

# Energy Cascade in Reconnection: 3<sup>rd</sup> Order Dynamics

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# Theory (Exact laws in Incompressible Hall MHD)

$$\frac{\partial S}{\partial t} + \nabla_l \cdot (\mathbf{Y} + \mathbf{H}) - 2A = 2\nu \nabla_l^2 S_u + 2\eta \nabla_l^2 S_b - 4\epsilon \quad \text{Hellinger et al. ApJL (2018)}$$

Assumptions made:  
Statistical homogeneity

$$4A = \nabla_l \cdot \mathbf{H} \quad \text{Ferrand et al. ApJ (2019)}$$

Increments  $\longrightarrow \delta \mathbf{u} = \mathbf{u}(\mathbf{x} + \mathbf{l}) - \mathbf{u}(\mathbf{x})$

Structure functions:

➤ Second order:  $S_b = \langle \delta \mathbf{b} \cdot \delta \mathbf{b} \rangle = \langle |\delta \mathbf{b}|^2 \rangle \quad S_u = \langle \delta \mathbf{u} \cdot \delta \mathbf{u} \rangle = \langle |\delta \mathbf{u}|^2 \rangle \quad S = S_u + S_b$

➤ Third order  $\mathbf{Y} = \langle \delta \mathbf{u} (|\delta \mathbf{u}|^2 + |\delta \mathbf{b}|^2) - 2\delta \mathbf{b} (\delta \mathbf{u} \cdot \delta \mathbf{b}) \rangle \quad \mathbf{H} = \langle 2\delta \mathbf{b} (\delta \mathbf{j} \cdot \delta \mathbf{b}) - \delta \mathbf{j} (|\delta \mathbf{b}|^2) \rangle$

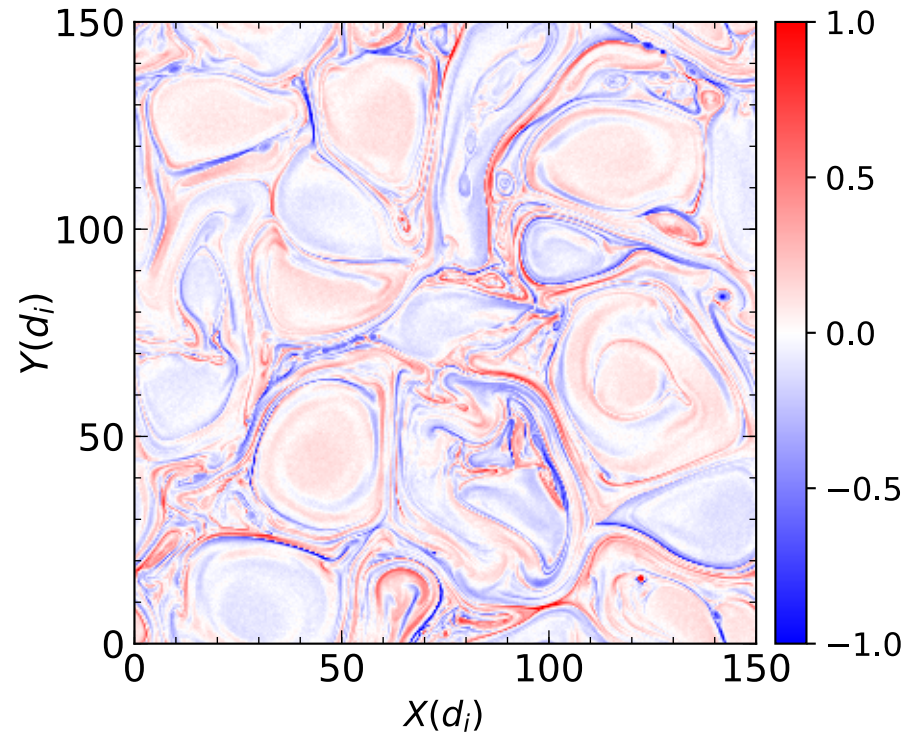
$$\mathbf{A} = \langle \delta \mathbf{j} \cdot \delta [(\mathbf{b} \cdot \nabla) \mathbf{b}] \rangle$$

No viscosity ( $\nu$ ), resistivity ( $\eta$ )

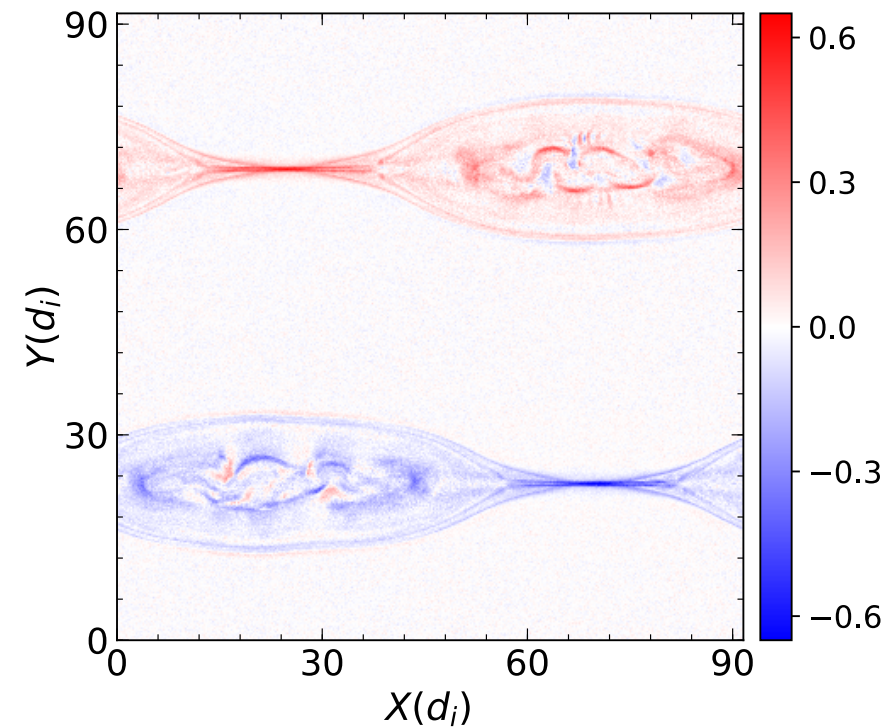
Cascade rate  $\epsilon = -\frac{1}{4} \frac{\partial S}{\partial t} + \frac{1}{4} \nabla_l \cdot \mathbf{Y} + \frac{1}{4} \nabla_l \cdot \mathbf{H} / 2$

# Fully kinetic Simulations: Turbulence Vs Reconnection

## TURBULENCE

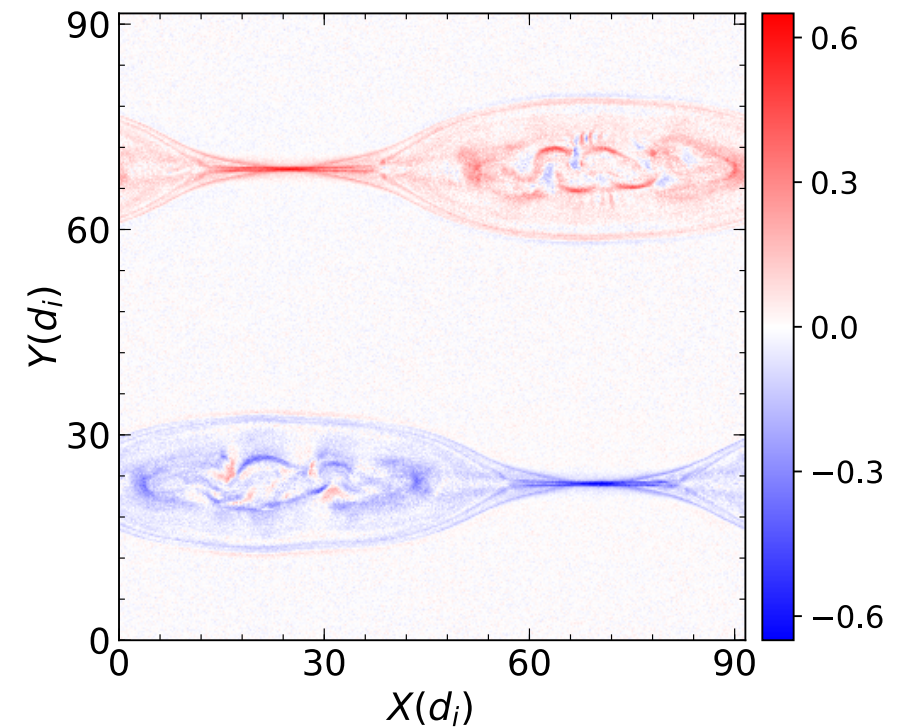
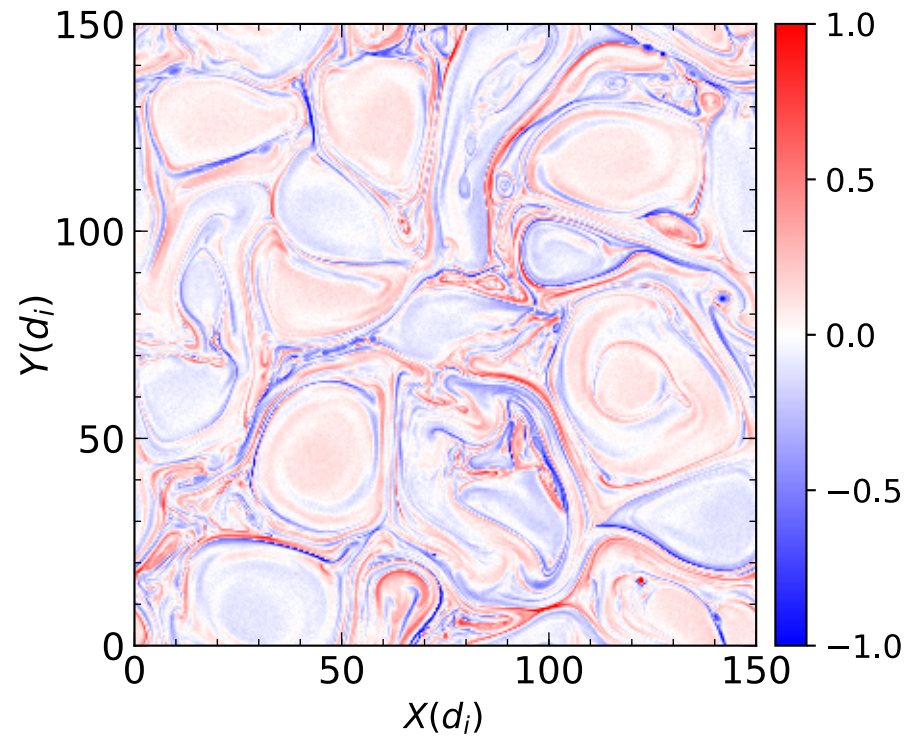


## RECONNECTION

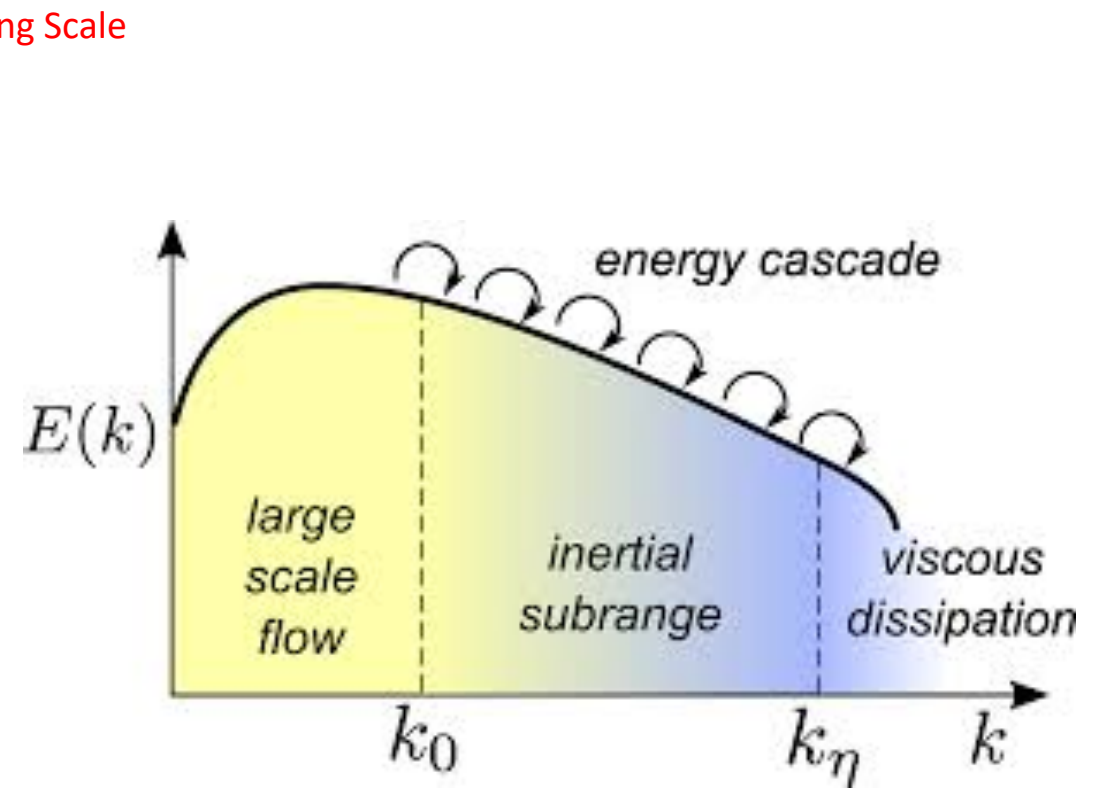
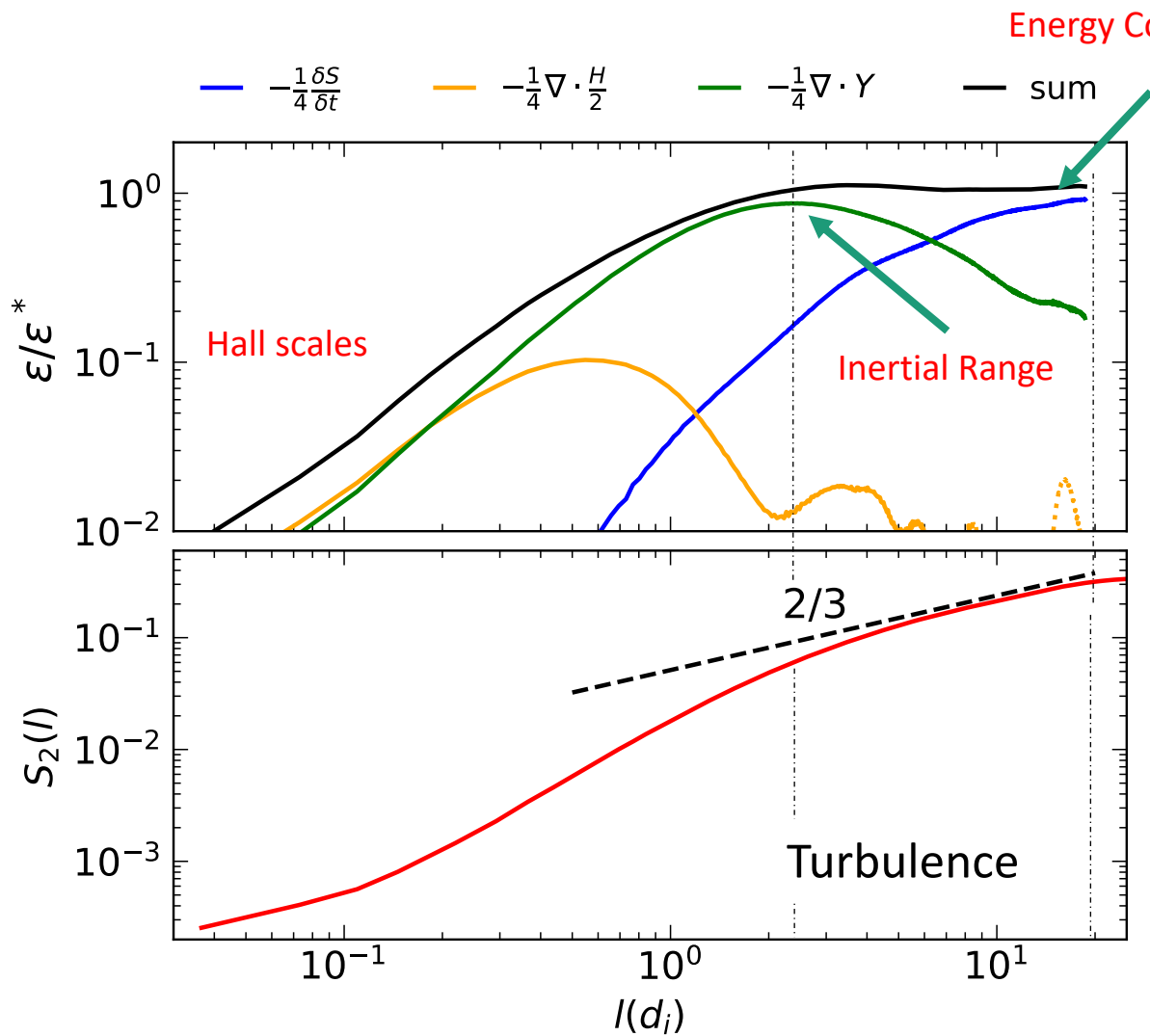


Out of plane current density for turbulence (left) when the mean square current is maximum, and for reconnection(right) when the reconnection is quasi-steady.

S.N.	$L_{\text{box}} [d_i]$	grids	$B_g$	$n_b$	$T_e/T_i$	$\Delta x$	$\delta b_{\text{rms}}$	$\delta u_{\text{rms}}$	Type	Initial Conditions	Boundary conditions
1	149.6	$4096^2$	$B_z=1$	1.0	0.3/0.3	0.0365	$1/\sqrt{10}$	$1/\sqrt{10}$	Turbulence	Fourier modes ( $2 \leq  k  \leq 4$ )	Periodic
2	91.6	$4096^2$	$B_z=0$	1.0	0.01/0.05	0.0223	$1/\sqrt{5}$	0	Reconnection	Double Harris current sheet	Periodic



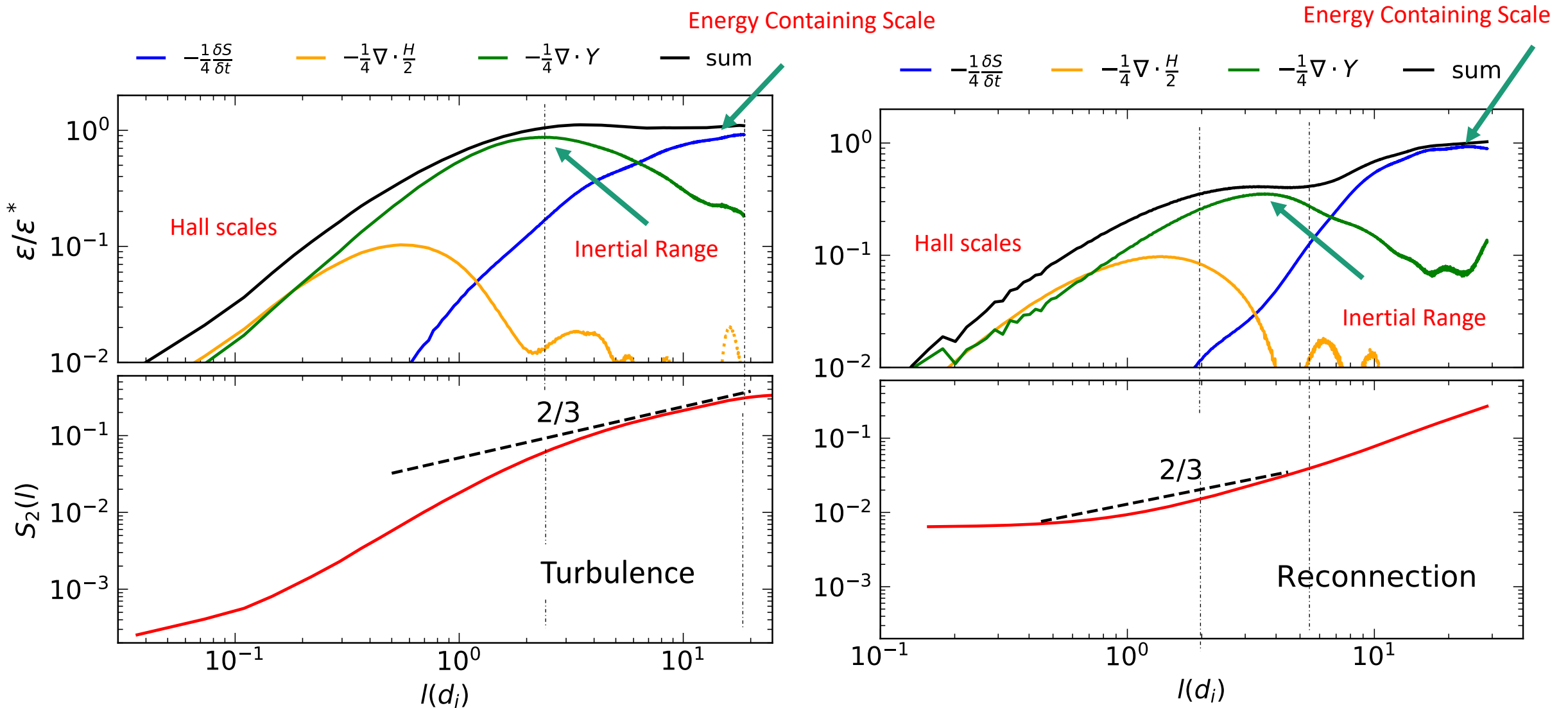
Out of plane current density for turbulence (left) when the mean square current is maximum, and for reconnection(right) when the reconnection is quasi-steady.



Narain et.al, ACM SIGGRAPH (2008)

<https://doi.org/10.1145/1457515.1409119>

Left: Cascade rate estimation for the turbulence simulation when current is maximum. Each term is normalized with the energy decay rate obtained from the magnetic and Ion flow energy. The bottom plot represents the total second order structure function. Right: Energy cascade process.



Cascade rate estimation for the turbulence simulation (left) when current is maximum, and reconnection simulation (right) when reconnection is quasi-steady. Each term is normalized with the energy decay rate obtained from the magnetic and Ion flow energy. The bottom plot represents the total second order structure function.

# Conclusion:

- Dynamics of third order law are similar for both turbulence and reconnection.
- Energy cascade rate is constant in the inertial range, where the divergence of the MHD energy flux in lag space dominates.
- Reconnection, like turbulence, is an energy cascade process.
- Is there a universal law governing reconnection and turbulence?