Analyses on the Geometrical Structure of Magnetic Field in the Current Sheet Based on Cluster Measurements

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Abstract: The geometrical structure of the magnetic field is a critical character in the magnetospheric dynamics. Using the magnetic field data measured by the Cluster constellation satellites, the geometrical structure including the curvature radius, directions of curvature and normal of the osculating planes of the magnetic field lines within the current sheet/neutral sheet have been investigated. The results are: (1) Inside of the tail neutral sheet (NS), the curvature of magnetic field lines points towards Earth, the normal of the osculating plane points duskward, and the characteristic half width (or the minimum curvature radius) of the neutral sheet is generally less than 2Re, for many cases less than 1600km. (2) Outside of the neutral sheet, the curvature of magnetic field lines pointed northward (southward) at the north (south) side of NS, the normal of the osculating plane points dawnward, and the curvature radius is about 5Re~10Re. (3) Thin NS, where the magnetic field lines have the minimum of the curvature radius less than $0.25\,R_E$, may appear at all the local time between LT 20hr and 4 hr, but thin NS occurrs more frequently near to midnight than that at the dawnside and duskside. (4) The size of the NS is dependent on substorm phases. Generally, the NS is thin during the growth and expansion phases and grows thick during the recovery phase. (5) For the one-dimensional NS, the half thickness and flapping velocity of the NS could be quantitatively determined. Therefore, the differential geometry analyses based on Cluster 4-point magnetic measurements open a window for visioning the three-dimensional static and dynamic magnetic field structure of geomagnetosphere.

1. Introduction

The geometrical configuration of magnetic field lines (MFLs) plays crucial roles on the magnetospheric dynamics, especially in the current sheet of magnetotail. Charged particles with gyroradius much less than the local spatial scale of the magnetic field are trapped and move along the filed lines. If the magnetic field is inhomogeneous with gradient and curvature, the trapped plasma particles will also perform magnetic gradient and curvature drift, which may produce significant current, e.g., the ring current in the inner magnetosphere. The particle velocity of curvature drift, as well as the resulted current, is reciprocal to the curvature radius of the MFLs. The geometrical configuration of MFLs also controls the non-adiabatic and chaotic motions of particles having gyroradius comparable to or larger than the characteristic scale of the magnetic

field. In a thin current sheet, these kinds of particles may perform diverse motions, e. g., the serpentine motion [Speiser, 1965]. The curvature parameter κ, which is the squared ratio between the minimum curvature $R_{c\min}$ of the magnetic filed and the maximum Larmor radius $\rho_{\max}$ of particles, *i.e.*, $\kappa = (R_{c\min} / \rho_{\max})^{1/2}$, has been put forward as a threshold to determine the motion types of charged particles in the current [Büchner and Zelenyi, 1987, 1989]. For $\kappa >> 1$, the particles perform adiabatic motions with their magnetic moment conserved; however, when $\kappa \le 1$, the motion of the particles will become non-adiabatic and some chaotic processes may appear. Furthermore, the spatial configuration of MFLs can also significantly affect the properties of waves and instabilities in plasmas [Mikhailovskii, 1992; Roux *et al*., 1991; Pu *et al*., 1997]. Presently it is believed that a thin current sheet with $\kappa \le 1$ favorites the occurrence of the tearing mode, current instability, even the current disruption and particle energization [Burkhart, 1992b; Lakhina, 1993; Mitchell *et al*., 1990; Lui *et al*., 1992; Wang *et al*., 1990; Liu and G. Rostoker, 1995].

Therefore it is important to know the actual geometrical configuration of MFLs in the tail current sheet (CS) and also its dynamical evolution. Assuming a special type of structure, such as the most popular 1-d Harris model [Harris, 1962], the thickness and other features can be determined based on single or two-satellite measurements. By measuring the electron precipitation into ionosphere caused by pitch angle scattering in the near-Earth current sheet, Sergeev *et al.* (1990) have found that the half thickness of the CS near the end of the growth phase is about $0.1\,R_E$. Considering the cross tail current during the growth phase is mainly contributed by the electron curvature drifting, Mitchell *et al*. (1990) have estimated the curvature radius of the CS based on the magnetic and plasma measurements and thus obtained the characteristic half thickness of CS. Lui (1993) has developed a procedure to calculate the global properties of the CS based on the local magnetic plasma and current density measurements by one single satellite, in which one dimensional magnetic configuration of the CS has been assumed. Based on ISEE-3, magnetic field and plasma data, Pulkkinen *et al*. (1993) have revealed the average thickness of the deep tail current sheet was $2.45\,R_E$. McComas *et al*. (1986) have first used the ISEE 1 and ISEE 2 two-satellite magnetic field measurements to deduce the current density distribution in the CS as well as the CS thickness. Assuming the magnetic field in the CS varies linearly from the northern boundary to the southern boundary of the CS, Sergeev *et al*. (1993) have determined the half thickness of the CS based on data from two satellites (ISEE 1 and ISEE 2); the minimum variance analysis method [Sonnerup and Cahill, 1967] has been applied to calculate the normal vector of the CS. It wasfound that the CS half thickness were thinning from ~3000km to ~800km just before the substorm onset. Assuming a one dimensional Harris model of the CS magnetic field, Sanny *et al.* (1994) have investigated the global structure of the CS and its temporal variation with the ISEE 1 and ISEE2 measurements; the lobe magnetic field strength B_L was estimated by the pressure balance condition. They have found that, for the CDAW 6 substorm on March 22, 1979, the CS at $X \approx -13R_E$ was approximately exponentially thinning with its half thickness decreasing from

~5 R_E to only ~1 R_E during the growth phase, and during the expansion phase after the onset the current sheet may keep with a half thickness as small as about 1000km. Zhou *et al.* (1997a, b) have investigated in details on the average structure of the current sheet and revealed that the current sheet could be twisted significantly due to the existence of IMF By. Regarding the configuration of current sheet/neutral sheet, many researchers have also performed numerical and theoretical explorations [Birn, *et al.*, 1977; Baker and McPherron, 1990; Pulkkinen, *et al.*, 1994; Burkhart, *et al*., 1992; Sitnov, 2000, and the references therein].

The Cluster II mission has made it a reality to directly deduce the 3 dimensional structure of the magnetic field in magnetosphere from the four-point simultaneous magnetic measurements [Balogh, *et al*., 2001]. Some methods have been developed to determine the spatial configuration and velocity of a discontinuity plane, wave vector, the gradient of magnetic field and current density [Dunlop, et al., 1990; Mottez and Chanteur, 1994; Dunlop and Woodward, 1998; Chanteur, 1998; Harvey, 1998]. However, presently there is still no special stress on drawing the geometrical structure of MFLs in magnetosphere based on the Cluster four-point magnetic observational data. In this investigation, we would develop an approach to calculate the curvature vector and normal of the osculating plane of magnetic filed lines based on Cluster magnetic measurements so that the geometrical structure of MFLs in the magnetosphere could be revealed. Here we would mainly focus on deducing the geometrical structure of MFLs of the tail CS from the Cluster FGM observations during July through October 2001.

2. Method for Determining the Geometrical Structure of Magnetic field

In order to determine the local geometrical structure of one magnetic field line, we need to know its curvature $\vec{\rho}_c$ and the normal $\vec{N}$ of its osculating plane (some necessary and important concepts of differential geometry are described in Appendix A). Fig 1 shows the configuration of one magnetic field line and the relationship between its unit tangential vector $\vec{b}$, curvature $\vec{\rho}_c$ and normal $\vec{N}$ of the osculating plane. Therefore, the local three dimensional structure of one magnetic field line is demonstrated by five parameters, i.e., the curvature radius R_c, the polar and azimuthal angles (θ_c, φ_c) and (θ_N, φ_N) of the curvature vector and the normal of the osculating plane, respectively.

To calculate the curvature, the formula (A1) is expanded as

$$\rho_{cj} = B^{-2} B_i \nabla_i B_j - B^{-4} B_j B_i B_l \nabla_i B_l , \quad (1)$$

where the subscript Latin index i, j or l (=1,2 and 3) denote the three components(x, y and z).

After obtaining the magnetic field $\vec{B}_c$ and its gradient $(\nabla\vec{B})_c$ at the mesocentre of the four Cluster satellites (mesocentre's definition refers to Appendix B), we can calculate the curvature $\vec{\rho}_c$

and curvature radius R_c at the mesocentre. Presently, there are two approaches that can be applied for deducing the local gradient of magnetic field, i. e., the least square minimization [Harvey, 1998] and the linear interpolation [Chanteur, 1998]. It could be verified that these two methods yield equivalent results [Chanteur and Harvey, 1998]. The difference between them is that the least square minimization method could include the solenoidal condition of the magnetic field. In order to take this advantage, we have used the least square minimization method in this research. In the appendix B, a new form of the action S, which is somewhat different from that of Harvey (1998), has been constructed so as to deduce the expressions of the magnetic field $\vec{B}_c$ and its gradient $(\nabla\vec{B})_c$ at the mesocentre of the Cluster tetrahedron. $\vec{B}_c$ is just the average of the magnetic field $\vec{B}_\alpha$ (α =1,2,3,4) at the positions of the four satellites as expressed as Eq. (B5). The obtained formula (B6) of the gradient $(\nabla\vec{B})_c$ at the mesocentre is equivalent to that of Harvey (1998), but rather simpler and helpful for reducing the calculation error.

Also, during Cluster CS/NS crossing from July 1 through October 31, 2001, the tetrahedron of Cluster is nearly a regular one. Thus it is proper to use the least square minimum method to calculate the gradient and curvature of magnetic field $\vec{B}$.

We may discuss the error of the calculations. Generally, the physical errors are rather small, $\Delta B/B \le 0.01$ and $\Delta r/2L \le 0.01$ [Dunlop *et al.*, 1990], where ΔB and Δr are the errors of magnetic field and Cluster position, respectively, and L is the maximum characteristic size of Cluster tetrahedron. When Cluster crossing CS, L is about 1600km $\approx \frac{1}{4}R_E$. Apart from the above two physical errors, the calculation method for the curvature may cause a truncation error. In Appendix B, we have made a linear approximation and omitted the second order term in Eq. (B3); this would produce a truncation error of the curvature as well as that of the radius. Compared with the truncation error, the contribution of the physical errors to the total errors of curvature and curvature radius can be neglected. Appendix C has offered an estimate on the truncation errors of the curvature and curvature radius, which are at the 2nd order of $L/(2R_c)$. When $R_c \le L/2$, the above method can only give the upper limit of R_c. Presently there is still no known method for calculating R_c when $R_c \le L/2$. The proper method needs to be developed in the future.

We can also obtain the directional feature of CS/NS boundary layer from the Cluster FGM 4 point magnetic measurements. In this research, the three axis of the GSM coordinates are denoted as X, Y and Z, while the three axis of the boundary layer (BL) coordinates of CS/NS are denoted as x, y and z. If the CS/NS has a steady structure, it is reasonable to regard that the y axis of the NS BL coordinate system is along the direction of the current density and the z axis is along the normal of the CS/NS. (Note that the y component of magnetic field in the NS BL

coordinates is considered constant as shown in Appendix D.) The current density is

$$\vec{J} = \mu_0^{-1} \nabla \times \vec{B}, \qquad (2)$$

which can be determined by using the formula (B5) of the gradient $(\nabla \vec{B})_c$ in Appendix C. The normal $\vec{n}$ of the CS/NS boundary layer can be regarded as pointing along the direction of the gradient of the magnetic pressure p_B, *i.e.*,

$$\vec{n} = \nabla p_B / |\nabla p_B|$$

$$= B_j \nabla B_j / |B_j \nabla B_j|. \qquad (3)$$

This formula can be applied for locally determining the normal of the CS/NS boundary layer based on the 3–d magnetic measurements of Cluster. The z axis of the NS BL coordinates is along the normal of the CS/NS. The components and gradients of magnetic field, B_j and ∇B_j, in Eq. (2) and (3) should be that in the mesocentre.

By the way, from the measured gradient of magnetic field $\nabla \vec{B}$ we can only draw three independent physical vectors, *i.e.*, the curvature $\vec{\rho}_c$ of MFLs, the curl of magnetic field $\nabla \times \vec{B}$ and the gradient of magnetic pressure $\nabla p_B = \mu_0^{-1} B_j \nabla B_j$.

3. Geometrical Structure of the Magnetic field of CS Deduced from Cluster Observations

In this exploration, the Cluster satellite position data with an one minute resolution is generated by the Hungarian Data Centre, magnetic field data with a 4-sec resolution is from Cluster FGM measurement.

The plasma sheet crossings of Cluster during July through October of 2001 have been surveyed. The curvature direction and curvature radius of MFLs within current sheet/neutral sheet have been calculated.

3.1 General Features

Figures 2 and 3 show the direction of the curvature, curvature radius, the direction of osculating plane and the magnetic field in GSM coordinates for the Cluster CS crossings during Sept. 17 and Sept. 29 of 2001, respectively, when the CS has no strong flapping. The CS crossing intervals are about 2 hours.

It is known that in the center region of CS there is a thin layer with small magnetic field strength, which is called neutral sheet (NS) [Ness, 1965; Cowley, 1972; Lui, 1978; Fairfield, 1980]. In this research, we would conveniently define the NS is the region within the CS where

$|B_X| \le B_N$, here B_N is the northward component of the magnetic field.

(1) Within NS

The curvature radius is very small, generally less than $2R_E$. The polar angle θ_c is about between 45° and 135°. The azimuthal angle φ_c is near to 0° or 360°. So that the curvature of the MFLs points earthward. As for the osculating plane, the azimuthal angle φ_N of its normal is near to 90°, and the polar angle θ_N is generally less than 90° due to the appearance of minus B_Y component within the NS. The observational features of NS can be expressed as the following.

$R_c \le 2R_E$,

$45^\circ \le \theta_c \le 135^\circ$,

$\varphi_c \approx 0^\circ \quad or \quad 360^\circ$, for the NS.

$\theta_N \le 90^\circ$,

$\varphi_N \approx 90°$.

(2) *In the north (south) lobe and north (south) CS*

In the north (south) lobe and north (south) CS, the curvature radius is about 5~10 R_E, the polar angle θ_c of the curvature is near to 0° (180°), the azimuthal angle φ_c is near to 90° or 270°. Thus the curvature of the MFLs points northward (southward), which is not in agreement with that predicted by the Harris model. The azimuthal angle φ_N of the binormal of the osculating plane is about 270°, and the polar angle θ_N is about 90° for the north (south) lobe and north (south) CS.

$R_c \approx 5 \sim 10R_E$,

$\theta_c \approx 0^\circ (180^\circ)$,

$\varphi_c \approx 90^\circ \quad or \quad 270^\circ$, for the north (south) lobe and north (south) CS.

$\theta_N \approx 90^\circ$,

$\varphi_N \approx 270°$.

(3) *In the transition layers between NS and north (south) CS*

There are two special transition layers between the NS and north (south) CS, where the

curvature radius grows larger rapidly, indicating that the curvature radius is rather large and the field lines are almost straight.

The common features of the curvature and normal of the osculating planes of the MFLs in the CS have been schematically demonstrated in Fig. 4. The figure 4 can be used to conveniently determine whether the region Cluster crossing is the NS, or the north (south) CS and north (south) lobe.

The general geometrical configuration of MFLs in the CS is illustrated in Fig. 5. It should be noted that not only during active period, such as the growth, expansion and recovery phases, but also when the magnetotail is quiet, CS always has this kind of geometrical structure. The observational results as shown above indicate that the magnetic field in the lobes and also CS outside NS can not be well modeled by Harris configuration. Some previous investigations have also shown that the actual CS/NS could have certain deviations from the Harris model , e.g., the distant tail CS might have double-peaked current density in some cases [Hoshino et al., 1996].

3.2 CS / NS Flapping Phenomena

CS/NS flapping are frequently occurring phenomena in magnetosphere [Lui, *et al.*, 1978]. For static CS/NS, the Cluster crossing would generally spend about 1-3 hours, as shown in Fig. 2 and 3. However, during CS/NS flapping, Cluster may enter and exit CS repeatedly for a number of times, and the crossing intervals can be only about 10 minutes during which the structure of CS would not have much large change. This would facilitate investigating the features of the CS/NS, such as the spatial size of NS. Fig. 6 shows a series of CS flapping events occurring on August 5, 2001.

During one complete NS crossing of Cluster, generally there is one minimum of R_c, i.e., $R_{c\min}$, which appears at the center of the NS. $R_{c\min}$ is an important parameter that may roughly reflect the characteristic thickness of NS. In the NS flapping event during UT (hrs) 17:30-18:00 on August 5, 2001, the minimum curvature radius is $R_{c\min} \approx 0.19 R_E$ at UT (hrs) 17.74, indicating the NS is very thin during this crossing event. Based on calculating $R_{c\min}$, we may find the variation properties of the NS size during CS flapping.

3.3 Variation of NS Size during Substorms

The thickness of NS is dependent of substorm phases. This investigation has found that generally NS is thin during the growth phase and expansion phase, and grows thick during the recovery phase. During August 5, 2001, the CS is strongly flapping and Cluster have crossed the NS several times as illustrated in Fig. 6, while a substorm with a maximum AE index of $1500\,nT$ is developing. Fig. 6 has shown the calculated curvature and normal of osculating planes of the MFLs of the CS in this period. Table 1 shows the variation of the NS characteristic thickness during a series of Cluster crossing events on August 5, 2001. On the onset during UT (hrs) 13.9-

14.0 and the expansion phase during UT (hrs) 14.0-14.5, $R_{c\min}$ of the NS is less than $0.6R_E$. On the recovery phase of the substorm during UT (hrs) 14.5-15, $R_{c\min}$ of the NS is larger than $0.8R_E$. However, the NS becomes thin again at the end of the recovery phase during UT (hrs) 15.4-15.5. During the last three crossings at UT (hrs) 16.6, 17.2 and 17.7-17.8, the NS is very thin, while the IMF is about southward as observed by ACE satellite and the magnetosphere is under the growth phase of the next substorm. ISEE 1/2 and other missions have also observed similar phenomena of substorm evolution processes [Sergeev *et al.* ,1990; Sergeev *et al.* ,1993; Sanny *et al.* ,1994; Zhou *et al.*,1997(a, b)].

Table 1. The calculated minimum curvature radius of the NS during a series of Cluster crossing on Aguest 5, 2001. The crossing types and corresponding substorm phases have also been shown.

Entry Time (UT, hrs)	Crossing Type	$R_{c\min}$ (Re)	Substorm Phase
13.9-14.0	N-N	0.5	Onset
14.15	N-N	0.1	Expansion
14.3-14.5	N-N	0.6	Expansion
14.7	N-N	1.1	Recovery
14.8	N-N	0.8	Recovery
15.4-15.5	N-S	0.2	End of Recovery
16.6	S-S	<0.1	Growth Phase
17.2	S-N	0.15	Growth Phase
17.7-17.8	N-S	0.19	Growth Phase

3.4 Dependence of NS Characteristic Thickness on the Local Time

The observation of Cluster shows that the half width of NS is dependent of the local time or the GSM Y dimension. During the Cluster crossing from LT04 (GSM $Y \approx -16R_E$) on July 1 to LT22 on Oct 30(GSM $Y \approx 16R_E$) (see the demonstration of the apogee of Cluster in Fig. 7), both the thin and thick NS can be observed. It is noted that in the dawn side and dusk side far away from the mid night, there still are thin NS. For example, during the Cluster crossing on July 27, NS $R_{c\min}$ can be as small as 0.2Re, while during the Cluster crossing on Oct 20, NS $R_{c\min}$ can be as small as 0.1Re. Nevertheless, there are more thin NS crossings during the midnight local time than that at the down or dusk sides. The thin NS with $R_{c\min} \le 1600km \approx 0.25R_E$ have appeared during the following dates of 2001:

July 1, 3, 5, 10, 20, 24 and 29;
August 3, 5, 7, 10, 12, 15, 17, 19, 22, 24, 27, 29 and 31;
September 5, 10, 12, 14, 15, 17, 19, 22, 24, and 26;
October 1, 4, 6, 8, 11, 13, 15 and 20.

This Cluster observation result is in agreement with the common understanding regarding the

CS thickness dependence on the local time [Bame, *et al.*, 1967; Meng and Mihalov, 1972; Crooker, 1977; Birn, *et al.*, 1977].

3.5 Quantitatively Determining the NS thickness

Commonly, the neutral sheet (NS) is regarded as a thin layer embedded in the centric region of current sheet (CS), and the magnetic field strength is rather small.

In this research, NS is considered to be the region with $-B_n \le B_x \le B_n$, or $|B_x| \le B_n$. B_x, B_y and B_z are the x, y and z components of the magnetic field in the NS BL coordinates, respectively, while B_n is the normal component.

Generally, B_x is approximately linear to z. Let $B_x = \pm B_n$ at $z = \pm h$, then

$$B_x = B_n \frac{z}{h}, \qquad (4)$$

here h is the characteristic half thickness of NS. The properties of the curvature and curvature radius of the MFLs in the NS have been shown in the appendix B. Generally the half thickness h is less than the minimum curvature radius $R_{c\min}$ when there is a dawn-dusk component B_y. If there is no B_y component, then the half thickness h is just equal to the minimum curvature radius $R_{c\min}$.

We may investigate the Cluster crossing event during UT (hrs) 17.7-17.8 on August 5, 2001 shown in Fig. 6. The magnetic field has been transformed into the NS boundary coordinates, where the y component $B_y \approx 2nT$, the normal component $B_n = |B_z| \approx 2.5nT$, as illustrated in Fig. 8. (It is noted that the normal of the NS, as defined as the gradient of the magnetic pressure, points northward (southward) of the NS at the northern (southern) part of the NS except near the center of the NS. At the center of the NS, the gradient of the magnetic pressure points earthwards.) Thus the constant $b = B_y / B_n \approx 0.8$. The minimum curvature radius is $R_{c\min} \approx 0.19R_E$ as shown in Fig. 7. Therefore, the half thickness of the NS can be obtained as $h = (1+b^2)^{-1} R_{c\min}$ $\approx (1+0.8^2)^{-1} \cdot 0.19R_E \approx 0.12R_E$.

We may further calculate the flapping velocity of the NS relative to Cluster. (The velocity of the Cluster satellites near the apogee as Cluster crossing the tail NS is about $1.1km/s$ and may be omitted compared with the flapping velocity of NS.) As illustrated in Fig. 7, when $t_1 = 17.74hr$, $R_c = R_{c\min} \approx 0.19R_E$; when $t_1 = 17.718hr$ or $t_2 = 17.763hr$, $R_c \approx \sqrt{8}R_{c\min}$. Therefore, based on the conclusion in Appendix D we may know that, during the interval

$\Delta t = t_2 - t_1 = 0.045hr$, the NS have been moving over a distance $\Delta z = 2h \approx 0.24R_E$ along the NS normal direction. Then the flapping velocity of the NS along the normal direction relative to the Cluster satellites should be

$$V_F = \frac{2h}{\Delta t}$$
$$\approx 9km/s , \qquad (5)$$

Therefore, for some cases, the half thickness of the NS can be quantitatively determined based on Cluster 4 point magnetic measurements, and the flapping speed along the NS normal may further be obtained. Still, it should be mentioned that in many situations the NS actually is under wave disturbance and has no regular 1-d structure but 3-d complicated geometric configuration; then the thickness of the NS could be characterized by the measured minimum curvature radius of MFLs, just as applied in Section 3.3 and 3.4.

4. Summaries and Discussions

In this research, we have investigated the geometrical structure of the magnetic field in the tail current sheet / neutral sheet based on the Cluster 4-point magnetic measurements. An approach has been developed to deduce the curvature and normal of the osculating plane of the MFLs in magnetotail. Further more, we have applied the magnetic geometrical analyses for exploring the variations of the NS thickness with the substorm phases, the dependence of the NS thickness on the local time and quantitatively determining the half thickness and the flapping speed of the NS with a one dimensional structure.

We have obtained the geometrical features of the MFLs in the CS /NS as summarized as bellows.

(1) Within the NS, the curvature of the MFLs points towards the earth, or the MFLs curve towards the earth; the osculating plane of the MFLs has a normal pointing duskside. The curvature radius of the NS MFLs is generally less than $2R_E$.

(2) In the northern (southern) lobes and the northern (southern) CS outside of the NS, the curvature of the MFLs points northwards (southwards), or outward of the CS; the osculating plane of the MFLs points dawnside. The curvature radius of the NS MFLs is about 5~10 R_E .

(3) Between the NS and the northern (southern) CS, there are two transition layers, where the curvature radius of the MFLs is much large and the MFLs are almost straight.

The observational properties of the MFLs in the NS / CS / Lobe have been illustrated in Fig. 4 and the deduced geometrical configuration demonstrated in Fig. 5.

We have investigated in details the CS / NS flapping phenomena during July 1 through October 31. For convinience we have used the minimum of the curvature radius, $R_{c\min}$, as the characteristic half thickness of the NS.

It is found that characteristic thickness of the NS is varying with substorm phases. The NS is

very thin during the growth phase and expansion phase with $R_{c\min}$ less than about $0.5R_E$. The NS grows thick during the recovery phase with $R_{c\min}$ larger than about $0.8R_E$. Approaching the end of the recovery phase, the NS again becomes thin.

The analysis results have shown that the thickness of the NS is dependent of the local time. It has been revealed that thin NS with $R_{c\min}$ <$0.25R_E$ may possibly appear at all the local time between LT 20hr and 4 hr. Nevertheless, near to the midnight local time there are more thin NS crossings than that at the dawnside and duskside. So that thin NS tends to occur near to midnight.

Finally, we have made explorations on quantitatively determining the half thickness of the NS when the NS approximately has a one-dimensional configuration. The NS BL coordinates have been applied, which are determined based on the Cluster FGM three-dimensional measurements. Deduction in Appendix B shows that the half thickness of the NS is dependent of the minimum curvature radius and the ratio between the y and z components of the magnetic field in NS BL coordinates. For the NS flapping event during UT (hrs) 17.7-17.8 on August 5, 2001, the calculated half thickness is about $0.12R_E$. Further more, the flapping velocity of the NS could be obtained. For the above flapping event, the NS flapping speed along the NS normal is about $9km/s$.

In conclusion, the geometry analyses based on the Cluster 4-point magnetic measurements makes it reality to vision the static and dynamic three dimensional magnetic structure of geomagnetosphere.

Appendix A: Some Basic Concepts of Differential Geometry

Generally the local geometrical structure of one curve may be specified by its curvature vector and osculating plane. The curvature and curvature radius of one curve reflect the intensity and spatial size of the curving. Locally, one curve lies in its osculating plane and can be fitted by a circle arc with a radius same as its curvature radius.

The curvature of one magnetic field line can be defined as

$$\vec{\rho}_c = \frac{\partial \vec{b}}{\partial s} = (\vec{b} \cdot \nabla)\vec{b}, \qquad \text{(A1)}$$

where s is the arc length of the magnetic field line; $\vec{b}$ is the unit vector of magnetic field $\vec{B}$, $\vec{b} = \vec{B}/B$. The curvature vector $\vec{\rho}_c$ points inward of the magnetic field line as defined here. Making differential of the condition $\vec{b} \cdot \vec{b} = 1$ yields

$$\vec{b} \cdot \frac{\partial \vec{b}}{\partial s} = \vec{b} \cdot \vec{\rho}_c = 0, \qquad \text{(A2)}$$

Thus the curvature vector $\vec{\rho}_c$ is perpendicular to the unit tangential vector $\vec{b}$.

The curvature radius is the reciprocal value of the curvature ρ_c, i. e.,

$$R_c = \frac{1}{\rho_c}, \qquad \text{(A3)}$$

Another important quantity for determining the geometrical structure of MFLs is the normal of the osculating plane, denoted as $\vec{N}$ here. The osculating plane of one curve at one point is the plane the curve locally lies in. The osculating plane of one magnetic field line can be determined by its unit tangential vector $\vec{b}$ and curvature vector $\vec{\rho}_c$. The normal of the osculating plane $\vec{N}$ is defined as

$$\vec{N} = \frac{\vec{b} \times \vec{\rho}_c}{\left|\vec{b} \times \vec{\rho}_c\right|}. \qquad \text{(A4)}$$

The curvature $\vec{\rho}_c$ and the normal of the osculating plane $\vec{N}$ may determine the local geometrical structure of the MFLs.

Appendix B: Deducing the Formula of the Magnetic Field and its Gradient at the Mesocentre of Cluster Tetrahedron

Here the least square minimum method [Harvey, 1998] is used here.

At one specific time, the 4 satellites of Cluster at 4 different positions $\vec{r}_\alpha$ (α =1,2,3,4) can obtain the measurement of the magnetic field $\vec{B}_\alpha$ (α =1,2,3,4). The mesocentre of the four identical satellites is the center of mass of them, which is at the position

$$\vec{r}_c = \frac{1}{4}\sum_{\alpha=1}^{4} \vec{r}_\alpha , \qquad \text{(B1)}$$

Here we use the barycentre coordinates for convenience, i.e.,

$$\vec{r}_c = 0 , \qquad \text{(B2)}$$

Magnetic field $\vec{B}_\alpha$ at one satellite may be expanded as

$$B_{\alpha i} = B_{ci} + (\partial_j B_i)_c (r_{\alpha j} - r_{cj}) + \mathrm{O}(r_{\alpha j} - r_{cj})^2$$

$$= B_{ci} + G_{ij} r_{\alpha j} + \mathrm{O}(r_{\alpha j})^2 , \qquad \text{(B3)}$$

Where $G_{ij} = (\partial_j B_i)_c$ is the gradient of the magnetic field at the mesocentre of the Cluster tetrahedron.

We have constructed a new action for deducing the formula of the magnetic field $\vec{B}_c$ and its gradient $(\nabla \vec{B})_c$ at the mesocentre, which has the following form

$$S = \frac{1}{4}\sum_{\alpha=1}^{4} [(B_{\alpha i} - B_{ci}) - G_{ij} r_{\alpha j}]^2 + 2\lambda G_{ll} , \quad \text{(B4)}$$

The second term at the right hand side of the above equation is to ensure the free convergence of magnetic field [Harvey, 1998].

To minimize S, we need $\delta S = 0$, which yields

$\delta S / \delta B_{ci} = 0$, $\delta S / \delta G_{ij} = 0$, $\delta S / \delta \lambda = 0$.

Subsequently, we may get

$$B_{ci} = \frac{1}{4}\sum_{\alpha=1}^{4} B_{\alpha i} \ , \qquad \text{(B5)}$$

$$G_{ij} = G^{0}{}_{ij} + \lambda R^{-1}{}_{ij} \ , \qquad \text{(B6)}$$

where,

$$G^{0}{}_{ij} = \frac{1}{4}\sum_{\alpha=1}^{4} B_{\alpha i} r_{\alpha k} R^{-1}{}_{kj} \ , \qquad \text{(B7)}$$

The volume tensor $R_{kj} = \frac{1}{4}\sum_{\alpha=1}^{4} r_{\alpha k} r_{\alpha j}$, and the Lagrangian multiplier λ is

$$\lambda = -G^{0}{}_{ii} / R^{-1}{}_{ii} \ . \qquad \text{(B8)}$$

The formula (B7) of the gradient is simpler than that of Harvey (1998). It can be verified that the expression for the gradient of magnetic field obtained here is equivalent to that of Harvey (1998). The special advantage of the formula (B7) is that the errors caused by magnetic and position measurements could be reduced.

Appendix C: An Estimate of the Truncation Error of the Curvature

In order to obtain an estimate of the truncation error of the curvature, we may just simplify the Cluster tetrahedron by two satellites that are at the same magnetic field line and have a distance between them of the characteristic size L, as illustrated in Fig. 9. It is assumed that the magnetic field line within the spatial scale of L has a constant curvature radius R_c.

According to the definition, the formula of the curvature is

$$\rho_c = \frac{\left|\vec{b}_2 - \vec{b}_1\right|}{\Delta s} \ , \qquad \text{(C1)}$$

where Δs is the arc length of the magnetic field line between the two satellites, $\vec{b}_2$ and $\vec{b}_1$ are the unit vectors of the magnetic field at the two satellites, as illustrated in Fig. 9. If θ is the angle between $\vec{b}_2$ and $\vec{b}_1$, then $\Delta s = R_c \cdot 2\theta$.

In calculating the curvature based on multiple satellites, the arc length Δs in (C1) has been replaced by the line segment between the satellites. Thus the calculated curvature may be approximated by

$$\tilde{\rho}_c = \frac{\left|\vec{b}_2 - \vec{b}_1\right|}{L} \ , \qquad \text{(C2)}$$

Then

$$\frac{\tilde{\rho}_c}{\rho_c} = \frac{\Delta s}{L} = \frac{R_c \cdot 2\theta}{L}$$

$$= \frac{2R_c \cdot \sin^{-1}(L/2R_c)}{L}, \qquad \text{(C3)}$$

The error of the curvature is

$$\frac{\Delta\rho_c}{\rho_c} = \frac{\tilde{\rho}_c - \rho_c}{\rho_c} = \frac{\sin^{-1}(L/2R_c) - L/2R_c}{L/2R_c}$$

$$\approx \frac{1}{6}(L/2R_c)^2. \qquad \text{(C4)}$$

Due to $R_c = 1/\rho_c$, the error of the curvature radius is

$$\frac{\Delta R_c}{R_c} = \frac{\Delta\rho_c}{\rho_c} \approx \frac{1}{6}(L/2R_c)^2. \qquad \text{(C5)}$$

Therefore the error of the curvature as well as that of the curvature radius is at the two order of $L/2R_c$.

Appendix D: Curvature of Magnetic Field in CS/NS

The components of magnetic field $\vec{B} = B_x\vec{e}_x + B_y\vec{e}_y + B_z\vec{e}_z$ of CS/NS in NS boundary coordinates may be expressed as

$$B_x = \eta(z)B_n, \qquad \text{(D1)}$$

$$B_y = B_{y0} = Const., \qquad \text{(D2)}$$

$$B_z = B_n = Const.. \qquad \text{(D3)}$$

Within the NS, $\eta(z)$ is about linear to z, i.e.,

$$\eta(z) = \frac{z}{h}, \qquad \text{(D4)}$$

where h is defined as the half thickness of NS.

We may denote $b = B_{y0}/B_n$. The total magnetic field is

$$B = (B_x{}^2 + B_y{}^2 + B_z{}^2)^{1/2} = B_n(\eta^2 + b^2 + 1)^{1/2},$$

(D5)

The curvature of $\vec{B}$ is

$$\rho_{cj} = (\delta_{jx}B^{-2}B_z - B^{-4}B_jB_xB_z)\partial_z B_x, \qquad \text{(D6)}$$

The 3 components of curvature are

$$\rho_{cx} = (1+b^2)(1+\eta^2+b^2)^{-2}\eta', \qquad \text{(D7)}$$

$$\rho_{cy} = -b\eta(1+\eta^2+b^2)^{-2}\eta', \qquad \text{(D8)}$$

$$\rho_{cz} = -\eta(1+\eta^2+b^2)^{-2}\eta', \qquad \text{(D9)}$$

where η' is the derivative of $\eta(z)$ on z.

The value of the curvature is

$$\rho_c = ({\rho_x}^2 + {\rho_y}^2 + {\rho_z}^2)^{1/2}$$

$$= (1+b^2)^{1/2}(1+\eta^2+b^2)^{-3/2}\eta'. \qquad \text{(D10)}$$

The curvature radius is

$$R_c = (1+b^2)^{-1/2}(1+\eta^2+b^2)^{3/2}/\eta'.$$

(D11)

For the NS, the value of the curvature and the curvature radius are

$$\rho_c = (1+b^2)^{1/2}(1+\eta^2+b^2)^{-3/2}h^{-1},$$

(D10')

$$R_c = (1+b^2)^{-1/2}(1+\eta^2+b^2)^{3/2}h.$$

(D11')

At the center of the NS, $B_x = \eta(z)B_n = 0$, both of B and R_c have the minimum values as

$$B_{\min} = B_n(1+b^2)^{1/2}, \qquad \text{(D12)}$$

$$R_{c\min} = (1+b^2)h. \qquad \text{(D13)}$$

Generally, within the CS

$$R_c / R_{c\min} = (1+b^2)^{-3/2}(1+\eta^2+b^2)^{3/2}(\eta' h)^{-1},$$

(D14)

and within the NS

$$R_c / R_{c\min} = (1+b^2)^{-3/2}(1+\eta^2+b^2)^{3/2}.$$

(D14')

Therefore we can get

$$|z| = R_{c\min}[(R_c / R_{c\min})^{2/3} - 1]^{1/2}(1+b^2)^{-1/2},$$

for the NS. (D15)

Formula (D15) can be used for determine the Cluster crossing distance along the normal of

NS (or the z direction) as NS is rapidly flapping. E. g., when $R_c / R_{c\min} = 2\sqrt{2}$, $|z| = R_{c\min}(1+b^2)^{-1/2}$.

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Captions

Fig.1 Illustration on the relationship between the unit tangential vector $\vec{b}$, curvature vector $\vec{\rho}_c$ and the normal $\vec{N}$ of the osculating plane of one magnetic field line in GSM coordinates. The direction of the curvature $\vec{\rho}_c$ is determined by the polar and azimuthal angles (θ_c, φ_c), while the direction of normal $\vec{N}$ determined by (θ_N, φ_N). The polar angles θ_c and θ_N are from the axis Z, and the azimuthal angles φ_c and φ_N are from the axis X.

Fig. 2 The CS crossing event during September 17, 2001. The first panel shows the direction of the curvature of the MFLs; the second panel shows the curvature radius; the third panel shows the direction of the normal of the osculating plane of the MFLs; the last panel shows the three components and strength of the magnetic field. The GSM coordinates are used in this figure.

Fig. 3 The CS crossing event during September 29, 2001. The form of the figures and the instruction are the same as Fig. 2.

Fig. 4 Schematic illustration of the variation of the geometrical structure of the MFLs in CS when Cluster are crossing PS from the north lobe to south lobe. It is assumed there is no strong flapping of PS.

Fig. 5 Schematic illustration of the magnetic geometrical structure of CS deduced from Cluster 4-point measurements.

Fig. 6 A series of CS flapping events occurring on August 5, 2001. The form of the figures and the instruction are the same as Fig. 2.

Fig. 7 Schematic illustration of the position of the apogee of Cluster during July-Oct, 2001. The apogee of Cluster is moving from dawnside to duskside smoothly, about 2 hours of local time per month. The Cluster apogee is about the position of one NS/CS crossing event observed at the corresponding time.

Fig. 8 The y and z components of magnetic field in NS BL coordinates in the NS flapping event during UT (hrs) 17:70-17:78 on August 5, 2001.

Fig. 9 Demonstration on the cause of the truncation error of the curvature of one magnetic field line.