Using Modal Decomposition to Study Beating Patterns of Solar Cycle Data

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Abstract

The periodic appearance and equator-ward migration of sunspots on the solar surface, the telltale sign of the 22-year solar cycle, is a process that does not operate symmetrically between the northern and southern hemispheres. We characterize the similarities and differences between the two hemispheres using a Fourier modal decomposition of the sunspot area record for each cycle. For this purpose we use long-term spot area data available since 1878. Our goals are first to see how well the individual cycle shapes can be described in terms of a few modes, second to characterize the asymmetries and symmetries between different hemispheres in terms of the interference of these individual modes, and third to establish whether or not any long term trends are evident when the data is viewed in this way. More specifically, we are interested in trends that might aid in the development of predictive capability for future cycles. We analyze the resulting amplitudes and phase shifts between cycles and hemispheres and find that individual cycles can be well represented by as few as 5 modes. Cycles with a high maximum total area tend to have a large variation in strength of harmonics, and correspondingly cycles with a small maximum total area tend to have little variation in strength of harmonics. A large difference between the amplitude of the fundamental mode in the north and south does not necessarily correspond to a large difference between north and south at higher harmonics.

Introduction and Motivation

Solar sunspot data, collected since 1878, has a periodic appearance but the Northern and Southern hemispheres do not match in phase and amplitude. It seems reasonable to assume that they communicate as there is no consistently leading hemisphere in both phase and amplitude and there is physical connection between the North and South through magnetic field lines. We asked if this communication is apparent in "beating" patterns from the North and South. We also asked if the North and South have systematic "beating" patterns, and if so whether we can predict patterns in the ascending cycle 24 from long-term patterns.

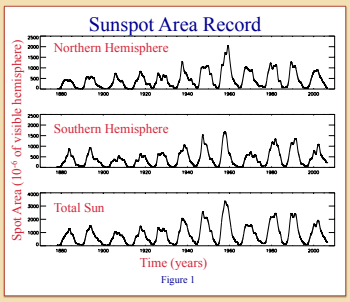

Figure 1

Mathematical Approach, Fourier Analysis of Individual Cycles

Characterize function with Fourier modes:

$$\mathcal{F}(t) = \sum_{j=0}^{N} \left[a_j \cos\left(\frac{j\pi t}{\tau}\right) + b_j \sin\left(\frac{j\pi t}{\tau}\right) \right]$$

Orthogonality relations:

$$a_j = \frac{1}{\tau}\int_0^{\tau} dt \mathcal{F}(t) \cos\left(\frac{j\pi t}{\tau}\right)$$

$$b_j = \frac{1}{\tau}\int_0^{\tau} dt \mathcal{F}(t) \sin\left(\frac{j\pi t}{\tau}\right)$$

If we express a_j and b_j as:

$$a_j = A_j \cos\delta_j$$

$$b_j = -A_j \sin\delta_j$$

Amplitude and phase come out to be:

$$A_j = \sqrt{a_j^2 + b_j^2}$$

$$\delta_j = \tan^{-1}\left(\frac{-b_j}{a_j}\right).$$

Final form

$$\mathcal{F}(t) = \sum_{j=1}^{N} A_j \cos\left(\frac{j\pi t}{\tau} + \delta_j\right)$$

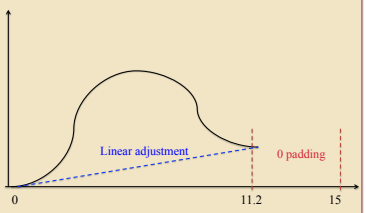

Figure 2a

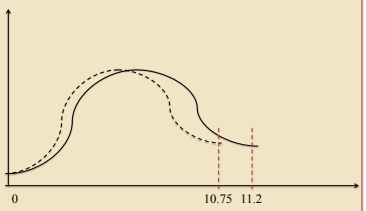

Figure 2b

Adjusted Cycles

- To effectively analyze Fourier modes, all cycles need the same fundamental mode
- The first method adds 0s, whereas the second method changes the length of the cycle
- Both methods preserve shape
- First method preserves timing
- Bottom method preserves minima level

Results and Analysis

Analysis of Power Spectra

- We computed power spectra of the full time series
- Most of the power is at the very low frequencies or around 11 years
- We can reconstruct the time series using a few low frequencies and power near 11 years
- The residual shows no apparent pattern

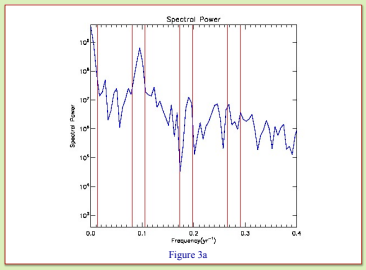

Figure 3a

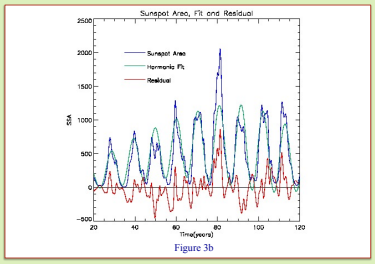

Figure 3b

Analysis of Individual Cycles

- We represented each cycle with 10 modes (see fig. 4a)
- Goodness of fit is calculated for each hemisphere by the following equation, in which f(x) is the original function and g(x) is the reconstruction:

$$X^2 = \sum_{i=1}^{n} \frac{(f_n(x) - g_n(x))^2}{n}$$

- 5 modes well represents individual cycles (see goodness of fit in fig 4b)

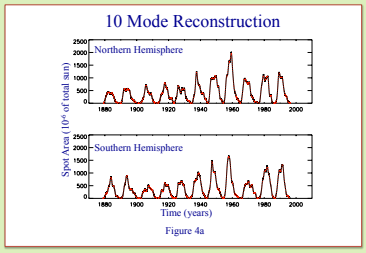

Figure 4a

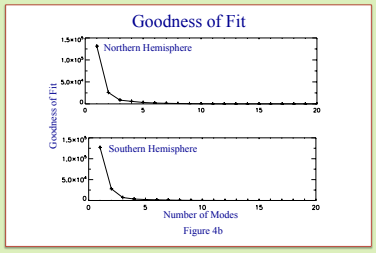

Figure 4b

Amplitude and Phase

- We examined the power of each mode, with respect to that of the fundamental
- The relative strength of higher harmonics shows no discernible correlation with cycle strength
- The 0th and first harmonic are well correlated between hemispheres
- Higher harmonics are not well correlated between hemispheres
- The first and second harmonics reveal a lead or lag between the N and S phases which switch back and forth within a few cycles

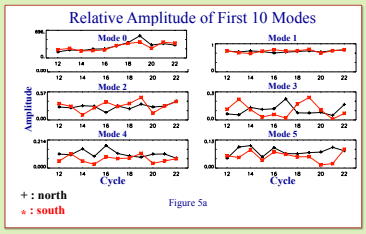

Figure 5a

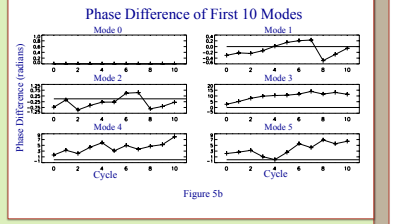

Figure 5b

Conclusions

- **All cycles can be well represented by as few as 5 modes**
- **The relative strength of higher harmonics shows no discernible correlation with cycle strength**
- **The strength of the mean (mode 0) and the fundamental (mode 1) are well correlated between hemispheres**
- **A large variation between the amplitude of the fundamental mode in the north and south does not necessarily correspond to a large difference between north and south at higher harmonics**
- **The fundamental (mode 1) and first harmonic (mode 2) reveal a lead or lag between the N and S phases which switch back and forth within a few cycles**
- **We have done the first method of adding 0s; the second method is in progress but showing similar results**

References:

- Bracewell, R.N. (2000), *The Fourier Transform and its Applications* (3rd ed.), Boston: McGraw-Hill
- Dikpati, M, Gilman, P.A., Toma, G, Ghosh, S.S., P.A.: 2007, *ApJ*, **245**

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