

SHOW ME THE MATH TITAN'S ESCAPE VELOCITY

What is escape velocity?

Escape velocity is the speed necessary for an object to completely escape the gravity of a large body such as a moon, planet or star. By using the conservation of energy and setting an object's kinetic energy equal to its negative gravitational potential energy, one can solve for velocity.

$$KE = -PE$$

$$\frac{1}{2}mv_e^2 = -\frac{GMm}{r}$$

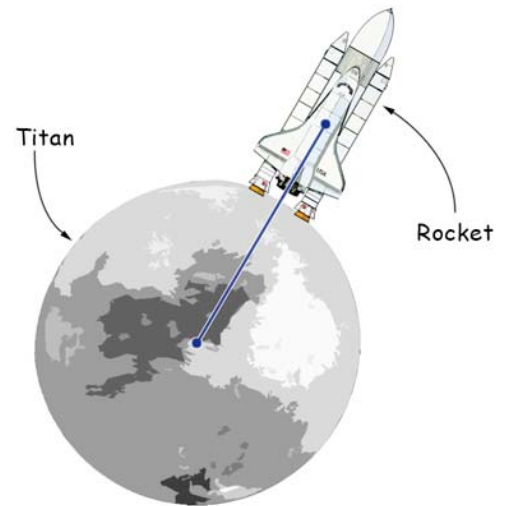
G = gravitational constant

$$G = 6.67 * 10^{-11} \frac{m^3}{kg * s^2}$$

M = mass of the central body

m = mass of the escaping body

r = distance between the central body and the escaping body (at point of escape). r is measured from the center of the central body to the center of the escaping body.



We can use the radius of the planet or body for 'r' if the escaping body begins from the ground. For example, the radius of the Earth can be used for 'r' if the escaping body begins from the ground.

Solving for the velocity, the *m*'s cancel out and the equation for the escape velocity of any object is:

$$v_e = \sqrt{\frac{2GM}{r}}$$

Titan's escape velocity:

$$M_{Titan} = 1.35 * 10^{23} kg$$

$$r_{Titan} = 2.575 * 10^6 m$$

$$v_e = \sqrt{\frac{2GM_{Titan}}{r_{Titan}}}$$

$$v_e = \sqrt{\frac{2 * (6.67 * 10^{-11} \frac{m^3}{kg * s^2}) * (1.35 * 10^{23} kg)}{2.575 * 10^6 m}}$$

$$v_e = \sqrt{6.99 * 10^6 \frac{m^2}{s^2}}$$

$$v_e = 2645 \frac{m}{s}$$

$$v_e = 2.65 \frac{km}{s}$$

This value is the same for all objects escaping from Titan's surface, regardless of mass. A hydrogen atom and the space shuttle have the same escape velocity from Titan; the only difference is in the amount of input energy needed to accelerate each to the escape velocity.

Think about throwing a bowling ball and a baseball above your head. It's harder to throw the bowling ball because it is so much heavier. In fact, it takes much more energy to throw the bowling ball than it does to throw the baseball. However, the bowling ball *can* be thrown to the same height as the baseball with enough energy. The same is true for atoms and space shuttles – each can escape if given the right amount of energy.

Why doesn't Titan's atmosphere have hydrogen?

Since a hydrogen molecule is lighter than a nitrogen molecule ($m_{hydrogen} \approx \frac{1}{14} m_{nitrogen}$), it takes less energy to get each hydrogen molecule to the necessary escape velocity for Titan.

$$m_H = 1.0079 \frac{g}{mol}$$

$$m_N = 14.0067 \frac{g}{mol}$$

$$KE_H = \frac{1}{2} m_H v_e^2 = \frac{1}{2} (1.0079 \frac{g}{mol}) * (\frac{1kg}{1000g}) * (2645 \frac{m}{s})^2 = 7051 \frac{kg * m^2}{s^2}$$

$$KE_N = \frac{1}{2} m_N v_e^2 = \frac{1}{2} (14.0067 \frac{g}{mol}) * (\frac{1kg}{1000g}) * (2645 \frac{m}{s})^2 = 97991 \frac{kg * m^2}{s^2}$$

$$KE_H \approx \frac{1}{14} KE_N$$

Individual hydrogen atoms have about 14 times more kinetic energy than individual nitrogen molecules. As a result, more hydrogen was able to escape from Titan's atmosphere while most of the nitrogen did not. Thus, Titan's atmosphere is made primarily of nitrogen.