstable when \( \mathbf{v}_{ni} > \mathbf{a} \).  

Assuming that \( \mathbf{v}_{ni} > \mathbf{a} \), we find, using (18),  

\[
\nu^2 = \left( \frac{M_i}{M_e} \right) C_s^2 \frac{1}{1 + k^2 \lambda_{De}^2} 
\]  

(20)  

\[
\frac{R_m}{bB_i} = \frac{2}{\pi} \left( \frac{\lambda_{De}}{\lambda_{De}^*} \right) \frac{1}{1 + k^2 \lambda_{De}^2} 
\]  

(21)  

\[
\beta_e = \frac{8}{3} \pi \mathbf{v}_e \mathbf{B}_e \mathbf{B}_i. 
\]  

The estimate (21) is probably accurate up to a numerical factor of order unity [Galeev, 1975]. We estimate the parameter dependence of the resistive-dispersive transition by substituting \( \nu^2 \) calculated using upstream parameter values, into the upstream stationary point differential equation that takes finite electron and ion inertia and resistivity into account, assuming quasi-neutrality [Coroniti, 1971]:  

\[
R_e^\perp B_i^\perp = R_i^\perp B_e^\perp + R_m^\perp B_m^\perp = \mathbf{D}(U_i^\perp, \mathbf{B}_i) 
\]  

(22)  

\[
R_e^\perp B_i^\perp + R_i^\perp B_e^\perp + R_m^\perp B_m^\perp = (1 - C_f) \mathbf{D}(U_i^\perp, \mathbf{B}_i) 
\]  

(23)  

where the upstream ion inertial scale length \( \lambda_{De} \) is  

\[
\lambda_{De} = \left( \frac{m_p}{e} \right) \epsilon / |\mathbf{E}_i| 
\]  

(24)  

6.3. Resistive Dispersive Transition  

We may estimate the anomalous magnetic Reynolds number, \( R_m^\perp \), as follows. For the shock geometry used in (6) and (11), Ohm's law reduces to  

\[
\mathbf{J}_e = \mathbf{E} + \mathbf{J}_e \approx \mathbf{E} \mathbf{E}_i \mathbf{B}_i \mathbf{B}_e. 
\]  

(20)  

\[
\text{Assuming that } \mathbf{E} \mathbf{B}_i < \mathbf{E} \mathbf{B}_i \mathbf{B}_e \text{, we find, using (18),} 
\]  

\[
\nu^2 = \left( \frac{M_i}{M_e} \right) C_s^2 \frac{1}{1 + k^2 \lambda_{De}^2} 
\]  

(19)  

\[
\frac{R_m^\perp}{bB_i} = \frac{2}{\pi} \left( \frac{\lambda_{De}}{\lambda_{De}^*} \right) \frac{1}{1 + k^2 \lambda_{De}^2} 
\]  

(21)  

choosing an \( \alpha^* = \text{ spatial dependence reduces}(22)-(24) \) to a quartic, one of whose four solutions corresponds to an upstream whistler that is resistively damped as it propagates away from the shock. We then seek the conditions for which \( R_m^\perp > 1 \). The whistler radiated by the steepening shock would then be damped after it propagates one wavelength upstream, and the shock transition would be monotonic. Our procedure is therefore based on the assumption that the nonlinear scale length and the wavelength of the upstream phase-standing wave are comparable. The formal results are not valid above the critical Mach number, because the additional dissipation needed has not been taken into account in calculating the whistler damping length. Figure 10 shows the curve \( k = \beta \mathbf{e} \mathbf{L} \) in a polar plot whose radial coordinate is the fast Mach number and whose polar angle is the shock normal angle. The critical Mach number is also shown. Each quadrant corresponds to a different value of \( C_f/C_s^2 \) upstream. We assumed that \( \epsilon / \mathbf{B}_i \mathbf{B}_i = 1 \). In general, quasi-perpendicular shocks should be dispersive and quasi-perpendicular shocks resistive. The range of \( \lambda_{De} \), for which the shock is resistive increases with increasing Mach number, and for a given Mach number, the resistive \( \lambda_{De} \) range decreases with increasing \( C_f/C_s^2 \) upstream.  

Melott and Greenstadt [1984] have summarized the existing data on the resistive-dispersive transition in sub-
critical bow shocks. They calculated the resistive and dispersive scale lengths using Gudkova's (1986) estimate of ion sound anomalous resistivity and measured values of the density, \(n_0\), and \(\nu_i\). They found that for shocks with upstream wave periods in the dispersive regime, the resistive scale length exceeded the ion sound resistive scale length, whereas, with one exception, monotonic shocks corresponded to most solar wind shocks, the more perpendicular shocks will be dispersive. The resistive computations presented here are strictly only for large \(\varepsilon\), unchanged by the strong field regime or, equivalently, when the shock is followed for longer than a gyropar, the reflected ions turn around in the upstream magnetic field and gain energy from the transverse flow electric field. The upstream ions decelerate through the shock on their second encounter create a superthermal ring distribution downstream. Recent numerical simulations indicate that a self-consistent ion reflection shock can exist in a quasi-neutral plasma without Debye length structure (Leroy, 1983). Since the simulated shocks resemble typical bow shocks in several important ways, we review the physics that went into, and came out of, these simulations (section 7.3). In section 7.3, we review these bow shock data analyses which were specifically designed to test the theory of ion reflection shocks. In section 7.4, we discuss the range of parameters for which an ion reflection shock is expected.

7.1. Introductory Remarks

Nearly all bow shocks are supercritical. Nearly all quasi-perpendicular shocks are also supercritical and resemble the one in the top panel of Figure 11, rather than the resistive or dispersive profiles predicted by fluid theory. The fluid model of a foot overshoot-undershoot cycle downstream. Instead of a leading whistler wave train with an ion inertial scale length, the overshoot-undershoot resembles a trailing wave train whose scale length is an ion Larmor radius. Classical fluid theory had suggested the new physics required beyond the supercritical transition. Ions would reflect from the Debye length electrostatic potential layer that would develop above the critical Mach number. If the upstream magnetic field were weak, reflected ions would free stream away from the shock. Ion instabilities induced by the relative streaming of incoming and reflected ions might provide instability viscosity which would decelerate the incoming flow and regulate the size of the potential jump. Many early experiments (Paul et al., 1965, 1967; Killebecker et al., 1975; Sagre and Martone, 1971) and simulations were effectively in the weak field regime, because they were completed in less than one upstream ion gyropar. In the strong field regime or, equivalently, when the shock is followed for longer than a gyropar, the reflected ions turn around in the upstream magnetic field and gain energy from the transverse flow electric field. The upstream ions decelerate through the shock on their second encounter create a superthermal ring distribution downstream. Recent numerical simulations indicate that a self-consistent ion reflection shock can exist in a quasi-neutral plasma without Debye length structure (Leroy, 1983). Since the simulated shocks resemble typical bow shocks in several important ways, we review the physics that went into, and came out of, these simulations (section 7.3). In section 7.3, we review these bow shock data analyses which were specifically designed to test the theory of ion reflection shocks. In section 7.4, we discuss the range of parameters for which an ion reflection shock is expected.

7.2. Supercritical Quasi-Perpendicular Shocks

7.2.1. Schematic Trajectories of Reflected Ions

The ions and electrons interacted by means of an ion hybrid code, with kinetic ions and resistive computations presented here are strictly only for large \(\varepsilon\), unchanged by the strong field regime or, equivalently, when the shock is followed for longer than a gyropar, the reflected ions turn around in the upstream magnetic field and gain energy from the transverse flow electric field. The upstream ions decelerate through the shock on their second encounter create a superthermal ring distribution downstream. Recent numerical simulations indicate that a self-consistent ion reflection shock can exist in a quasi-neutral plasma without Debye length structure (Leroy, 1983). Since the simulated shocks resemble typical bow shocks in several important ways, we review the physics that went into, and came out of, these simulations (section 7.3). In section 7.3, we review these bow shock data analyses which were specifically designed to test the theory of ion reflection shocks. In section 7.4, we discuss the range of parameters for which an ion reflection shock is expected.

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7.3. Observations of Ion Reflection Bow Shocks

In this section, we summarize those bow shock data analyses which were specifically designed to test the theory of ion reflection shocks. Earlier measurements had found a second peak in the ion distribution downstream of supercritical shocks. These were reflected in the upstream magnetic field. They reduced the center-of-mass velocity of the upstream ion distribution, and as they turned in the upstream magnetic field, their Lorentz field added to the longitudinal electric field. These effects contributed to the formation of a potential overshoot in the shock ramp, which effectively insulated the downstream region from the upstream space distribution. Reflected ions in the foot, a ring distribution in the overshoot, and gradual downstream ion phase space distribution. Reflected ions in the foot, a ring distribution in the overshoot, and gradual downstream ion phase space distribution. Reflected ions in the foot, a ring distribution in the overshoot, and gradual downstream ion phase space distribution.
The top panel shows the overshoot amplitude, defined as $A = B_2 - B_1$, where $B_2$ is the maximum magnetic field in the overshoot and $B_1$ is the downstream magnetic field, as a function of the ratio of the fast Mach number to the critical Mach number. The use of ISEE 1 and 2 measurements permitted an accurate calculation of spatial scale lengths. The bottom panel shows the dependence of the foot thickness upon the Mach number ratio. Livingstone et al. [1984] generalized the observations of oblique shocks Woods [1971] and Phillips and Robson's [1972] estimate of the foot thickness for perpendicular shocks, assuming that upstream ions specularly reflect from the main shock ramp. The foot thicknesses were normalized to the distance $d$ along the shock normal at which a reflected ion turns back to the shock. Subcritical shocks had neither a foot nor an overshoot, while supercritical shocks had both. The foot thicknesses scaled as $d$ and were independent of Mach number and other shock parameters. The overshoot thicknesses scaled as the reflected ion Larmor radius based on the upstream magnetic field.

7.4. Parameter Space for Ion Reflection Shocks

7.4.1. Range of $\theta_{\text{sh}}$. For 90° shocks, all reflected ions are turned back into the shock. However, for oblique shocks some reflected ions recross the shock and some escape upstream, depending upon the ions' Larmor phase angles at the point of reflection. The fraction of the ions that can escape upstream increases with decreasing $\theta_{\text{sh}}$, and most escape for $\theta_{\text{sh}} < 45°$. [Phillips and Robson, 1972; Edmiston et al., 1982; Leroy and Winske, 1983]. Thus, ion reflection shocks would be quasi-perpendicular, consistent with the fact that these ions find shocks with an overshoot and a foot only for $\theta_{\text{sh}} < 45°$.

7.4.2. The second critical Mach number. We argued section 5 that an electrostatic ion sound subshock is expected to form above the Mach number $M_{\text{c2}}$, yet the ion reflection shocks studied in section 7.2 occur in a quasi-neutral plasma. In this section, we discuss the possibility that the ion sound subshock and the ion reflection shock occur in distinct Mach number ranges. There has to be enough shock-heated ions approaching the shock surface from downstream to initiate a reflection shock. Leroy et al. [1983] suggested that the downstream flow speed must equal the thermal speed $C_{\text{th}}$, which exceeds the (fast) critical Mach number $M_{\text{c2}}$ of the shock. The foot thickness is independent of shock parameters when $M > M_{\text{c2}}$.

shocks ($43° < \theta_{\text{sh}} < 88°$) are summarized in Figure 12. The top panel shows the overshoot amplitude, defined as $A = B_2 - B_1$, where $B_2$ is the maximum magnetic field in the overshoot and $B_1$ is the downstream magnetic field, as a function of the ratio of the fast Mach number to the critical Mach number. The overshoot amplitude and foot normalization are defined in the text. Error bars indicate typical uncertainties in overshoot amplitude, foot thickness, and Mach number ratio (Livingstone et al., 1984). Subcritical shocks have neither a foot nor an overshoot. The overshoot amplitude increases suddenly in the range $M_{\text{c1}} < M < M_{\text{c2}}$. The foot thickness is independent of shock parameters when $M > M_{\text{c2}}$.

The subshock is difficult to observe at the bow shock, because high time resolution potential and ion distribution functions are required. Moreover, Figure 13 shows that for typical solar wind parameters ($\beta_1 = 1$, $T_e/T_i = 0.5$, 1, and 3), the second critical Mach number upon upstream shock parameters for $0 < T_e/T_i < 4$. The upper left-hand panel contours the dependence of the ratio of the first and second critical Mach numbers upon $T_e/T_i$ and the upstream shock normal angle for an upstream electron plus ion $eta_1 = 1$. The other three panels contour the second critical Mach number as a function of $\beta_1$ and $\theta_{\text{sh}}$, for $T_e/T_i = 0.5$, 1, and 3.

A laboratory experiment by Echevarria et al. [1971] found an isomagnetic potential jump with $\sim 1000V$ scale length between the first critical Mach number and a Mach number of 4.5-5.5. This isomagnetic jump is presumably the ion sound subshock predicted by two-fluid theory. Manheimer and Spicer [1985] review other laboratory evidence for an electrostatic subshock between the first and second critical Mach numbers. These laboratory experiments typically have small $\beta_1$ upstream, so that the second critical Mach number is relatively large. The subshock is difficult to observe at the bow shock, because high time resolution potential and ion distribution functions are required. Moreover, Figure 13 shows that for typical solar wind parameters ($\beta_1 = 1$, $T_e/T_i = 0.5$, 1, and 3), the first and second critical Mach numbers differ by about 50%, so that high-precision Mach number estimates are required to determine which regime the shock is in. Echevarria [1983] has reviewed the low shock evidence favoring the existence of the ion sound subshock.

Manheimer and Spicer [1985] argue that the dissipation between the first and second critical Mach numbers is due to "longitudinal resistivity"; basically, that the ion and electron flow velocities parallel to the shock...
normal are coupled by an interaction with the ion sound branch. The magnetic field bends and damps in the small-amplitude limit. At the second critical Mach number, the electrostatic subshock is completely damped, and the ion reflection shock becomes nearly neutral. While some of the reflected ions are expected between the two critical Mach numbers, ion reflection is strong enough to dominate the ion dynamics. This leads to a second critical Mach number.

7.4.3. A third critical Mach number? A heuristic argument suggests that ion reflection cannot supply all the dissipation needed for steady high Mach number shocks. The internal energy density approaches a limit of 2/3 of the upflow energy flow density downstream of the strong shocks (assuming γ = 5/3). The state downstream of ion reflection shocks consists of heated electrons, a fraction (1 - N) of compressed thermal ions, and a fraction N of gyrating ions with speeds of about 1.7 times the upstream speed. Given this, one might postulate that well above the second critical Mach number the density of the downstream energy resides in the gyrating ions. Thus, the energy density downstream would be about 1.6 p_{\text{ex}}/N. Thus, a must exceed 2/3 for the strong shock limit to be satisfied. However, simulations [Leroy et al., 1981, 1987], laboratory experiments (Chodura, 1973), and bow shock observations [Paschmann et al., 1981] all find that N, which presumably is self-consistently regulated, is roughly 0.2. Furthermore, simulations [Leroy et al., 1981, 1982; Forslund et al., 1984] also indicate that the ion reflection shock becomes unstable at a critical Mach number above the second critical Mach number most of the downstream energy resides in the gyrating ions, the energy density downstream would be about 1.6 p_{\text{ex}}/N. Thus, a must exceed 2/3 for the strong shock limit to be satisfied. However, simulations [Leroy et al., 1981, 1987], laboratory experiments (Chodura, 1973), and bow shock observations [Paschmann et al., 1981] all find that N, which presumably is self-consistently regulated, is roughly 0.2. Furthermore, simulations [Leroy et al., 1981, 1982; Forslund et al., 1984] also indicate that the ion reflection shock becomes unstable at a critical Mach number above the second critical Mach number.

8. Bow Shock and Interplanetary Shock Observations 8.1. Introductory Remarks Observational studies of the dependence of bow shock structure, and of the region upstream of the bow shock, on solar wind parameters have shown that the magnetic profiles of quasi-parallel shocks are much broader and more complex than quasi-perpendicular profiles. Their magnetic field appears to pulsate between upstream and downstream values on spatial scales that are far below the current fraction of an earth radius (Greenstadt et al., 1970). It is often difficult using magnetic data alone to determine where the quasi-parallel "shock" is, since the current region often extends far beyond this characteristic region (Greenstadt et al., 1984). Field-aligned beams of energetic electrons are found nearest the leading edge of the shock, where the magnetic field is compressed (Bierwirth et al., 1973, 1983; K. Anderson et al., 1978; R. Anderson et al., 1981). The magnetic field angular distributions become progressively more diffuse with distance downstream of the shock region, and the typical energies decrease in a pattern consistent with the swelling back of electron trajectories by the solar wind electric field (D. Anderson et al., 1979). At lower energies the electron heat flux in the shock region is often directed upstream away from the bow shock, reversing the normal direction of the solar wind electron heat flux. The ion reflection shock ion distribution mirrors that of the electrons. Few keV field-aligned beams are found at the leading edge of the ion foreshock. Further downstream, so called "intermediate" ion distributions are spread in energy and pitch angle (Greenstadt et al., 1980; Bonoli and Moreno, 1981a,b) which extend to energies of several hundred keV (Scholer et al., 1979; Ipsch et al., 1981a,b), compatible with the energies achieved by Fermi acceleration in interplanetary shocks (Lee, 1982a). Phase-bunched "gyrating" ion beams are often observed deep within the shock (Dargiolo et al., 1981; Sentman et al., 1982).

The upstream solar ion energy density can exceed that of the interplanetary field by as much as a factor 5 (Ipsch et al., 1981). More significantly, because of the solar wind is decelerated and deflected when it enters the interplanetary field, the solar wind speed should decrease to the thermal sound speed. Thus, the third critical Mach number should be defined by a steady flow and the sound speed based upon the thermal pressure alone.

The electron and ion velocity distribution functions evolve with time down the shock. A significant fraction of the reflected ions have only a small part of the ion sound wave normal angle at the point where the particles first escape upstream. The energetic electron and ion beams originate from the quasi-perpendicular zone of the bow shock. Energetic electrons can be accelerated by instabilities generated by the reflected ions in the magnetic foot of a supercritical quasi-perpendicular shock (Papadopoulos, 1980). Furthermore, as the downstream ion ring distribution is thermalized and isotropized, some energetic ions will be scattered onto trajectories that reconnect the curved bow shock surface from behind. These ions that cross that shock can form the ion beams that are observed to stream from the quasi-perpendicular zone of the bow shock (Tsuneta et al., 1981).

The diffuse distributions certainly appear to escape from the quasi-perpendicular zone of the bow shock. Because of the inherent ambiguity, earth foreshock measurements cannot conclusively settle whether the apparent diffuse electron and ion beams that originate from the quasi-perpendicular zone of the bow shock. However, the electron distribution is thermalized and isotropized, some energetic ions will be scattered onto trajectories that reconnect the curved bow shock surface from behind. These ions that cross that shock can form the ion beams that are observed to stream from the quasi-perpendicular zone of the bow shock (Tsuneta et al., 1981).

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assign uniquely the phenomena observed upstream to a particular shock normal angle. Nonetheless, it appears that the quasi-parallel, quasi-perpendicular transition occurs suddenly near $\alpha_{\text{th}} = 45°-50°$.

The large-amplitude waves upstream also blur the relationship between waves and particles and the parameters of the bow shock. For example, when the local shock normal angle based on the averaged upstream magnetic field is near $45°$, the instantaneous shock normal angle may oscillate between the quasi-perpendicular and quasi-parallel regimes, thereby making the local shock structure and the distribution of escaping particles unsteady (Grosnestadt, this volume).

8.4. Structure Upstream of Interplanetary Shocks

Since the radii of curvature are 250–2500 times larger than the bow shock's, interplanetary shocks should reveal what is intrinsic to quasi-parallel structure. However, detections of the classical foreshock signatures—superthermal ions and magnetohydrodynamic wave-hours before an interplanetary shock have been difficult to relate to the shock, not only because the solar wind normally has energetic ions and is magnetically turbulent, but also because the global shock and interplanetary field geometry is difficult to ascertain. Our increasingly complete understanding of how shock upstream phenomenology has helped to clarify the shock association, since MHD turbulence that is accompanied by other upstream signatures can now be related to interplanetary shocks.

The first evidence that quasi-parallel interplanetary shocks have large foreshocks came from a study of upstream ion sound fluctuations (Kennel et al., 1962). Ion sound fluctuations, whose spectrum was similar to that upstream of the bow shock, extended hundreds of earth radii upstream of quasi-parallel interplanetary shocks. They were not found upstream of quasi-perpendicular interplanetary shocks. Shortly thereafter, magnetometer studies (Russell et al., 1983; Tsurutani et al., 1983; Kennel et al., 1984a, b; Vinas et al., 1984) revealed that MHD waves similar in period and amplitude to those upstream of the bow shock occur upstream of quasi-parallel interplanetary shocks. The measurements of superthermal electrons and ions upstream of interplanetary shocks have been discussed by Gosling et al. (1983, 1984) and Tsurutani and Lin (1983). Thus, it appears that the foreshock is intrinsic to quasi-parallel shocks.

9. Quasi-Parallel Shocks

9.1. Theories of Parallel Shocks

The first parallel shock theory (Parker, 1961) visualized the shock layer as consisting of two counterstreaming ion beams which would be firehose unstable when the upstream $\beta_i$ is high—a remarkably precocious forecast.

The first parallel shock theory to incorporate classical steepening arguments is due to Moissee and Sagdeev (1963). When $C_T^2/C_s^2 > 1$, the parallel fast mode is an ion sound wave which will steepen until it reaches Debye length scales. Reflection of upstream ions would then lead to an irreversible ion sound wave train which accomplishes the shock transition. Moissee and Sagdeev (1960) went on to argue that, in the absence of collisions, shock compression would increase only the temperature parallel to the magnetic field, so that if $\beta_i$ were high enough, a firehose instability would grow on the downstream thermal anisotropy. This suggested motivated Kennel and Sagdeev (1967), Kennel and Petschek (1968), Hesstvedt and Sagdeev (1969), and Gruzinov and Sagdeev (1970) to develop a theory of low Mach number parallel firehose shocks in very high $\beta_i$ plasmas (see also Sagdeev, 1979). At low Mach numbers, relaxation of the ion anisotropy through the growth of Alfven waves can provide for a shock transition. Auer and Voigt's (1973) numerical calculations confirmed the general features of this theory, but indicated that an ion sound subshock was required at higher Mach numbers (Jackson, 1984). A recent simulation of a 10° shock with an Alfven Mach number of 4 showed that the downstream thermal anisotropy relaxed to firehose marginal stability via the growth of Alfven waves (Ken and Swift, 1982) but, because the code assumed quasi-neutrality, could not have found an electrostatic substructure.

The above firehose shock models do not pay attention to upstream structure, either a standing whistler wave train (Ken and Swift, 1982; Quest et al., 1983) or the injection of energetic particles into the foreshock. Lee (1982, 1983a) considered the idealized case in which a thin, parallel planar shock injects a monoenergetic ion beam into a broad foreshock. He then computed the growth rate of parallel propagating Alfven waves by the resonant analog of the firehose instability (Kennel and Swift, 1963), the spatial decay of the ion beam due to quasi-linear pitch angle scattering, and the subsequent ion Fermi acceleration by shock compression. Lee's (1982, 1983a) theory is the foreshock analog of the firehose shock models discussed above.

In summary, nearly, all theories of quasi-parallel shock structure agree that long-wavelength MHD turbulence is central to the dissipation in both the shock and foreshock. 9.2. Escape of Superthermal Ions Upstream of Quasi-Parallel Shocks

The fact that quasi-parallel shocks allow significant access upstream of ions that have interacted with the shock seems to be their primary observational characteristic, since the upstream waves can be derived from the ions. The types of ion distributions observed upstream—reflected, "intermediate," "diffuse"—reflect both how they are generated and how they interact with upstream turbulence, and sophisticated studies are presently underway to unravel these detailed interrelationships (Schwartz et al., 1983).

Plate 1. The two-fluid quasi-neutral dispersion relation for oblique propagation. The left-hand and right-hand panels sketch the two-fluid quasi-neutral dispersion relation obtained by Pursimo and Kennel (1985) for $C_T^2 < C_s^2$ and $C_T^2 > C_s^2$, respectively. The fast, intermediate, and slow branches are indicated by red, blue, and green lines, respectively. The dashed and dash-dotted lines indicate the flow speed upstream of fast shocks (both parallel and downstream of subcritical (left) and supercritical (right) shocks, respectively. In principle, a whistler wave can stand upstream of both $C_T^2 < C_s^2$ and $C_T^2 > C_s^2$ oblique shocks. When the shock is subcritical (left), an almost electrostatic wave on the whistler resonance cone can phase-stand downstream. It is possible for a dispersive mode on the intermediate branch to phase-stand downstream of supercritical shocks. A Dobyla length structure (not included in the right-hand panel) might also be part of the downstream structure of supercritical oblique shocks.
MHD waves are found downstream of the ion foreshock’s leading edge. The ion foreshock is downstream of the leading edge of the electron foreshock. Large-amplitude, long-wavelength perpendicular zone of the bow shock, but because they propagate more slowly than electrons, the leading edge of magnetic field (solid blue lines) and the bow shock. Superthermal ions (red) escape upstream from the quasi-perpendicular zone of the bow shock (yellow) near the point of tangency between the upstream stream. The condition a “foreshock critical Mach number,” at which significant catching ions becomes significant. The right-hand panel contours the dependence of this critical shock normal angle upon the upstream fast Mach number of 4. The critical shock normal angle depends weakly on \( \beta_i \) and somewhat more strongly on \( T_e/T_i \).

We have already pointed out that when the upstream shock normal angle is less than 45°, most reflected ions escape upstream. However, quasi-parallel shocks also cannot confine heated ions downstream. A downstream ion can free stream along the magnetic field and catch the shock if its parallel velocity \( V_p \) satisfies \( U_1 = V_p \cos \theta_{NB} \), where \( \theta_{NB} \) is the downstream shock normal angle. Edmiston et al. [1982] estimated the superthermal ion flux upstream by assuming that shock-catching ions are transmitted back through the shock, conserving their magnetic moments. The upstream fluxes will be maintained if the loss region in the downstream ions phase space is continuously refilled by wave-particle scattering.

It is clear that no particles can escape upstream of a perpendicular shock, and that the escaping flux will increase with decreasing upstream shock normal angle. Edmiston et al. [1982] found that near \( \beta_i \approx 45° \) the flux of upstream ions suddenly becomes comparable to that observed. Thus, this mechanism can account for the rapid change between quasi-parallel and quasi-perpendicular behavior.

To conform to the spirit of this paper, we will construct a “foreshock critical Mach number,” at which significant fluxes of downstream ions are expected to escape upstream. The condition \( U_1 = 3C_s \cos \theta_{NB} \), defines a rough threshold Mach number at which the number of shock-catching ions becomes significant. The right-hand panel of Figure 14 contours the dependence of the foreshock critical Mach number upon the upstream \( \beta_i \) and shock normal angle, assuming \( T_e = T_i \) downstream. To the right of each curve, \( U_1 = \frac{3C_s}{3C_s - 1} \), and the escaping flux will be small. For the Mach number range appropriate to the bow shock, the escaping flux turns on at a critical shock normal angle. The left-hand panel of Figure 14 contours the dependence of this critical shock normal angle upon the upstream \( \beta_i \) and the downstream \( T_e/T_i \), assuming the upstream fast Mach number is 4.

A more sophisticated view of the ion transport across the shock has been put forth by Eichler [1979] and Edmiston et al. [1982], who argued that the scattering mean free path is proportional to the ion Larmor radius and is therefore energy dependent. In such a case, we would observe the low-energy ion “temperature” to jump across a thin “shock,” whereas we would find that energetic ions free stream through the “shock” and only scatter upstream and downstream. Far upstream, we would divide the ion distribution into a low-energy part and a distinct superthermal component. The entire region would be filled with hydromagnetic waves over the broad wavelength range required to resonate with both thermal and superthermal ions.

**Plate 2. Foreshock schematic** [Tsurutani and Rodriguez, 1981]. Energetic electrons escape upstream from the quasi-perpendicular zone of the bow shock (yellow) near the point of tangency between the upstream magnetic field (solid blue lines) and the bow shock. Superthermal ions (red) escape upstream from the quasi-perpendicular zone of the bow shock, but because they propagate more slowly than electrons, the leading edge of the ion foreshock is downstream of the leading edge of the electron foreshock. Large-amplitude, long-wavelength MHD waves are found downstream of the ion foreshock’s leading edge.
with collisionless shocks which are considered to be infin-itely thin and as such the shock structure is therefore as-sumed to be relatively unimportant. Looked at in this fashion, shocks can accelerate particles in several ways. Ions whose motions are decelerated by the magnetic field can be trapped by gyrophase resonances and trapped particles can subsequently be released. Shock fronts are magnetic, a fact which may be explained by the finite extent of the bow shock. Either a given magnetic field line is connected to the shock front, or the particles diffuse across the mag-netic field into field lines which no longer contain the shock [Eichler et al., 1984]. Since this effect limits the number of shock crossings a particle can have and, therefore, the energy to which it can be accele-rated. The field line connection time is larger for interplanetary shock fronts than for the bow shock, so the first-order Fermi mechanism will have longer to operate. The en-ergic ion fluxes theoretically should increase exponen-tially, approaching a steady, planar shock, maximize at the shock, and hold approximately constant downstream—features characteristic of ESP events. The accelerated ions should be, and are, essentially isotropic in the shock frame. The test particle limit discussed in section 10.1 may therefore be misleading. Wentzel [1971], Eichler et al. [1977, 1982], and Drury and Volk [1981] included the upstream cosmic rays in the calculation of the structure of the shock front. They assumed that the energetic cosmic rays diffuse spatially with a long characteristic spatial length, and that the thermal plasma is subject to unspecified dissipation due to microturbulence. Their calculations reveal the gas dynamic jump conditions when no energetic particles are present. On the other hand, if the upstream cosmic ray intensity is nonzero and the sonic Mach number exceeds about 10, the entire shock transition takes place in the cosmic rays without a dissipation zone in the thermal plasma. For lower sonic Mach numbers, there must be both a cosmic ray fore­shock and a local plasma subshock—the situation which should pertain to typical solar system shocks. In summary, the distinction between quasi-parallel and quasi-perpendicular shocks is beginning to emerge from recent studies of energetic particles associated with in­terplanetary shocks. Quasi-parallel shocks appear to pro-duce the largest fluxes of diffusively accelerated protons.

10.4. Self-Consistent Foreshock Models

Lee and Li [1982] and Lee et al. [1985] have reviewed the approximation of energetic particles and shocks in the heliosphere. Interplanetary shocks near 1 AU are accom­plished by ESP events [Kolka et al., 1981], "shock formation" events [Sarris and Reinhard, 1981], and "postshock enhancement" [Gosling et al., 1980]. As mentioned above, ESP events are upstream events, whereas the energies of a few keV occurring for a few hours prior to shock passage. Shock spike events are impulsive energy en­hancements associated with local plasma subshocks. In this section, we concentrate on recent stud­ies that relate the properties of accelerated particles to shock parameters. Intrinsic characteristics of particle acceleration mechanisms are closely related to magnetic field properties on scales that are comparable to the distance between the shock and the particle's gyrophase. The shock parameters that are measured by ESP events can be divided into the shock normal angle, the shock density compression ratio, and the angular coordinate is the shock normal angle. The shock density compression ratio (top left quadrant) and Alfvén Mach number (bottom left) are reduced, because part of the RH relations are satisfied in the cosmic ray fore­shock. Similarly, magnetic field reconnection in the fore­shock increases the shock normal angle relative to that of the entire structure (lower right). The ratio of downstream cosmic ray to total pressure is probably an overestimate.

In summary, the distinction between quasi-parallel and quasi-perpendicular shocks is beginning to emerge from recent studies of energetic particles associated with interplanetary shocks. Quasi-parallel shocks appear to produce the largest fluxes of diffusively accelerated protons. In summary, the distinction between quasi-parallel and quasi-perpendicular shocks is beginning to emerge from recent studies of energetic particles associated with interplanetary shocks. Quasi-parallel shocks appear to produce the largest fluxes of diffusively accelerated protons.
Before we can arrive at a comprehensive theory that computes the energetic particle intensity and spectrum as a function of shock parameters, we must understand how particles that are originally part of the thermal plasma begin to be accelerated. Present energetic particle diffusion calculations start with a source of "seed" particles which can either be in the upstream flow or be injected at a subshock. It matters not for the final spectral index whether the seed particles are injected far upstream [Axford et al., 1971; Blandford and Ostriker, 1978] or at the subshock [Lee, 1968, 1980]. However, the energetic particle intensity will depend upon the nature of the source and therefore upon the shock normal angle as well as the Mach number. It now seems clear that seed particles are thermal ions that interact with the subshock once on their way to participating in the Fermi process. In the case of the bow shock, these are the few keV "upstream" ions that are reflected from or transmitted through the shock.

10.8. Supernova Shocks

The discoveries that most of the volume of the interstellar medium is in a hot low-density phase and that the composition of galactic cosmic rays is that of the interstellar medium and not of material recently processed in supernova explosions have revived the notion that supernova shocks Fermi accelerate the cosmic rays directly out of the interstellar medium. MHD shocks can produce the observed galactic cosmic ray energy spectrum. The density, temperature, and magnetic field in the hot interstellar medium are similar to those in the solar wind, and the Mach numbers of the supernova shocks at the phase when they accelerate the most cosmic rays are similar to those of solar system shocks. Thus, in addition to their intrinsic interest, studies of collisionless shocks in the solar system are directly relevant to the plasma physics of supernova shock acceleration.

11. Concluding Remarks

This review has focused on the critical Mach numbers at which collisionless shock structure changes. Figure 16, a Friedrichs diagram for collisionless shocks, summarizes, for $C_1^* = 0.1, C_2^*$ upstream, the dependence upon the upstream shock normal angle of the first (solid line), second (dashed), and subshock (dot-dashed) critical Mach numbers, and the ion sound resistive-dispersive transition for subcritical shocks. We assumed $T_2 = T_1$ in computing the second and foreshock critical Mach numbers. The radial coordinate is the fast Mach $M = T_2/T_1$ in computing the second and foreshock critical Mach numbers. The whistler critical Mach number and the critical Mach number discussed in section 10 are not shown. We hope that the use of such collisionless shock Friedrich diagrams will facilitate rigorous understanding of the dependence of shock structure upon upstream plasma parameters.

The staggering variety of collisionless shock structures predicted by theory and found in experiments over the past 25 years reflects the richness of contemporary plasma physics. Understanding collisionless shocks, the simplest of all nonlinear flow configurations, has required merging sophisticated concepts from nonlinear fluid physics with microscopic plasma physics, and, at all times, an equal opportunity to parameter dependencies. The next 25 years of collisionless shock research promise to be as fruitful as the past 25 years, as we extend our understanding to higher $p$ and higher Mach number fast shocks, to slow shocks, and to relativistic shocks, and find further applications to the plasmas in the laboratory, at the sun and in the solar system, and in astrophysics.


Some Macroscopic Properties of Shock Waves in the Heliosphere

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In situ plasma and magnetic field observations demonstrate the existence of collisionless shocks associated with spatial inhomogeneities or temporal variations in the solar wind and with solar wind-planetary interactions. Remote observations suggest that similar shocks occur in association with solar activity in the solar corona. This tutorial will be focused on the formation and propagation of such shock waves in the heliospheric plasma. I will draw upon simple theoretical models (both analytic and numerical) of these phenomena to illuminate the basic physical processes controlling shock formation and propagation in the interplanetary medium.

1. Introduction

"Kernel [this volume] describes physical processes that can account for the existence of shock fronts in plasmas where Coulomb collisions are extremely rare. The thickness of such a collisionless shock (or more precisely, the spatial scale over which the entropy of plasma flowing through the shock front is increased) depends upon the detailed nature of those processes. From a macroscopic point of view, in which the plasma flow in a physical system is considered on a spatial scale much larger than the thickness of any shocks it may contain, these details are unimportant. Such a flow can be described using the methods of fluid dynamics or magnetohydrodynamics without specific knowledge of, or reference to, the actual nature of the shock mechanism. The physical properties of the plasma on the two sides of the shock "layer" can be related by mass, momentum, energy, and magnetic conservation laws to yield the well-known [e.g., Carlson and Sonett, 1966; Burlaga, 1971] Rankine-Hugoniot relations.

The existence of collisionless shocks was first suggested as an explanation of the sudden commencement of some geomagnetic storms; a shock was postulated at the leading edge of a plasma cloud ejected from the sun by a solar flare [Gold, 1955] despite the objection that the material in interplanetary space must be so tenuous that the time required for Coulomb collision lengths were astronomically large. Interplanetary observations have since confirmed the existence of such shock waves propagating outward through the solar wind. These observations have also revealed the existence of large-amplitude variations in solar wind speed that correspond to the "corotating streams" suggested by other studies of geomagnetic activity. These streams are usually not preceded by shocks near the orbit of earth but are observed to steepen and form shock fronts farther out in the solar system. Thus nature affords us, in the solar wind, the opportunity to illustrate general behavior and lead to the capability of "guessing" the behavior of real, physical systems. I will attempt to follow this path by illustrating the formation and propagation of shock waves in the heliosphere through examples based on the simplest possible quantitative model that contains the physics basic to..."