Problem 1
The reduced distribution function of a plasma with two electron beams is given by a counter-streaming Cauchy distribution

$$F_o(v_z) = \frac{C}{2\pi}\left[\frac{1}{C^2 + (v_z - U)^2} + \frac{1}{C^2 + (v_z + U)^2}\right] \tag{1}$$

a) Show that the distribution function is double humbed only if $C < \sqrt{3}U$
b) Make a plot by mapping the imaginary p axis onto the complex D plane (Nyquist procedure) for three cases: 1) $C < U$; 2) $C = U$; and 3) $C > U$.

Problem 2
Show that the 1D Kappa distribution

$$F_0(v_z) = \frac{\Gamma(\kappa + 1)}{\Gamma(\frac{1}{2})\Gamma(\kappa + \frac{1}{2})\kappa^{\frac{1}{2}}C}\left[1 + \frac{v_z^2}{\kappa C^2}\right]^{-(\kappa+1)} \tag{2}$$

a) becomes a Gaussian as $\kappa \to \infty$, and
b) it approaches a Cauchy distribution for small values of κ

Problem 3
a) Assuming a plasma with electron temperature $T_e = 3$ eV, ion temperature of $T_i = T_e/100$, and a plasma density of $n_p = 10^6$ cm^{-3}, estimate to within an order of magnitude how long it would take for $r = 5$ μm radius dust grain to become fully charged?
b) Write a small code to follow the time-dependent charging ($dQ/dt = I_i - I_e$) to verify your order of magnitude estimate.