Before leaving the photosphere, we should note that it’s not really in time-steady equilibrium like we assumed (see Cravens, Chapter 5.3):

- **Granulation:** We see the tops of the convection cells, with diameters of about 1000 km. Cells: brighter ... lanes between cells: darker.

- **Supergranulation:** We also see magnetic fields collect themselves into larger patterns, with sizes of about 30,000 km. Are they just big convection cells? We’re not sure. *(up/down flows very hard to see!)*

- **Sunspots:** Galileo saw them; they’re as big as supergranules, but monolithic and dark. *Very* strong magnetic fields suppress convection in them. In UV & X-ray, they are ultra-bright active regions.

- Other features (flares, spicules, filaments, prominences, coronal holes) will be discussed later, since their main manifestation is “higher up.”

I’ll address 3 questions about granulation:

(1) Why are the “cells” bright and the surrounding “lanes” dark?

---

**Solar Granulation: Continuum Intensity**

The cells show rising convective blobs, which are hotter than surroundings.

If in pressure equilibrium, they’re also lower density (and thus more buoyant).

Hotter $\rightarrow$ higher $B_{\nu}(T)$.

Hotter by about 100 K (2% of $T_{\text{eff}}$), & brighter by about 8%.

<table>
<thead>
<tr>
<th>Aside: If $\delta I \approx 0.08$ between 2 neighboring features, what’s the residual $\delta I$ integrated over the whole solar disk?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{granules}} \approx \frac{A_{\text{disk}}}{A_{\text{granule}}} \approx 2$ million. Thus, $\langle \delta I \rangle \approx \frac{\delta I}{\sqrt{N}} \approx 6 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

which, believe it or not, *Kepler* can detect on other stars! It can see fluctuations down to $\sim 1 \times 10^{-5}$ (10 ppm).

On the Sun, gas rises in the centers of the cells, diverges out at the top, then “collects” (converges, gets denser, & **cools down**) in the intergranular lanes, which show downward velocities.

<table>
<thead>
<tr>
<th>Exception: In some spectral bands (e.g., the “G-band” at about 430 nm [violet]) the dark lanes contain many tiny bright points. They’re so small (50–100 km, i.e., 5%–10% of a granule!), they remain fuzzy even in best images from one-meter-class solar telescopes. Naming is evolving...</th>
</tr>
</thead>
<tbody>
<tr>
<td>(MBPs) magnetic</td>
</tr>
<tr>
<td>(GBP) G-band</td>
</tr>
<tr>
<td>(IBPs) intergranular</td>
</tr>
<tr>
<td>bright points</td>
</tr>
</tbody>
</table>

but they are jostled/buffetted around by the granular motions. We’ll come back to them soon.
(2) Why is the typical granule diameter about 1000 km?

Several explanations exist. One basic one is that as the hot/rising blob expands, it cools (adiabatically? not quite). That can’t go on forever. It eventually cools enough for \( P \)-equilibrium to re-establish it as denser, and that gas then descends.

However, another explanation is mainly geometric: Let’s consider a \(~\)circular granule cell as the top of a cylinder:

\[
h \approx H = \frac{k_B T}{\mu m_{\text{Hg}}}
\]

and for \( T \approx 6000 \text{ K} \), and \( \mu = 0.6 \) (ionized, below photosphere), \( h \approx 300 \text{ km} \).

Also, consider mass flux balance between blob gas coming up from the bottom and the gas oozing out the sides. (Nothing comes up through the top.)

Mass flux conservation: \( \rho u A = \text{constant} \), and we can consider \( \rho \) constant all through the cylinder. Thus, \( uA \) (from below) = \( uA \) (through the sides):

\[
uz(\pi r^2) = ux(2\pi rh) \quad r = h \left( \frac{2ux}{uz} \right)
\]

Lastly, we can note that the upward convective velocity \( uz = v_c \) gets faster as we approach the surface. It’s capped (saturated) at the sound speed \( c_s \).

\( \rightarrow \) and so is the sideways outflow speed \( ux \)!

Therefore, if \( ux \approx uz \), then \( r \approx 2h \).

Thus, \( r \approx 600 \text{ km} \), so the diameter is about 1200 km! Pretty close.

..........................................................
(3) What are the MBPs? They’re thin flux tubes of strong magnetic field (and thus lower density, since they have high $P_{\text{mag}}$).

Why do the strong fields gather in the downflow lanes? i.e., why to the MBPs appear there, and not in the bright cells?

If the field is initially weak and uniform, it gets carried up into the photosphere with the rising blobs (since $\beta \gg 1$).

Then the field lines get dragged from cell-centers into the lanes, along with the flow. $\mathbf{B}$ collects there and intensifies into “thin flux tubes.”

Strong-$\mathbf{B}$ tubes don’t sink back down in the lanes, since they have low $\rho$ & are more buoyant than surroundings! (full “circulation” is prevented)

Everything beyond granulation involves the magnetic field. How do we measure it in the photosphere?

Zeeman splitting:

We’ve seen that spectral lines correspond to bound electrons transitioning from one level to another, with a specific (quantized) change in energy $\Delta E$.

We measure it as a frequency ($\Delta E = h\nu_0$).

But when atoms are bathed in a magnetic field, there’s another key frequency that may interfere with $\nu_0$: the Larmor frequency.

To understand it, we’ll have to dip a toe into Chapter 3.2 of Cravens and think about the motion of single charged particles in a uniform $\mathbf{B}$.

Recall Lorentz force... $\mathbf{F} = m \frac{d\mathbf{v}}{dt} = q (\mathbf{v} \times \mathbf{B})$ (for $\mathbf{E} = 0$).

Define $\mathbf{B} = B_z \hat{e}_z$ as constant along $z$-axis.
The 3 components of the Lorentz equation of motion are:

\[
\begin{align*}
\frac{dv_x}{dt} &= \frac{q}{m} (v_y B_z) = \Omega v_y \\
\frac{dv_y}{dt} &= \frac{q}{m} (-v_x B_z) = -\Omega v_x \\
\frac{dv_z}{dt} &= 0 \quad \text{i.e., } v_z = \text{constant.}
\end{align*}
\]

and we define

\[
\Omega = \frac{q|B|}{m} = \begin{cases} 
\text{Larmor frequency} \\
\text{cyclootron frequency} \\
\text{"gyrofrequency"}
\end{cases} = \text{constant, if } B = \text{constant.}
\]

Take note that there is no Lorentz magnetic force parallel to \(B\). Particles moving along the field act like there’s no \(B\)-field at all.

But what about the equations for \(v_x\) and \(v_y\)? Coupled O.D.E.’s!

One trick for solving them is to take \(d/dt\) of both sides, and then substitute from the other...

\[
\begin{align*}
\frac{d^2v_x}{dt^2} &= \Omega \frac{dv_y}{dt} = -\Omega^2 v_x \quad \rightarrow \quad \ddot{v}_x + \Omega^2 v_x = 0 \\
\frac{d^2v_y}{dt^2} &= -\Omega \frac{dv_x}{dt} = -\Omega^2 v_y \quad \rightarrow \quad \ddot{v}_y + \Omega^2 v_y = 0
\end{align*}
\]

The solutions are sinusoids, but the original 2 equations show that \(v_x\) must be 90° out of phase with \(v_y\).

Thus, if \(v_x(t) = v_\perp \sin(\Omega t)\)

then \(v_y(t) = v_\perp \cos(\Omega t)\)

and... \(v_z(t) = v_\parallel\)

where both \(v_\parallel\) and \(v_\perp = \sqrt{v_x^2 + v_y^2}\) are constants.

Thus, the kinetic energy of the particle,

\[
E_K = \frac{1}{2} m |v|^2 = \frac{1}{2} m \left(v_\parallel^2 + v_\perp^2\right) = \text{constant, too!}
\]

i.e., a magnetic field “accelerates” particles by changing their direction, but it doesn’t speed them up or slow them down.
Why? $\mathbf{F}$ is always perpendicular to $\mathbf{v}$, so $\mathbf{F}$ does no net work on the particle ($\text{work} = \mathbf{F} \cdot \mathbf{v}$).

To get particle position $(x, y, z)$ versus time, we must integrate...

$$
\begin{align*}
x(t) &= \int dt \, v_x(t) = x_0 + \frac{v_\perp}{\Omega} \left(1 - \cos \Omega t\right) \\
y(t) &= \int dt \, v_y(t) = y_0 + \frac{v_\perp}{\Omega} \sin \Omega t \\
z(t) &= \int dt \, v_z(t) = z_0 + v_\parallel t
\end{align*}
$$

where these have been normalized so that the positions are $(x_0, y_0, z_0)$ at $t = 0$.

Define the gyroradius $r_\perp = \frac{v_\perp}{\Omega}$.

Positively charged particles have $\Omega > 0$, and thus their gyro-motion is left-hand polarized (i.e., use “left-hand rule” with thumb pointing along $\mathbf{B}$).

Negatively charged particles have $\Omega < 0$, and thus their gyro-motion is right-hand polarized.

In all plasmas that we’ll be dealing with $r_\perp$ is tiny in comparison to all other length-scales of the system.

Thus, if $v_\parallel \neq 0$, charged particles travel helical paths aligned with the field.... but if you “blur your eyes,” particles mainly just flow along $\mathbf{B}$.

And since + and − charged particles go in opposite senses, there’s net current $\mathbf{J}$ associated with gyro-motion (when particle-particle collisions are weak).

Note units: $\Omega =$ frequency in radians/sec... we often also see

$$
\nu_L = \frac{\Omega}{2\pi}
$$

as the Larmor frequency in cycles/sec.

The above was for individual charged particles (e.g., free electrons).

What about electrons that are bound in atomic orbitals?
Although the old “Bohr model” of an atom is wrong (i.e., nucleus = sun, electrons = orbiting planets), it’s still useful to think about electron orbitals having a unique spin (intrinsic, quantized angular momentum).

Very roughly, the orbital frequency of a bound electron is related to its energy level. When an atom is placed in a B field, it acts like a compass, aligning its spin axis along the field’s direction.

(not exactly... there’s a discrete # of quantized orientations)

But some atoms are aligned with B (↑↑), and some end up anti-aligned (↑↓).

Thus, some transitions have \[ \Delta E_+ = h(\nu_0 + \nu_L) \]
and others have \[ \Delta E_- = h(\nu_0 - \nu_L) \]
and some are unaffected \[ \Delta E_0 = h\nu_0 \]

Usually, \( \nu_L \ll \nu_0 \).

So, when \( B = 0 \), the photons are emitted with just \( \nu_0 \), and with a random assortment of polarization directions (light = wave, too!).
But for $\mathbf{B} \neq 0$,

- the $\Delta E_+$ photons have a higher frequency, and a bit of extra right-hand circular polarization, and
- the $\Delta E_-$ photons have a lower frequency, and a bit of extra left-hand circular polarization.

This is measurable as **Zeeman splitting** of spectral lines, and the frequency split is $\propto \nu_L$, which lets us measure the line-of-sight projected component of $\mathbf{B}$.

---

**Sunspots** were the 1st place on the Sun that Zeeman splitting was observed: fields are strong ($B \sim 0.1–0.3$ T).

Sunspots are dark in the photosphere, but bright (in UV and X-rays) higher up in the corona. Why?

In the photosphere, dark = cool (lower $T$, so lower $B_\nu(T)$).

So why are sunspots **cooler**? Two connected reasons:

1. The strong $\mathbf{B}$ field suppresses convection, thus the hot blobs can’t rise & transport heat from interior to the upper layers.

2. Total pressure equilibrium with surrounding atmosphere. Because $P_{\text{mag}}$ is higher, $P_{\text{gas}}$ is lower, and sunspots have lower $T$ at their $\tau \approx 2/3$ “surface.”

Why are they bright in X-rays? We’ll cover that when we talk about coronal heating.
The total solar luminosity varies with solar cycle, only by about 0.1%, but surprisingly it’s brighter at sunspot maximum.

Why? Surrounding most dark sunspots are slightly brighter, subtle patches called faculae (Latin for “little torches”).

More spots → more faculae, but faculae are spread out over larger areas than the spots, so they “win” the intensity war:

Why are faculae bright?

(1) Sunspots suppress convection, but the thermal energy “underneath” has got to go somewhere! Some of it is re-directed around the sunspot.

(2) Other geometry effects: B-field causes dips in the atmosphere, and off to the side we can sometimes see the bright “walls” as faculae (esp. for $\mu < 1$).
The Solar Magnetic Field

We’ve talked about it already quite a bit, but now we’ll concentrate on how the interior/surface field manifests itself above the surface.

In interior & photosphere, $\beta \gtrsim 1$, but in corona, it’s $\ll 1$ so that the plasma “passively” flows along the “rigid” field lines.

**Problem:** It’s very difficult to measure $B$ above the surface. We essentially have the photosphere, *some* measurements in the chromosphere, and then nothing until spacecraft “magnetometer” probes in the solar wind ($r > 60 R_\odot$).

However we can *infer* the direction of the field from observations of coronal loops & streamers...

Why do we believe these observed “strands” follow field lines?

Remember *Larmor gyro-motions.* The Lorentz force has no effect on particles moving parallel to the field, but constrains them to gyrate perpendicularly to the field.

Thus, things like heat conduction, wave energy transport, etc., are much more *efficient* parallel to the field. Plasma parcels “talk to each other” along the field, but not so much to their neighboring flux tubes.

The brighter loops & strands are connected to denser/hotter regions at their *footpoints.* Efficient transport makes those field lines “light up” more than their neighbors.
We can also use our knowledge of \( \mathbf{B} \) at the photosphere to **extrapolate** what’s going on higher up. It depends on how the polarities are distributed on the surface:

It’s never *purely* poloidal or toroidal. It’s always a mix of the two, and even the “field-free” regions in between strong patches contain a distribution of salt-and-pepper tiny concentrations of \( \mathbf{B} \).

The geometry of the field lines that **connect** these regions depends on how far one has to go to “find” enough opposite polarity to connect with...

Eventually, the solar wind accelerates, and you can see from eclipse images that the dipole-like field gets **stretched** out by the radially outflowing plasma (up there, \( \beta > 1 \) again!).

But in the low corona \( r \lesssim 2 R_\odot \), the flow speed \( \mathbf{u} \approx 0 \), so momentum conservation is hydrostatic:

\[
- \nabla P + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B} = 0
\]
See Cravens 6.3.1–6.3.2.

In the corona, $\beta \ll 1$, so the $\nabla P$ term is negligibly small compared to magnetic forces.

Also, when working out the numbers for coronal values of $\rho g$ and $J \times B$, it turns out that (near the surface) gravity is often negligible, too.

Thus, in the corona, $J \times B \approx 0$ (“force-free fields”)

There are 2 ways to make $J \times B = 0$:

$J = 0$ or $J$ parallel to $B$

We will see that $J \parallel B$ occurs for twisted strands of magnetic field (“flux ropes”), which occur in prominences, filaments, and CMEs.

Most of the volume of the corona obeys the first condition, $J = 0$, which in MHD means that

$\mu_0 J = \nabla \times B = 0$.

Remember that $\nabla \times \nabla \psi = 0$ for all scalar functions $\psi$, so this means that we can express this kind of magnetic field as a gradient of a potential function, $B = -\nabla \psi$.

(The minus sign is arbitrary, but it’s the usual convention.)

This is called a potential field, and it’s seen to be sort of a “ground state” (lowest magnetic-energy state) of a magnetic field. Add any nonzero $J$ to the system – say, by twisting it up – and you add to the total magnetic energy.

Remember that $\nabla \cdot B = 0$ for all magnetic fields, so this means that the potential obeys

$\nabla \cdot \nabla \psi = \nabla^2 \psi = 0$ (Laplace’s equation)

Cravens (section 6.3.2) goes through solutions to Laplace’s equation for the spherical domain $r > R_\odot$, when we specify $B(\theta, \phi)$ at the photospheric boundary $r = R_\odot$. 

9.12
Recall the spherical harmonics $Y_{\ell m}(\theta, \phi)$ for stellar pulsations. One can express any solution for the potential $\psi(r, \theta, \phi)$ as a sum of spherical harmonics, where each of them is multiplied by a radial fall-off function, i.e.,

$$\psi(r, \theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} a_{\ell m} \left( \frac{R_\odot}{r} \right)^{\ell+1} Y_{\ell m}(\theta, \phi)$$

where the $a_{\ell m}$ coefficients are computed from the lower boundary condition. When we have this solution, we then take $B = -\nabla \psi$ to get the actual field.

- $\ell = 0$ monopole field $\psi \propto r^{-1}$ $B$ drops off as $1/r^2$
- $\ell = 1$ dipole field $\psi \propto r^{-2}$ $B$ drops off as $1/r^3$
- $\ell = 2$ quadrupole field $\psi \propto r^{-3}$ $B$ drops off as $1/r^4$
- $\ell = 3$ octupole field $\psi \propto r^{-4}$ $B$ drops off as $1/r^5$

Higher $\ell$ → more complex the structure in $\theta, \phi$
→ faster drop-off of $|B|$ with increasing height.
→ the most compact coronal loops have shortest heights.

Thus, the solar dipole component “survives” to largest distances.

Of course, true magnetic monopoles don’t exist. If $B$ points outward in a spherically symmetric way, then $\nabla \cdot B \neq 0$, which violates Maxwell’s equations.

(Even though it is a solution to Laplace’s equation.)
However, a **split-monopole** can exist: + polarity in one hemisphere, − polarity in the other... and with $|B| \propto 1/r^2$.

The solar wind stretches out the surviving dipole into something like this.

The penalty is that at the equator, $\nabla \times B \neq 0$, and thus $J \neq 0$ in this thin **current sheet**. (Here, it’s *not* a potential field.)

Eventually, the solar wind accelerates, so all of the other terms in the momentum equation (including $Du/Dt$) become important again.

Although a full solution to the 3D MHD equations is needed to solve for $B(r, \theta, \phi)$, in practice a lot can be done with a simple approximation.

Assume the field is potential inside a sphere of radius $r \approx 2.5 R_\odot$, and that it’s stretched out radially (like a split monopole, $B_r \propto 1/r^2$) outside that sphere. This **potential-field source surface** (PFSS) model is useful for predicting what parts of the corona “connect” to distant parts of the solar wind.
At the forefront of “space weather” research, a lot of work is being done to figure out what kinds of solar wind come from classical (stretched-dipole) helmet streamers, versus what kinds come from weirder kinds of pseudo-streamers (with stretched quadrupole & octupole components):

Coronal Hole: low expansion, fast wind

Streamer: rapid expansion, slow wind

Pseudo-streamer: “squashed” expansion, slow wind (?)

Lastly, as we move on to talk about the solar wind, we should sum up the multi-scale nature of the “open” magnetic field...

Inter-granular MBPs are jostled, and $|B|$ gets weaker as you go up from photosphere. Fragmented flux tubes eventually merge together.

Much of the field collects into “network” lanes between supergranules, and it expands into a “canopy” as you go up from there.

Eventually, in the corona the field expands to fill the volume and follows the largest-scale multipole components of the entire Sun. The dipole is stretched out by the solar wind.