Solutions

ASTR/ATOC 3720
Homework Assignment #1
Due: Tues. Jan. 28 in class

Show your work, as partial credit will be given for your thought processes, even if you don't get to the correct answer at the end! Work the problems on a separate sheet of paper please. Be neat and organized! We can't give credit for answers we can't read!

1. The "Solar Constant" measured in space at the orbit of Earth is 1368 Watts/m^2. If the Sun were a lightbulb, calculate what "Wattage" light bulb it would be? (Hints: the Sun is emitting energy in all directions; and 1 AU = 1.5×10^{11} m).

2. Given that Saturn's distance from the Sun is 9.52 AU and it absorbs 37% of the sunlight that falls on it, what is the effective (or equilibrium) temperature of the planet?

3. The mean distance of Mars from the Sun is 1.524 AU, but the planet is actually in an elliptical orbit with an eccentricity of 0.093. Given that the albedo of Mars is 0.16, what is the effective (equilibrium) temperature at both the perihelion (closest) and aphelion (farthest) points on Mars' orbit?

4. Calculate the scale heights of the atmosphere near the surfaces of Mars and Venus. Assume that the atmospheres of Mars and Venus are essentially all CO_2 and the temperature at Mars is 223 K and Venus is 750 K. Note, a periodic table, and the tables at the back of NSS (the New Solar System) will help.

5. What is the percentage change of the acceleration due to gravity (g) between the surface of the Earth and 100 km altitude above the surface? Estimate the error in the pressure determination at 100 km altitude using the barometric equation with g held constant.

6. Calculate the adiabatic lapse rates for Mars, Venus, and Jupiter.

HW#1 Solutions

① At 1 AU: $S_0 = 1368\ W/m^2$

But this is through a unit area and the Sun emits in all directions, so we need to integrate over the entire area of a sphere with radius 1 AU to capture all the energy leaving the Sun.

$$S_0 * 4\pi R^2 = (1368\ W/m^2)(4\pi)(1.5\times10^{11}\,m)^2$$
$$= 3.87\times10^{26}\ \text{watts}$$

So the Sun is like a 3.87×10^{26} watt bulb!

② $T_e = \left[\dfrac{S_0}{D^2}\dfrac{(1-\alpha)}{4\sigma}\right]^{1/4}$

For Saturn: $D = 9.52$ AU

$1-\alpha = 0.37$ (note: problem says "absorbs" 37% and albedo = reflected fraction, so $1-\alpha$ = absorbed fraction)

$\sigma = 5.67\times10^{-8}\ \frac{W}{m^2K^4}$

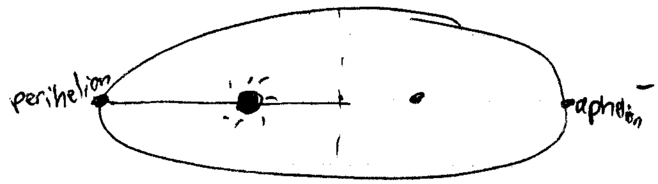
$$T_e = \left[\frac{1368\ W/m^2}{(9.52)^2}\frac{(0.37)}{4(5.67\times10^{-8}\frac{W}{m^2K^4})}\right]^{1/4}$$

$$T_e = (2.46\times10^{7}\,K^4)^{1/4}$$

$$T_e = 70.4\ K$$

HW #1 ②

③ Elliptical Orbit :



- Sun is at one focus.
- Perihelion is closest pt
- Aphelion is farthest pt.

Mean Distance = Semimajor Axis = a = 1.524 AU

$R_{perihelion} = a(1-\text{eccentricity}) = (1.524\,AU)(1-0.093) = 1.382\,AU$

$R_{aphelion} = a(1+\text{eccentricity}) = 1.666\,AU$

Albedo $= \alpha = 0.16$

$$T_{e\,perihelion} = \left[\frac{S_0}{R_{peri}^2}\frac{(1-\alpha)}{4\sigma}\right]^{1/4} = \left[\frac{1368\,W/m^2}{(1.382)^2}\frac{(1-0.16)}{4(5.67\times10^{-8}\,W/m^2K^4)}\right]^{1/4}$$

$= 227\,K$

$$T_{e\,aphelion} = \left[\frac{S_0}{R_{aphelion}^2}\frac{(1-\alpha)}{4\sigma}\right]^{1/4} = \left[\frac{1368\,W/m^2}{(1.666)^2}\frac{(1-0.16)}{4(5.67\times10^{-8}\,W/m^2K^4)}\right]^{1/4}$$

$= 207\,K$

④ Scale Height

$$H = \frac{kT}{\bar{m}g}$$

Mars: $T = 223\,K$

$g = 3.69\,m/sec^2$ (from table on p387 of NSS)

Mars atmosphere mostly CO_2, which has a molar mass of 44 g/mole $\left(\begin{array}{l} C = 12\,g/mole \\ +O = 16\,g/mole \\ +O = 16\,g/mole \end{array}\right)$

$$\bar{m} = (44\,g/mole)\times\left(10^{-3}\frac{kg}{g}\right)\left(\frac{1\,mole}{6.022\times10^{23}\,molecules}\right) = 7.3\times10^{-26}\,kg$$

$$H_{Mars} = \frac{(1.38\times10^{-23}\,J/K)(223\,K)}{(7.3\times10^{-26}\,kg)(3.69\,m/sec^2)} = 11{,}400\,m = \boxed{11.4\,km}$$

Venus: $T = 750\,K$

$g = 8.87\,m/sec^2$

$\bar{m} = 7.3\times10^{-26}\,kg$ (atmosphere is CO_2 also)

$$H_{Venus} = \frac{(1.38\times10^{-23}\,J/K)(750\,K)}{(7.3\times10^{-26}\,kg)(8.87\,m/sec^2)} = 16{,}000\,m = \boxed{16.0\,km}$$

Oops! See next page for #5!

⑥ From class: $\frac{dT}{dz} = -\frac{g}{c_p}$

& for a diatomic gas $c_p = \frac{7}{2}\frac{k}{\bar{m}}$

and for a triatomic gas $c_p = \frac{9}{2}\,k/\bar{m}$

Mars: main gas is CO_2 a triatomic gas

$$\bar{m}_{CO_2} = \frac{(44\,g/mol)(10^{-3}\,kg/g)}{6.022\times10^{23}\,molecules/mole} = 7.3\times10^{-26}\,kg$$

triatomic

$$c_p = \left(\frac{9}{2}\right)\left(\frac{1.38\times10^{-23}\,J/K}{7.3\times10^{-26}\,kg}\right) = \cancel{850}\ 851\,\frac{J}{kg\,K}$$

$g = 3.69\,m/sec^2$ (from table in book)

$$\frac{dT_{mars}}{dz} = -\frac{3.69\,m/sec^2}{851\,\frac{J}{kg\,K}} = -4.33\times10^{-3}\,K/m$$

$$\boxed{\frac{dT_{mars}}{dz} = -4.3\,K/km}$$

Venus: also CO_2 so $c_p = 851\,\frac{J}{kg\,K}$

$g = 8.87\,m/sec^2$

$$\frac{dT_{venus}}{dz} = -\frac{8.87\,m/sec^2}{851\,J/kg\,K} = -1.04\times10^{-2}\,K/m = \boxed{-10.4\,K/km}$$

Jupiter: gas is H_2 a diatomic gas

$$\bar{m}_{H_2} = \frac{(2\,g/\cancel{mole})(10^{-3}\,kg/\cancel{g})}{(6.022\times10^{23}\,molecule/\cancel{mole})} = 3.3\times10^{-27}\,kg$$

diatomic

$$c_p = \left(\frac{7}{2}\right)\left(\frac{1.38\times10^{-23}\,J/K}{3.3\times10^{-27}\,kg}\right) = 14{,}500\ J/kg\,K$$

$$g = 23.12\ m/sec^2 \text{ (from book)}$$

$$\frac{dT_{Jupiter}}{dz} = -\frac{23.12\ m/sec^2}{14500\ \frac{J}{kg\,K}} = -1.6\times10^{-3}\ K/m$$

$$\boxed{\frac{dT_{Jupiter}}{dz} = -1.6\ K/km}$$

5 $g(z) \propto \frac{1}{R^2}$ where R = distance from center of the planet

so in terms of altitude $R = R_0 + z$
where R_0 is the radius of the earth (6378 km)

$$\frac{g(z)}{g(0)} = \frac{\frac{1}{(R_0+z)^2}}{\frac{1}{R_0^2}} = \left(\frac{R_0}{R_0+z}\right)^2 = \frac{1}{\left(1+\frac{z}{R_0}\right)^2}$$

using $z = 100$ km and $R_0 = 6378$ km

$$\frac{g(100\text{ km})}{g(0)} = \frac{1}{\left(1+\frac{100}{6378}\right)^2} = \frac{1}{(1+0.0157)^2} = 0.969$$

So the percent change in gravity from the surface to 100 km is

$(1-0.969)\times 100\% = \boxed{3.06\%}$, about 3%

To estimate error in p due to error in g calculate barometric eqn with and without g constant:

$$p = p_0 e^{-\frac{z-z_0}{H}}$$

call $p_c = p_0 e^{-\frac{100}{H_c}}$ the pressure at 100 km with g constant

and $p_A = p_0 e^{-\frac{100}{H_A}}$ the pressure at 100 km with actual g

$$\frac{p_c}{p_A} = \frac{\cancel{p_0} e^{-\frac{100}{H_c}}}{\cancel{p_0} e^{-\frac{100}{H_A}}} = e^{-\frac{100}{H_c} + \frac{100}{H_A}}$$

But $H_A = \frac{kT}{mg_A}$, $H_c = \frac{kT}{mg_c}$

Using T at surface and g at surface we know (derived in class)

$$H_c = \frac{(1.38\times10^{-23}\,\mathrm{J/K})(288\,\mathrm{K})}{(4.8\times10^{-26}\,\mathrm{kg})(9.8\,\mathrm{m/sec^2})} = 8.5\times10^{3}\,\mathrm{m} = 8.5\,\mathrm{km}$$

Assuming ~~that is true~~ T is same at 100 km get

$$H_A = \frac{(1.38\times10^{-23}\,\mathrm{J/K})(288\,\mathrm{K})}{(4.8\times10^{-26}\,\mathrm{kg})[0.969 * 9.8\,\mathrm{m/sec^2}]} = 8.7\,\mathrm{km}$$

$$\frac{p_c}{p_A} = e^{-\frac{100}{8.5}+\frac{100}{8.7}} = e^{-0.27} = 0.76$$

$\Rightarrow$ $\boxed{\approx 24\%}$ error using constant g instead of actual g in pressure eq'n.

Either solution ok as is any reasonable assumed Temp for 100 km

If we use real T for thermosphere at 100 km

$T \approx 200\,\mathrm{K}$

then $H_c = 5.9$ km

$H_A = 6.1$ km

$$\frac{p_c}{p_A} = e^{-\frac{100}{5.9}+\frac{100}{6.1}} = e^{-0.55} = 0.57$$

$\Rightarrow$ $\approx 43\%$ error using constant g instead of actual g.