

Problem 1

For Gaussian wave packet

$$f(x) = e^{-a_0^2 x^2/2 + ik_0 x} \quad , \quad (1)$$

where a_0 and k_0 are constants.

a) Calculate the Fourier transform $\mathcal{F}(f(x))$.

b) Show that $\langle \Delta x^2 \rangle \langle \Delta k^2 \rangle = 1/4$, where the mean squares are defined as

$$\langle \Delta x^2 \rangle = \frac{\int_{-\infty}^{\infty} x^2 f(x) f^*(x) dx}{\int_{-\infty}^{\infty} f(x) f^*(x) dx}$$

$$\langle \Delta k^2 \rangle = \frac{\int_{-\infty}^{\infty} (k - k_0)^2 f(k) f^*(k) dk}{\int_{-\infty}^{\infty} f(k) f^*(k) dk}$$

Problem 2

Show that $f(x) = \mathcal{F}^{-1}(\mathcal{F}(f(x)))$.