Using Modal Decomposition to Study Beating Patterns of Solar Cycle Data

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Solar activity
Solar butterfly diagram and spot-area cycle

Most likely a hydromagnetic dynamo operating inside the Sun governs the solar activity cycle.
Solar dynamo mechanism

(i) Generation of toroidal field by shearing of a pre-existing poloidal field through differential rotation ($\Omega$-effect)

(ii) Re-generation of poloidal field by lifting and twisting a toroidal flux tube by helical turbulence ($\alpha$-effect)

(iii) Flux-transport by meridional circulation
Solar dynamo

- Sunspots are thought to form from strong toroidal flux tubes which rose to the surface due to their magnetic buoyancy.

- Equatorward migration of sunspot-belt is explained by an equatorward propagation of the subsurface toroidal fields by the dynamo wave and equatorward return flow of meridional circulation.
Spot-area cycle in North & South hemisphere and total cycle

- **Northern Hemisphere**
- **Southern Hemisphere**
- **Total Sun**

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Spot Area ($10^{-6}$ of visible hemisphere)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1880</td>
<td>0</td>
</tr>
<tr>
<td>1900</td>
<td>0</td>
</tr>
<tr>
<td>1920</td>
<td>0</td>
</tr>
<tr>
<td>1940</td>
<td>0</td>
</tr>
<tr>
<td>1960</td>
<td>0</td>
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<tr>
<td>1980</td>
<td>0</td>
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<tr>
<td>2000</td>
<td>0</td>
</tr>
<tr>
<td>1910</td>
<td>0</td>
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<tr>
<td>1930</td>
<td>0</td>
</tr>
<tr>
<td>1950</td>
<td>0</td>
</tr>
<tr>
<td>1970</td>
<td>0</td>
</tr>
<tr>
<td>1990</td>
<td>0</td>
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</tbody>
</table>

The data shows fluctuations in the spot area over time, with peaks and troughs indicating the cycle's activity.
• Can we characterize communication between the North and South?
• Do the North and South cycles have any systematic “beating” patterns?
• If yes, can we predict the “beating” patterns of the ascending cycle 24 from the long-term patterns in the past?
Total amplitude heavily depends on the phase and amplitude of North cycle and South cycle.

Knowledge of “beating” patterns would help understand the cross-talk between the two hemispheres. Hemispheric cross-talk, like solar cycle amplitude and shape, plays important role in influencing certain aspects of space weather, such as geoeffectiveness.
Approach: Simple Fourier Analysis

Prescribe the cycles in the form:

Cast them in Fourier modes:

Orthogonality relations:

If we express $a_j$ and $b_j$ as:

Amplitude and phase come out to be

Final form

$$Cy_n(t) = \frac{a_{n0}}{2} + \sum_{k=1}^{l} a_{nk} \left( \sin^2(k \omega_n t + \delta_{nk}) \right),$$

$$Cy_s(t) = \frac{a_{s0}}{2} + \sum_{k=1}^{l} a_{sk} \left( \sin^2(k \omega_s t + \delta_{sk}) \right),$$

$$F(t) = \frac{a_0}{2} + \sum_{j=1}^{N} \left[ a_j \cos \left( \frac{j \pi t}{\tau} \right) + b_j \sin \left( \frac{j \pi t}{\tau} \right) \right]$$

$$F(t) = \sum_{i=0}^{N} \left[ a_j \cos \left( \frac{j \pi t}{\tau} \right) + b_j \sin \left( \frac{j \pi t}{\tau} \right) \right]$$

$$a_j = \frac{1}{\tau} \int_{0}^{\tau} dt F(t) \cos \left( \frac{j \pi t}{\tau} \right)$$

$$b_j = \frac{1}{\tau} \int_{0}^{\tau} dt F(t) \sin \left( \frac{j \pi t}{\tau} \right)$$

$$A_j = \sqrt{a_j^2 + b_j^2}$$

$$\delta_j = \tan^{-1} \left( \frac{-b_j}{a_j} \right).$$

$$F(t) = \sum_{j=0}^{N} A_j \cos \left( \frac{j \pi t}{\tau} + \delta_j \right)$$
A look at the whole time series

- Most of the power is at the very low frequencies or around 11 years
- There is no apparent pattern in the residual modes
Two Main Approaches to Individual Cycles

• ‘Expansion’:
  – Adding 0s till you reach fundamental
  – Needs a fundamental initially bigger than any individual cycle (15 years used)
  – Every cycle has the same number of ‘time’ points

• ‘Stretching/Compression’:
  – Changing the size of the interval between time points to give make the difference between beginning time and end time the same
  – Fundamental is the average fundamental of each cycle (10.75 years use here)
  – Every cycle has a different number of ‘time’ points
Reconstructed cycles

- 10 modes (including 0th) used
- Fitting individual cycles rather than all together
- This is obtained using expansion method
- Mode 0 is mean
- Mode 1 (fundamental) is 15 years
- Mode 2 is 7.5 years
- Mode 3 is 5 years
- Etc.
$X^2$ (‘Goodness of Fit Estimation for North and South’)

$$X^2 = \frac{1}{n} \sum_{i=1}^{n} (f_i - g_i)^2$$

Where $f$ is the original function and $g$ is the function reconstructed by modes.
Amplitude pattern in North vs South

+ : north
* : south
Amplitude, Accounting for Fundamental Amplitude

Mode 0

Mode 2

Mode 4

Mode 6

Mode 8

Mode 1

Mode 3

Mode 5

Mode 7

Mode 9

+ : north

* : south

Original Amplitude, For Comparison
Phase pattern in N & S

Phase difference:

\[ \delta_n - \delta_s \]

- Positive indicates that south is leading north
- Negative means that north is leading south
Results

• Individual Cycles can be well-represented by as few as 5 modes.
• The relative strength of higher harmonics shows no discernible correlation with cycle strength
• The 0th and first harmonic are well correlated between hemispheres
• The first and second harmonics reveal a lead or lag between the N and S phases which switch back and forth within a few cycles
Reconstructed Cycles

Fit of individual cycles in NH

Fit of individual cycles in SH
Summary

- Individual Cycles can be well-represented by as few as 5 modes.
- The relative strength of higher harmonics shows no discernible correlation with cycle strength.
- The 0th and first harmonic are well correlated between hemispheres.
- The first and second harmonics reveal a lead or lag between the N and S phases which switch back and forth within a few cycles.
- Future goal: We have analyzed the expansion method which preserves timing, and now we would like to finish analysis of stretched method that preserves the minima level.
References:


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Questions?
Amplitude pattern
Phase pattern