

# HW#3 Solutions

## ASTR/ATOC 3720: Homework Assignment #3

Due: Tues. Feb 25 in class

Show your work, as partial credit will be given for your thought processes, even if you don't get to the correct answer at the end! Work the problems on a separate sheet of paper please. Be neat and organized! We can't give credit for answers we can't read! You may work in groups, but turn in your own individual answers. Remember: **RTDQ** (Read the D\*\*\* Question)!

1. In class we calculated a total optical depth for the Earth of 0.6 using our "slab" model of the atmosphere and a  $T_e=255\text{K}$  and  $T_g=288\text{K}$ . Assume that of that total optical depth,  $\text{CO}_2$  accounts for a partial optical depth of 0.2.
  - a. What happens to  $T_g$  if we double the amount of  $\text{CO}_2$  in the atmosphere and keep  $T_e$  the same?
  - b. What happens to  $T_g$  if we increase the amount of  $\text{CO}_2$  in the atmosphere to 50 times the current amount, keeping  $T_e$  the same?
  - c. What happens to  $T_g$  if we double the amount of  $\text{CO}_2$  in the current atmosphere, but allow the cloud cover to increase, thereby changing the earth's albedo to 0.45?
  
2. Assume that the pressure at the surface of Earth over a land mass is 1013 millibars and the pressure at the surface over a neighboring large body of water is also 1013 millibars, but the temperature at the surface of the land is 300 K, while the temperature at the surface of the water is 290K.
  - a. Assuming that the temperature remains constant in the air above the land and water, calculate the pressures at an altitude of 3km above the land and at 3km above the water.
  - b. Which way will the wind be blowing at 3km?
  - c. Which way will the wind be blowing at the surface?

①  $\tau_{\text{Total}} = 0.6$   
 $T_e = 255\text{K}$   
 $T_g = 288\text{K}$

$\tau_{\text{Total}} = \tau_{\text{CO}_2} + \tau_{\text{other}}$  ; recall optical depths are additive

$\tau_{\text{CO}_2} = 0.2$  ,  $\tau_{\text{other}} = 0.6 - 0.2 = 0.4$

a) Double the  $\text{CO}_2$ :

Recall the equation for optical depth, applied to  $\text{CO}_2$

$$\tau_{\text{CO}_2} = \int \sigma_{\text{CO}_2} n_{\text{CO}_2} ds$$

By doubling the  $\text{CO}_2$ , the only thing that changes in the integral is the  $\text{CO}_2$  density ( $n_{\text{CO}_2}$ ), and it only doubles

$$n'_{\text{CO}_2} = 2n_{\text{CO}_2}$$

$$\Rightarrow \tau'_{\text{CO}_2} = \int \sigma_{\text{CO}_2} 2n_{\text{CO}_2} ds = 2 \int \sigma_{\text{CO}_2} n_{\text{CO}_2} ds = 2\tau_{\text{CO}_2}$$

so the optical depth doubles for  $\text{CO}_2$ .

From class or GBW have the relationship

$$T_g^4 = (1 + \tau) T_e^4 = (1 + \tau_{\text{Total}}) T_e^4 = (1 + \tau_{\text{CO}_2} + \tau_{\text{other}}) T_e^4$$

$$T_g'^4 = (1 + \tau'_{\text{CO}_2} + \tau_{\text{other}}) T_e^4$$

Since the energy from the Sun doesn't change  $T_e$  doesn't change, and since  $\text{CO}_2$  is the only change then  $\tau_{\text{other}}$  doesn't change (was  $\tau_{\text{other}} = \tau_{\text{Total}} - \tau_{\text{CO}_2} = 0.6 - 0.2 = 0.4$ )

So the new  $T_g'$  for doubling  $CO_2$  is:

$$T_g' = [(1 + 2(0.2) + 0.4)]^{1/4} (255K)$$

$$T_g' = 295K$$

By doubling the  $CO_2$ , we've raised the mean ground temperature from 288K to 295K!

b) Increase  $CO_2$  by 50 times:

$$\tau'_{CO_2} = 50 \tau_{CO_2} = 10$$

$$T_g' = [1 + 10 + .4]^{1/4} (255K)$$

$$T_g' = 469K \quad \text{Very hot!}$$

c) Double  $CO_2$ , but ~~allow~~ allow albedo to increase to 0.45

$$\text{Doubling } CO_2 \Rightarrow \tau'_{CO_2} = 2\tau_{CO_2} = 2(.2) = 0.4$$

Increasing albedo  $\Rightarrow \alpha' = 0.45$  means  $T_e$  will change.

$$T_e = \left[ \frac{S_0 (1 - \alpha)}{D^2 4 \sigma} \right]^{1/4}$$

(without increased clouds had  $\alpha = 0.3$  and  $\Rightarrow T_e = \left[ \frac{(1368 \frac{W}{m^2}) (1 - 0.3)}{(1AU)^2 4 (5.67 \times 10^{-8} \frac{W}{m^2 K^4})} \right]^{1/4} = 255K$ )

Need to re-calculate  $T_e$  with new albedo

$$T_e' = \left[ \frac{(1368 \frac{W}{m^2}) (1 - 0.45)}{(1AU)^2 4 \times 5.67 \times 10^{-8} \frac{W}{m^2 K^4}} \right]^{1/4}$$

$$T_e' = 234K$$

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Next re-calculate  $T_g$  with new  $T_{CO_2}$  and new  $T_e$ :

$$T_g' = [1 + \underbrace{0.4}_{T_{CO_2}} + \underbrace{0.4}_{T_{other}}]^{1/4} \underbrace{273}_{T_e'} \text{ K}$$

$$T_g' = 271 \text{ K}$$

Doubling  $CO_2$ , but increasing clouds actually results in a temperature decrease at the surface! (Just below freezing in this case!)

②  $P_{land} = 1013 \text{ mb}$ ,  $T_{land} = 300 \text{ K}$   
 $P_{water} = 1013 \text{ mb}$ ,  $T_{water} = 290 \text{ K}$

a) Since the temp. is assumed to stay constant we can use the barometric equation to calculate the pressures:

$$P_{3km} = P_{0km} e^{-\frac{3km}{H}}$$

So we need to calculate the scale heights over land and water

$$H_{land} = \frac{1/2 T_{land}}{\bar{m} g} = \frac{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{(4.8 \times 10^{-26} \text{ kg})(9.8 \text{ m/sec}^2)} = 8.8 \text{ km}$$

$\bar{m}$  = mean molecular mass of air =  $(28.9 \text{ g/mole}) \times (10^{-3} \text{ kg/g})$   
 (78%  $N_2$ , 21%  $O_2$ , 1%  $Ar$ )  $(6.022 \times 10^{23} \text{ molecules/mole})$

$$H_{water} = \frac{1/2 T_{water}}{\bar{m} g} = \frac{(1.38 \times 10^{-23} \text{ J/K})(290 \text{ K})}{(4.8 \times 10^{-26} \text{ kg})(9.8 \text{ m/sec}^2)} = 8.5 \text{ km}$$

Plugging back into the barometric equation:

$$P_{land}(3km) = P_{land}(0km) e^{-\frac{\Delta z}{H_{land}}}$$

$$P_{land}(3km) = (1013 mb) e^{-\frac{3km}{8.8km}}$$

$$P_{land}(3km) = 720 mb$$

$$P_{water}(3km) = (1013 mb) e^{-\frac{3km}{8.5km}}$$

$$P_{water}(3km) = 711 mb$$

b) Since the surface pressure is the same over land and water, but the pressure at 3km is ~~low~~ higher over land than water in our case, we expect that pressure forces would cause a wind blowing from land to ~~the~~ water at 3km.

c) Since the air is moving from land to water at 3km, we need a return flow so we'd expect at the surface the wind would blow from water to land (a 'sea breeze').