The Chromosphere and Corona

Now we can talk about what happens to the plasma above the photosphere.

We’ve talked about two kinds of MHD fluid motions down below:

- **Oscillations** (p-modes) that in general are “trapped” below the surface because the Sun is a finite-sized cavity;

- **Convective blobs** that rise and fall beneath the surface, but are stopped at the “glass ceiling” of the photosphere because the conditions turn back to convectively stable there.

However, the photosphere is not a rigid ceiling... it’s got a gradual fall-off inρ and T. Thus, there may be some leakage of kinetic/magnetic energy of these motions, up through the photosphere.

The still-unsolved “coronal heating problem” is to figure out exactly how these leaked motions transfer enough of their energy into heat (i.e., an increase in thermal energy) in order to make $T \sim 10^6$ K above the surface.

This addition of heat means that the photospheric radiative equilibrium is no longer true. Let’s go back to the gray atmosphere to see what happens...

Recall the zeroth moment of the equation of transfer,

$$\frac{dH}{dz} = -\kappa \rho (J - B) = 0$$

where $\kappa$ is here just the contribution from absorption (not scattering), and we set the whole thing equal to zero because of our assumption that flux is conserved through the thin photosphere.

If you remember, $H = F_{rad}/4\pi$, and the flux conservation equation was just a plane-parallel version of the full interiors version

$$\nabla \cdot F_{rad} = \rho \epsilon$$

where we assumed $\epsilon = 0$ far away from the nuclear-burning core.

As another refresher,

$$\epsilon = \text{energy generation rate, in } \frac{J}{s \text{ kg}}$$

$$\rho \epsilon = \text{energy generation rate, in } \frac{J}{s \text{ m}^3} \equiv Q$$
We should be careful about the sign conventions, though.

In interiors, \( Q > 0 \) corresponded to energy generation in the “photon gas” \( \rightarrow \) i.e., a net creation of photons.

We will eventually care about what is heating the particle gas, when radiation is involved. In that case, we’ll want to define

\[ Q > 0 \text{ (gas heating)} \quad \text{comes from absorbing/destroying photons} \]
\[ Q < 0 \text{ (gas cooling)} \quad \text{comes from net emission of photons (“radiative losses”)} \]

Thus, the signs are flipped. We want to define the net heating rate of the particles (due to photon creation/destruction) as

\[ Q_{\text{rad}} = -\nabla \cdot \mathbf{F}_{\text{rad}} = 4\pi\kappa\rho(J - B) \]

Let’s keep going with the gray atmosphere...

\[ B = \frac{\sigma T^4}{\pi} \]
\[ J = \frac{3\sigma T_{\text{eff}}^4}{4\pi} \left( \tau + \frac{2}{3} \right) \]

In the photosphere, we assumed that \( Q_{\text{rad}} = 0 \), so the condition of \( J = B \) gave us \( T(\tau) \). However, this isn’t true in the chromosphere and corona.

Let’s continue to explore how \( Q_{\text{rad}} \) behaves high above the photosphere.

Way up here, \( \tau \ll 1 \), so we can estimate

\[ J \approx \frac{\sigma T_{\text{eff}}^4}{2\pi} \quad \text{and let’s also define} \quad T_{\text{rad}}^4 = \frac{T_{\text{eff}}^4}{2} \quad \text{(i.e.,} \quad T_{\text{rad}} \approx 0.84T_{\text{eff}}) \]

where \( T_{\text{rad}} \) is called the radiative equilibrium temperature. Why call it that?

When \( \tau \ll 1 \),

\[ Q_{\text{rad}} = 4\kappa\rho\sigma(T_{\text{rad}}^4 - T^4) \]

and when \( T = T_{\text{rad}} \), there’s no radiative heating or cooling.

On the other hand, if \( T \) is somehow boosted to values above \( T_{\text{rad}} \), then \( Q_{\text{rad}} < 0 \) and the system wants to cool down.

If \( T \) is somehow decreased below \( T_{\text{rad}} \), then \( Q_{\text{rad}} > 0 \) and the system wants to heat up.

This is a thermally stable situation. If \( Q_{\text{rad}} \) were the only “right-hand-side” source of heating or cooling in the thermal energy conservation equation, then the gas would be driven toward radiative equilibrium (\( T = T_{\text{rad}} \)).
However, there are OTHER sources/sinks on the right-hand side of the thermal energy equation. Recall, for no motions \((u = 0)\), we wrote it as

\[
\frac{\partial U}{\partial t} = \sum (H - C) - \nabla \cdot F_{\text{cond}}
\]

where \(U\) is the thermal energy density of the gas. Note I also moved over the heat conduction flux term to the right-hand side, and changed the variable-name a bit to make it agree with all the other “fluxes” \((F)\).

In the chromosphere and corona, there are really two heating/cooling terms: one from radiation \((Q_{\text{rad}})\) that can be positive or negative, and one from the source of “coronal heating” \((Q_{\text{heat}})\) that’s always positive.

If we want to talk about time-steady equilibrium solutions, then we can say that \(\partial U/\partial t \to 0\), and the balance of heating/cooling terms on the right looks like:

\[
Q_{\text{heat}} + Q_{\text{rad}} - \nabla \cdot F_{\text{cond}} = 0
\]

\[
Q_{\text{heat}} + Q_{\text{rad}} + \nabla \cdot (K \nabla T) = 0
\]

where \(K = K_0 T^{5/2}\) for heat conduction in an ionized plasma, and \(K_0 \approx 8 \times 10^{-12}\) (in SI units) is the “Spitzer conductivity”.

Our goal is to find the \(T\) at which this balance occurs.

More about \(Q_{\text{rad}}\)...

When \(\tau \ll 1\), most sources of opacity obey the approximate form \(\kappa \propto \rho T^n\), but we should remember that specific spectral lines (or other properties of a specific ionization stage of a given element) exist only for a finite range of \(T\).
Thus, we collapse all of the $\kappa$ contributions into a single “radiative cooling function” $\Lambda(T)$, i.e.,

$$Q_{\text{rad}} = -4\kappa\rho\sigma T^4 \left( 1 - \frac{T_{\text{rad}}^4}{T^4} \right) = -\rho^2 \Lambda(T) \left( 1 - \frac{T_{\text{rad}}^4}{T^4} \right)$$

where the term in parentheses is $\approx 1$ because the chromosphere & corona usually exhibits $T \gg T_{\text{rad}}$. Adding up opacities from all elements...

When $\Lambda$ is increasing as a function of $T$, it’s **stable:**

- increase $T$
  - $\downarrow$
  - $\Lambda \uparrow$, so **more** cooling
  - $\downarrow$
  - $T$ decreases

When $\Lambda$ is decreasing as a function of $T$, it’s **unstable:**

- increase $T$
  - $\downarrow$
  - $\Lambda \downarrow$, so **less** cooling
  - $\downarrow$
  - $T$ keeps increasing

10.4
In the chromosphere & corona, $Q_{\text{rad}} < 0$ so radiation cools the plasma.

But in order to maintain the high temperatures we see, there must be either some “actual” heating ($Q_{\text{heat}} > 0$) or some heat conduction flux $F_{\text{cond}}$ that points INTO the region that we are studying. Recall:

$$Q_{\text{heat}} + Q_{\text{rad}} - \nabla \cdot F_{\text{cond}} = 0$$

We’ll look at a few of these regions, and solve for $T$. Let’s first summarize which of the 3 terms are important in the various regions:

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- Radiative cooling goes away at $r \gg R_\odot$ because $Q_{\text{rad}} \propto \rho^2$.
- Conduction is important only for a hot, ionized, & low density plasma (recall, $K \propto T^{5/2}$).

We already covered the photosphere, where radiation “rules alone” (i.e., $Q_{\text{rad}} = 0 \rightarrow$ radiative heating balances radiative cooling).

In the chromosphere, there’s some other source of heat ($Q_{\text{heat}}$) that competes against radiative cooling, such that equilibrium $T$ is slightly higher than $T_{\text{rad}}$.

What is that source of heat? **Unknown!** (it’s similar to the coronal heating problem... most believe they’re both really just one big problem with 2 faces)

The challenge: convert some other local source of energy into heat.
If you look “down” just a bit, there’s a huge pool of available energy: **convective overturning motions** (kinetic & magnetic).

How much energy density is needed?

- **Photosphere**: $U_{\text{kinetic}} \sim \rho v^2 \sim (2 \times 10^{-7} \, \text{g/cm}^3) (0.5 \, \text{km/s})^2 \approx 50 \, \text{J/m}^3$

- **Low corona**: $U_{\text{thermal}} \sim n k_B T \sim (10^{10} \, \text{cm}^{-3}) k_B (2 \, \text{MK}) \approx 0.2 \, \text{J/m}^3$

Only a small fraction of this energy (about 1%) needs to “leak” up above the photosphere and “randomize” itself into heat.

Note that, in the photosphere, the magnetic energy is comparable to the kinetic energy ($U_{\text{mag}} \sim U_{\text{kinetic}}$).

Why haven’t we figured out exactly HOW that happens?

- The chromosphere & corona are:
  - ⇒ highly structured & complex (in space)
  - ⇒ intermittent & bursty (in time)
  - ⇒ optically thin (so all that structure is summed up → blurred!)

- Telescopes still haven’t resolved the fundamental “flux tubes.”

- Theorists gone wild! Lots of ideas... difficult to choose which one is right.
  - ⇒ maybe different theories apply to different regions?

  (least active) Coronal Holes $\rightarrow$ Quiet-Sun Loops $\rightarrow$ Active Regions $\rightarrow$ Flares & CMEs (most active)

Usually, going left to right also means going from weaker to stronger **magnetic fields** (also: potential $\rightarrow$ increasingly twisted).
There are several main schools of thought:

**Waves:** If convective motions at surface are FAST, the upper atmosphere can only “respond” by having the perturbations propagate up in the form of waves. Eventually, waves are damped out by collisions or other effects.

**Nanoflares:** If convective motions at surface are SLOW, the upper atmosphere has time to rearrange itself. Field lines twist & braid themselves into a quasi-static state, building up lots of $\mathbf{J}$. Eventually, twists are “too much,” and $\mathbf{B}$ relaxes to a simpler state via lots of little *magnetic reconnection* bursts.

**Interchange reconnection:** Magnetic bipoles are continuously emerging from interior (pushed up by convection?). When they encounter opposing fields, there’s more reconnection. Closed fields may open up and shoot out hot plasma as *jets.*
Many of the proposed heating mechanisms consist of 3 parts:

- *Transport* the convective energy into upper atmosphere.
- *Store* energy in either motions or magnetic fields... preferably on small length & time scales.
- *Randomize* that stored energy, which turns it into heat. Collisional effects (viscosity, conductivity, resistivity) are popular, but there are weird “collisionless” wave-particle resonances, too.

My own favorite heating mechanism is **MHD turbulence:**

- It starts like in the wave picture, but then the waves spontaneously break up into smaller and smaller eddies/whirls.
- On tiny scales, the eddies undergo “bursty” reconnection like in the nanoflare picture (i.e., lots of tiny ‘current sheets’).
- In a way, it doesn’t matter what the actual randomization process is, since there’s a $\sim$constant flux of energy going from large eddies to small eddies. If you know how much energy goes in, then that same amount must come “out” as heat!

$\Rightarrow$ Turbulence *does* seem to provide the right amount of $Q_{\text{heat}}$ needed to heat the chromosphere and corona!
In any case, \( Q_{\text{heat}} \) depends on the local properties of the atmosphere: in general, \( \rho, T, u, B \), and the size/time scales of the structures being buffeted.

Many of those dependences have been “rolled up” into density. For example,

\[
Q_{\text{heat}} = C \rho^\alpha
\]

where some theories say:

\[
\alpha = \begin{cases} 
-1/2 & \text{sound waves} \\
+1/2 & \text{MHD turbulence} \\
+1 & \text{shocks or curr. sheets}
\end{cases}
\]

Remember that for thin flux tubes,

\[
\rho \propto \exp \left( -\frac{z}{H} \right) \quad \text{and} \quad B \propto \exp \left( -\frac{z}{2H} \right)
\]

so, \( B \propto \rho^{+1/2} \)

and this means that the \( \alpha = +1/2 \) case implies \( Q_{\text{heat}} \propto B \).

\( \Rightarrow \) This is typically what people expect for “magnetic heating.”

\( \Rightarrow \) (Active regions are heated more than quiet regions, etc.)

This brings us back to the heating/cooling/conduction balance,

\[
Q_{\text{heat}} + Q_{\text{rad}} - \nabla \cdot F_{\text{cond}} = 0
\]

In the chromosphere, we’ll find that \( T \) is too low for heat conduction to be important (recall \( F_{\text{cond}} \propto T^{5/2} \nabla T \)). Thus, it’s just externally-imposed (turbulent?) heating balanced by radiative cooling...

\[
Q_{\text{heat}} \approx -Q_{\text{rad}} \quad \Rightarrow \quad C \rho^\alpha = \rho^2 \Lambda(T)
\]

and we can write

\[
\Lambda(T) \propto \rho^{\alpha-2} \quad (\text{which is } \rho^{-2.5} \text{ to } \rho^{-1})
\]

which means that as we go up in height, \( \rho \) goes down, and thus \( \Lambda(T) \) must be always increasing as we go up.

\[
\begin{array}{c}
\text{\( \Lambda(T) \)} \\
\text{10^4 \quad 10^5 \text{ K}}
\end{array}
\]

\[
\begin{array}{c}
\text{\( T \)} \\
\text{\( \rho \) decreases}
\end{array}
\]

height
The chromosphere is heated steadily & gradually... as long as we’re on the stable side of the curve... i.e., as long as \( \partial \Lambda / \partial T > 0 \).

But when we reach the peak (call it \( \Lambda_{\text{max}} \)), there’s no more available cooling!

Once the cooling can’t keep pace with the heating any more, we get a catastrophic runaway to hot, coronal temperatures.

For the Sun, this is the sharp transition region (TR) that occurs at a height of \( \sim 2000 \) km (or only 0.003 \( R_\odot \)) above the photosphere.

We’ll look at the energy balance in the corona soon, but it’s worthwhile to first summarize how we know what we know observationally.

**Chromosphere:** “color layer” first seen around rim during eclipses.
**Red:** enhanced H\( \alpha \) emission (\( \lambda = 656 \) nm), but other emission lines seen, too.

The supergranular network is seen clearly in images of chromospheric lines:
• **Network**: patchy lanes/vertices between cells, with strong **B** field. Jutting up from base, dense thin strands follow the field. On disk: dark *mottles* ... Off limb: bright *spicules*.

• **Internetwork**: cell centers, filled with rapidly evolving “grains” ($\neq$ granules), more like 1000–3000 km “bright points” at the intersections of spider-web-like emission strands. Probably *shocks* driven by *p*-mode oscillations that leak up from below photosphere.

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• The **corona** emits most of its photons in X-ray & UV range. It’s mostly an *emission spectrum*, but it’s hard to get a spectrum at every “pixel” in an image!
  
  − Line intensities tell us how much “stuff” is there at a given $T$ (lines only exist for limited range of $T$ when its ion is $\sim$dominant)
  − Line widths tell us about the ion temperature (possibly $\neq T_e$)
  − Offset central wavelengths tell us if plasma is *Doppler* shifted

• Some X-ray/UV telescopes have *filters* to screen out all but a narrow range of $\lambda$’s. The goal is to find narrow ranges where the emission lines come from only $\sim$1 ion. Thus, **narrow-band images** show us coronal plasma at a given temperature (high spatial & time resolution).

• In eclipses (or in special “occulted” *coronagraphs*), we see very faint visible-light emission. The photons we see start as photospheric $I(\mu)$, and a tiny fraction of them are scattered (by about 90$^\circ$) into our line-of-sight.

  $$\text{# of photons scattered } \propto \int dx \: n_e$$  
  (where $x =$ line of sight direction).
Back to energy balance...

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We’ll skip the full solution for the plasma properties in the vicinity of the TR, since it involves all 3 heating/cooling terms. It’s mathematically nasty...

It was worked out by Rosner, Tucker, & Vaiana (RTV) in 1978. Genius paper, but they didn’t really describe how they did their math! Others have explained it much better; see Markus Aschwanden’s book *Physics of the Solar Corona* (ebook on CU Chinook).

What about the energy balance in the low corona? Once we get up into regions where \( T \gtrsim 10^6 \text{ K} \), we saw that radiative cooling is no longer able to put a stop to what’s going on with the heating.

Thus, the low corona is a balance between heating & conduction.

\[
Q_{\text{heat}} \approx \nabla \cdot \mathbf{F}_{\text{cond}}
\]

and we’ll see that this also produces a sharp TR.

First, let’s note that as we go up from chromosphere to corona, \( T \) increases drastically. Thus, so does the scale height \( H \propto kT/mg \).

Thus, \( \rho(z) \) flattens out.

Because \( Q_{\text{heat}} \propto \rho^\alpha \), this means that the heating rate can be assumed to be approximately constant in the low corona.
Also, because we’re still close to the solar surface, let’s continue to use Cartesian (plane-parallel) coordinates, and

$$Q_{\text{heat}} = -K_0 \frac{d}{dz} \left( T^{5/2} \frac{dT}{dz} \right) = \{\text{constant}\}.$$ 

Let’s say we’re looking at a coronal loop, with maximum height $L$. We know that $T(z)$ increases in the corona, and we can reasonably guess that $T = T_{\text{max}}$ at $z = L$. If we define new variables,

$$x = \frac{z}{L} \quad \text{and} \quad y = \left( \frac{T}{T_{\text{max}}} \right)^{7/2}$$

then the differential equation becomes

$$\frac{d^2y}{dx^2} = -\frac{7L^2Q_{\text{heat}}}{2K_0T_{\text{max}}^{7/2}} \equiv -\xi = \{\text{constant}\}.$$ 

There are some assumptions to make about boundary conditions, and those conditions tell us that $\xi$ must be $\approx 2$. With that, the solution for $y(x)$ is a piece of a concave-down parabola:

$$y(x) = 1 - (x - 1)^2$$

If we transform back to $T$, we have to take $y$ to the power $2/7$, which brings values of $0 < y < 1$ closer to 1.

$$\implies \quad \text{Downward conduction from the corona makes the TR sharp!}$$

So even if the chromosphere’s “loss of cooling” (at $\Lambda_{\text{max}}$) wasn’t as catastrophic as it is, we would still see a sharp TR!
Also: \( \xi = 2 \) means that we can solve for \( T_{\text{max}} \) as a function of loop length \( L \) and heating rate \( Q_{\text{heat}} \),

\[
T_{\text{max}} \propto L^{4/7} Q_{\text{heat}}^{2/7}
\]

Double the heating, and you get only an increase of \( 2^{2/7} \approx 1.22 \) in \( T_{\text{max}} \).

Conduction acts as a kind of a “thermostat” that smooths out the heating.

Let’s just look at one more region: the outer corona; i.e., heights more than about \( 1 R_\odot \) above the surface.

Let’s follow Sidney Chapman’s derivation (from early 1950s) that also kept on assuming (naïvely, we know now!) that it’s all hydrostatic; \( u = 0 \).

At large distances, the “input” of energy from coronal heating decreases down to almost nothing, so all that’s left in our energy balance equation is conduction:

\[
\nabla \cdot \textbf{F}_{\text{cond}} = 0 \ .
\]

Out at these larger distances, let’s assume spherical symmetry (i.e., the PFSS has stretched out the field into a split-monopole). Thus,

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 K_0 T_5^{5/2} \frac{dT}{dr} \right) = 0 .
\]

The quantity in big parentheses is constant, so this means that

\[
T_5^{5/2} \frac{dT}{dr} = C \frac{dr}{r^2} .
\]

Integrating that out, we find that \( T \) now decreases with increasing distance. Assuming \( T \to 0 \) as \( r \to \infty \), we get \( T(r) \propto r^{-2/7} \).

In other words, heat is deposited at some height in the mid-corona, and it is conducted “out” from there (both up and down).

\[
\begin{align*}
T & \quad T_{\text{max}} \\
\text{conduction} & \\
\rightarrow & \quad r^{-2/7}
\end{align*}
\]

Again, conduction smears out the thermal energy.