Examining the Impact of Prandtl Number and Heat Transport Models on Convective Amplitudes

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Some Definitions

• Convection describes the transport of heat by the motion of a fluid.

• Convective Amplitude refers to the root mean squared velocity ($v_{rms}$) of convection.

• Recall: Kinetic Energy (KE) $\propto v_{rms}^2$.
Why Study it?

• Convection: IT’S EVERYWHERE!!!

The Rayleigh number describes heat transfer between a hot and cold plate.

\[ R \downarrow a = a \Delta T g D^3 / \mu r \kappa^2 \]

http://www.mis.mpg.de/applan/research/rayleigh.html
Other reasons

- Magnetic Field Generation
- Coronal Mass Ejections
- Solar Flares
What is the Prandtl Number?

• Ratio of the viscous diffusivity to the thermal diffusivity, where:
  
  • $v =$ viscous diffusivity
  
  • $k =$ thermal diffusivity

So (mathematically) $Pr = \frac{v}{k}$

Which is inversely proportional to the Rayleigh number

$Ra = \frac{a \Delta T g D^3}{Pr \kappa^2}$
• Typically, we expect to see an increasing KE with an increasing Rayleigh number and increasing $\frac{1}{\kappa}$
2 parts to Convection:

- Small scale surface convection
- Large scale deep convection
- Scales too far apart for any computer to replicate
Pleiades
NASA's fastest supercomputer

The Navier Stokes Equations under an anelastic constraint

• The Navier-Stokes equations describe the relationship among velocity, pressure, temperature, and density within a moving fluid (i.e., convection)

Based on the conservation equations:
• Conservation of Mass
• Conservation of Momentum
• Conservation of Energy

• Anelastic constraint:
  \[ \nabla \cdot (\rho \mathbf{v}) = 0 \]
Fluxes in the Sun

- Heat transport can be described with the fluxes:
  - Radiative
  - Enthalpy & Kinetic Energy
  - Conductive

The sum of these fluxes adds to constant:

\[ \frac{L_\odot}{4\pi r^2} \quad \text{(or) } F_\odot \]

\[ L_\odot = \text{Luminosity of the Sun} \]

http://www.astronomy.ohio-state.edu/~pogge/TeachRes/Ast162/Stars/index.html
Fluxes in the Sun

Conservation of Energy:

\[ \rho T \frac{\partial S}{\partial t} = - \nabla \cdot (F_{\text{rad}} + F_{\text{e}} + F_{\text{k}} + F_{\text{cond}}) \]

Problem: \( F_{\text{cond}} \propto -\kappa \frac{dS}{dr} \rightarrow \) As \( \kappa \) decreases, \( v_{\text{rms}} \) increases, and no longer matches observations of the sun (\( \kappa_{\text{sun}} \sim 10^{17}, \kappa_{\text{model}} \sim 10^{12} \))

Recall: \( F_{\text{cond}} \) represents small scale convection
For Example...
Conclusion: We might need a new model to replace $F_{\text{cond}}$. 
New model

- We add a new flux, the granulation flux, to $F_{\text{cond}}$
  $F_g = F_\odot (0.5+0.5\tanh x)$
- Where $x = r - r_g/d_g$
  ($r_g$ and $d_g$ are constants)
- $0.5+0.5\tanh x$ satisfies the boundary conditions:
  \[
  \lim_{x \to 0} F_g = 0 \quad \text{and} \quad \lim_{x \to \infty} F_g = F_\odot
  \]
- Advantage: $F_g$ is not proportional to $-\kappa dS/dr$
  so we can lower $\kappa$ without raising the velocity
Comparing the new vs. the old

- KE vs time
- Flux balance
- Power spectra
- Convection structure
- $u_{\text{rms}}$ vs. $1/\kappa$
KE vs. Time

- Without $F_{\downarrow}g$
- With $F_{\downarrow}g$

![Kinetic Energy vs. Time for changing values of kappa](image1.png)

![Kinetic Energy vs. Time for changing values of kappa](image2.png)
Flux Balances

- Without $F_{\downarrow\sigma}$

- With $F_{\downarrow g}$

Note: $F_{\downarrow g}$ has been added to $F_{\downarrow rad}$
Flux Balances

- Without $F_{\parallel g}$

- With $F_{\parallel g}$
Flux Balances

- Without $F\downarrow g$
- With $F\downarrow g$

Avg flux balance for kappa = $4\times10^{12}$

![Graph showing energy flux through the shell vs. radius for different flux components with and without $F\downarrow g$.]
Power Spectra

- Without $F_{\downarrow g}$
- With $F_{\downarrow g}$

$\text{Wave number} = \frac{2\pi}{L}$, Where $L$ is the length of an eddy
Convection Structure

- Without $F \downarrow g$

- With $F \downarrow g$

- Colored bars show $v_{r.m.s.}$ value ($cm/s$)
Convection Structure

- Without $F \downarrow g$
  - $\kappa = 6 \times 10^{12}$
    - $F = 0.93$
    - $F = 0.91$
    - $F = 0.87$

- With $F \downarrow g$
  - $\kappa = 6 \times 10^{12}$
    - $F = 0.93$
    - $F = 0.91$
    - $F = 0.87$
Convection Structure

• Without $F \downarrow g$

• With $F \downarrow g$
Recall: \( R \downarrow a \propto KE \propto 1/\kappa \)
Results:

- Opposite from expected

Why?
Prandtl Number Effect Tests

• Recall $P\downarrow r = \nu/\kappa$

• Ran tests varying $\nu$ and $\kappa$ to observe the Prandtl number effect on convective amplitude
- Models correspond when Prandtl number is constant
• We can plot against the $v_{rms}$ (convective amplitude) instead of KE to get a better look at the Prandtl Number Effect.

• The convective amplitude (i.e., $v_{rms}$) decreases with increasing Prandtl number.
Summary

• The structure and amplitude of deep convection is similar for both models of surface convection considered (conduction and fixed-flux)
  - Suggests that deep convection is not very sensitive to the details of the surface convection

• The Prandtl number makes a big difference on the convective amplitude!
  - As kappa is decreased, holding Pr constant, vrms increases
  - As kappa is decreased, holding nu constant (increasing Pr), vrms decreases! (surprise!)
  - The second situation (increasing Pr) is very promising because vrms in current convection simulations might be too big and the real kappa of the Sun is very small
What’s next?

• Testing the following Hypothesis:
  
  • $T^\uparrow$, in the middle of the convection zone, increases as you decrease $\kappa$.

  • Where $F\downarrow e \propto v\downarrow rms T^\uparrow \approx F\downarrow \odot$

  • If true, explains decreasing $v\downarrow rms$ with increasing Prandtl number
Questions?
Flows in the Sun

- The Solar Dynamo generates magnetic fields from the flows:
  - Convection
  - Differential rotation
  - Meridional Circulation

Fusion
mass $\rightarrow$ radiation & thermal energy
Convection
thermal Energy $\rightarrow$ kinetic energy
Dynamo
kinetic energy $\rightarrow$ magnetic energy
• Tips: highlight important words in different colors
The whole story?

Maybe not...

This is exciting!

Disagreeing Observations

Something new under the Sun?