Physical models of the geospace radiation environment

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Abstract

A goal of predictive models of the space radiation environment is to provide advanced knowledge of significant variations in the highly energetic particle populations that form the Earth’s radiation belts. The global geomagnetic field models that result from the Center for Integrated Space-weather Modeling (CISM) effort will provide a necessary input, a real-time description of the dynamic variation of the electromagnetic fields in the magnetosphere, for conducting detailed simulations of the radiation belts. In this work, we describe the issues and techniques that CISM will use to provide a physical space radiation model. An analysis of the global field configurations typical of the magneto hydrodynamic (MHD) models used in CISM suggest that much of the radiation belt modeling can proceed under a guiding center particle approximation, whereby individual, non-interacting test particles are used to track the aggregate dynamics of the radiation belts. This technique provides a relatively simple means of both conducting the necessary simulations, and coupling the relevant codes with other elements of the CISM project. The guiding center equations used to track the particles are based on a phase-space conserving approximation that conserves energy in regions of high curvature. Examples of test particle simulations with the MHD fields are given, both in the initial trapped particle population, and among the energetic particles that form the plasma sheet in the tail. These simulations suggest that multiple radiation belt models running simultaneously in the framework provided by CISM can be combined to provide an overall picture of the energetic particle environment. Finally, analysis of the spectral properties of the fields suggests how an alternate approach to modeling the global radiation belts, solution of the appropriate transport equations, might be advanced through the CISM effort.

Keywords: Magnetosphere; Radiation belts; Particle acceleration; Numerical modeling; MHD

1. Introduction

The radiation belts consist of energetic protons and electrons trapped in electromagnetic drifts encircling the Earth, with energies ranging from a few hundred keV to several MeV and beyond. The belts are largely field-aligned structures forming a torus about the Earth, and are well-organized in terms of the global magnetic field configuration. Fig. 1 indicates the spatial extent and energy range of protons and electrons from the NASA AE/AP empirical model, a static model based on long-term average observations throughout the inner magnetosphere. The fluxes are plotted in terms of energy and the “L-parameter”, a quantity based on magnetic field configuration which radially orders trapped particle motion. In a dipole magnetic field, L indicates the equatorial crossing distance of a given field line,
measured in $R_E$ (1 $R_E \sim 6370$ km) from the center of the Earth. The protons are largely confined to the inner regions of the magnetosphere, from a few hundred kilometers above the surface of the Earth extending to 2–3 $R_E$ from the Earth’s center, and can contain particles with energies exceeding several 10’s of MeV. The electrons, by contrast, are typically divided between two regions of the magnetosphere: an inner-zone belt extending to 2–2.5 $R_E$ from the center of the planet, and an outer zone, extending from perhaps 3 to 7–9 $R_E$.

Between these two zones of energetic electrons lies the “slot region”, a region of space kept relatively free of energetic electrons by their interaction with high-frequency waves (Kennel and Petschek, 1966). The outer zone is highly time-variable, exhibiting variations in fluxes over orders of magnitude, and occurring on time scales ranging from a few minutes to time scales commensurate with the solar cycle (Baker et al., 1986).

The high particle energies found in the radiation belts represent a particular hazard to human technology in
space. For example, variations in the electron radiation environment of a spacecraft can lead to failure via deep dielectric charging of spacecraft components (Vampola, 1987; Shea et al., 1992; Wrenn, 1995; Baker et al., 1994, 1998a). Geosynchronous orbit, an important region for communications and weather satellites, lies at 6.6 $R_E$ and typically well within the outer zone. The 4.2 $R_E$, 55° inclination orbits of the GPS constellation of spacecraft likewise carry them through the heart of the outer radiation zone. With humanity’s increasing dependence on space-based technology for communications, navigation, weather prediction, and a variety of other economic and geopolitical purposes, there is clearly a need to effectively model the radiation environment. In fact, at present static models for the radiation belts do exist (e.g., the NASA AE and AP environment of a spacecraft can lead to failure via deep dielectric charging of spacecraft components. Geosynchronous orbit, an important region for communications and weather satellites, lies at 6.6 $R_E$ and typically well within the outer zone. The 4.2 $R_E$, 55° inclination orbits of the GPS constellation of spacecraft likewise carry them through the heart of the outer radiation zone. With humanity’s increasing dependence on space-based technology for communications, navigation, weather prediction, and a variety of other economic and geopolitical purposes, there is clearly a need to effectively model the radiation environment. In fact, at present static models for the radiation belts do exist (e.g., the NASA AE and AP environments of a spacecraft). Sturrock and CRRESPRO (Meffert and Gussenhoven, 1994; Brautigam and Bell, 1995), etc., providing useful information regarding the average radiation conditions that might be seen over the lifetime of a spacecraft. However, they are of limited use in predicting the effects of the sometimes-drastic temporal variations present in a storm-time magnetosphere, and do not include contributions from solar energetic particles (except perhaps in a temporally averaged sense). Magnetohydrodynamic (MHD) simulations of the magnetosphere, by contrast, provide sufficient temporal resolution to catch the dynamics of a storm, but the cold plasma approximation made in obtaining the ideal MHD equations (e.g., Sturrock, 1994) does not allow specification of the energetic particle populations in the magnetosphere or solar wind that contribute to the space radiation environment.

There are a variety of processes at work within the belts that effect energization and transport of radiation belt particles in near-Earth space. Dynamic impulses from the coupled solar wind/magnetosphere system can lead to substantial reconfiguration of the radiation belts on time scales of a few minutes. Magnetospheric waves in the ULF (mHz) frequency range provide a reservoir of energy capable of accelerating particles on time scales of a few hours to several days (Baker et al., 1998c; Rostoker et al., 1998). Wave/particle interactions at frequencies commensurate with the particle gyration time may also play a role in the overall dynamics of the radiation belts (for example, Horne and Thorne, 1998; Summers et al., 1998), and mechanisms involving a variety of waves interacting in combination have similarly been suggested (e.g., Liu et al., 1999). Observational evidence suggests that multiple independent mechanisms may be acting simultaneously in different regions of the magnetosphere (O’Brien et al., 2003).

A number of methods for modeling the dynamical variations within the radiation belts have been developed as knowledge of the physics affecting the belts has advanced. Dynamic empirical models, ranging in complexity from numerical adjustments of a static model to match time-varying observations (Moore and Baker, 1999), to the application of more sophisticated models involving linear and nonlinear filter techniques (Vassiliadis et al., 2002; Rigler et al., 2004) and neural networks (Koons and Gorney, 1991). A physically based technique, application of Fokker–Planck (Schulz, 1996) and Boltzmann (Fok and Moore, 1997) transport equations has been used to evolve global models of the radiation belts under a variety of quiet and dynamic magnetospheric conditions (e.g., Bourdarie et al., 1996; Brautigam and Albert, 2000; Fok et al., 2001; Fei et al., 2003). However, the relevant transport coefficients for these simulations depend on both the static and dynamic details of the fields controlling the particle motion (Elkington et al., 2003; Fei et al., 2003) and are not typically well-known (Riley and Wolf, 1992). Perhaps the most direct approach to modeling radiation belt dynamics is to computationally track the motion of individual test particles in model magnetic and electric fields. This technique has the advantage of being direct and based on well-known physical principles, but still requires that the fields guiding the particle motion be specified in some detail.

One of the goals of CISM is to provide, in the context of a coupled series of numerical models extending from the Sun to the Earth, a global, time-evolving model of the complete geomagnetic environment. The information provided by such a model may be used as input for physical models of the radiation belts, providing dynamical models of the space radiation environment. The means of modeling evolution of the radiation belts is proposed to be done through direct test particle simulations in the model electric and magnetic fields produced by the CISM project. The use of dynamic spectral analyses of the fields to produce accurate transport coefficients will also be explored. The aim of the radiation belt component of the CISM project will be to develop a flexible but general radiation belt model, capable of handling a variety of field interactions occurring over a range of time scales.

2. Test particle simulations in the magnetosphere

Test particle simulations, in the context of radiation belt studies, use the charged particle equations of motion to directly track the evolving trajectory of energetic particles moving in model electric and magnetic fields. Such studies have been used to both elucidate the physical processes affecting charged particles in the magnetosphere (Elkington et al., 2003), and to describe the bulk evolution of radiation belts under a range of geomagnetic conditions (Hudson et al., 1997, 2001). For example, Li et al. (1993) used a relatively simple analytic
field model to investigate the effects of the March 24, 1991 storm sudden commencement on the outer electron belts. The remarkable agreement between the simulation results of Li et al. (1993) and satellite observations of the event (see, for example, Blake et al. (1992)) gave credity to the notion of test-particle simulations as a valid way of modeling the evolution of outer zone electron fluxes, particularly at short time scales. The technique was extended by Hudson et al. (1996) and Elkington et al. (2002), who used the results from global MHD simulations of the magnetosphere to track the bulk evolution of the radiation belts. In these studies, results from the Lyon–Fedder–Mobarry MHD code (Fedder and Lyon, 1995; Lyon et al., 1998), driven principally by upstream solar wind conditions measured by the \( L_1 \) spacecraft ACE or WIND, were used to evolve test particle simulations of the radiation belts in realistic geomagnetic conditions (Hudson et al., 1997, 1999; Elkington et al., 1999). Fig. 2 shows the results of an MHD/particle simulation conducted by Elkington et al. (2002) corresponding to the March 1991 SSC described above. The simulation captured the prompt acceleration and enhancement of the outer radiation belts, including the formation of a new belt at \( R_E \approx 2.5 \) with a peak in the energy spectrum between 12 and 15 MeV.

Test particle simulations of energetic particles have also been carried out in the plasma sheet and near-Earth tail. Such particles, acted on by substorms or convective forces, may be transported Earthward into the trapping region, potentially providing a source population of radiation belt particles. Li et al. (1998) and Sarris et al. (2002) have used an analytic model, similar to that used in the SSC study described above, to evolve energetic particles moving under the influence of a model substorm injection front. Birn et al. (1997) and Kim et al. (2000) similarly examined the detailed dynamics of plasma sheet ions and electrons moving under the influence of an idealized substorm onset, using a physically based magnetohydrodynamic (MHD) simulation of the near-Earth tail. Though the field model used there only encompassed the tail regions of the magnetosphere and did not include an inner magnetospheric simulation, the Kim et al. (2000) study suggested that electrons injected into the Earthward boundary of the simulation may serve as a viable source of trapped radiation belt particles during geomagnetic storms. More recently, Elkington et al. (2004) used global fields from the LFM MHD model to simulate energetic protons injected from the tail and inside geosynchronous during a substorm occurring during the March 31, 2001 superstorm.

The energy spectrum observed within the radiation belts, and exhibited in Fig. 1, is typically very steep power law, with particle numbers dropping quickly with energy (Li et al., 1993, for example). Consequently, particles at the high energies of interest in space weather forecasting do not contribute significantly to the overall energy density of the particles and fields within the magnetosphere. Since the capability of a population of particles to modify the fields that guide their motion is
proportional the energy density of that population (Dessler and Parker, 1959; Sckopke, 1966), the particles of interest in these simulations can be considered non-interacting in that they make no significant contribution to the global fields that guide their drift. This lack of feedback among particles and between the particle simulation and the model fields makes the problem naturally very parallel, with relatively large problems becoming tractable under the application of massively parallel supercomputing architectures.

2.1. Particle motion in the magnetosphere

The motion of a charged particle moving under the influence of an electric or magnetic field is given by the Lorentz equation. In a dipole magnetic field, the magnetic component of the Lorentz force will result in a charged particle displaying three characteristic types of motion: gyration of the charged particle about the local magnetic field line, bounce motion along the local field line, and a drift of the average particle position about the Earth.

Table 1 indicates the relevant time scales for each of the three types of trapped particle motion in the magnetosphere, for both energetic protons and electrons. The time scales are seen to be widely separated in value. Each of the three types of motion have an associated adiabatic invariant, a quantity which is approximately conserved under conditions where the fields vary slowly in time and space when considered on the scale of the relevant particle motion. For example, changes in the geomagnetic field configuration occurring on the order of a few minutes might violate the third adiabatic, $\Phi$, while conserving the first and second invariants, $M$ and $J$. Such a situation is of considerable practical interest in radiation belt research, as variations in the geomagnetic field on time scales that violate $\Phi$ are not uncommon. In as much as the third invariant can be related to the radial ordering coordinate $L$ by

$$\phi = -\frac{2\pi B_0}{L R_E},$$

(Roederer, 1970), while

$$M = \frac{p^2}{2m_0 B}$$

where $B$ is a function of $L$, then we see that violating the third invariant while conserving the first will necessarily lead to a change in energy as the particle moves in $L$ to regions of higher or lower magnetic field strength. In Eqs. (1) and (2), $B_0$ is the geodipole moment ($\sim 30500 nT R_E^2$), $p_\perp$ is the particle momentum perpendicular to the local magnetic field, and $m_0$ the particle rest mass.

Conservation of one or more adiabatic invariants can frequently be used to advantage when modeling the motion of test particles in the magnetosphere, as described below.

2.2. Guiding center equations

Numerically resolving the full motion of a trapped test particle will generally involve computationally advancing the position of a particle at time intervals much smaller than the gyropreiod of the particle. This can lead to considerable computational expense when one wishes to track the evolution of the belts in aggregate over a multi-day period, an operation which can require hundreds of thousands or millions of particles in a statistically significant sample. The problem is compounded for electrons, where gyro time scales can result in computational time steps on the order of a few tens of microseconds.

However, under circumstances where fields change on time scales that are relatively slow, such that one or more adiabatic invariants is conserved, one may average over the corresponding motion. For example, in cases where the first adiabatic invariant, $M$, is conserved, we may average over the gyromotion and track the particle in terms of the motion of its so-called “guiding center,” the average position of the particle taken on time scales large compared to its gyropreiod but small in terms of its bounce or drift motion. Expressions using various approximations have been given to describe the drift motion of a particle, for example Northrop (1963). Under CISM, guiding-center simulations will be undertaken using the equations derived under a phase-space Lagrangian formulation, as expressed in Brizard and Chan (1999):

$$\frac{dr}{dt} = \frac{\mathbf{E}}{B^*_i} + \frac{M c \hat{b} \times \nabla B}{q_i B^*_i} + \frac{p^2_i}{\gamma m_0 B^*_i} \mathbf{B}^*,$$

with

$$\frac{dp_\perp}{dt} = \frac{B^*_i}{B^*_i} \left(qE - \frac{M}{\gamma} \nabla B \right).$$

Here $\gamma$ is the relativistic correction factor, $\hat{b}$ is a unit vector in the direction of the magnetic field evaluated at

<table>
<thead>
<tr>
<th>Invariant, motion</th>
<th>Time scale (s)</th>
<th>Electron</th>
<th>Proton</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, gyro</td>
<td>$10^{-3}$–$10^{-4}$</td>
<td>$10^{-1}$–$10^{-2}$</td>
<td></td>
</tr>
<tr>
<td>$J$, bounce</td>
<td>$10^{-1}$</td>
<td>$10^0$</td>
<td></td>
</tr>
<tr>
<td>$\Phi$, drift</td>
<td>$10^2$–$10^3$</td>
<td>$10^2$</td>
<td></td>
</tr>
</tbody>
</table>
the particle guiding center, and

$$\mathbf{B}^* = \mathbf{B} + \frac{c \mathbf{p}_b}{q} \nabla \times \mathbf{b}$$

(5)

$$\mathbf{B}_i^* = \mathbf{b} \cdot \mathbf{B}^* = B \left( 1 + \frac{c \mathbf{p}_b}{q B} \cdot \nabla \times \mathbf{b} \right).$$

(6)

In Eq. (3), the first term on the left hand side represents the average effect of an external electric field, the so-called "\(E \times B\) drift"; the second term the "grad-\(B\)" drift, which results from magnetic field gradients perpendicular to the local magnetic field; and the third term (in combination with the mirror force, Eq. (4)) describes both the effects of the "curvature drift," which induces motion perpendicular to the local magnetic field as a result of the non-inertial forces acting in the rest frame of the particle, as well as the time-dependent motion of the particle parallel to the local magnetic field. The form of \(\mathbf{B}^*\) results from a transformation to the guiding center frame, and ensures conservation of phase-space volume along a trajectory (Brizard and Chan, 1999).

Under circumstances where it is appropriate to look strictly at a particle’s gyro-averaged motion, i.e. where \(M\) is conserved, the guiding center approximation outlined in Eqs. (3) and (4) will result in a substantial reduction in the number of points at which the particle position needs to be calculated. Table 1 suggests that the number of computations under the guiding center approximation could be reduced by a factor of \(\sim 10^2 \sim 10^3\) for electrons, and up to \(10^2\) for protons, as compared to following the full Lorentz motion in each case. The significance of the Brizard and Chan (1999) guiding center equations, in contrast to expressions given by other authors, will be discussed below.

2.2.1. The equatorial approximation

Many processes that act on those particles mirroring near the equatorial plane act similarly on those particles that mirror some distance off the equator. For example, quantitative modifications to the description of radial diffusion in a dipole magnetic field increase as a particle’s mirror point moves further off the equator (Schulz and Lanzerotti, 1974), but the qualitative effect of diffusion acting on either equatorially mirroring or off-equatorial particles is largely the same. Further, particle distributions in the radiation belt are often peaked toward 90° equatorial pitch angles, especially in the inner magnetosphere (Lyons and Williams, 1975a, b; Selesnick and Blake, 2002; Önsager et al., 2004). In such circumstances, an equatorial approximation to the guiding center equations may give a sufficiently accurate picture of the overall dynamics of the radiation belts. Under equatorial conditions, \(\nabla_{\parallel}\) and \(p_{\parallel}\) are both ideally zero, and the problem represented by Eqs. (3) and (4) reduces from a second-order differential equation in three dimensions to a 2d, first-order problem

$$\dot{r} = \frac{E_{\parallel}}{B} \pm \frac{Mc}{q r B} \mathbf{b}$$

$$\dot{\phi} = -\frac{E_{\perp}}{r B} + \frac{Mc}{q r B} \mathbf{b}$$

(7)

The equatorial approximation has been used in global simulations of radiation belt dynamics to examine short-timescale events, such as the effect of an SSC acting on the radiation belts over the course of a few minutes (Li et al., 1993; Hudson et al., 1997), to quantitatively tracking the evolution of the radiation belts over the course of geomagnetic storms lasting multiple days (Hudson et al., 2001; Elkington et al., 2002). As indicated by Table 1, the time step required to computationally track the motion of equatorial particles is reduced by up to 2 orders of magnitude over following the bounce motion under the guiding center approximation, and by approximately 6 orders of magnitude over following the full Lorentz motion. Under these circumstances, and using presently available computer architectures, tracking millions of test particles in real time becomes a computationally tractable problem. In the parallel computational environment envisioned for CISM, relatively large equatorial simulations may be run simultaneously with smaller scale (but more computationally complex) 3d simulations. By combining results from each method using an appropriate weighting scheme (Section 3.3), a quantitative picture of the global evolution of the belts may be obtained.

2.2.2. Test particle simulations in 3d

In some circumstances, important dynamics occur out of, or explicitly as a result of, particle motion out of the equatorial plane. For example, Kim et al. (2000) found that those particles most likely to be trapped in the inner magnetosphere as the result of a substorm injection were those where one or both of the first two adiabatic invariants were altered in the course of the injection process. To capture the dynamics of such particles, the CISM magnetosphere/radiation belt model must be capable of tracking off-equatorial as well as equatorially bound motion.

The value of the Brizard and Chan (1999) phase-space Lagrangian guiding center expressions is demonstrated in Figs. 3 and 4. In each figure, a 1 MeV geosynchronous electron is traced in a T89 model magnetic field (Tsaganenko, 1989). In the top panels, the equatorial (left) and \(x\)–\(z\) projection (right) of the particle motion is plotted, while the bottom panels plot the equatorial position of the particle and the particle energy as a function of time. In Fig. 3, the particle motion is traced using guiding-center expressions based on a truncation of small-order effects from a Taylor-expansion of the...
Lorentz equations (for example, Northrop, 1963). The “smallness” parameter used in such calculations, $\varepsilon$, is defined as the ratio of the particle gyromotion to the characteristic distance over which the fields change, and is discussed in more detail in Section 2.3. Fig. 4 tracks the electron motion using Eqs. (3) and (4).

Since the field model in Figs. 3 and 4 is static and does not include electric fields, we expect that energy will be conserved through the course of the drift motion. In Fig. 3, a small but noticeable change in energy occurs in the tail region, where magnetic field line curvature is high. This is a result of the truncation of higher order terms from the equations of motion in regions where $\varepsilon$ increases. In Fig. 4, by contrast, energy is well-conserved throughout the course of the drift motion. This effect, while small when measured over the single drift orbit illustrated in Figs. 3 and 4, might be expected to increase for particles at higher $L$ values and in the tail, where field line curvature may be more pronounced. Further, in the dynamic field models produced by CISM, the presence of magnetospheric electric fields might be expected to further modify the energy conservation characteristics of the drift.

![Fig. 3. Guiding center motion in a static T-89 magnetosphere, showing the non-conservation of energy inherent in $\varepsilon^2$ guiding center equations in regions of relatively high magnetic field curvature.](image1)

![Fig. 4. Guiding center motion in a static T-89 magnetosphere using guiding center equations obtained under a phase-space Lagrangian formulation. This method conserves energy throughout the particle drift.](image2)
equations, and the time-varying nature of the field may lead to potential for systematically increasing errors when many particles are run over long periods of time.

Solution of the 3d expressions is computationally much more expensive than tracking the motion of particles in the equatorial approximation. This is a result of both the reduced time step required to resolve the bounce motion of the particles (Table 1), as well as the increased complexity of the equations of motion. We might expect to be able to run far fewer particles in the 3d case for a given number of computer cycles, in contrast to tracking only equatorially bound particles. However, in the coupled-code environment that defines CISM, it is easy to conceive of 2d and 3d versions of the radiation belt code running independently and in parallel. The 2d code could be initially used to provide quick views of the radiation belt evolution, or to track the aggregate motion where a large number of particles is required. The 3d version might be run with fewer particles to compensate for the large increase in computational expense in tracking each test particle, but be available to capture relevant dynamics and evolution out of the equatorial plane. Results from each code might be combined (if properly weighted, see Section 3.3) to give a global picture of the radiation belt dynamics at reasonable computational expense.

2.3. Adiabaticity and the guiding-center equations

Implicit in the derivation of Eqs. (3) and (4) is the assumption that the particle being tracked conserves its first adiabatic invariant, $M$. A particle will remain adiabatic so long as the fields in the particle’s frame of reference change slowly over the course of its gyromotion (Northrop, 1963), implying that the fields vary slowly in both space and time when compared to the particle’s gyroradius and drift period. Quantitatively, we must have

$$\varepsilon_\perp = |\rho \nabla_\perp (\ln B)| \ll 1,$$

$$\varepsilon_1 = |\rho \nabla_1 \hat{b}| \ll 1,$$

and

$$\varepsilon_t = |T_g \hat{c} (\ln B)| \ll 1.$$

A particle which fails any of conditions (8)–(10) cannot be tracked using Eq. (3), and must instead be tracked using the full Lorentz motion. We use the stable trapping criterion proposed by Chirikov (1987) and stop using the guiding center approximation on a particle when

$$\frac{e^{-1/\varepsilon}}{\varepsilon} \approx \frac{1}{40},$$

or when $\varepsilon \leq 0.187$.

In a dipole magnetic field, $\varepsilon_\perp$ for an equatorial particle of a given $M$ will increase as one moves further from the center of the Earth. This is a result of the $L^{-3}$ dependence of the dipole magnetic field strength contrasting with the $L^{-4}$ dependence of its radial gradient. Fig. 5 shows the lines of constant first adiabatic invariant for equatorial particles a dipole magnetic field of strength 30500 nT at the Earth’s surface. The hatched areas indicate those regions where the guiding center approximation breaks down according to Eq. (11). For protons, there are large regions of $L$–$W$ space where Eq. (3) is not valid, due to the relatively large gyroradius of the protons as compared to the electrons. Comparing with Fig. 1, we see that the

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![Fig. 5](image_url)
guiding center approximation is valid when modeling the trapped inner-zone protons. However, in the case of proton populations at higher energy or $L$-shells than the average populations plotted in Fig. 1 (for instance, the solar protons often accompanying coronal mass ejections (Hudson et al., 1995)), the guiding center approximation breaks down. The guiding center approximation for the electrons, on the other hand, appears to be valid at all energies and $L$-shells likely to be of interest in the inner magnetosphere.

In regions where external components to the geomagnetic field become comparable to the internal dipole field (for example at high $L$-values or in the tail), the applicability of the guiding center equations is not so clear. The currents responsible for the external contributions to the field might act to either increase or decrease the local field gradients and curvatures. Currents at the magnetopause, for example, might be expected to smooth out perpendicular gradients in the geomagnetic field, while currents in the tail typically increase gradients parallel to the geomagnetic field. To investigate the regions where we might be able to appropriately conduct radiation belt simulations in the guiding center approximation, we may plot the adiabaticity parameter parallel and perpendicular to the local magnetic field in terms of particle energy using

$$e_{\perp} = \frac{\nabla \times B}{qcB^2} \sqrt{W^2 + 2Wm_0c^2},$$

and

$$e_{||} = \frac{(B \cdot \nabla) B}{qcB^2} \sqrt{W^2 + 2Wm_0c^2}.$$  \(12\)

$$13\)

Fig. 6 shows the value of $e<0.187$ in the GSM equatorial plane for 200 keV protons. Here $e$ was calculated using global fields from the LFM MHD model (Fedder and Lyon, 1995; Lyon et al., 1998) corresponding to a nominally quiet period a few hours prior to the onset of the March 31, 2001 sudden commencement. The value of $e$ is depicted on the color axis and is shaded gray in those regions where $e$ exceeds the Chirikov criterion 0.187. Superimposed are contours of constant magnetic field strength in the equatorial plane. Although these keV particles satisfy the guiding center condition throughout most of the magnetosphere, the top plot indicates that the guiding center approximation will break down in the distant tail and at the magnetopause. Comparing the top and bottom plots of Fig. 6, it appears that the perpendicular gradients in the field are more of a concern in tracking guiding-center motion than is parallel field line curvature.

In Fig. 7, we contrast the perpendicular adiabaticity parameter for 1 MeV protons with that for 1 MeV electrons. The period depicted here is one of that exhibited significant driving and distortion of the fields within the MHD simulation, corresponding to a growth phase of a substorm during the March 31, 2001 storm event (Baker et al., 2002). While the electrons are susceptible to non-adiabatic behavior in reconnection regions in the tail and at the magnetopause, it appears that the guiding center approximation may be used with electrons throughout the inner magnetosphere and near-Earth tail, even under conditions of extreme driving by the solar wind. Protons at this energy, by contrast, are largely limited to use of the guiding center approximation in the inner magnetosphere.
In the execution of MHD/particle simulations of the radiation belts, the guiding center approximation is checked for each particle at each time step. This avoids the accidental inclusion of non-adiabatic behavior in very dynamic periods, for example, or when a particle trajectory takes it into a reconnection region in the tail or near the magnetopause. Particles failing the adiabaticity criterion may either be removed from the simulation, or the trajectory calculation continued using the full Lorentz motion of the particle.

3. MHD/particle simulations: practical implementation

3.1. Time-stepping, grid interpolation, and coupling to the CISM magnetosphere model

Numerical solution of the equations of motion in the current implementation of the particle code is accomplished using a 4(5) Runge–Kutta as described by Cash and Karp (1990). This technique efficiently compares the 4th- and 5th-order numerical solution at each time step to make an estimate of the local truncation error (LTE). The solver then uses this information to adjust to size of the time step to either larger or smaller values, such that the largest possible time step is used that maintains an LTE under a specified tolerance. While this tends to maximize the computational efficiency and accuracy of the particle code, it might present some complication when coupling the code to external magnetic field models with independent time steps and output cadence.

Fortunately, the lack of feedback from the particle code to the magnetospheric model (Section 2) removes some of this complication. At present, the MHD simulation is run independently of the radiation belt code, and the results of the MHD run are moved to an independent grid. In a separate step the particle code is run, and the fields interpolated from the independent grid to the particle position in the course of tracing its motion. Such an approach might be used in the situation of coupled codes. Each code may be run simultaneously and in real time, but with the magnetospheric models running 1–2 exchange steps ahead of the radiation belt code, and the field results being stored in memory, in a coupling process (see Wiltberger et al., 2004, this issue), or on disk in the intervening time. Second, the fact that the radiation belt code is naturally very parallel allows us to easily scale the radiation belt problem to match the efficiency of other codes. That is, selection of a larger or smaller number of processors for a given number of particles allows us to reduce or lengthen the particle codes’ run time, relative to the other coupled CISM models.

In spite of the parallel nature of the problem, implementing the code in a parallel computing architecture does require some forethought. For example, particles that find themselves near the noon magnetopause during a sudden commencement event will likely require more computations to resolve the effect of the SSC, compared to particles in the midnight regions and closer to the Earth. Likewise, particles that mirror some distance from the equatorial plane are going to require more calculations near the mirror points, and thus be computationally more involved than those particles with near 90° pitch angle. Properly distributing the computational load across a given architecture therefore requires some knowledge of the physical situations likely to be encountered by the particles, with appropriate distribution of the particles across processing elements.

Both the 2d and 3d versions of the particle code interpolate the fields from a grid in space and time to the actual test particle position at each time step. The
relative simplicity of the 2d grid allows us to use a quadratic interpolation scheme in space, and leads to efficient time step evaluation in the solve engine. However, the added computational expense of directly applying the quadratic interpolation used in the 2d situation to the 3d code negates the benefit of the variable time step, so a linear interpolation in space is presently used instead. This, in turn, precludes the use of the 4(5) method, which requires that the derivatives at the cell interfaces likewise be smooth (Birn et al., 1997). Therefore, at present, the 3d code is run with a fixed time step, chosen to be small enough to minimize the global truncation error for particles running in known field configurations, e.g. a dipole. Finding more efficient, higher order interpolation schemes for the 3d code is an ongoing effort in the CISM radiation belt project.

3.2. Initial particle distribution

The present implementations of the CISM radiation belt codes distribute particles initially in space in a fashion similar to that shown schematically in Fig. 8. Here particles are given an initial position in the magnetosphere corresponding to constant radial, azimuthal, and in the 3d case, latitudinal increments. Typical 2d MHD/particle simulations of trapped populations use an azimuthal spacing of 3° or less, and a radial spacing of 0.1 \( R_E \). This distribution is of course scalable, depending on statistical needs and the availability of computational resources.

The constant angular distribution in Fig. 8 clearly leads to an increase in test particle density in the inner magnetosphere. This bias can easily be removed in a post-processing step, described below. Other test particle simulations found in the literature have used techniques to create a uniform initial distribution of particles to avoid the potential biasing indicated in Fig. 8. For example, the substorm simulations conducted by Sarris et al. (2002) used a random number generator to choose initial particle locations within the simulation domain. For a sufficiently large numbers of particles, a relatively uniform initial distribution of particles is achieved, without the low-L biasing inherent in Fig. 8. However, in present implementations of the radiation belt code, the capability of specifying a particular initial distribution is retained so that a single code can be used to easily used to simulate both specific particle populations (e.g. a ring of particles starting at a particular radial distance or mirroring at a particular latitude), as well as examine the aggregate dynamics of the belts in simulations comprising particles with a range of initial conditions.

The distribution in energy space must also be considered. For example, the steep energy spectrum exhibited by electrons in Fig. 1 implies that the keV-energy component of the radiation belts will be substantially larger than that of the higher energy particles. Correspondingly, most energetic particle detectors aboard spacecraft have better resolution at low energies. Finally, motion of more energetic particles typically requires more computational effort to resolve than lower energy particles. To allow sufficient resolution of the dynamics at low energies, while balancing the computational load with the higher energy particles, present implementations of the CISM particle code use a logarithmically increasing energy increment. This results in more particles at low energy with correspondingly increased energy resolution, while still allowing the dynamics of the high-energy component of the radiation belts to be efficiently tracked within the same code.

To accurately model the evolution of the space radiation environment, one may need to consider not only the sources and means of particle energization acting on the initially trapped population of particles, but also the sources of particles that might serve to populate the belts. One potential source of particles might be energetic protons and electrons within the plasma sheet, injected and trapped in the inner magnetosphere as a result of substorm activity or the convective
action of the magnetosphere. In Fig. 9, we show initial results from a coupled MHD/particle simulation of the injection of plasma sheet electrons into the inner magnetosphere during the March 31, 2001 storm main phase. Here new 60 keV electrons are added to the simulation as the particle code is running, injected from a boundary region 20 $R_E$ downtail at 15 s intervals. The effect of magnetospheric convection moves particles Earthward into regions of stronger magnetic field, increasing the particle energy through conservation of $M$. Some portion of these newly injected particles move across the stable trapping boundary and into the radiation belts, where radial diffusion acts to heat them to higher energies. Through this process the keV electrons in the tail are heated to energies exceeding 1 MeV in the inner magnetosphere, suggesting that plasma sheet electrons may provide a significant contribution to the storm-time MeV radiation belt population. This simulation effectively represents a boundary region 20 $R_E$ downtail at 15 s intervals. To remove the biasing that results from this aspect of the distribution, each particle must first be given a weight scaling as $r_i/r_0$, where $r_i$ is the initial radial position of the particle in question and $r_0$ is some arbitrary reference distance (the inner boundary of the simulation, for example).

Likewise, biasing can result from the choice of initial energy distributions. In the initial distribution described in Section 3.2, we had particle energy varying logarithmically (for example at 5% increments), appropriate for a steeply decreasing flux at higher energies. For this type of energy distribution, each particle with initial energy $W_0$ would therefore be given a second weight

$$Q_W = \left( \frac{W_{\text{inc}}}{W_0} \right)^n,$$

where $W_{\text{inc}}$ is the energy increment ($=1.05$ for an energy increment of 5%), $W_0$ is an arbitrary reference energy, and

$$n = \frac{\log(W_i/W_0)}{\log W_{\text{inc}}}.$$  

Next, we wish to recognize that in these simulations each test particle is actually representative of an ensemble of particles with similar initial positions and energies. As the ensemble of particles represented by a given test particle moves adiabatically into regions of higher or lower magnetic field strength, the energy will vary through conservation of $M$. The change in flux, $j,$

Fig. 9. Snapshot of plasma sheet electrons convecting Earthward from 20 $R_E$, a portion of which become trapped in the inner magnetosphere as part of the outer radiation belts. This figure demonstrates how particles from an assumed outside source may be introduced into the test particle simulations.
associated with this change in energy is given by Liouville’s Theorem for trapped particles, which states that the phase space density, \( f_p = j/p^2 \), is constant along the dynamical path of the particles (Roederer, 1970). For particles moving in the equatorial plane and conserving their first invariant, we can account for this change in flux as particles move through the magnetosphere by applying an additional weighting factor \( Q_L = B_f/B_i \), where \( B_f \) is the guiding center magnetic field at some later time of interest.

Once all initial biasing has been removed and Liouville’s theorem used to account for conservation of phase space density, we can assign a differential flux to each particle based on its initial position and energy and some static model of the radiation belts. In the work of Li et al. (1993), for example, an analytic radiation belt model is used with fluxes increasing with \( L \) throughout the magnetosphere. A more physically accurate distribution might be obtained using the AE-8 or AP-8 models illustrated in Fig. 1, for instance (Vette, 1991), or the more recent CRRES-ELE or CRRES-PRO radiation belt models (Meffert and Gussenhoven, 1994; Brautigam and Bell, 1995). Work on empirical models of the radiation belts, such as being conducted under the auspices of the CISM program (Vassiliadis et al., 2002; Rigler et al., 2004), might likewise be used to provide global radiation belt flux and spectral estimates for initializing the physical radiation belt models.

Finally, if the model flux function is given by \( j(r_i, W_i) \), the total weight given each test particle in the flux mapping procedure is then

\[
Q_{\text{tot}} = r_i \left( \frac{W_{\text{inc}}}{W_0} \right)^\alpha B_f/B_i j(r_i, W_i).
\]

After applying the given flux weighting to each particle, the results of the simulation can be scaled to match instrument geometrical factors and efficiencies (for example, Fig. 1 of Blake et al. (1992)) for direct comparison to observed fluxes.

### 3.4. CISM and transport simulations of the radiation belts

While the bulk of this paper has described methods and presented results pertaining to test particle simulations conducted within the framework of CISM, work conducted in parallel with this effort will attempt to better understand and advance alternate methods of modeling the radiation belts. For example, simulations involving solution of the Fokker–Planck equations for the trapped particles, as described in Schulz (1996), may provide an alternate method of efficiently modeling the evolution of the radiation belts. Here the distribution function of the trapped energetic particles is written as a function of the three adiabatic invariants, and the differential equation tracking the evolution of the distribution solved in time. This method has the advantage of being able to model high-frequency effects that are not presently contained within the MHD approximation; however, the principle shortcoming of the transport approach is that the transport coefficients corresponding to various magnetospheric processes are not well-determined. For example, the various transport coefficients that govern diffusion in the third invariant, or, equivalently, \( L \), depend not only on the static configuration of the field, but also the power spectral density of wave activity at various multiples of the drift frequency corresponding to the mode structure of the driving waves (Schulz and Lanzerotti, 1974; Elkington et al., 2003; Fei et al., 2003). Similar constraints and unknowns hold for variations in the first and second invariants. At present, accurate statistical maps of the global, time-dependent geomagnetic wave activity do not generally exist, providing a severe constraint in the use of the Fokker–Planck equations in quantifying evolution of the radiation belts.

Figs. 10 and 11 indicate how analysis of the CISM magnetospheric models might provide understanding leading to better-defined particle transport coefficients. In Fig. 10, we plot azimuthal wave power at a particular local time during a simulation of the September 24–26, 1998 geomagnetic storm. Here wave power is plotted on the color axis vs. radial distance and frequency. Features to note include a clear decrease in wave activity as we move away from the magnetopause and into the inner magnetosphere, and a similar decrease in power with increasing frequency. Inasmuch as radial diffusion depends on the wave power at multiples of the particle drift frequency, Fig. 10 suggests widely different rates of diffusion may be applicable in different regions of the magnetosphere and at different energies.

In Fig. 11, we plot the integrated ULF wave power as a function of position within the equatorial magnetosphere, for the same simulation and period of time as shown in Fig. 10. The figure illustrates some of the variations in the global mode structure of the ULF wave power which drive radial transport in the radiation belts, indicating, for example, a difference in dominant azimuthal modes at various radial distances. Figs. 10 and 11 together suggest some of the complexity involved in accurately generating the coefficients associated with radiation belt transport.

Lacking the global spacecraft coverage required to provide a similar picture of magnetospheric wave power based on measurements, global maps from simulations such as illustrated in Figs. 10 and 11 may provide an initial statistical basis for constructing empirical models of wave activity. These empirical wave maps might then be used to generate accurate coefficients to be applied in transport simulations of the radiation belts.
4. Concluding remarks

This paper describes the major issues and techniques involved in providing a global physical description of the space radiation environment, as part of the CISM effort in space weather forecasting.

Analyses of the magnetic field configurations present in the LFM MHD code, which will form the backbone of the CISM magnetospheric simulations, suggest that the bulk of the modeling effort may be carried out in the guiding center approximation. This is depicted in Fig. 6 for a nominally quiet period and in Fig. 7 for a considerably more active period of simulation. Electrons, in particular, are amenable to simulation via the guiding center approximation, although energetic protons will be more restricted under this model. However, appropriate codes running in parallel will maximize the modeling domain, populations, and physical situations that might be included in the simulations. For example, particles failing the adiabaticity test in a guiding center code might serve as input for an appropriately coupled Lorentz solver, perhaps running in parallel on another architecture. Fig. 9 likewise suggests how two guiding center codes may be coupled together within the framework of CISM. The results of the plasma sheet code, with a dynamically injected source of particles as depicted in Fig. 9, may be coupled with the output of a simulation evolving the initially trapped radiation belt particles suggested by Fig. 8. Each code may be run simultaneously, receiving identical field information from the magnetospheric codes. The parallel, non-interacting nature of the simulations provide a significant amount of flexibility in modeling the overall radiation environment.

Figs. 10 and 11 indicate the information that might be gained through spectral analyses of the output of the CISM magnetospheric simulations. Knowledge of the spectral properties of the waves within the magnetosphere are typically required in constructing the transport coefficients required for Fokker–Planck...
simulations of the radiation belts. Such analyses will therefore advance a complimentary method of modeling the global structure of the radiation belts.

The coupled solar-geomagnetic field models that will be the product of CISM provide an ideal framework for conducting the global radiation belt simulations described in this paper. The non-interacting nature of the technique will significantly reduce the effort required to integrate these codes with other elements of the CISM modeling effort. Such simulations will not only provide physical insight into the processes and dynamics of the radiation belts, but can provide a valuable product, knowledge of the evolving state of the space radiation environment, to the space weather community.

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