

## ASTR/ATOC 3720: Homework Assignment #5

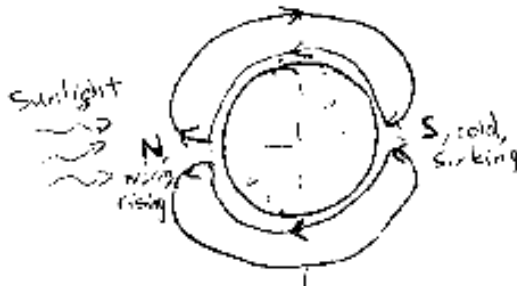
Due: Thur. May 1 in class

1. The obliquity of Uranus is about  $98^\circ$  and its orbital period is about 84 years. This means that the planet is essentially on its side and the seasons are many years long. Draw pictures and explain what (and why) you'd expect the general global circulation patterns to look like in the following cases (hint: think Hadley):
  - a. During the period when the north pole is pointed almost directly and continuously at the Sun (northern summer).

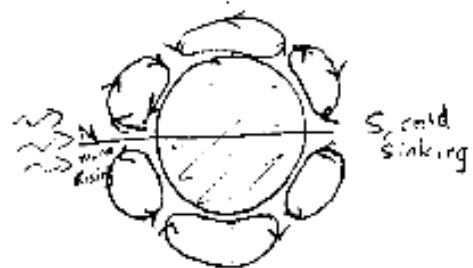
*Since the Sun is directly over the north pole for a long time, you'd expect the north pole to be warmer, so air would rise there and the south pole to be colder, so air would sink there. Therefore, you'd expect air to rise at the north pole, travel towards the south pole, sink at the south pole, and return towards the north pole lower down in the atmosphere. With the addition of coriolis forces, this single large cell may break up into smaller cells, but still you would have rising at the north pole and sinking at the south pole.*

Neptune N. Pole Summer:

W/o Coriolis:



With Coriolis:



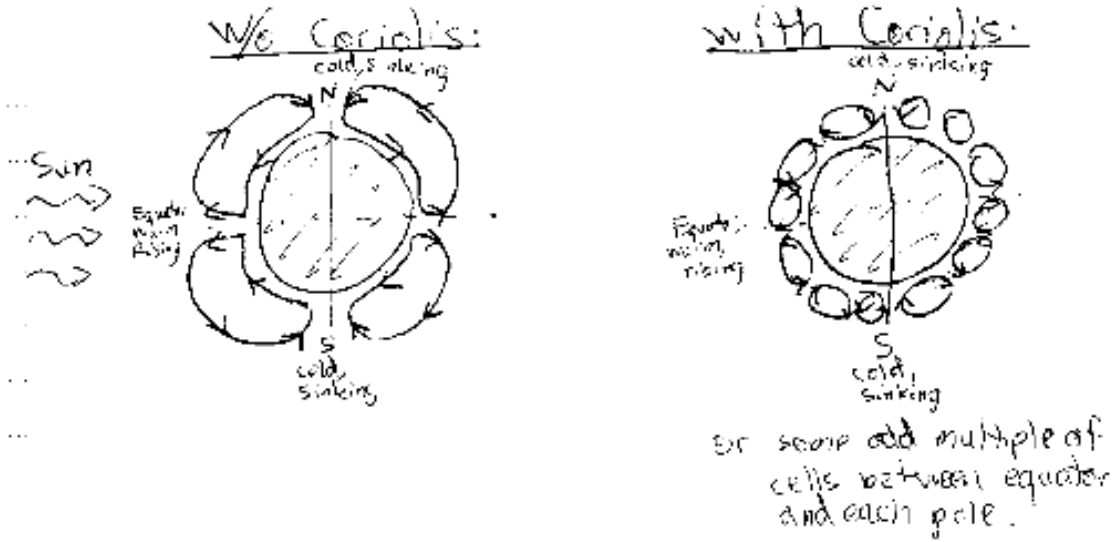
or some odd multiple of cells.  
between poles

- b. When the Sun is overhead at the equator (equinox).

*This is the more classic case, like on Earth. You'd expect the equator to be warmer and both poles to be colder. This would set up two cells with air rising at the equator, travelling away from the equator towards the poles, sinking at each pole, then returning towards the equator deeper in the atmosphere. With the addition of coriolis forces, these two cells may break up into smaller cells,*

but you would still have rising at the equators and sinking at both poles.

## Neptune Equinox:



2. Pluto's orbit has an eccentricity of 0.244 and a semi-major axis (mean distance from the Sun) of 39.2 AU.

- a. Calculate the the perihelion and aphelion distances for Pluto.

$$r_{\text{perihelion}} = a \cdot (1 - e) = (39.2 \text{ AU}) \cdot (1 - 0.244) = 29.6 \text{ AU}$$

$$r_{\text{aphelion}} = a \cdot (1 + e) = (39.2 \text{ AU}) \cdot (1 + 0.244) = 48.8 \text{ AU}$$

- b. Calculate the effective temperatures for Pluto at both perihelion and aphelion, assuming that the planet's albedo is 0.5.

$$T_{\text{eff, perihelion}} = \left[ \frac{S_0 \cdot (1 - A)}{D^2 \cdot 4 \cdot \sigma} \right]^{1/4} = \left[ \frac{(1368 \text{ Watts/m}^2) \cdot (1 - 0.5)}{(29.6)^2 \cdot 4 \cdot (5.67 \cdot 10^{-8} \text{ Watts/m}^2\text{K}^4)} \right]^{1/4} = 43.1 \text{ K}$$

$$T_{\text{eff, aphelion}} = \left[ \frac{(1368 \text{ Watts/m}^2) \cdot (1 - 0.5)}{(48.8)^2 \cdot 4 \cdot (5.67 \cdot 10^{-8} \text{ Watts/m}^2\text{K}^4)} \right]^{1/4} = 33.5 \text{ K}$$

- c. The temperature change due to the eccentric orbit of Pluto probably causes much of its atmosphere to freeze out on the surface, increasing its albedo to as high as 0.70. Calculate a new aphelion effective temperature based on this new albedo.

$$T_{eff, aphelion} = \frac{(1368 \text{ Watts/m}^2) \cdot (1 - 0.7)}{(48.8)^2 \cdot 4 \cdot (5.67 \cdot 10^{-8} \text{ Watts/m}^2 \text{K}^4)} = 29.5 \text{K}$$

Which is about 4K colder than the temperature found without changing the albedo.

3. Titan is a satellite of Saturn. It has an albedo of 0.20. (You'll find most of the numbers you need from your lecture notes and the tables in the back of NSS.)

- a. Calculate the effective temperature of Titan.

$$T_{eff, Titan} = \frac{S_0 \cdot (1 - \alpha)}{D^2 \cdot 4 \cdot \sigma} = \frac{(1368 \text{ Watts/m}^2) \cdot (1 - 0.2)}{(9.5)^2 \cdot 4 \cdot (5.67 \cdot 10^{-8} \text{ Watts/m}^2 \text{K}^4)} = 85.5 \text{K}$$

- b. The actual surface temperature of Titan is 94 K. Using the "slab" model, determine the optical depth of Titan's atmosphere.

$$\tau = \frac{T_{surface}}{T_{effective}} - 1 = \frac{94 \text{K}}{85.5 \text{K}} - 1 = 0.46$$

- c. Calculate the acceleration due to gravity at the surface of Titan.

$$g_{Titan} = \frac{G \cdot M_{Titan}}{r_{radius}^2} = \frac{(6.67 \cdot 10^{-11} \frac{\text{Newtons} \cdot \text{m}^2}{\text{kg}^2}) \cdot (1.34 \cdot 10^{23} \text{kg})}{[(2575 \text{km}) \cdot (1000 \text{m/km})]^2} = 1.35 \text{m/sec}^2$$

- d. Calculate the adiabatic lapse rate for Titan, assuming that the atmosphere is almost completely made of N<sub>2</sub>.

$$\text{Mass of } N_2 \text{ molecule: } m_{N_2} = \frac{2 \cdot (14 \text{g/mole}) \cdot (10^3 \text{kg/g})}{6.022 \cdot 10^{23} \text{ molecules/mole}} = 4.65 \cdot 10^{-26} \text{kg}$$

Specific Heat for N<sub>2</sub> (a diatomic gas):

$$c_p = \frac{7}{2} \cdot \frac{k}{m_{N_2}} = \frac{7}{2} \cdot \frac{(1.38 \cdot 10^{-23} \text{Joules/K})}{(4.65 \cdot 10^{-26} \text{kg})} = 1039 \frac{\text{Joules}}{\text{kg} \cdot \text{K}}$$

Adiabatic Lapse Rate:

$$\frac{dT}{dz} = -\frac{g}{c_p} = -\frac{(1.35 \text{m/sec}^2)}{(1039 \frac{\text{Joule}}{\text{kg} \cdot \text{K}})} = -1.3 \cdot 10^{-3} \text{K/m} = -1.3 \text{K/km}$$

- e. Assuming a surface temperature of 94 K and a "dry" adiabatic lapse rate up to the tropopause at 42 km, what would you expect the temperature to be at the tropopause?

*The change in temperature over 42 km is:*

$$\Delta T = \frac{dT}{dz} \cdot \Delta z = (-1.3 \text{ K/km}) \cdot (42 \text{ km}) = -54.6 \text{ K}$$

*So the temperature at 42 km is:*

$$T_{42 \text{ km}} = T_{0 \text{ km}} + \Delta T = 94 \text{ K} + (-54.6 \text{ K}) = 39.4 \text{ K}$$

- f. The actual tropopause temperature on Titan is around 71 K. Pressures and temperatures in Titan's atmosphere are high enough that methane can exist as a liquid, solid, or gas. Explain then why the temperature at 42 km is warmer than what a dry adiabatic lapse rate would give.

*The lapse rate is not "dry" adiabatic, but "wet" adiabatic because methane can condense out to a liquid when it reaches its saturation vapor pressure. This is just like water on Earth condensing out when air is lifted upwards. The process of condensation liberates heat, which means that for a "wet" atmosphere the temperature drops off more slowly as you go upwards from the surface than it does for a "dry" atmosphere.*