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THE ROYAL SOCIETY

MMS SWT

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# The Influence of Plasma Conditions on Electric Fields in Magnetosheath Turbulence

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# Turbulent Electric Fields

- Electric fields are required to describe how B evolves
- **Generalised Ohm's law** describes electric field in collisionless plasma

$$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \frac{1}{n_e e}\mathbf{j}\times\mathbf{B} - \frac{1}{n_e e}\nabla\cdot\mathbf{P}_e + \frac{m_e}{n_e e^2}\left[\nabla\cdot\left(\mathbf{uj} + \mathbf{ju} - \frac{\mathbf{jj}}{n_e e}\right) + \frac{\partial\mathbf{j}}{\partial t}\right]$$

$E_{MHD}$ controls **large** scales

$E_{Hall}$ controls **sub-ion** scales

$E_{Pressure}$ subdominant throughout

$E_{Inertia}$ controls **sub-electron** scales → typically low power

$$\mathbf{u} = \frac{m_i \mathrm{n_i} \mathbf{v}_i + m_e \mathrm{n_e} \mathbf{v_e}}{m_i \mathrm{n_i} + m_e \mathrm{n_e}} \qquad \mathbf{j} = e(\mathrm{n_i}\mathbf{v_i} - \mathrm{n_e}\mathbf{v_e})$$



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# Generalised Ohm's Law: Components

- Contributions to the electric field come from background, linear and nonlinear
- Interplay between different effects is complex

$$\mathbf{E} = -[\mathbf{u}_0 \times \mathbf{B}_0 + \underbrace{\delta\mathbf{u} \times \mathbf{B}_0 + \mathbf{u}_0 \times \delta\mathbf{b}}_{\text{Linear}} + \underbrace{\delta\mathbf{u} \times \delta\mathbf{b}}_{\text{Nonlinear}}]$$

$$+ \frac{1}{n_e e}[\mathbf{j}_0 \times \mathbf{B}_0 + \underbrace{\mathbf{j}_0 \times \delta\mathbf{b} + \delta\mathbf{j} \times \mathbf{B}_0}_{\text{Linear}} + \underbrace{\delta\mathbf{j} \times \delta\mathbf{b}}_{\text{Nonlinear}}]$$

$$- \frac{1}{n_e e}\nabla \cdot [\mathrm{n}_{\mathrm{e0}}\mathbf{T}_{\mathrm{e0}} \underbrace{+ \delta\mathrm{n}_{\mathrm{e}}\mathbf{T}_{\mathrm{e0}} + \mathrm{n}_{\mathrm{e0}}\delta\mathbf{T}_{\mathrm{e}}}_{\text{Linear}} + \underbrace{\delta\mathrm{n}_{\mathrm{e}}\delta\mathbf{T}_{\mathrm{e}}}_{\text{Nonlinear}}]$$

- Each of these components can be measured by MMS
- Understanding interplay of terms tells us about what is influencing dynamics


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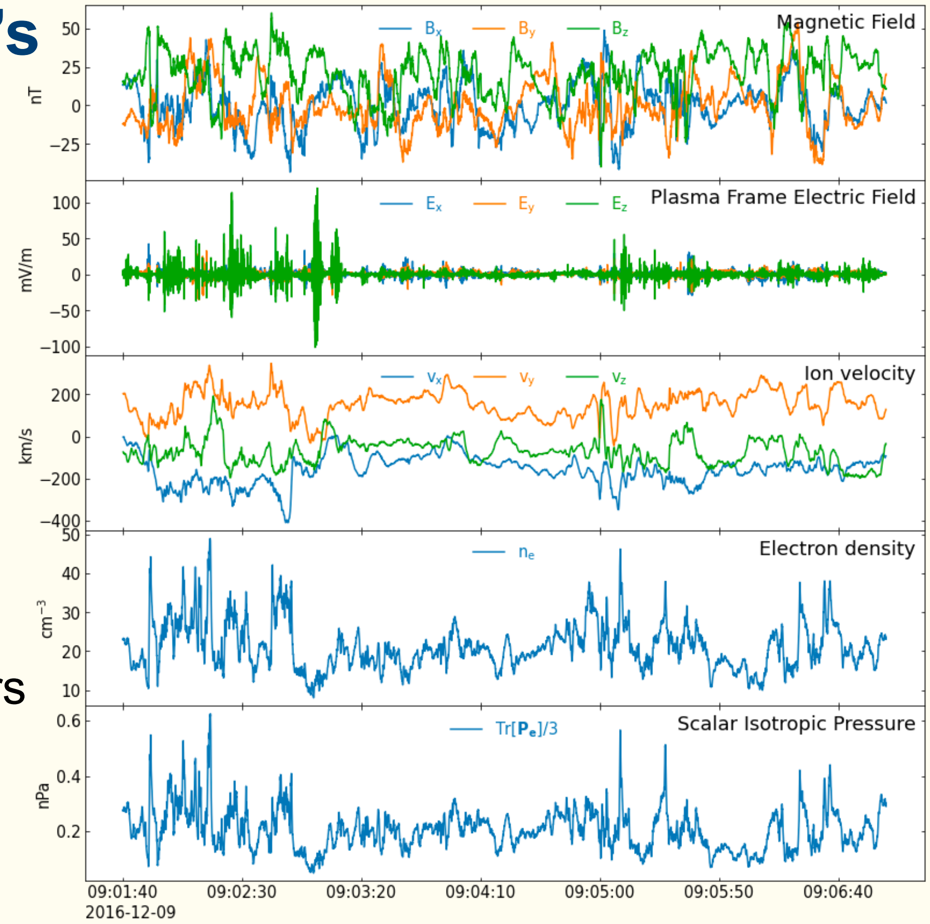
# Measuring Ohm's Law with MMS

Why MMS?

- High cadence plasma moments
- Multipoint measurements ($\nabla \cdot \mathbf{P}_e$)

58 Intervals of data used:

- From Stawarz+ (2022)*
- Situated in the Magnetosheath
- Strong fluctuations, steady parameters
- Taylor's hypothesis valid
- 3 – 43 minutes in length






*https://doi.org/10.1063/5.0071106

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# Spectra of Ohm's Law Terms

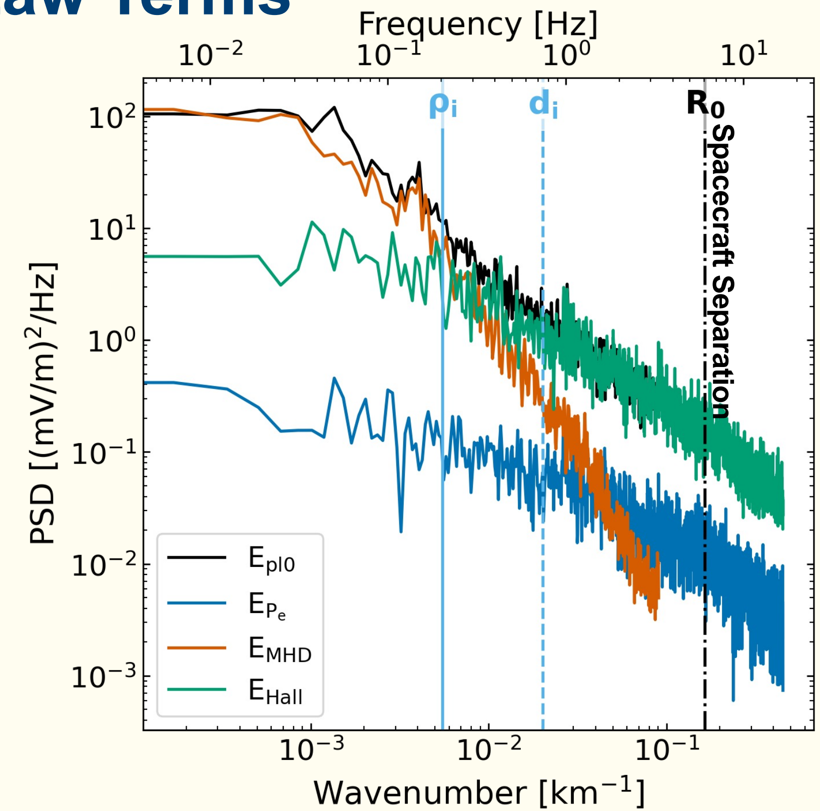
Scale dependence is noticeable:

- $E_{pl0}$ well described by dominant term
- $E_{MHD}$ dominates above ion scales
- $E_{Hall}$ dominates sub-ion scales
- $E_{Pe}$ tracks $E_{Hall}$; subdominant throughout

Two characteristics:

1. **Crossover** between $E_{MHD}$ and $E_{Hall}$
2. **Relative amplitude** of $E_{Hall}$ and $E_{Pe}$

Do these characteristics vary with plasma properties?



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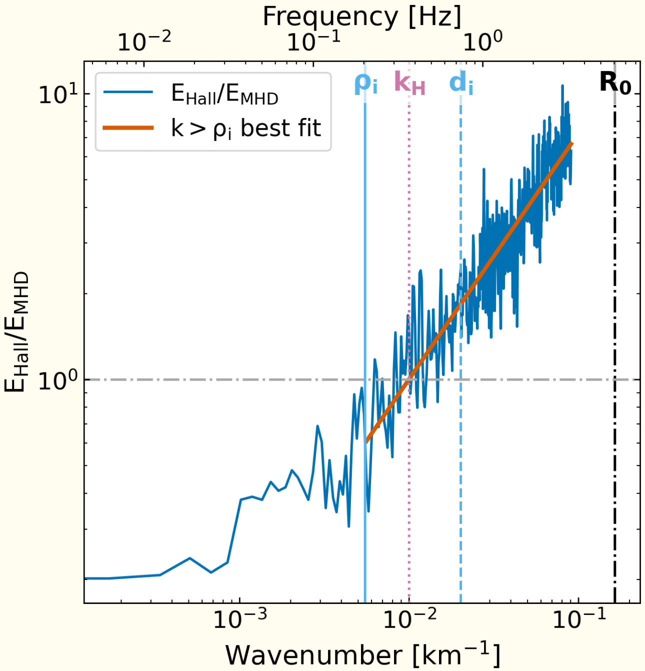
# Hall/MHD: The Hall scale

- Tells us where Hall term begins to control the dynamics




$$\frac{E_{Hall}}{E_{MHD}} \sim \frac{\left|\frac{1}{n_e e}\mathbf{j}\times\mathbf{B}\right|}{|-\delta\mathbf{u}\times\mathbf{B}|} \sim k d_i \frac{\delta b_A}{\delta u} \Rightarrow k_{Hall} \sim \frac{1}{d_i}\frac{\delta u}{\delta b_A}$$

$\frac{\delta u}{\delta b_A}$ itself is scale dependent…

What is the best way to evaluate $\frac{\delta u}{\delta b_A}$?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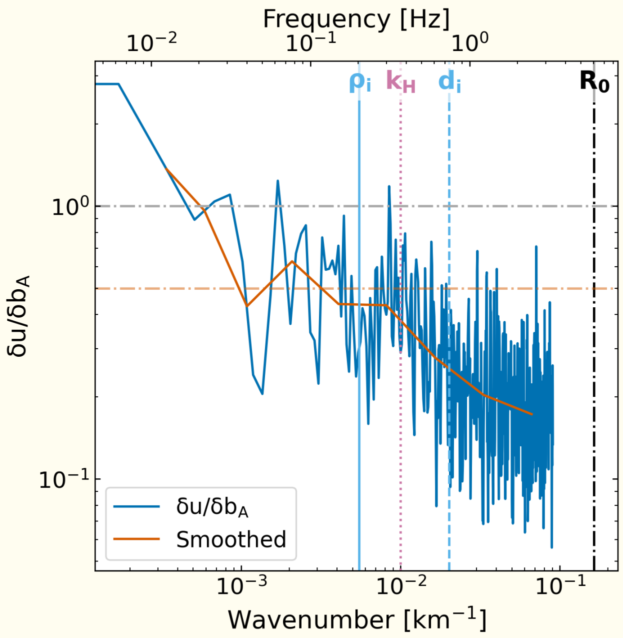
# Hall/MHD: Estimating the Hall scale




- Alfvénic: assume Alfvén waves; take $\frac{\delta u}{\delta b_A} = 1$
- RMS: Incorporates full range of $\frac{\delta u}{\delta b_A}$
- Ion scales: Contribution from around $k_{Hall}$
  - Typically $k_{Hall}$ ~ 0.5 $d_i$ in our intervals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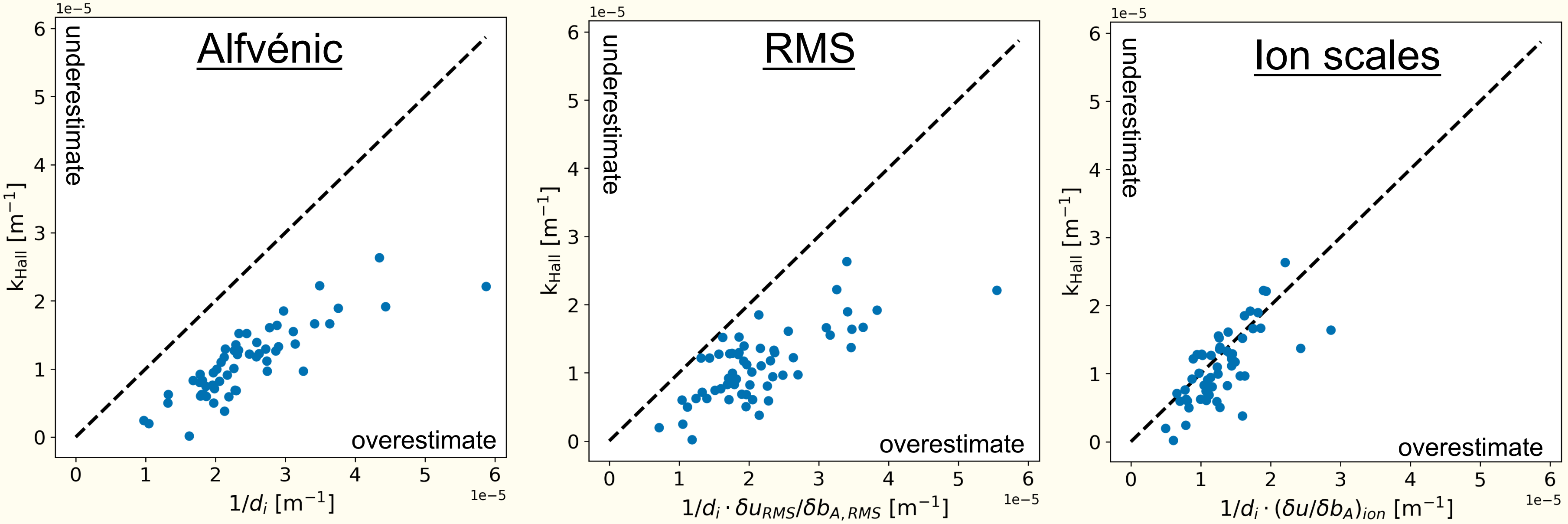
# Hall/MHD: Estimating the Hall scale

- Alfvénic and RMS fluctuations overestimate measured value
- Contribution to $\delta u/\delta b_A$ from ion scales gives best agreement



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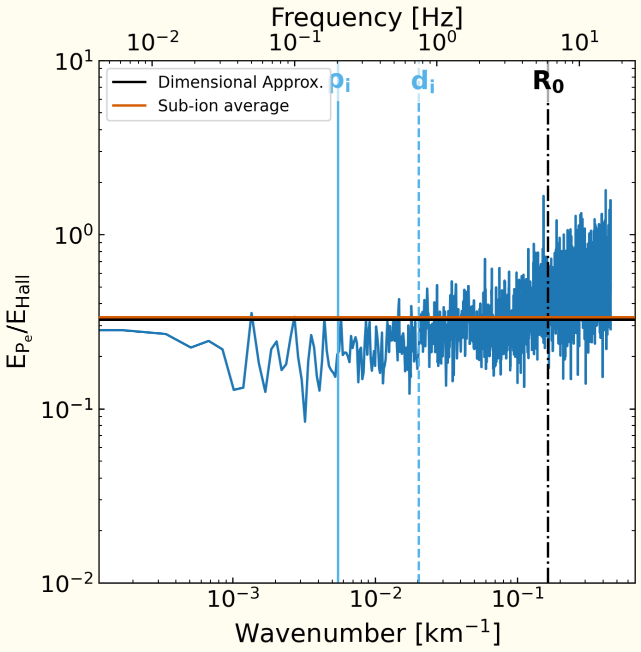
# Pressure/Hall Amplitude

- Tells us relative importance of Hall and Pressure terms




$$\frac{E_{Pe}}{E_{Hall}} \sim \frac{\left|\frac{1}{n_e e}\nabla\cdot\mathbf{P_e}\right|}{\left|\frac{1}{n_e e}\mathbf{j}\times\mathbf{B}\right|} \sim \frac{2\mu_0 P_{e0}}{B_0{}^2/2}\frac{\nabla\cdot\delta \mathrm{p_e}/P_{e0}}{\nabla\times\left({}^{\delta\mathbf{b}}/_{B_0}\right)\times\left({}^{\mathbf{B}}/_{B_0}\right)}$$

$$\boxed{\frac{\beta_e}{2}\frac{\delta \mathrm{p_e}/P_{e0}}{\delta \mathrm{b}/B_0}\frac{1}{\langle|\mathbf{B}|\rangle/B_0}}$$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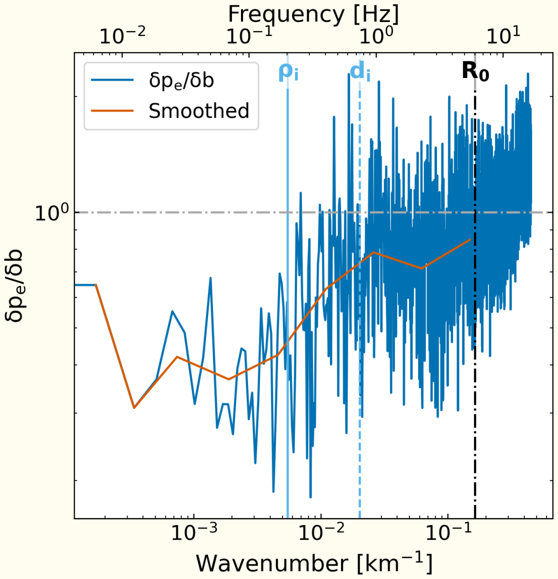
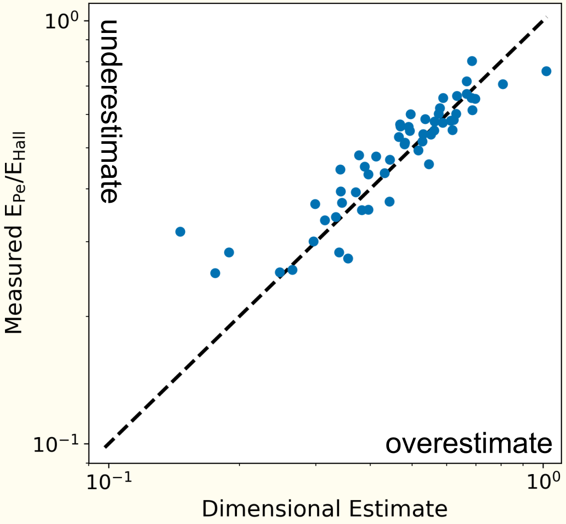
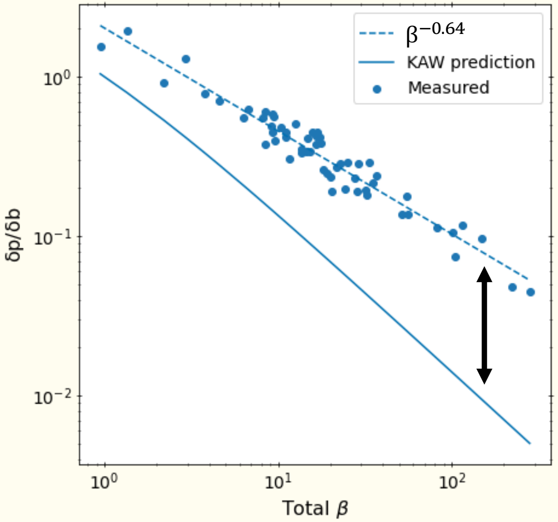
# Pressure/Hall Relative Amplitude




We evaluate contribution to $\frac{\delta p_e}{\delta b}$ in sub-ion range

KAW prediction* underestimates measured values



*doi:10.1088/0004-637X/777/1/41

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# Nonlinear / Linear Terms

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- We split each term into linear (wave-like) and nonlinear (scale transfer)

| Term | Linear | Nonlinear |
|---|---|---|
| MHD | $-\delta\mathbf{u}\times\mathbf{B}_0$ | $-\delta\mathbf{u}\times\delta\mathbf{b}$ |
| Hall | $\frac{1}{n_e e}\delta\mathbf{j}\times\mathbf{B}_0$ | $\frac{1}{n_e e}\delta\mathbf{j}\times\delta\mathbf{b}$ |
| Pressure | $-\frac{1}{n_e e}\nabla\cdot(n_{e0}\delta\mathbf{T}_e+\delta n_e\mathbf{T}_{e0})$ | $-\frac{1}{n_e e}\nabla\cdot(\delta n_e\delta\mathbf{T}_e)$ |

MHD & Hall ratios: $\dfrac{\delta b}{B_0}$

Pressure ratio: $\dfrac{\delta n\delta T_e}{\delta n T_{e0}+\delta T_e n_{e0}}$

Assuming $\delta n \gg \delta T_e$ $\Rightarrow$ $\dfrac{\delta T_e}{T_{e0}}$

Assuming $\delta n \ll \delta T_e$ $\Rightarrow$ $\dfrac{\delta n}{n_{e0}}$


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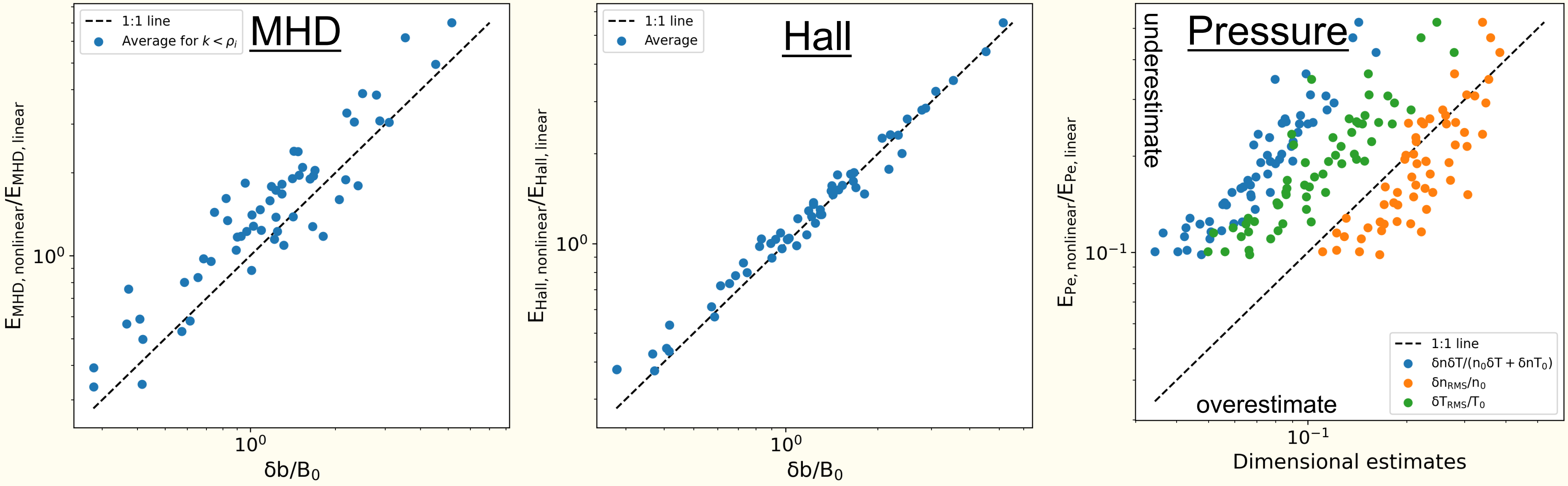
# Nonlinear / Linear Terms




- MHD and Hall well described by $\delta b_{RMS}/B_0$
- Pressure term approximation improved assuming dominant $\delta T_{RMS}$



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# Summary

**Results**

- Predicted interplay between MHD, Hall and Pressure terms via dimensional analysis estimates of 'Hall scale' and relative amplitude of $E_{Pe}/E_{Hall}$
  - Explored how to best estimate quantities used in dimensional analysis
- We find intervals where MHD and Hall terms dominated by either nonlinear or linear dynamics

**Ongoing work**

- Why is $\delta u/\delta b_A$ adjacent to Hall scales $\approx$0.5?
- How can we improve nonlinear/linear pressure term estimate?
- Investigating discrepancy with KAW prediction
- Linking structures in Ohm's law to velocity distributions (next slide)


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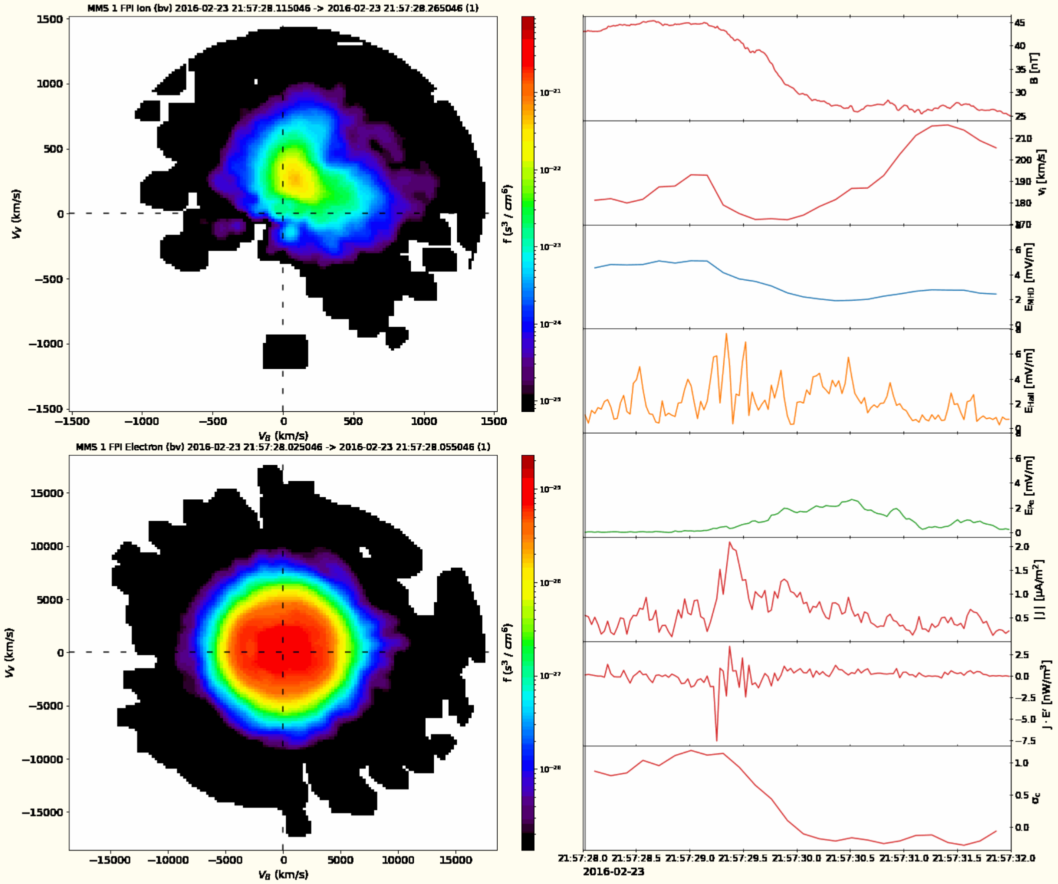
# Ongoing Work

How do physical structures affect velocity distributions?

Are terms in Ohm’s law associated with velocity structures?

What can we learn about dissipation etc from these relations?



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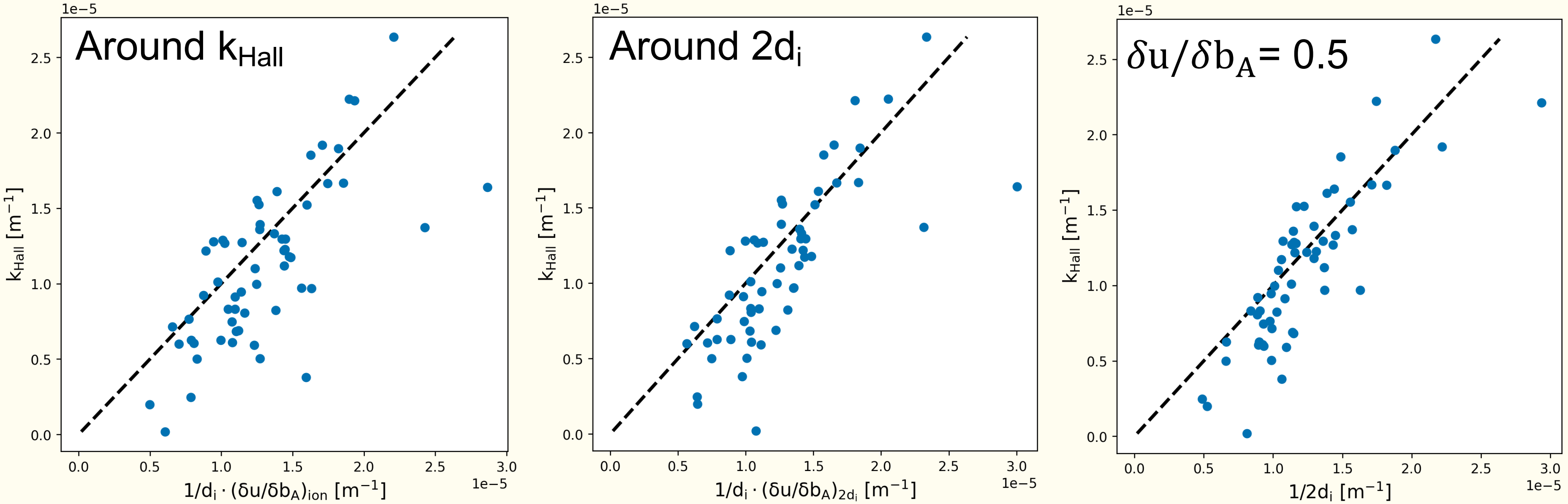
# EXTRA: Hall/MHD: Values of $\delta u / \delta b_A$

- If $k_{Hall}$ is not known, taking $\delta u / \delta b_A$ around $2d_i$ provides a good result




Empirically, $\frac{\delta u}{\delta b_A} \approx \frac{1}{2}$ for these intervals



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