Solutions

ASTR/ATOC 3720: Homework Assignment #2

Due: Tues. Feb 18 in class

Show your work, as partial credit will be given for your thought processes, even if you don't get to the correct answer at the end! Work the problems on a separate sheet of paper please. Be neat and organized! We can't give credit for answers we can't read! You may work in groups, but turn in your own individual answers. I suggest you use the Feb 13 classtime to gather in the classroom to work on these problems.

1. Imagine a lamp in the lab that can emit a horizontal beam of EUV light at a wavelength of 30.4 nm. The pressure of air in the lab is 1013 mb, the temperature is 288 K, and the ionization plus absorption cross section for N_2 at 30.4 nm is 2.340×10^{-17} cm^{-2} and for O_2 is 3.328×10^{-17} cm^{-2}. Assuming that air is made of 78% N_2 and 22% O_2 and nothing else, perform the following calculations:

   a. What are the optical depths of the N_2, O_2 and air in the lab at 30.4 nm?

   b. How far will the EUV light from the lamp travel before it is diminished to 5% of its original brightness?

   c. Repeat the calculation of part b) for a lab in Boulder which has a pressure of air of 840 mb.

2. In the Earth's atmosphere the number density of N_2 at 250 km altitude is about 8×10^{8} cm^{-3} and that for O_2 at the same altitude is 8.5×10^{7} cm^{-3} and for O it is 9×10^{8} cm^{-3}. Assume that the scale heights for these three constituents are constant with altitude above 250 km and are equal to 30.1 km for N_2, 26.4 km for O_2 and 49.2 km for O.

   a. Calculate the optical depth for ionization at 250 km for each of these three constituents at 30.4 nm assuming the cross sections are 1.170×10^{-17} cm^{-2} for N_2, 1.664×10^{-17} cm^{-2} for O_2, and 7.693×10^{-18} cm^{-2} for O.

   b. Assume that the flux of 30.4 nm solar light before entering the Earth's atmosphere is 1.0×10^{10} photons-cm^{-2}-sec^{-1}. Also assume that the sunlight enters the atmosphere vertically from the top and that the cross sections for absorption at these wavelength are equal to the cross sections for ionization. Calculate how much 30.4 nm sunlight is left at 250 km if no other processes besides absorption by and ionization of N_2, O_2, and O are occurring.

   c. Calculate the ionization rates (ionizations-cm^{-3}-sec^{-1}) for N_2, O_2, and O at 250 km due to the 30.4 nm flux in part b). Also calculate the total electron production rate (electrons-cm^{-3}-sec^{-1}) assuming one electron per each ionization.

   d. Repeat parts b) and c) assuming that the sunlight is entering the atmosphere at an angle of 30 degrees from the vertical.

70 pts total

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Due 2/18/03

①

ASTR/ATOC 3720 HW#2 Solutions

① $\lambda = 30.4\, nm$

$p_{Lab} = 1013\, mb$

$T_{Lab} = 288 K$

$\sigma_{N_2}(30.4nm) = 2.34 \times 10^{-17} cm^2$

$\sigma_{O_2}(30.4nm) = 3.328 \times 10^{-17} cm^2$

$["Air"] = 0.78[N_2] + 0.22[O_2]$ or $\frac{n_{N_2}}{n_{air}} = 0.78$, $\frac{n_{O_2}}{n_{air}} = 0.22$

10 a) $\tau \equiv \int_{s_0}^{s} \sigma(\lambda)\, n\, ds$

Since the light is travelling horizontally, the density isn't changing, so

$$\tau = \sigma(\lambda) \cdot n \cdot \Delta s$$

$$\tau_{air} = \tau_{N_2} + \tau_{O_2}$$

$$\tau_{N_2} = \sigma_{N_2} \cdot n_{N_2} \cdot \Delta s$$

$$\tau_{N_2} = \sigma_{N_2} \cdot (0.78\, n_{air})\, \Delta s$$

from ideal gas Law $p_{air} = n_{air} kT \Rightarrow n_{air} = \frac{p_{air}}{kT}$

$$n_{air} = \frac{1013\, mbar}{(1.38 \times 10^{-23} \frac{J}{K})(288K)} \cdot \underbrace{\frac{\left(10^{+2}\frac{Pa}{mbar}\right)\left(1\frac{N/m^2}{Pa}\right)}{\left(1\frac{N\,m}{Joule}\right)}}_{\text{unit conversions}} = 2.55 \times 10^{25} m^{-3}$$

so

$$\tau_{N_2} = (2.34 \times 10^{-17} cm^2)(0.78 \times 2.55 \times 10^{25} m^{-3})\left(10^{-4}\frac{m^2}{cm^2}\right) \cdot \Delta s$$

$$\boxed{\tau_{N_2} = 4.65 \times 10^{4}\, m^{-1} * \Delta s}$$

Similarly for O_2:

$$\tau_{O_2} = \sigma_{O_2} \cdot (0.22\, n_{air}) \cdot \Delta s$$

$$\tau_{O_2} = (3.328 \times 10^{-17} \cancel{cm^2})(10^{-4} \tfrac{m^2}{\cancel{cm^2}})(0.22 \times 2.55 \times 10^{25} m^{-3}) \cdot \Delta s$$

$$\boxed{\tau_{O_2} = 1.87 \times 10^{4} m^{-1} \times \Delta s}$$

And for "air"

$$\tau_{air} = \tau_{N_2} + \tau_{O_2} = (4.65 \times 10^{4} m^{-1}) \Delta s + (1.87 \times 10^{4} m^{-1}) \Delta s$$

$$\boxed{\tau_{air} = (6.52 \times 10^{4} m^{-1}) \times \Delta s}$$

b) Want $\frac{I}{I_0} = 0.05 \leftarrow$ 5%

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$$\frac{I}{I_0} = e^{-\tau_{air}} = 0.05$$

$$-\tau_{air} = \ln(0.05)$$

$$-(6.52 \times 10^{4} m^{-1}) \times \Delta s = \ln(0.05)$$

$$\Delta s = \frac{-\ln(0.05)}{6.52 \times 10^{4} m^{-1}}$$

$$\boxed{\Delta s = 4.6 \times 10^{-5} m = 46 \text{ micrometers!}}$$

Hardly any distance at all!
"Air" is very opaque to EUV light.

c) Repeat b) for $p_{air} = 840\,mb$ in Boulder

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Optical depth is directly related to density which is directly related to pressure (ideal gas)

$$\frac{\tau_{Boulder}}{\tau_{1013mb}} = \frac{n_{Boulder}}{n_{1013mb}} = \frac{p_{Boulder}}{p_{1013mb}} = \frac{840\,mb}{1013\,mb} = 0.829$$

$$\tau_{Boulder} = 0.829\,\tau_{1013mb}$$

So for $\frac{I}{I_0} = 0.05 = e^{-\tau_{Boulder}} = e^{-0.829\,\tau_{1013mb}}$

or $-0.829\,\tau_{1013mb} = \ln(0.05)$

$$-0.829\cdot(6.52\times10^{4}\,m^{-1})\cdot\Delta s_{Boulder} = \ln(0.05)$$

$$\Delta s_{Boulder} = \frac{-\ln(0.05)}{(0.829)(6.52\times10^{4}\,m^{-1})}$$

$$\boxed{\Delta s_{Boulder} = 5.5\times10^{-5}\,m = 55\text{ micrometers}}$$

Not much further in Boulder than at sea level!

(2) $z = 250\,km$

$n_{N_2} = 8\times10^{8}\,cm^{-3}$ $\quad H_{N_2} = 30.1\,km$

$n_{O_2} = 8.5\times10^{7}\,cm^{-3}$ $\quad H_{O_2} = 26.4\,km$

$n_{O} = 9\times10^{8}\,cm^{-3}$ $\quad H_{O} = 49.2\,km$

a) In the atmosphere for light from above

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$$\tau = \int_{z}^{\infty} \sigma n\,dz$$

Recall that for H constant

$$n = n_R e^{-\frac{(z-z_R)}{H}}$$

where R is a reference level, for this case $z_R = 250\ km$ where we know everything.

so:

$$\tau = \int_{z_R}^{\infty} \sigma n_R e^{-\frac{(z-z_R)}{H}} dz$$

with σ, n_0, and H constant:

$$\tau = \sigma n_R \left[(-H) e^{-\frac{(z-z_R)}{H}} \right] \Big|_{z_R}^{\infty}$$

$$\tau = -\sigma n_R H \left[e^{-\infty} - e^{0} \right]$$

$$\tau = -\sigma n_R H [0 - 1]$$

$$\tau = \sigma n_R H$$

So:

$$\tau_{N_2} = \sigma_{N_2} n_{N_2}(250km) \cdot H_{N_2} = (1.17\times10^{-17}\ cm^2)(8\times10^{8}\ cm^{-3})\frac{(30.1\ km)}{10^{-5}\ km/cm}$$

$$\boxed{\tau_{N_2} = 0.028}$$

$$\tau_{O_2} = \sigma_{O_2} n_{O_2} H_{O_2} = (1.664\times10^{-17}\ cm^2)(8.5\times10^{7}\ cm^{-3})(26.4\ km)(10^5\ \tfrac{cm}{km})$$

$$\boxed{\tau_{O_2} = 0.0037}$$

$$\tau_{O} = \sigma_{O} n_{O} H_{O} = (7.693\times10^{-18}\ cm^2)(9\times10^{8}\ cm^{-3})(49.2\ km)(10^5\ \tfrac{cm}{km})$$

$$\boxed{\tau_{O} = 0.034}$$

b) Optical depths add:

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$$\tau_{Total} = \tau_{N_2\,ion} + \tau_{N_2\,abs} + \tau_{O_2\,ion} + \tau_{O_2\,abs} + \tau_{O\,ion} + \tau_{O\,abs}$$

But $\tau_{N_2\,ion} = \tau_{N_2\,abs}$, $\tau_{O_2\,ion} = \tau_{O_2\,abs}$, $\tau_{O\,ion} = \tau_{O\,abs}$

so $\tau_{Total} = 2\tau_{N_2} + 2\tau_{O_2} + 2\tau_{O}$

$$\tau_{Total} = 2(0.028 + 0.0037 + 0.034)$$

$$\tau_{Total} = 0.1314$$

$$I_{250km} = I_0 e^{-\tau} = \left(1.0\times10^{10} \tfrac{ph}{cm^2\text{-}sec}\right) e^{-0.1314}$$

$$\boxed{I_{250km} = 8.77\times10^{9} \tfrac{ph}{cm^2\,sec}}$$

c) Ionization Rate = (Flux) * (Cross-Section) * (Number Density)

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$$J_i(z) = I(z)\,\sigma_i\,n_i(z)$$

Use flux left at altitude z, and density at z

$$J_{N_2} = \left(8.77\times10^{9} \tfrac{ph}{cm^2\,sec}\right)\cdot\left(1.17\times10^{-17}\,cm^2\right)\cdot\left(8\times10^{8}\,cm^{-3}\right)$$

$$\boxed{J_{N_2} = 82.1 \text{ ioniz}/cm^3\,sec}$$

$$J_{O_2} = \left(8.77\times10^{9} \tfrac{ph}{cm^2\,sec}\right)\left(1.664\times10^{-17}\,cm^2\right)\left(8.5\times10^{7}\,cm^{-3}\right)$$

$$\boxed{J_{O_2} = 12.4 \tfrac{ioniz}{cm^3\,sec}}$$

$$J_{O} = \left(8.77\times10^{9} \tfrac{ph}{cm^2\,sec}\right)\left(7.963\times10^{-18}\,cm^2\right)\left(9\times10^{8}\,cm^{-3}\right)$$

$$\boxed{J_{O} = 62.8 \text{ ioniz}/cm^3\,sec}$$

Total e^- production rate = (Total ionization rate) × (#e^- per ionization)

$$J_{e^-} = \left(J_{N_2} + J_{O_2} + J_O\right) \times 1 \frac{e^-}{ioniz}$$

$$J_{e^-} = \left(82.1 + 12.4 + 62.8 \frac{ioniz}{cm^3 sec}\right)\left(1 \frac{e^-}{ioniz}\right)$$

$$\boxed{J_{e^-} = 157.3 \frac{e^-}{cm^3 sec}}$$

d) For a slant incidence (angle θ from vertical) the optical depth is

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$$\tau_{slant} = \int_s \sigma n \, ds = \int_z \frac{\sigma n \, dz}{\cos\theta}$$

$$\tau_{slant} = \frac{\tau_{vert}}{\cos\theta}$$

$$\theta = 30^0$$

$$\tau_{slant} = \frac{0.1314}{\cos 30^0} = 0.1517$$

$$I_{slant} = I_0 e^{-\tau_{slant}} = \left(1 \times 10^{10} \frac{ph}{cm^2 sec}\right) e^{-0.1517}$$

$$\boxed{I_{slant} = 8.59 \times 10^9 \frac{ph}{cm^2 sec}}$$

$$J_{N_2} = \left(8.59 \times 10^9 \frac{ph}{cm^2 sec}\right)\left(1.17 \times 10^{-17} cm^2\right)\left(8 \times 10^8 cm^{-3}\right) = \boxed{80.4 \frac{ioniz}{cm^3 sec}}$$

$$J_{O_2} = \left(8.59 \times 10^9 \frac{ph}{cm^2 sec}\right)\left(1.664 \times 10^{-17} cm^2\right)\left(8.5 \times 10^7 cm^{-3}\right) = \boxed{12.1 \frac{ioniz}{cm^3 sec}}$$

$$J_O = \left(8.59 \times 10^9 \frac{ph}{cm^2 sec}\right)\left(7.963 \times 10^{-18} cm^2\right)\left(9 \times 10^8 cm^{-3}\right) = \boxed{61.6 \frac{ioniz}{cm^3 sec}}$$

$$J_{e^-} = \left(80.4 + 12.1 + 61.6 \frac{ioniz}{cm^3 sec}\right)\left(1 \frac{e^-}{ioniz}\right) = \boxed{154.1 \frac{e^-}{cm^3 sec}}$$