New Theoretical Developments on Energy Conversion via the Pressure-Strain Interaction

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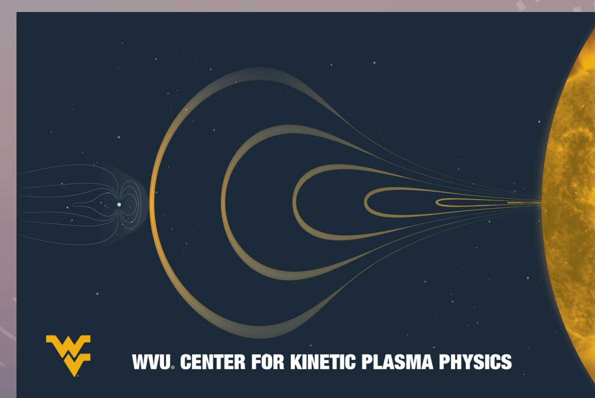

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Quantifying Energy Conversion

- **The bulk flow energy $\mathcal{E}_k = (1/2)mnu^2$ evolves as:**

$$\frac{\partial \mathcal{E}_k}{\partial t} + \boldsymbol{\nabla} \cdot (\mathbf{u}\mathcal{E}_k + \mathbf{u} \cdot \mathbf{P}) = (\mathbf{P} \cdot \boldsymbol{\nabla}) \cdot \mathbf{u} + n\mathbf{u} \cdot \mathbf{F} + \mathbf{R}_{\text{coll}}$$

- **The thermal energy $\mathcal{E}_{th} = 3\mathcal{P}/2 = \int (\frac{1}{2}mv'^2) f d^3v$, where $\mathcal{P} = (1/3)\text{tr}(\mathbf{P})$, evolves as:**

$$\frac{\partial \mathcal{E}_{th}}{\partial t} + \boldsymbol{\nabla} \cdot (\mathcal{E}_{th}\mathbf{u}) = -(\mathbf{P} \cdot \boldsymbol{\nabla}) \cdot \mathbf{u} - \boldsymbol{\nabla} \cdot \mathbf{q} + \dot{Q}_{\text{visc,coll}}$$

- $-(\mathbf{P} \cdot \boldsymbol{\nabla}) \cdot \mathbf{u}$ **is called <u>pressure-strain interaction</u> and it describes conversion of $\mathcal{E}_{th} \Longleftrightarrow \mathcal{E}_k$**
- **Pressure-strain interaction can be decomposed (Yang et al., 2017; 2017) as**

$$-(\mathbf{P} \cdot \boldsymbol{\nabla}) \cdot \mathbf{u} = -\mathcal{P}(\boldsymbol{\nabla} \cdot \mathbf{u}) - \Pi_{jk}\mathcal{D}_{jk}$$

"Pressure dilatation": $-\mathcal{P}(\boldsymbol{\nabla} \cdot \mathbf{u})$

"Pi-D", aka "collisionless viscosity": $-\Pi_{jk}\mathcal{D}_{jk}$

$P_{jk} - \mathcal{P}\delta_{jk}$ = deviatoric pressure tensor

$$\frac{1}{2}\left(\frac{\partial u_j}{\partial r_k} + \frac{\partial u_k}{\partial r_j}\right) - \frac{1}{3}\delta_{jk}(\nabla \cdot \mathbf{u})$$

= traceless strain rate tensor

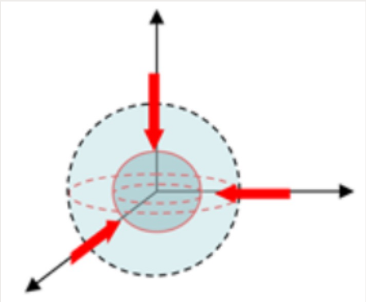
- **A puzzle – Pi-D is collisionless viscous heating (Yang et al., 2017), but it can be negative! How?!? What does it mean?!?**

Physics of Pressure Dilatation and Pi-D

- **The physical picture of the pressure-strain interaction was described in the fluid description by Del Sarto et al. (2018)**

Pressure dilatation $-\mathcal{P}(\boldsymbol{\nabla} \cdot \mathbf{u})$

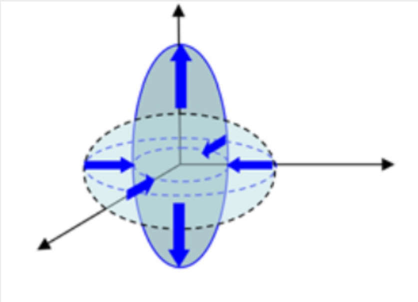
- Denotes **compressive heating or expansive cooling** of a plasma of pressure $\mathcal{P} = (1/3)\mathrm{tr}(\mathbf{P})$
- The volume of a fluid element changes



Del Sarto et al. (2018)

Pi-D $-\Pi_{jk}\mathcal{D}_{jk}$

- Denotes **incompressible heating/cooling**
- Purely a deformation
- The volume of a fluid element does not change

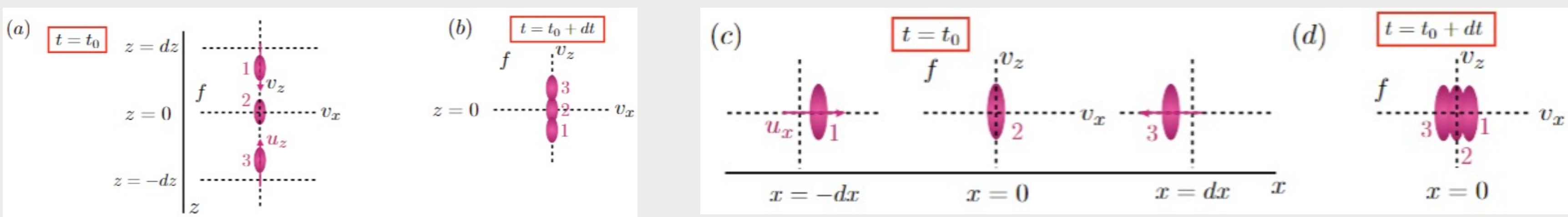


Del Sarto et al. (2018)

- **Note, the physical picture has more to it; will be discussed (Cassak and Barbhuiya, PoP, submitted)**

A Thought Experiment

- To understand the sign and interpretation of Pi-D, we need a *kinetic* description (*Cassak & Barbhuiya, PoP, submitted*)
- Consider a distribution function that is everywhere bi-Maxwellian with $P_{||} > P_{\perp}$
  - Suppose there is 1D converging flow in the parallel direction – we know this must give compressible heating! It does [$-(\mathbf{P} \cdot \boldsymbol{\nabla}) \cdot \mathbf{u}$ > 0] , and a simple calculation shows Pi-D > 0
  - Suppose there is converging flow in the perpendicular direction – we know this must give compressible heating too! It does [$-(\mathbf{P} \cdot \boldsymbol{\nabla}) \cdot \mathbf{u}$ > 0], but a simple calculation shows Pi-D < 0!?!
- Thus, Pi-D < 0 does not imply cooling! One can't tell if there is heating from Pi-D alone!
- The sketches below describe the physics in the kinetic description



Cassak and Barbhuiya (PoP, submitted)

- Note, Pi-D contains contributions from converging/diverging flow, like $-\mathcal{P}(\boldsymbol{\nabla} \cdot \mathbf{u})$ does

Alternative Decomposition

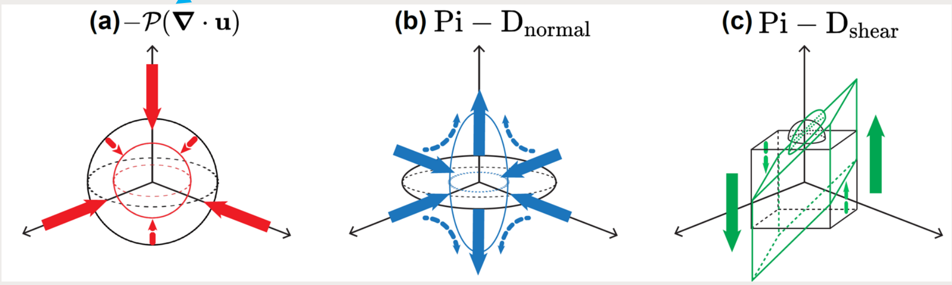
- **Since both pressure dilatation and part of Pi-D contain information about converging/diverging flows, we propose an alternative decomposition of the pressure-strain interaction (Cassak and Barbhuiya, PoP, submitted)**
  - **Break Pi-D into its <u>normal</u> and <u>shear</u> parts**
- **PDU is the expected generalization of compression/expansion:**

$$\mathrm{PDU} = -\mathcal{P}(\nabla \cdot \mathbf{u}) + \mathrm{Pi} - \mathrm{D}_{\mathrm{normal}} = -\left(P_{xx}\frac{\partial u_x}{\partial x} + P_{yy}\frac{\partial u_y}{\partial y} + P_{zz}\frac{\partial u_z}{\partial z}\right)$$

- **For the bi-Maxwellian example, PDU is positive for both parallel and perpendicular compression, agreeing with intuition**
  - **Pi-D_{normal} is the correction to compression/expansion by thinking of a distribution as being Maxwellianized**

PDU, pure converging/ diverging flow — **Pi-D_{shear}, pure flow shear**

$$-(\mathbf{P}\cdot\nabla)\cdot\mathbf{u} = \underbrace{-\mathcal{P}(\nabla\cdot\mathbf{u}) - \mathbf{\Pi}:\boldsymbol{\mathcal{D}}_{\mathrm{normal}}}_{\text{PDU}} \underbrace{- \mathbf{\Pi}:\boldsymbol{\mathcal{D}}_{\mathrm{shear}}}_{\text{Pi-D}_{\text{shear}}}$$



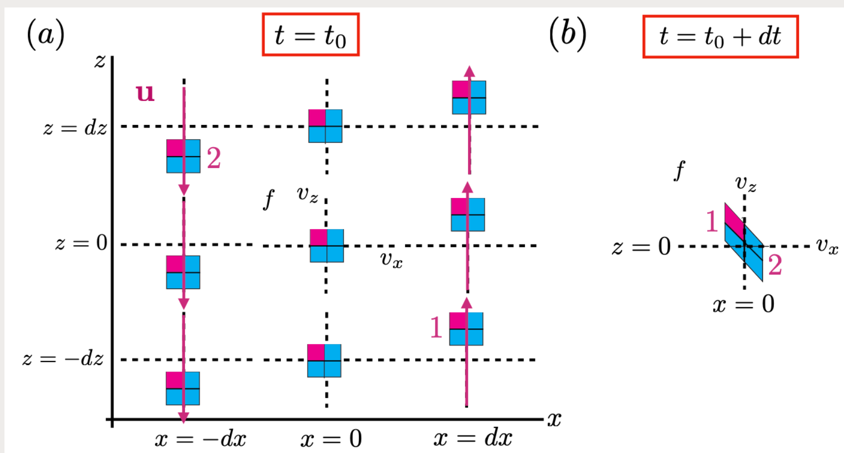
Cassak and Barbhuiya (submitted)

Kinetic Physics of Pi-D_{shear}

- **What is Pi-D_{shear} and what determines its sign?**

$$\begin{aligned} \mathrm{Pi} - \mathrm{D}_{\mathrm{shear}} &= -(2\Pi_{xy}\mathcal{D}_{xy} + 2\Pi_{xz}\mathcal{D}_{xz} + 2\Pi_{yz}\mathcal{D}_{yz}) \\ &= -\left[P_{xy}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) + P_{xz}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) + P_{yz}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)\right] \end{aligned}$$

  - **It's the kinetic analog of shear viscosity from fluid mechanics**
  - **Kinetically, there needs to be a velocity shear and an off-diagonal pressure tensor element**
- **In the example below, $P_{xz} < 0$, $dv_z/dx > 0$, and there is positive Pi-D_{shear}**



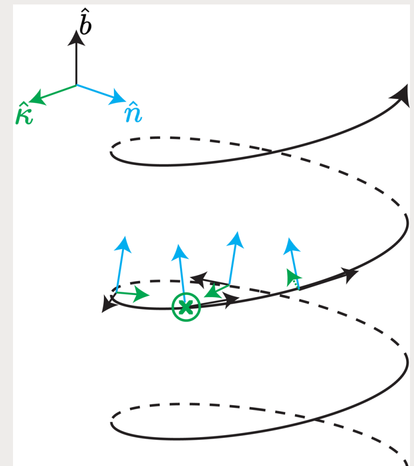
Cassak and Barbhuiya
(PoP, submitted)

Pressure-Strain Interaction in Field-Aligned Coordinates

- **For a magnetized plasma, the dynamics are not necessarily aligned with a Cartesian coordinate system**
- **It is useful to employ a coordinate system locally aligned with the field at each point in space (*Cassak, Barbhuiya and Weldon, PoP, submitted*):**
  - $\hat{\mathbf{b}}$ Magnetic field direction
  - $\hat{\boldsymbol{\kappa}}$ Curvature direction = $\dfrac{(\hat{\mathbf{b}}\cdot\boldsymbol{\nabla})\hat{\mathbf{b}}}{|(\hat{\mathbf{b}}\cdot\boldsymbol{\nabla})\hat{\mathbf{b}}|}$
  - $\hat{\mathbf{n}}$ Binormal direction = $\hat{\mathbf{b}}\times\hat{\boldsymbol{\kappa}}$
- **In field-aligned coordinates,** $\mathbf{u} = e_\alpha u_\alpha, \mathbf{P} = P_{\alpha\beta} e_\alpha e_\beta, \boldsymbol{\nabla} = \mathrm{e}_\alpha \nabla_\alpha$
- **The pressure-strain interaction then becomes**

$$-(\mathbf{P}\cdot\boldsymbol{\nabla})\cdot\mathbf{u} = \underbrace{-P_{\alpha\beta}(\nabla_\alpha u_\beta)}_{\text{Flow gradients}} \underbrace{- P_{\alpha\beta} u_\delta[\mathbf{e}_\beta\cdot(\nabla_\alpha \mathbf{e}_\delta)]}_{\text{Geometrical terms}}$$

Flow gradients **Geometrical terms**



Greek letters go from $\hat{\mathbf{b}}, \hat{\boldsymbol{\kappa}}, \hat{\mathbf{n}}$

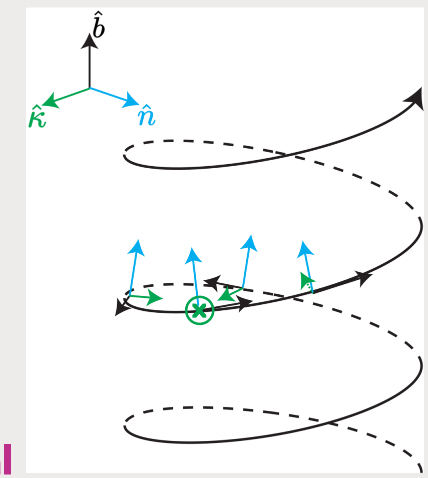
Pressure-Strain Interaction in Field-Aligned Coordinates

- **Can solve for pressure-strain analytically in field aligned coordinates (*Cassak, Barbhuiya and Weldon, PoP, submitted*):**

$$
\begin{aligned}
-PS_1 &= -P_{\|}(\nabla_{\|} u_{\|}), \\
-PS_2 &= -P_{\kappa\kappa}(\nabla_{\kappa} u_{\kappa}) - P_{nn}(\nabla_n u_n), \\
-PS_3 &= -P_{\kappa b}(\nabla_{\kappa} u_{\|}) - P_{nb}(\nabla_n u_{\|}), \\
-PS_4 &= -P_{b\kappa}(\nabla_{\|} u_{\kappa}) - P_{bn}(\nabla_{\|} u_n) - P_{\kappa n}(\nabla_{\kappa} u_n) - P_{n\kappa}(\nabla_n u_{\kappa}), \\
-PS_5 &= u_{\kappa}\left(P_{\|} + P_{b\kappa} W_{\kappa} + P_{bn} W_n\right)\kappa = u_{\kappa} P_{b\alpha} W_{\alpha} \kappa \\
-PS_6 &= -u_{\kappa}\left(P_{bn} + P_{\kappa n} W_{\kappa} + P_{nn} W_n\right)\tau = -u_{\kappa} P_{n\alpha} W_{\alpha} \tau \\
-PS_7 &= -u_{\|}\left(P_{b\kappa} + P_{\kappa\kappa} W_{\kappa} + P_{\kappa n} W_n\right)\kappa = -u_{\|} P_{\kappa\alpha} W_{\alpha} \kappa \\
-PS_8 &= u_n\left(P_{b\kappa} + P_{\kappa\kappa} W_{\kappa} + P_{n\kappa} W_n\right)\tau = u_n P_{\kappa\alpha} W_{\alpha} \tau,
\end{aligned}
$$

Flow gradients ($-PS_1$ to $-PS_4$)

Geometrical terms ($-PS_5$ to $-PS_8$)



$$
\begin{aligned}
W_b &= 1, \\
W_{\kappa} &= (\hat{\boldsymbol{\kappa}} \cdot \nabla_{\kappa} \hat{\mathbf{b}})/\kappa, \\
W_n &= -(\hat{\boldsymbol{\kappa}} \cdot \nabla_n \hat{\mathbf{n}})/\tau
\end{aligned}
$$

- **The geometry of the magnetic field line is completely described by the curvature and the torsion; the torsion is a measure of how much the magnetic field is not confined to a plane**

Curvature $\kappa = |(\hat{\mathbf{b}} \cdot \nabla)\hat{\mathbf{b}}|$

Torsion $\tau = -\hat{\boldsymbol{\kappa}} \cdot \nabla_{\|} \hat{\mathbf{n}}$

Physical Interpretation

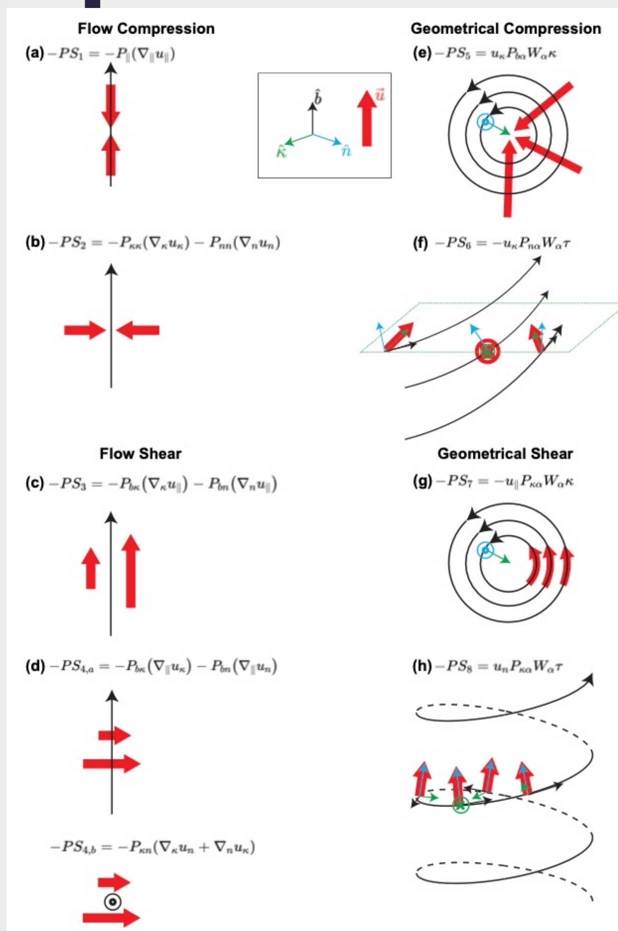
Parallel flow changing in parallel direction

Perpendicular flow changing in the same perpendicular direction

Parallel flow changing in the perpendicular directions

Perpendicular flows changing in the parallel direction

Perpendicular flow changing in the other perpendicular direction



Curvature flow compressing due to field line curvature (quantified by κ)

Curvature flow compressing due to field line non-planarity (quantified by τ)

Parallel flow shearing due to field line curvature (quantified by κ)

Normal flow shearing due to due to field line non-planarity (quantified by τ)

(Cassak, Barbhuiya and Weldon, PoP, submitted)

Conclusions

- **We present a kinetic interpretation of the pressure-strain interaction, and an alternative decomposition of it (*Cassak and Barbhuiya, PoP, submitted*)**

$-\mathcal{P}(\boldsymbol{\nabla}\cdot\mathbf{u})$ and $\mathrm{Pi}-\mathrm{D}$	PDU and $\mathrm{Pi}-\mathrm{D}_{\mathrm{shear}}$
describe **compressible** and **incompressible** physics	describes **bulk flow convergence** and **bulk flow shear** physics
essentially is **isotropic compression** and **deformation**	combines **isotropic compression** with **normal deformation**, separating out **shear deformation**
Pi-D can be **negative for converging flow** (associated with heating); a counterintuitive result	PDU is **positive for converging flow** (associated with heating); an intuitive result

- **We present a decomposition of the pressure-strain interaction in magnetic field-aligned coordinates (*Cassak, Barbhuiya and Weldon, PoP, submitted*)**
  - Decomposition gives flow compression and shear, geometrical compression and shear
- **We use these results to study energy conversion in a PIC simulation of reconnection (Barbhuiya and Cassak, PoP, submitted) – next presentation**