Line Profile Morphology with Rapid, Differential Rotation

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1. Objective

Our project aims to diagnose differential rotation through a careful analysis of spectral absorption lines. An increased knowledge of differential rotation in other stars will present us with a better understanding of stellar formation and evolution.

2. Background

Our project uses models from the Self-Consistent Field Method (SCF) of Jackson, MacGregor, and Skumanich 2004 (HAO). These models use an effective potential P that incorporates both the gravitational potential M_{grav} and centrifugal potential M_{cent}.

$\Sigma = \Sigma(r\sin 2)$

$M(\Sigma) = M_{cent}(r)$

$P(r) = M_{grav}(r) + M_{cent}(r)$

For the model to converge, the angular momentum must be a function of the distance from the axis of rotation and, therefore, effective potential P must increase monotonically with spherical radius. The temperature T(P), pressure P(P), and density Δ(P) are all functions of the effective potential. As such, concentric level surfaces which are topologically equivalent to a sphere must exist for the SCF models to converge.

$P = P(P(r))$ $\quad \Delta = \Delta(P(r))$ $\quad T = T(P(r))$

so, $P = P(r)$ $\quad \Delta = \Delta(r)$ $\quad T = T(r)$

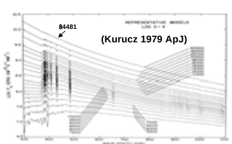
Concentric Level Surfaces

As a result, the photosphere is a level surface in the SCF models. We impose a temperature gradient using von Zeipel's gravitational darkening (von Zeipel, H. 1924., MNRAS)

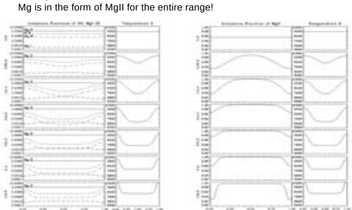
$T_{eff} \sim (M_{grav})^{1/4}$

3. What Line Are We Using?

We are using the Mg II 84481 doublet. It exists as the $3d^2D$ to $4f^2F$ transition. It is a prominent absorption line for a *vast* range of temperatures with an oscillator strength of 0.95.

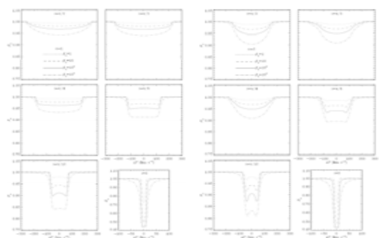


Above, you will notice that the Mg 84481 line is prominent for an extremely vast range of temperatures. Below are the ionization fractions for different models. Over 92% of Mg is in the form of MgII for the entire range!

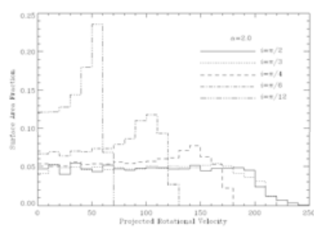


4. What do these profiles look like and *why*?

Our profiles develop a 'wing' structure as the inclination angle decreases. This 'wing' structure is also increased with more severe differential rotation.



Above, profiles are shown for different degrees of differential rotation with changing inclination angle. Below, the projected rotational velocity versus surface are is plotted for a differentially rotating model.

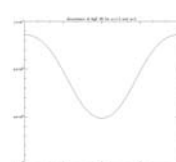


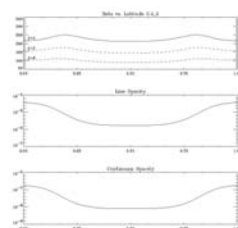
5. What modifications were made to the profile code?

The code was optimized by using a more sophisticated algorithm for the Voigt function (Humlíček 1981) and by exploiting inherent symmetries. The profile code was also modified by deriving a latitude-dependent opacity ratio $\exists_o(2)$.

$\exists_o(2) = 6_\Lambda{}^l(2) \,/\, 6_\Lambda{}^c(2)$

The continuum opacities were estimated using Rosseland mean opacities from both Kurucz 1993 and OPAL 1995. The MgII abundances were derived from a series of Saha equations. The abundance of the MgII $3d^2D$ was derived using a Boltzmann distribution.





Above is the abundance of MgII in our lower level of the transition for one model. To the right is $\exists_o$ and the line and continuum opacities.

Surprisingly, $\exists_o$ remains relatively constant while both of the opacities vary logarithmically. With this $\exists_o(2)$ dependent on latitude, we were ready to begin Principal Component Analysis (PCA)

6. Principal Component Analysis

We are currently using Principal Component Analysis (PCA) to analyze the morphology of a vast range of profiles with varying inclination angles i, degrees of absolute and differential rotation ∀ and 0, and varying degrees of microturbulence >. PCA is a pattern recognition technique whose eigenprofiles we are using to look at the "principal components" of our stellar spectra. PCA involves performing singular value decomposition on a covariance matrix C from N profiles M_n (observation matrix X).

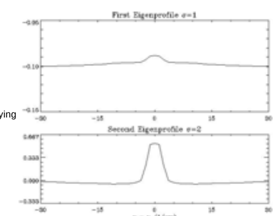
$$X = \begin{bmatrix} M_{11} & \cdots & M_{n1} & \cdots & M_{N1} \\ \vdots & & \vdots & & \vdots \\ M_{1n} & \cdots & M_{nn} & & \vdots \\ \vdots & & & \ddots & \\ M_{1N} & \cdots & \cdots & & M_{NN} \end{bmatrix}$$

$$C = XX^T = UEV^T$$

Where UEV^T is the singular value decomposition (SVD) of the covariance matrix C.

For the SVD of the covariance matrix C=UEVT, E contains eigenvalues $\Phi_1 \ldots \Phi_n$, with corresponding eigenprofiles in U, for which C can be *reconstructed*. PCAattempts to use only the largest eigenvalues and their corresponding eigenprofiles to reconstruct observational profiles. These largest components are the "Principal Components."

To the right is an example of our first two eigenprofiles while varying only inclination i, differential rotation ∀, and microturbulence >



7. Future Plans

To be a viable resource, the PCA models need to be expanded to include other masses, absorption lines, and von Zeipel coefficients. With this expanded parameter space, the PCA from our model should be able to diagnose differential rotation in oblate stars such as Achernar, Altair, Vega, Alderamin, and many more!

8. References and Acknowledgements

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