① $m_s \frac{d\bar{v}}{dt} = e_s \bar{E} \underbrace{- \nu_s m_s \bar{v}_s}_{\text{drag term}}$

a) FT

$-i\omega m_s \tilde{v}_s = e_s \tilde{E} - \nu_s m_s \tilde{v}_s$

$-i\omega m_s \tilde{v}_s \left(1 + \frac{i\nu_s}{\omega}\right) = e_s \tilde{E}$

"effective" mass $\boxed{m_s^* = m_s\left(1 + i\frac{\nu_s}{\omega}\right)}$

b) long. electron oscillations : $\omega^2 = \omega_p^2$

$\omega_p^2 = \sum_s \omega_{ps}^2 \xrightarrow[\text{ignore ions}]{} \omega_p^2 = \omega_{pe}^2 = \frac{n_e e^2}{\varepsilon_0 m_e^*}$

$\omega^2 = \omega_p^2 \frac{1}{1 + i\frac{\nu_e}{\omega}} \approx \omega_p^2\left(1 - \frac{i\nu_e}{\omega}\right)$ for $\nu_e \ll \omega$

$\omega = \omega_p\left(1 - \frac{i\nu_e}{\omega}\right)^{1/2} \approx \omega_p\left(1 - \frac{i}{2}\frac{\nu_e}{\omega}\right)$

$\omega = \omega_r + i\omega_i \Rightarrow \omega_r = \omega_p$

$\omega_i = -\frac{1}{2}\cancel{\frac{\omega_p}{\omega}}\nu_e$ $\quad \approx 1$

The oscillations are damped and $\boxed{\omega_i = -\frac{1}{2}\nu_e}$

the damping rate is $\sim \frac{1}{2}$ the coll. frequency!

HW 4 p2

c) $\omega^2 = \omega_p^2 + c^2 k^2$

$$\uparrow$$

$$\sum_s \omega_{ps}^2 = \sum_s \frac{n_s e_s^2}{\varepsilon_0 m_s} \qquad m_s \rightarrow m_s^*$$

$$\omega^2 = \sum_s \frac{\omega_{ps}^2}{1 + \frac{i\nu_s}{\omega}} + c^2 k^2$$

$$k^2 = \left(\frac{\omega}{c}\right)^2 \left[1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \frac{1 - i\nu_s/\omega}{\left(1 + \frac{i\nu_s}{\omega}\right)\left(1 - \frac{i\nu_s}{\omega}\right)}\right]$$

$$k^2 = \left(\frac{\omega}{c}\right)^2 \left[1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \frac{(1 - i\nu_s/\omega)}{(1 + \nu_s^2/\omega^2)}\right]$$

$$k^2 = \left(\frac{\omega}{c}\right)^2 \left[1 - \sum_s \frac{\omega_{ps}^2}{\omega^2 + \nu_s^2} + i \sum_s \frac{\omega_{ps}^2 \nu_s/\omega}{\omega^2 + \nu_s^2}\right]$$

$$\uparrow$$

$$(k_r + ik_i)^2 = k_r^2 + 2ik_ik_r + k_i^2$$

small damping $k_i \ll k_r$

$$k_r^2 \approx \left(\frac{\omega}{c}\right)^2 \left[1 - \sum_s \frac{\omega_{ps}^2}{\omega^2 + \nu_s^2}\right]$$

$$\Rightarrow k_r \approx \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2} \approx \frac{\omega}{c}\left(1 - \frac{\omega_p^2}{2\omega^2}\right) \checkmark$$

$$2k_ik_r = \left(\frac{\omega}{c}\right)^2 \sum_s \frac{\omega_{ps}^2 \nu_s/\omega}{\omega^2 \cancel{+ \nu_s^2}} \qquad k_r \approx \frac{\omega}{c}$$

$$\Rightarrow k_i \approx \frac{1}{2c} \sum_s \frac{\omega_{ps}^2 \nu_s}{\omega^2} \checkmark$$