Hale COLLAGE: Solar Observations ........................... Problem Set 1 (Due Feb. 2, 2016)

Please try to be neat when writing up answers. In cases where analytic calculations are called for, please show all intermediate steps, including any approximations you choose to make and any sketches you may need to illustrate what’s what. In cases where you do numerical computations or plots, there’s usually no need to submit your full source code. However, if you’re having trouble getting an answer that looks realistic, feel free to submit the code so we can debug.

In several of the problems below, you will need to estimate the visible-light brightness of the off-limb corona. Feel free to use the following approximate fit to the mean observed radial dependence:

\[
\frac{I_{\text{cor}}}{I_\odot} = \frac{1.82 \times 10^{-6}}{(r/R_\odot)^8} + \frac{2.65 \times 10^{-8}}{(r/R_\odot)^{2.22}}
\]

where \(I_{\text{cor}}\) is the specific intensity at \(\lambda \approx 500\) nm, \(I_\odot\) is the disk-center photospheric intensity, and \(r\) is the heliocentric radial distance of the observed “impact parameter” (i.e., distance measured in the plane of the sky).

1. **Are solar physicists “photon starved?”** It is sometimes difficult to grasp that high-resolution solar observations can face some of the same problems seen in night-time astronomy.

In all parts below, assume you’re using a telescope with a 1 m diameter circular mirror and a 1% efficiency for the overall system (i.e., only 1% of the photons entering the telescope make it to the detector), and that you are observing a 1 \(\mu\)m wide piece of the spectrum (spectral bandwidth) centered at 5000 \(\mu\)m. Also, feel free to assume that all stars are blackbodies.

(a) From our vantage point on Earth, how much brighter is the Sun than the star Sirius?

(b) Compute the number of photons collected by our assumed telescope for each of the following cases:

- One 0.1'' \(\times\) 0.1'' pixel of the solar photosphere, integrated over 0.1 second of observing time.
- One 1' \(\times\) 1' pixel of the solar corona, observed at \(r = 1.75 R_\odot\), integrated over 10 seconds of observing time.
- The entire star Sirius, integrated over 4 seconds of observing time.

(c) For the photospheric case above, 0.1 seconds may not be short enough. The Earth’s turbulent atmosphere fluctuates with frequencies around 100 Hz. Thus, ultra-quick exposures (\(\Delta t \approx 5\) ms) are often needed to avoid smearing due to seeing effects. Also, if you’re doing high-resolution spectropolarimetry, you may need to gather light in narrower wavelength bands (\(\Delta \lambda \approx 0.1\) \(\mu\)m), and the polarized signal may be roughly \(10^{-3}\) times the overall unpolarized intensity.

Using the same 0.1'' \(\times\) 0.1'' pixels, compute the number of polarized photons collected in each exposure. If the photons obey Poisson statistics, what is the relative noise level (i.e., standard deviation divided by mean signal)? If you were an observer, would you be satisfied with it?

You’ll have to look up a few quantities; please list your sources and assumptions.
2. Building an unpolarized beam. In the lectures, it was mentioned that the sum of \( N \gg 1 \) plane waves gives an “unpolarized” beam. In this problem you will demonstrate that this is the case for linear polarization.

(a) Construct a collection of \( N \) randomly oriented plane waves by successively “adding on” each wave to a running total for the Stokes parameters. Assume each wave component is in phase with all others (ensuring linear polarization), and that they all have equal \( E \)-field magnitudes. Use whatever random number generation mechanism you like for the transverse orientation angle \( \psi \) of each wave. As each wave is added on (and \( N \) is incremented by 1), compute the Stokes parameters \( I, Q, \) and \( U \) for this value of \( N \). Plot the fraction of linear polarization

\[
P = \frac{\sqrt{Q^2 + U^2}}{I}
\]

as a function of \( N \). Keep going until you get to \( N = 1000 \), and use a log-log scale.

(b) Repeat the above for at least 100 independent random trials, and compute the mean value of \( P \) for each value of \( N \). Plot that quantity versus \( N \) and discuss what you think it means, from a statistics standpoint.

3. Straight-edge solar occultation. In the lectures, we will learn that a linear occulting surface does a better job at reducing the diffracted light behind its “shadow” when the distance \( R \) (between occulter and observer) is larger. We would like to figure out the critical value of \( R \) needed to observe the white-light corona.

(a) For observers in the shadow of a straight-edge occultor (i.e., \( v_1 > 0 \) in the notation given in lectures), implement the full expression for the ratio of diffracted intensity \( I_{\text{diff}} \) to unobstructed intensity \( I_\odot \) in a computer code, using numerical library functions for the Fresnel integrals. Plot it as a function of \( v_1 \) alongside the following approximate solution,

\[
\frac{I_{\text{diff}}}{I_\odot} \approx \frac{1}{C + 2\pi^2 v_1^2}
\]

where you have to choose a value of \( C \) that does a good job. (In lectures, we assumed \( C = 0 \) for \( v_1 \gg 1 \).) Discuss how well your approximation does at matching the exact solution.

(b) Determine the range of \( v_1 \) values appropriate for observing the solar corona from its limb (i.e., \( r = R_\odot \) in the plane of the sky) out to a heliocentric distance of \( r = 20 R_\odot \). For this part, assume visible light (\( \lambda = 500 \text{ nm} \)) and an occulter-to-observer distance of \( R = 2 \text{ m} \).

(c) Given the observational fit for \( I_{\text{cor}}(r) \), solve for the critical occulter distance \( R_c \) at which \( I_{\text{diff}} = I_{\text{cor}} \). This quantity varies as a function of the on-sky observing distance \( r \), which you should vary between 1 and 20 \( R_\odot \) as in part (b).

(d) Redo the above calculation by solving for a critical occulter distance \( R'_c \) at which \( I_{\text{diff}} = 0.1 I_{\text{cor}} \). Why might this be a more realistic quantity? Plot both \( R_c \) and \( R'_c \) as a function of observing distance \( r \).

(e) Verify that it is realistic for us to see the corona during total solar eclipses. Also: If there were no other ways to reduce diffraction, could one observe the corona with an occultor attached to a telescope?