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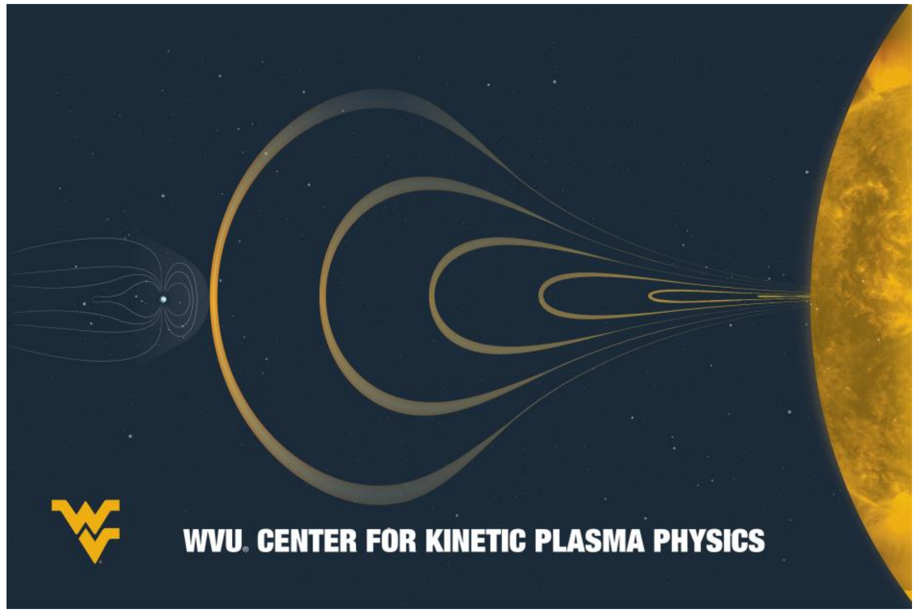
3D Anti-Parallel Reconnection Spreading in Non-Uniform Current Sheets and Application to the Near-Earth Magnetotail



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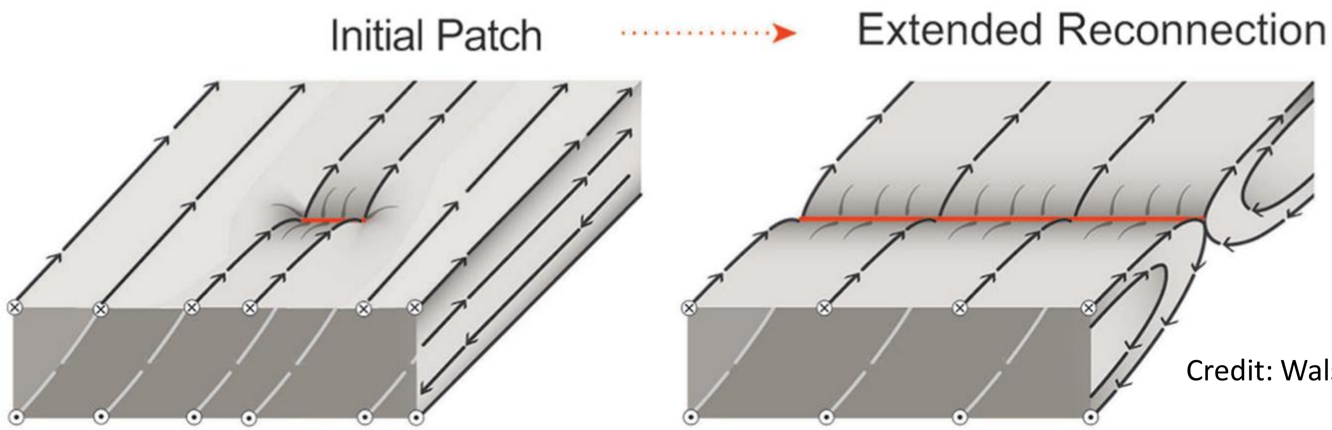
What Is Reconnection Spreading?

- Reconnection can begin as multiple x-lines with a fixed length reminiscent of BBF (Shay et al., 2003), or as a single x-line that spreads in the out-of-plane direction over time
- Shown below: anti-parallel reconnection initiates in a spatially localized region then spreads into an extended x-line, shown in red



Credit: Walsh et al. (2018)

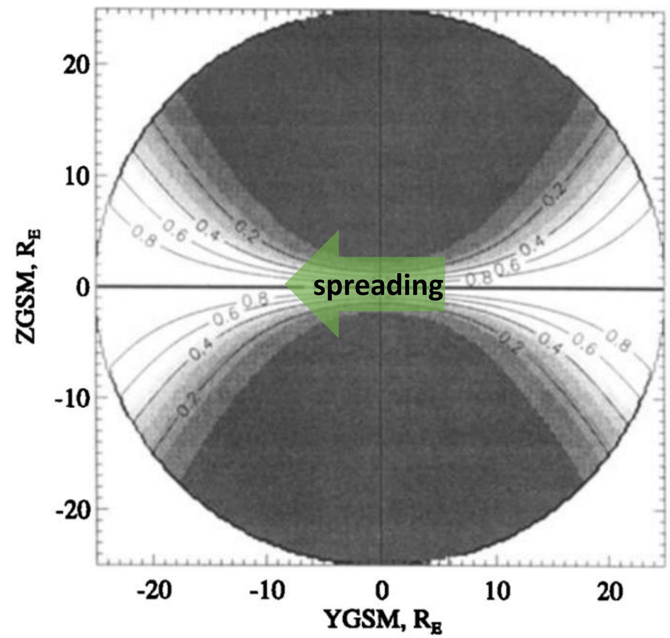
- X-line spreading occurs in: dayside magnetopause [Zou et al., 2017], magnetotail [Nagai et al., 2013], solar wind [Phan et al., 2006], as well as in the lab (VTF, MRX)
- Anti-parallel collisionless reconnection is understood to spread by way of current carriers due to the Hall effect (Arencibia et al., 2021 and references therein)
- Past studies have addressed spreading in uniform, ion scale current sheets

How does reconnection spread along a current sheet that varies in thickness along the propagation direction?

Spreading in Non-Uniform Current Sheets

- Current sheets in nature are not expected to have a uniform thickness as a function of the out of plane coordinate

- *In situ* observations of near-Earth cross-tail current sheet show the thickness can vary continuously from $< 3\ \mathrm{R_E}$ at midnight (in MLT) to $8\ \mathrm{R_E}$ at the magnetopause in a quiet magnetosphere - thinning down to $0.1\ \mathrm{R_E}$ at midnight to $1\ \mathrm{R_E}$ at the magnetopause at the end of a substorm growth phase
  - Shown: model of near-Earth magnetotail plasma sheet with a thickness that varies along the dawn-dusk direction ($\mathrm{Y_{GSM}}$) (Tsyganenko, 1998)

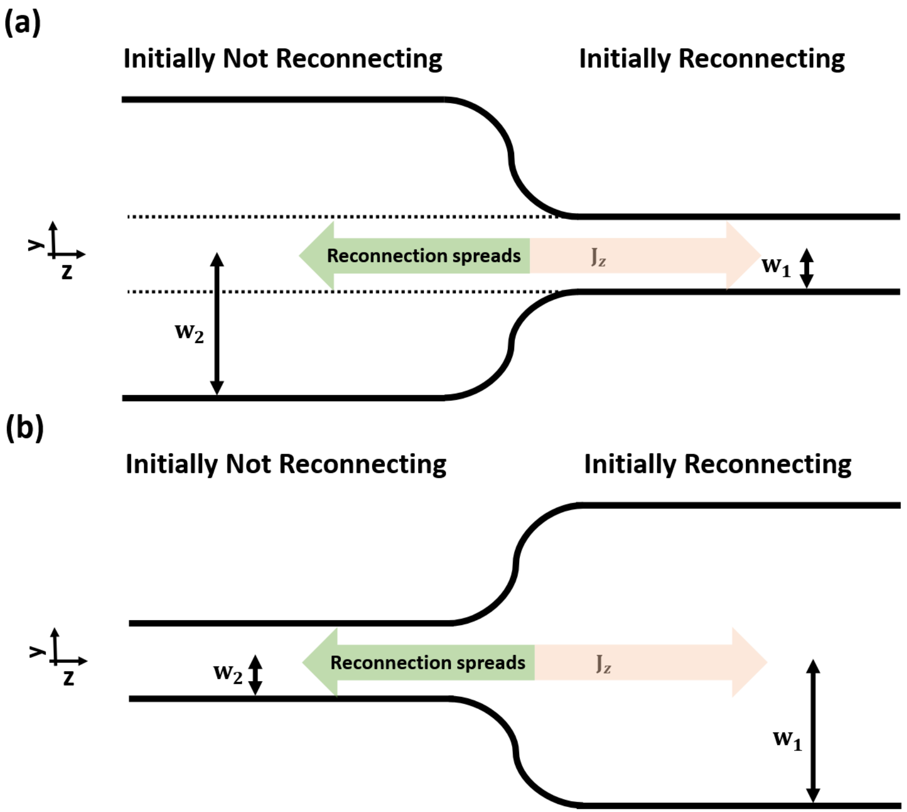


- In numerical simulations of reconnection spreading in non-uniform current sheets, we find a surprising result – the x-line spreading speed is slower than expected from existing knowledge (slower than Alfven and local current carrier speeds)

We determine a scaling for the anti-parallel reconnection spreading speed in current sheets of non-uniform thickness from first principles and confirm the result with two-fluid simulations (Arencibia et al., submitted, JGR, 2022)

How Spreading Occurs in Current Sheets of Non-Uniform Thickness

- Reconnection can **(a)** start in a thinner region sheet with half-thickness w_1 and spread to a thicker region with half-thickness w_2 or **(b)** vice versa
- **(a)** If anti-parallel reconnection spreads into a thicker current sheet , the incoming reconnected (perturbing) magnetic field in the thicker part remains collimated at the smaller thickness (dotted lines)
    - This reduces the “upstream” magnetic field participating in reconnection, called “embedding” (Shay et al., 2004)
- **(b)** This effect does not occur in thicker-to-thinner spreading; the whole width w_2 is perturbed and the full upstream field strength participates from the onset



Arencibia et al., submitted 2022

Scaling of Spreading Speed (Thin-to-Thick)

- Assuming the upstream field varies linearly in y (inflow direction) within the wider part of the sheet [Shay et al., 2004], we find $\mathrm{B_{up}} \sim \mathrm{B_0 w_1/w_2}$, where $\mathrm{B_0}$ is the upstream asymptotic field in both thin and thick regions

- The spreading speed in the wider region then becomes

$$\mathrm{v_{s2}} \sim \mathrm{c_A} \frac{\mathrm{w_1 d_i}}{\mathrm{w_2^2}} = \left(\frac{w_1}{w_2}\right) \mathrm{v_e},$$

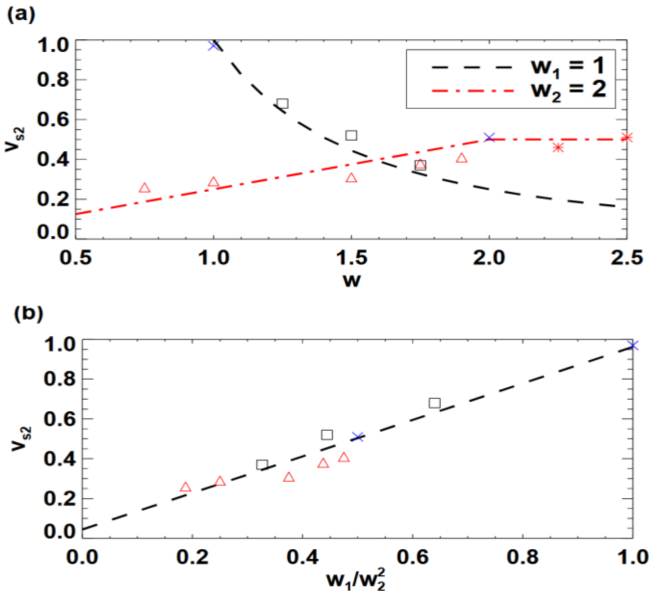
  thus slower than the prediction for a sheet of uniform half-thickness $\mathrm{w_2}$ (the local current carrier speed in the equilibrium sheet, $\mathrm{v_e} = \mathrm{c_A d_i / w_2}$) by a factor of $\mathrm{w_1/w_2}$, a key prediction of this theory and a departure from previous knowledge

- This implies the spreading speed in the wider region fundamentally depends not just on the local current sheet half-thickness $\mathrm{w_2}$, but there is also a “memory” effect of the current sheet half-thickness in the thinner region

- For reconnection spreading from a thicker into a thinner sheet, the incoming reconnected magnetic field perturbs the entire width of the thinner region, which then participates in reconnection from the beginning with the full asymptotic field strength $\mathrm{B_0}$
  - In this scenario, reconnection spreads at the local current carrier speed

Validation: Scaling of Spreading Speeds in Non-Uniform Sheets

- We confirm the scaling with 3D two-fluid simulations with Hall physics and electron inertia using a non-uniform Harris-type sheet with variable thickness profile
- Shown: (a) local spreading speed $\mathbf{v_{s2}}$ in the outer part of the current sheet (w_2) as a function of the current sheet half-thickness w, which represents either independent variables w_1 or w_2 and (b) $\mathbf{v_{s2}}$ vs. $\mathbf{w_1/w_2^2}$, the predicted dependence from the theory for thin-to-thick spreading
  - **Black squares** are for simulations with $w_1 = 1$ and variable w_2, **red triangles** are for simulations with $w_2 = 2$ and variable w_1, and **blue crosses** are for simulations with uniform current sheet thickness
  - Dashed line in (b) is least squares fit of these points with a functional form $v_{s2} = (0.919 \pm 0.082) w_1/w_2^2 + (0.044 \pm 0.041)$ showing excellent agreement with the theory



	(w_1, w_2)	Predicted v_{s2}	v_{s2}	Error
×	(1.0,1.0)	1.00	0.97	-3.1%
×	(2.0,2.0)	0.50	0.51	2.0%
△	(1.9,2.0)	0.48	0.41	-15.9%
△	(1.75,2.0)	0.44	0.38	-15.1%
△	(1.5,2.0)	0.38	0.31	-21.0%
△	(1.0,2.0)	0.25	0.29	13.8%
△	(0.75,2.0)	0.19	0.26	-27.9%
□	(1.0,1.25)	0.64	0.68	5.9%
□	(1.0,1.5)	0.44	0.52	14.5%
□	(1.0,1.75)	0.33	0.37	11.7%

Table 1. Results for 3D Hall MHD simulations in this study, labeled as ordered pairs (w_1, w_2) for simulations with current sheets that vary in half-thickness along the out-of-plane direction from a value of w_1 to w_2 in units of d_{i0}. v_{s2} is the reconnection spreading speed magnitude calculated in the region with half-thickness w_2

Gradually Varying Current Sheets

- The theory for non-uniform sheets can be generalized to study systems where the current sheet thickness $\mathrm{w(z)}$ varies gradually rather than abruptly
- We expect the instantaneous spreading speed simply depends on the local current sheet thickness $\mathrm{w(z)}$ but otherwise is as the abrupt spreading theory predicts:

$$\mathrm{v_s(z) \sim c_A \frac{w_1 d_i}{w^2(z)}} \quad \text{Thin-to-thick current sheet}$$

$$\mathrm{v_s(z) \sim c_A \frac{d_i}{w(z)}} \quad \text{Thick-to-thin current sheet}$$

  - For thinner-to-thicker spreading, it leads to embedded reconnection as in the abrupt case
- Since the spreading speed is continuously varying, it is more observationally relevant to predict the time scale τ of spreading across a current sheet of arbitrary current sheet thickness profile $\mathrm{w(z)}$:

$$\tau = \int d\tau = \int_{z_1}^{z_2} \frac{\mathrm{dz}}{\mathrm{v_s(z)}}$$

  - This integral can be carried out for thin-to-thick or thick-to-thin spreading for a specified $\mathrm{w(z)}$

Application: Near-Earth Magnetotail X-Line Spreading

- For this purpose, we assume a profile of the form: $\mathrm{w(z) = w_1 + (w_2 - w_1)\left(\frac{z}{\Delta z}\right)^{\alpha}}$ for a current sheet that increases monotonically in thickness from $\mathrm{w_1}$ to $\mathrm{w_2}$ over a scale $\Delta\mathrm{z}$, where α is a dimensionless parameter that can be chosen for a particular model current sheet
- In the direction of increasing thickness (i.e., midnight to dawn), the spreading timescale is

$$\tau = \frac{w_1 \Delta z}{c_A d_i}\left[1 + 2\frac{w_2/w_1 - 1}{\alpha + 1} + \frac{(w_2/w_1 - 1)^2}{2\alpha + 1}\right]$$

while in the direction of decreasing thickness (i.e., dusk to midnight) it is

$$\tau = \frac{w_1 \Delta z}{c_A d_i}\left(1 + \frac{w_2/w_1 - 1}{\alpha + 1}\right).$$

- Sample calculation for the pre-substorm near-Earth magnetotail: using $\mathrm{w_1 = 0.4R_E, w_2 = 1R_E, \Delta z = 15R_E, n{\sim}0.1cm^{-3}, c_A{\sim}1400km/s}$ and $\alpha = 2$ for a parabolic profile, assuming reconnection spreads from midnight to dawn, this gives a timescale $\tau{\sim}20$ minutes - it would be interesting to compare this prediction with observations for future work

Conclusions

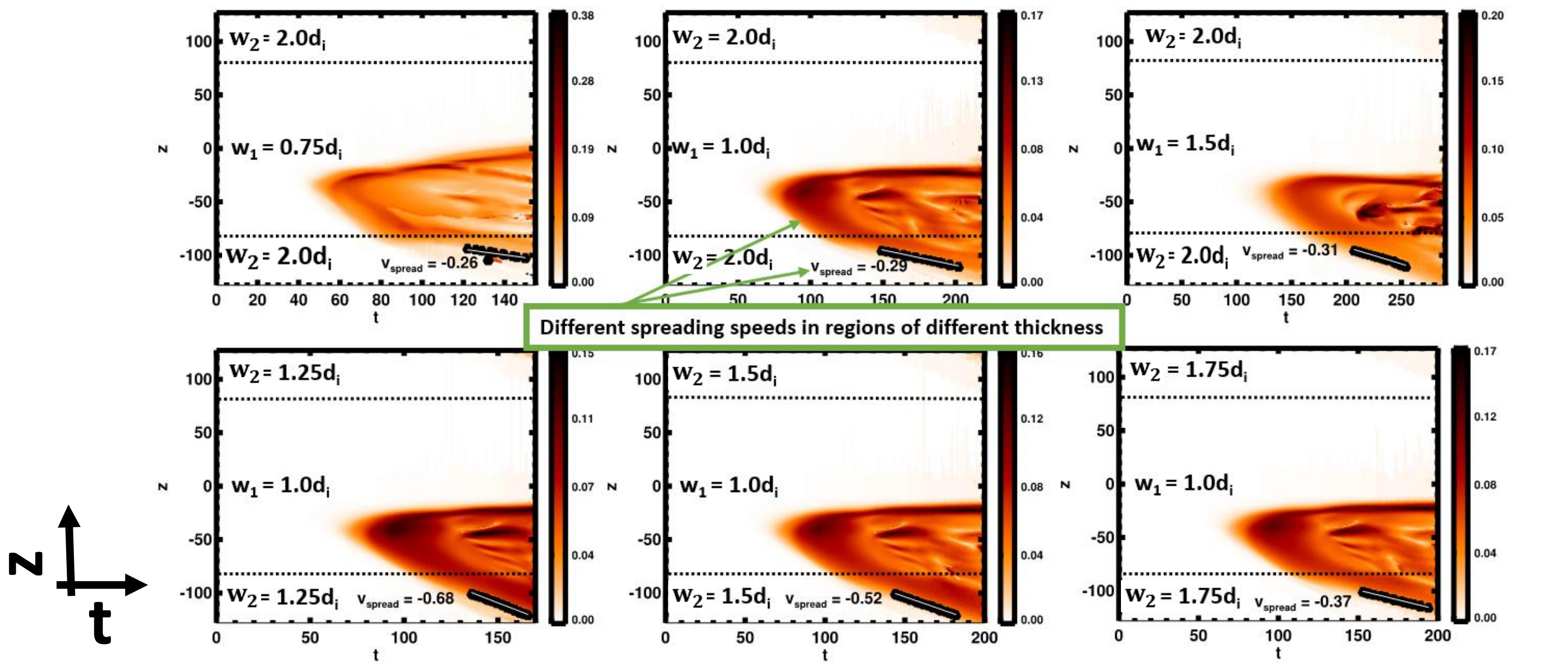
- We develop a first-principles scaling for 3D anti-parallel reconnection spreading in current sheets with varying thickness in the out of plane direction and confirm the theory with 3D two-fluid simulations **[Arencibia et al., submitted, JGR, 2022]**
- Spreading of anti-parallel reconnection from a thinner to a wider part of a current sheet proceeds at sub-Alfvenic and sub-current carrier speeds in the wider part of sheet, a key prediction of the theory and a departure from previous knowledge
- Spreading from a wider to a thinner part of the current sheet occurs at the local current carrier velocity, as expected in a sheet with uniform thickness
- Our results may be important for applications in the solar wind, solar flares, dayside magnetopause, magnetotail, and laboratory experiments for which current sheets are expected to be non-uniform in the out-of-plane direction

3D Simulations of Spreading from Thin-to-Thick Current Sheets

- Shown: time-distance stack plots of average reconnected field given by

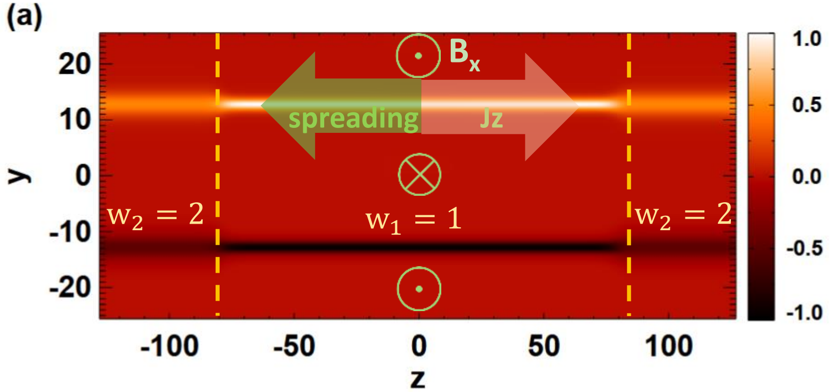
$$\tilde{B}_y(z,t) = \frac{|B_y(\tilde{x}+L, y_{cs}, z, t)| + |B_y(\tilde{x}-L, y_{cs}, z, t)|}{2},$$

  where $\tilde{x}$ is the location of the x-line in the plane specified by z at time t, and $\mathrm{L} \approx 2\mathrm{d_i}$ is the approximate half-length of the electron diffusion region (Arencibia et al., 2021), for six 3D anti-parallel two-fluid simulations with non-uniform thicknesses in which reconnection is initiated in a portion of the smaller half-thickness $\mathrm{w_1}$ and spreads into the region of larger half-thickness $\mathrm{w_2}$
- Spreading speeds are the slope of points at the times in which $\widetilde{\mathrm{B_y}} > 0.04$ (fast reconnection ≈ 0.1)



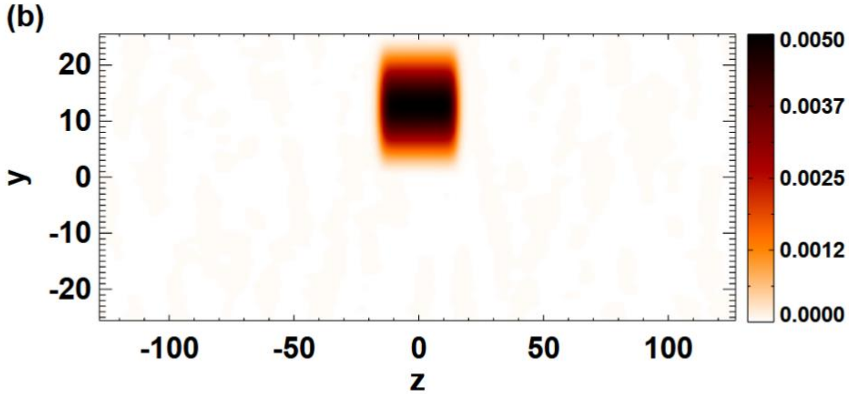
Numerical Validation

- We confirm the theoretical model for reconnection spreading with 3D two-fluid numerical simulations (F3D, Shay et al., 2004) with electron inertia and Harris-type current sheet equilibrium



- x = outflow, y = inflow, z = out-of-plane with system size $L_x \times L_y \times L_z = 102.4 \times 51.2 \times 256\ d_i$ and triply periodic boundary conditions
- Grid size 0.05 d_i (for xy plane) and 1 d_i (z direction)
- Electron mass = ion mass / 25, upstream β = 2
- Double Harris current sheet equilibrium with field:

$$B_{0x} = \tanh[(y + L_y/4)/w_0] - \tanh[(y - L_y/4)/w_0)] - 1$$



- Non-uniform current sheet thickness profile:

$$w_0(z) = \frac{w_1 + w_2}{2} + \frac{w_1 - w_2}{2}\left[\tanh\left(\frac{z + L_0}{w_z}\right) - \tanh\left(\frac{z - L_0}{w_z}\right) - 1\right]$$

with w_1 fixed and variable w_2 and vice versa

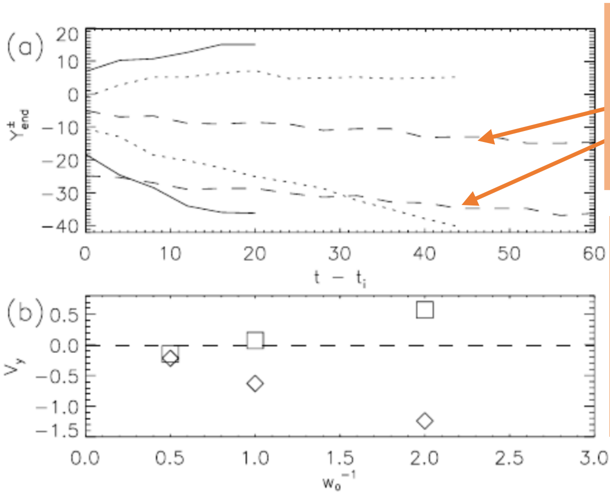
- The plots show:

(a) initial out-of-plane current density $J_z(y,z,t=0)$ for a sheet with $w_1 = 1, w_2 = 2$ (separated by dashed lines) and $L_0 = 80, w_z = 4\ d_i$

(b) The initial coherent perturbation in $B_y(x=0,y,z,t=0)$ localized to $|z|<15\ d_i$ in the upper current sheet that has 0.5% the amplitude of the background equilibrium field B_0

X-line Drifting vs. Spreading

- For current sheets with $w_0 \geq 2d_i$, we find whether the reconnection region merely drifts or broadens depends on reconnecting plane aspect ratio
- Shown: x-line advection in F3D simulation with $w_0 = 2d_i$ using square aspect ratio (Shay et al., 2003) vs x-line spreading with 2:1 ratio (Arencibia et al., 2021)



X-line advects at a fixed length for square aspect ratio

Figure 3. Expansion and propagation of finite length x-lines. In (a) the y positions of the ends of the x-line for differing w_0.: (solid) $w_0 = 0.5$, $t_i = 8.0$; (dotted) $w_0 = 1.0$, $t_i = 16.0$; and (dashed) $w_0 = 2.0$, $t_i = 96.0$, where t_i is the time required for the system to establish a strong x-line with $\mathbf{J}_y > 2.5$. In (b) the propagation velocities along y of the edges of the x-line for different w_0^{-1}: (squares) $dy^{+}_{\rm end}/dt$ and (diamonds) $dy^{-}_{\rm end}/dt$.

X-line grows in length in rectangular aspect ratio

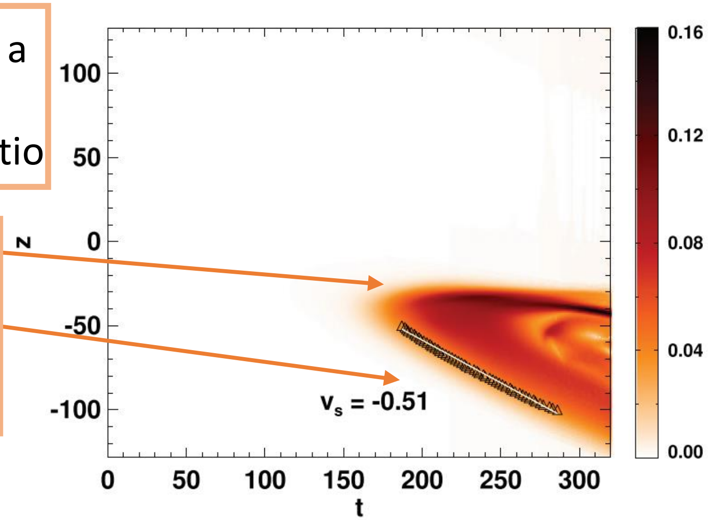


FIG. 5. Average reconnected magnetic field $\tilde{B}_y(z,t)$, defined in Eq. (16), as a function of out of plane position z and time t, for the anti-parallel reconnection simulation with $w_0 = 2$. The triangles mark where $\tilde{B}_y$ crosses over 0.04, and the white line gives the best fit of these points, giving the spreading speed v_s. This plot shows reconnection spreads, in contrast to the results in a square computational domain in which the x-line convects without spreading [Fig. 3(a) of Shay *et al.*[13]].