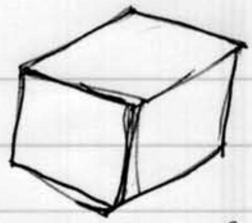
Atmospheric gas – meter^3



m^3

$$\rho = n\, m$$
$$= n \langle A \rangle m_p$$

ρ = density ($kg\,m^{-3}$)
m = mass of a gas molecule
$\langle A \rangle$ = average molecular mass (a.m.u.)
m_p = mass of a proton
n = N° molecules m^{-3}

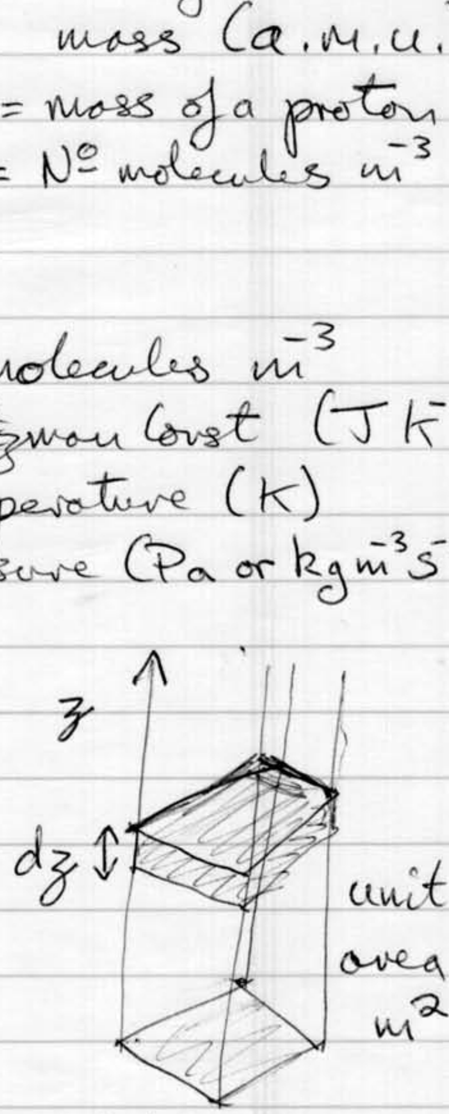
$\oplus$ $\rho \sim 1.2\ kg\,m^{-3}$
$n \sim 3 \times 10^{25}\ m^{-3}$
$\langle A \rangle \sim 29$ amu

Ideal Gas Law

$$P = n k T$$
$$= \frac{\rho}{m} kT$$
$$= \frac{\rho}{\langle A \rangle m_p} kT$$

n = N° molecules m^{-3}
k = Boltzmann Const ($J\,K^{-1}$)
T = Temperature (K)
P = Pressure (Pa or $kg\,m^{-3}s^{-2}$)

$$\rho = \frac{mP}{kT}$$



Baratropic Law

Balance of forces per unit area on slice of atmosphere dz thick

$$dP = -\rho g\, dz$$
$$= -\frac{mg}{kT} P\, dz$$

$g = \frac{GM}{R^2}$ ($m\,s^{-2}$)

$H = \frac{kT}{mg}$ (m) or km

Thermal Radiation

Stefan-Boltzmann Law – Power Emitted per Area $= \sigma T^4$

Wein's Law – $\lambda_{\text{max. emission}} \propto \frac{1}{T}$

Solar Flux

$$\text{Flux} = \frac{1368}{a} \text{ Watts m}^{-2} \quad a = \text{distance in AU}$$

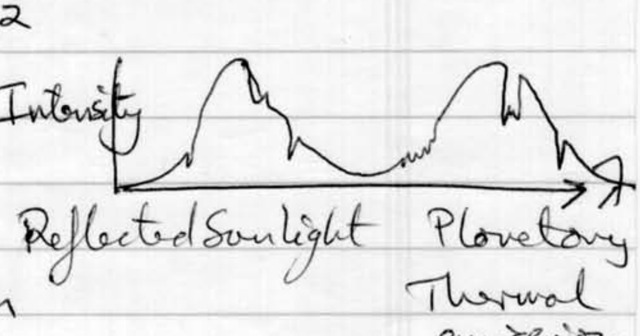
Tequilibrium

Power Absorbed = Power Emitted

$$\pi R^2 \, \frac{1368}{a^2} (1-A) = 4\pi R^2 \sigma T^4$$

$$T_{eq} = 280\text{K} \, (1-A)^{1/4} / \sqrt{a} \qquad A = \text{Albedo}$$

$$\text{Flux (Latitude)}_{\text{absorbed}} = \frac{(1-A)\,1368}{a^2} \cos(\theta)$$



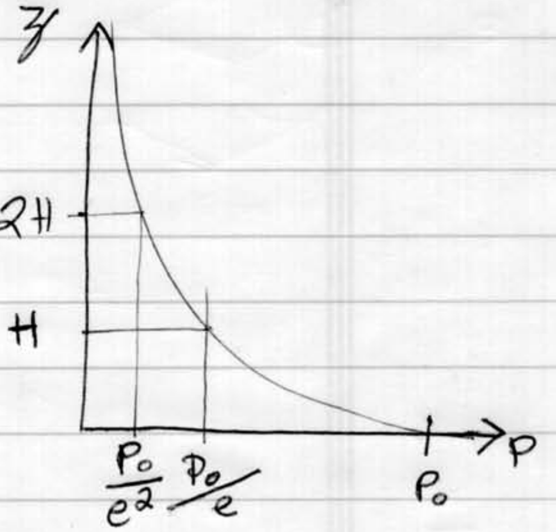
Spectroscopy

Absorption / emission
Lines / bands / continuum

Light → Atmosphere

dissociation, excitation, ionization.
Scattering, absorption, fluorescence.

$$\frac{dP}{P} = \frac{-1}{H}\,dz$$



$$\int_{P_0}^{P} \frac{dP'}{P'} = \frac{-1}{H}\int_0^z dz$$

$$\log P \Big|_{P_0}^{P} = \frac{-1}{H}\, z\Big|_0^z$$

$$\log P - \log P_0 = -\frac{1}{H}(z - 0) \Rightarrow P = P_0 \exp -z/H$$

$$n = n_0 \exp -z/H$$

$$\rho = \rho_0 \exp -z/H$$

Column Mass

$$M_c = \int_0^\infty \rho\, dz = \int_{P_0}^{0} -\frac{dP}{g} \qquad dP = -\rho g\, dz$$

$$= -\frac{1}{g}\int_{P_0}^{0} dP = -\frac{1}{g}\, P\Big|_{P_0}^{0} = \frac{P_0}{g} \quad (\mathrm{kg\,m^{-2}})$$

$$\text{Total Mass} = 4\pi R^2 P_0/g$$

$$n_0 = P_0/kT \qquad \rho_0 = mP_0/kT = \langle A\rangle m_p n_0$$

$$V_{escape} = \sqrt{2GM/R}$$

$1\,\mathrm{bar} \sim 1\,\mathrm{atm}$
$= 10^5\,\mathrm{Pa}$
$= 10^5\,\mathrm{kg\,m^{-3}\,s^{-2}}$