Linear eigenmode analysis of the tearing instability in electron-scale current sheet with nongyrotropic electron pressure

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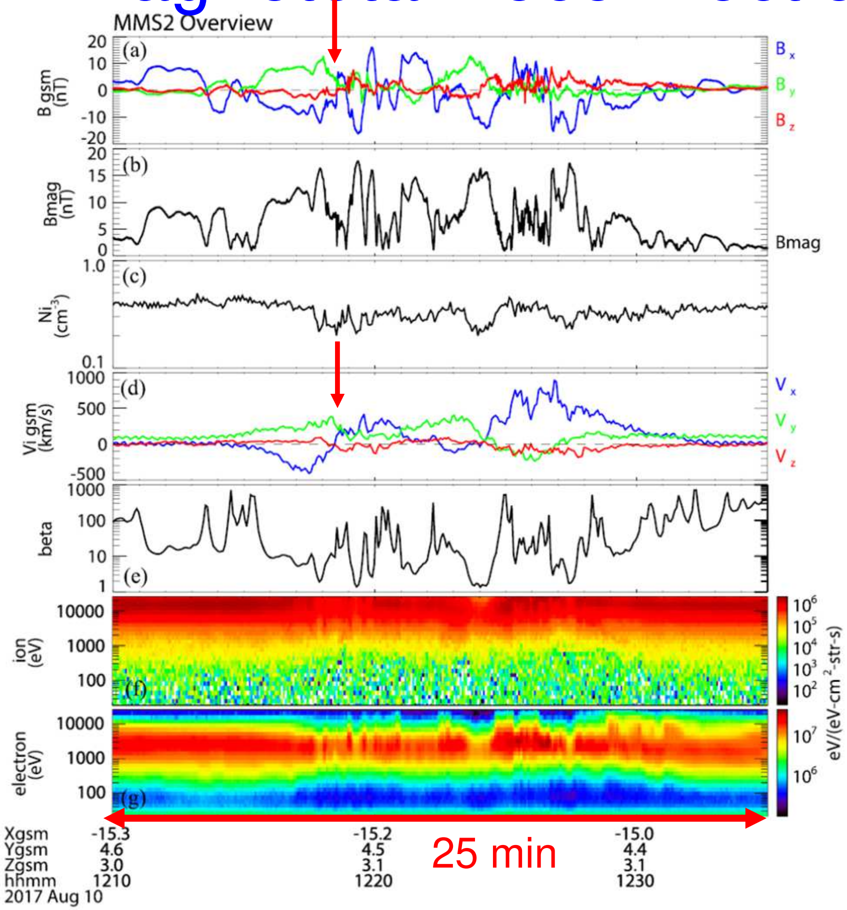
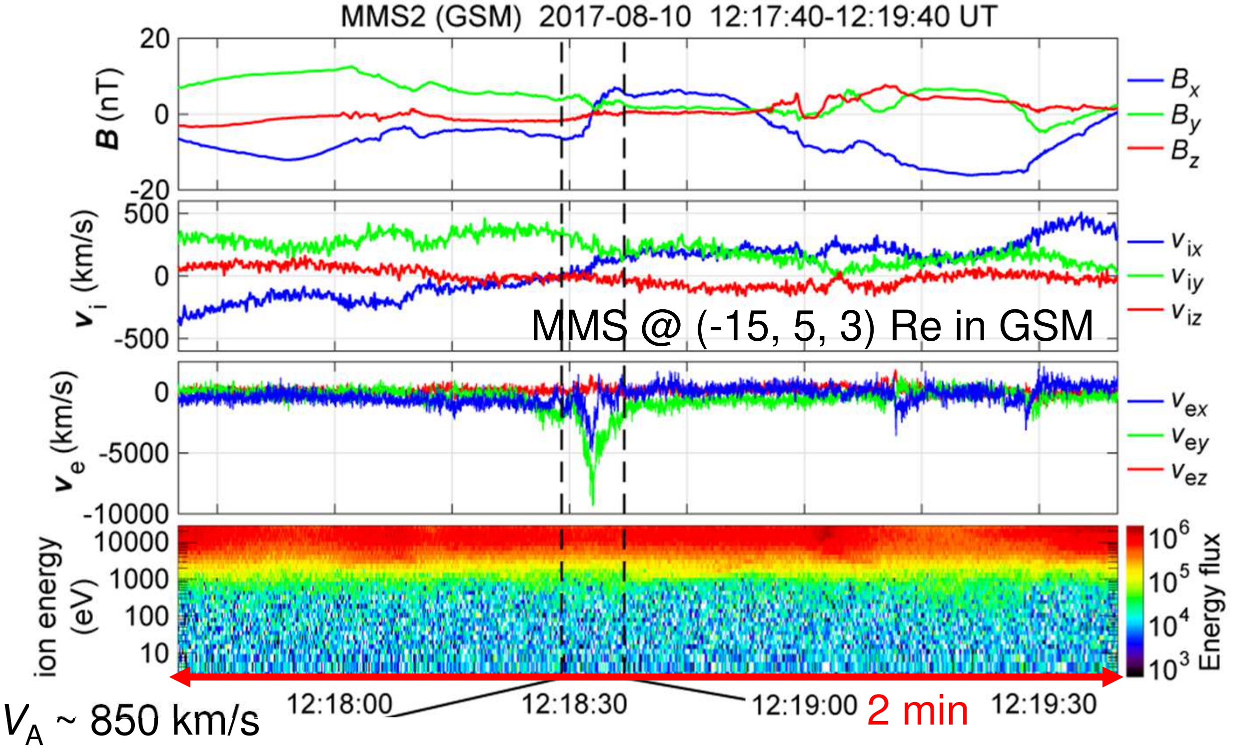
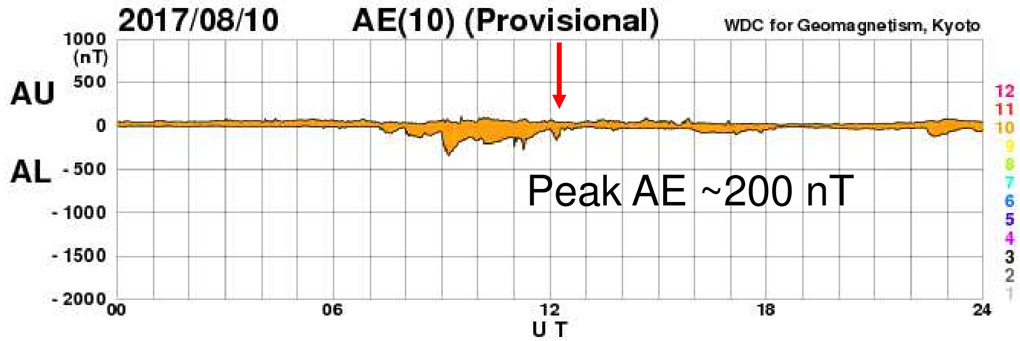
Magnetotail reconnection seen by MMS on 10 August 2017



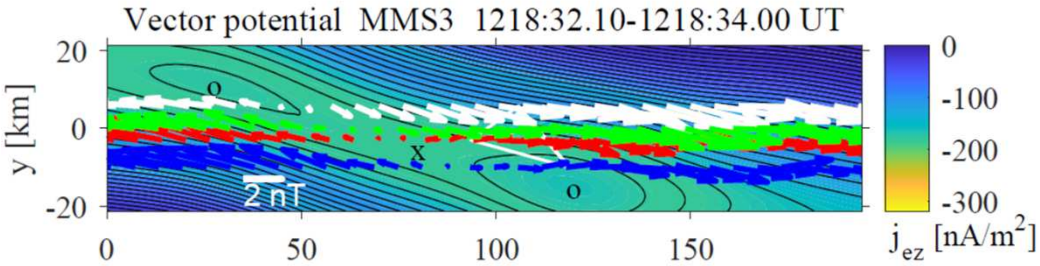


V_A ~ 850 km/s
B_0 ~ 15 nT

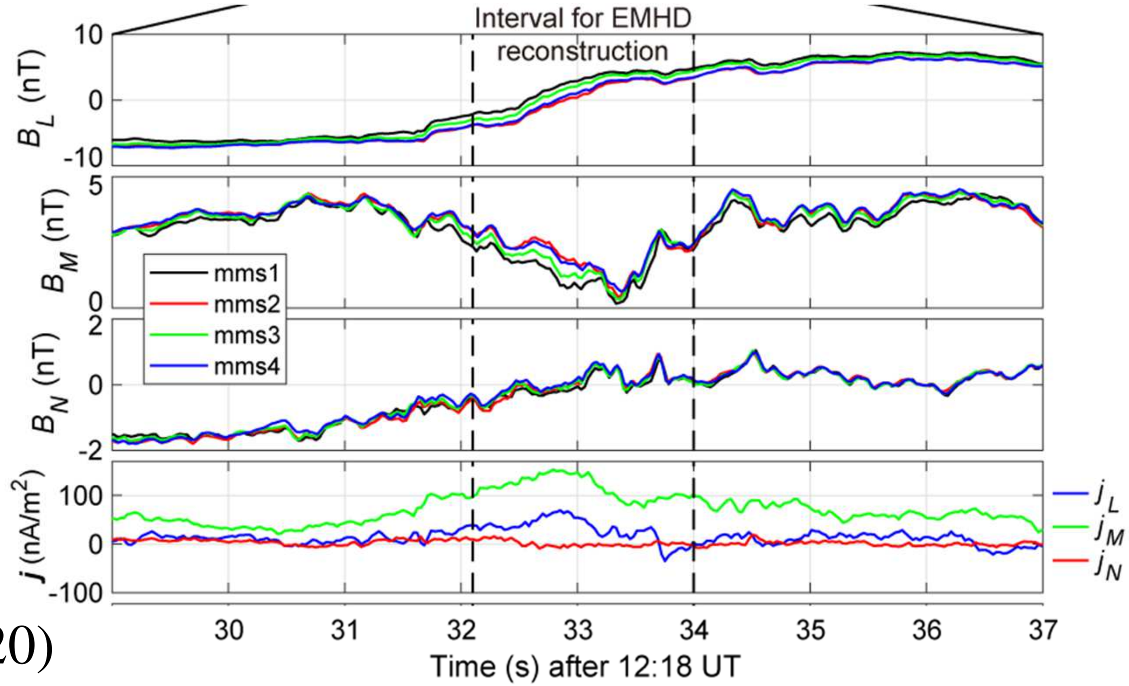


- Significant ***j**E*** (Zhou+, ApJ19; Li+, GRL20)
- Crescent-shaped electron distributions
- Super-Alfvenic electron flows

E-MHD & Polynomial reconstruction results



(Sonnerup+, JGR16)

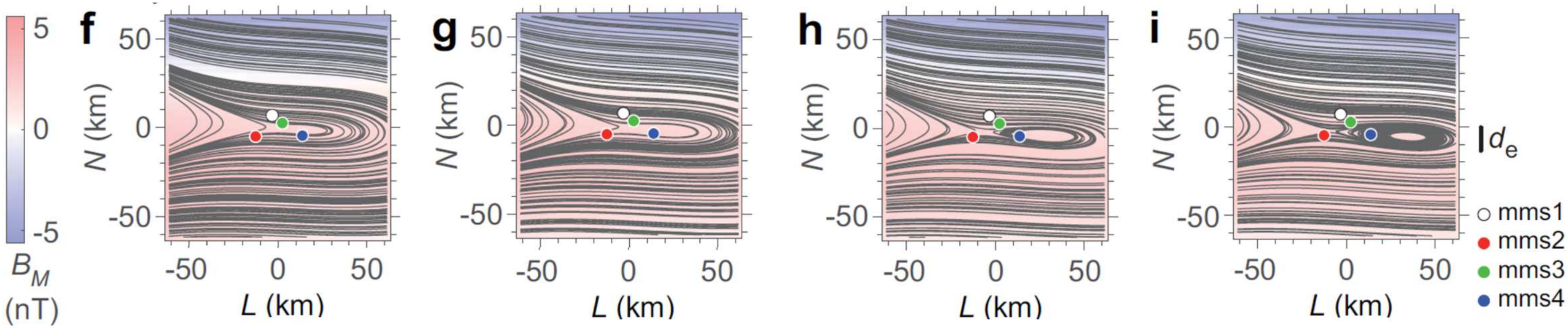


Spacecraft separation ~18 km

Electron inertial length d_e ~14 km

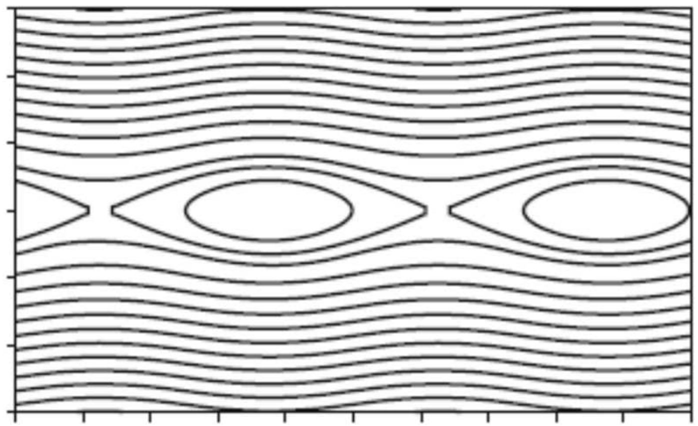
(Denton+, JGR20)

Time after 12:18 UT: 32.77 s 32.8 s 32.84 s 32.87 s



Motivation

- A growing electron-scale magnetic island was identified right in the electron diffusion region (EDR) of magnetotail reconnection (MMS observations of an electron-scale current sheet on 2017-08-10).
- The formation mechanism is not very clear, but may be an electron tearing instability.
- We revisit 2-D tearing instability in electron-scale current sheets, but now including a non-gyrotropic electron pressure effect (Hesse-Kuznetsova dissipation term) (Hesse+, SSR11).



Linear eigenmode analysis in electron-MHD framework

Generalized Ohm's law $$\mathbf{E} = -\mathbf{v}\times\mathbf{B} - \frac{\nabla\cdot\mathbf{P}}{ne} - \frac{m}{e}\left[\frac{\partial\mathbf{v}}{\partial t} + \nabla\frac{v^2}{2} - \mathbf{v}\times(\nabla\times\mathbf{v})\right]$$

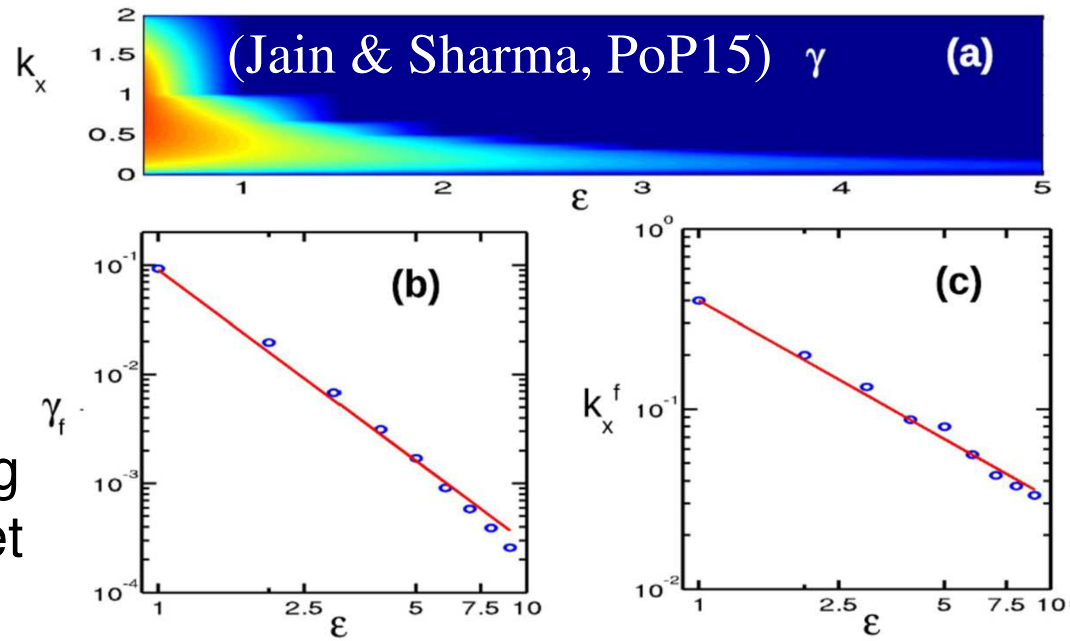
- In the linear eigenmode analysis, Faraday's law is linearized.
- If electrons are isotropic, the electron pressure tensor term does not contribute to the tearing instability.

Faraday's law

$$\frac{\partial}{\partial t}\left(\mathbf{B} - \frac{m}{\mu_0 ne^2}\nabla^2\mathbf{B}\right) = \nabla\times\left[\mathbf{v}\times\left(\mathbf{B} - \frac{m}{\mu_0 ne^2}\nabla^2\mathbf{B}\right)\right]$$

Ampere's law $$\mathbf{v} = -\nabla\times\mathbf{B}/(\mu_0 ne)$$

- The growth rate γ (normalized to ω_{ce}) and the wavelength (k_x) for the fastest-growing mode are both scaled by the current sheet half-thickness ε (normalized to d_e).



Linear eigenmode analysis in electron-MHD framework

- Hesse et al. (SSR, 2011) show that the reconnection electric field in the dissipation region (at the X-point) of steady 2D antiparallel reconnection can be expressed as:

$$E_y = -\frac{1}{n_{\mathrm{e}} e}\left(\frac{\partial P_{\mathrm{e}xy}}{\partial x} + \frac{\partial P_{\mathrm{e}yz}}{\partial z}\right) \approx \frac{1}{e}\frac{\partial v_{\mathrm{e}x}}{\partial x}\sqrt{2m_{\mathrm{e}}k_{\mathrm{B}}T_{\mathrm{e}}}$$

- The electron pressure tensor term is then non-negligible in the tearing instability.

Faraday's law

$$\frac{\partial}{\partial t}\left(\mathbf{B} - \frac{m}{\mu_0 n e^2}\nabla^2\mathbf{B}\right) = \nabla\times\left[\mathbf{v}\times\left(\mathbf{B} - \frac{m}{\mu_0 n e^2}\nabla^2\mathbf{B}\right)\right] - \frac{\sqrt{2mkT_e}}{e}\nabla\left(\frac{\partial v_x}{\partial x}\right)\times\hat{\mathbf{y}}$$

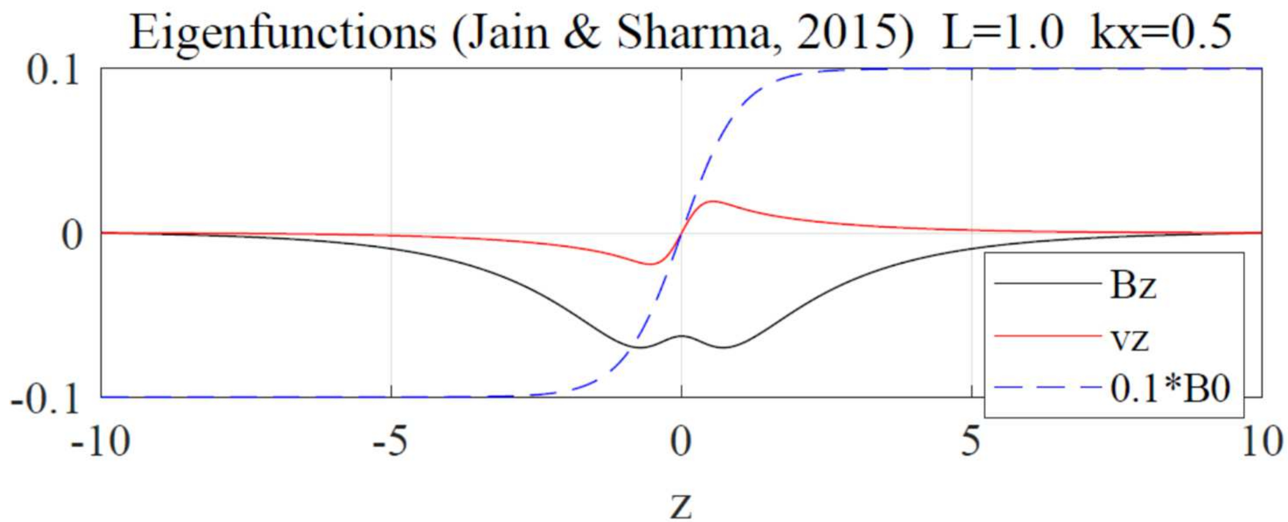
- If normalized to the upstream field intensity B_∞, $|\omega_{ce}|^{-1}$, electron inertial length, and electron Alfven speed,

$$\frac{\partial}{\partial t}(\mathbf{B} - \nabla^2\mathbf{B}) = \nabla\times[\mathbf{v}\times(\mathbf{B} - \nabla^2\mathbf{B})] - \sqrt{\beta_e}\nabla\left(\frac{\partial v_x}{\partial x}\right)\times\hat{\mathbf{y}}$$

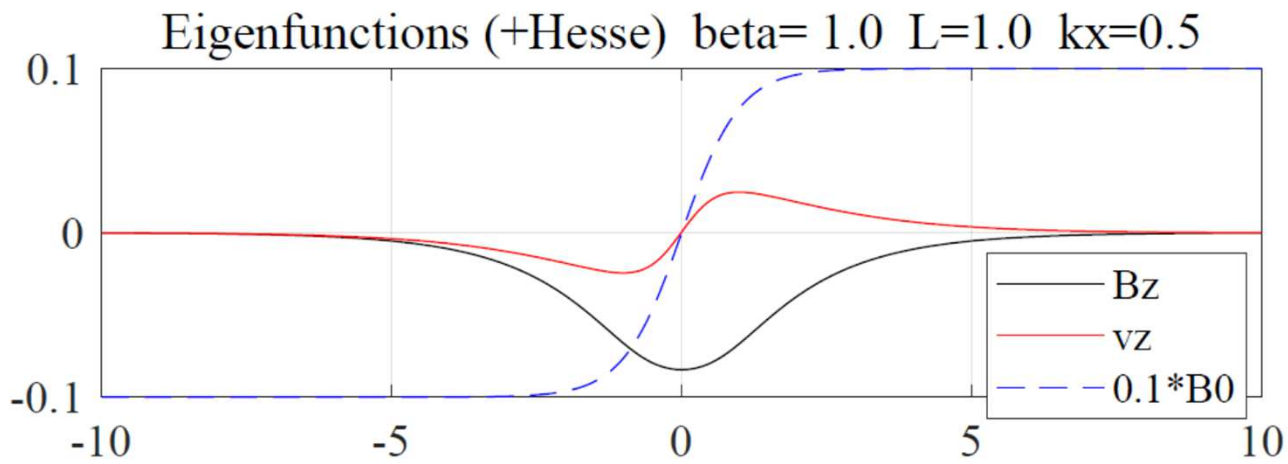
- Assuming uniform density & incompressible perturbation $\nabla\cdot\mathbf{v} = 0$, the above equation can be linearized for 2-D perturbations $\mathbf{B}_1 = \mathbf{B}(\mathrm{z})\exp(ik_x x - i\omega t)$ & $\mathbf{v}_1 = \mathbf{v}(\mathrm{z})\exp(ik_x x - i\omega t)$ and equilibrium profile $B_0(z) = \tanh(z/\varepsilon)$.
- Dependence of the growth rate γ on ε, β_e, and k_x can be investigated.

Eigen functions of Bz (reconnected field) & vz (inflow velocity)

No electron pressure



With non-gyrotropic electron effect (electron beta = 1 case)

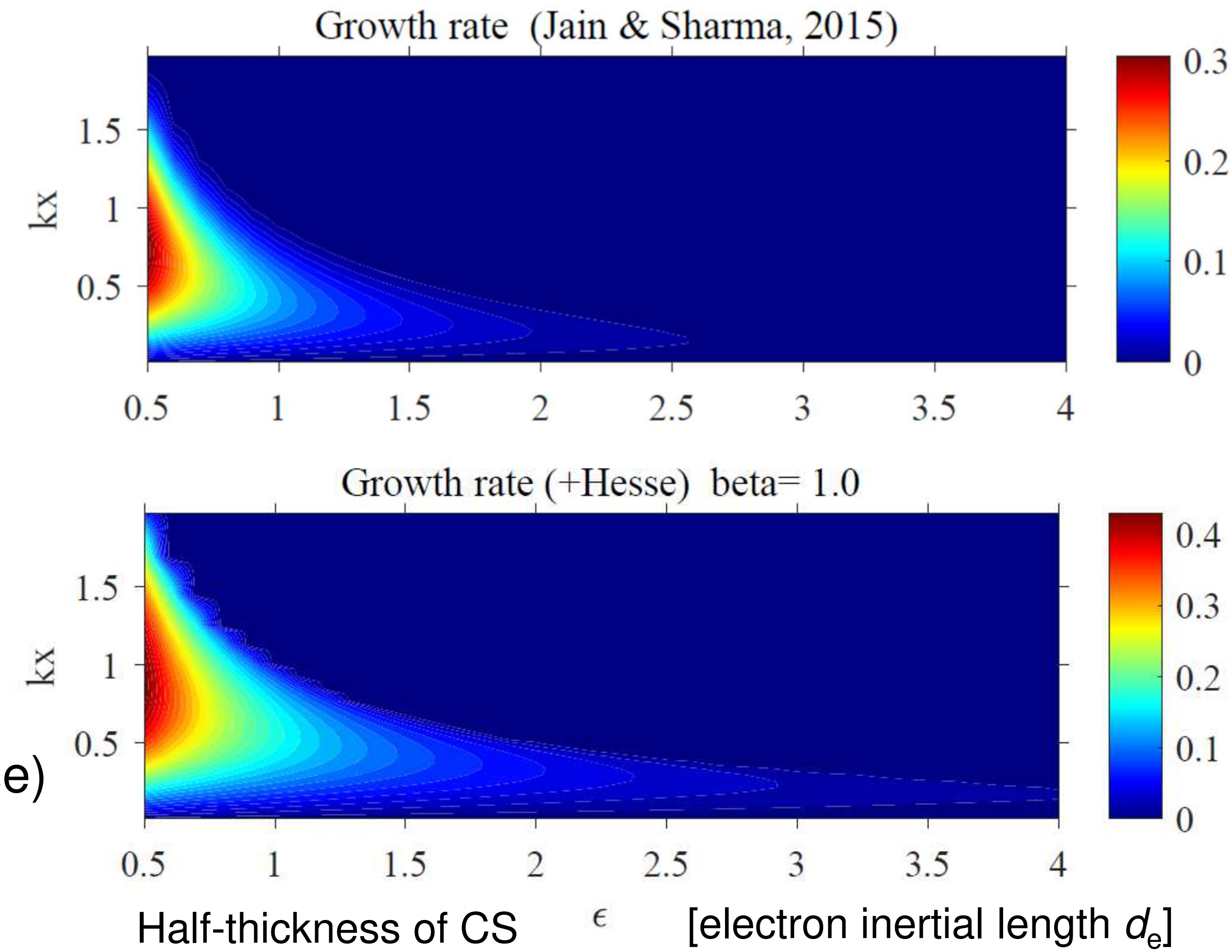


Distance from CS plane z [electron inertial length d_e]

Growth rate for electron beta = 1 case

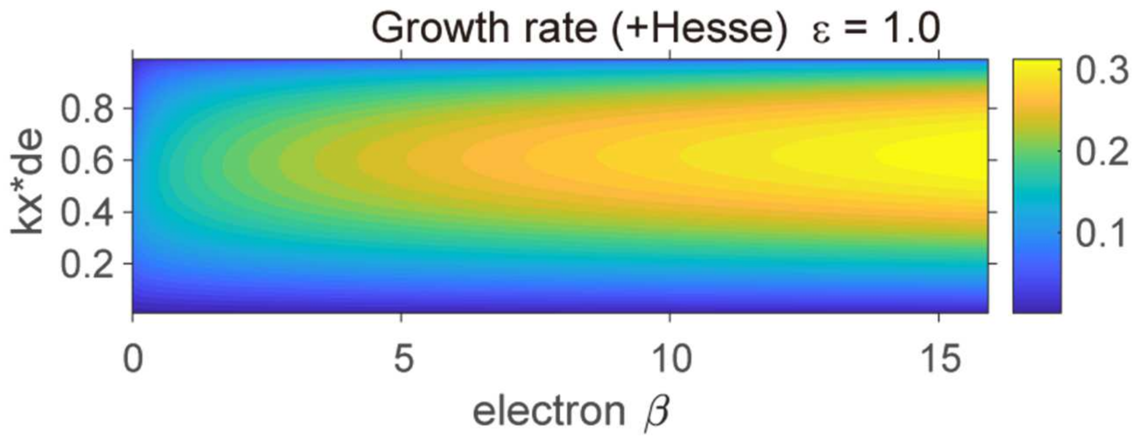
No electron pressure

With non-gyrotropic electron effect (electron beta = 1 case)



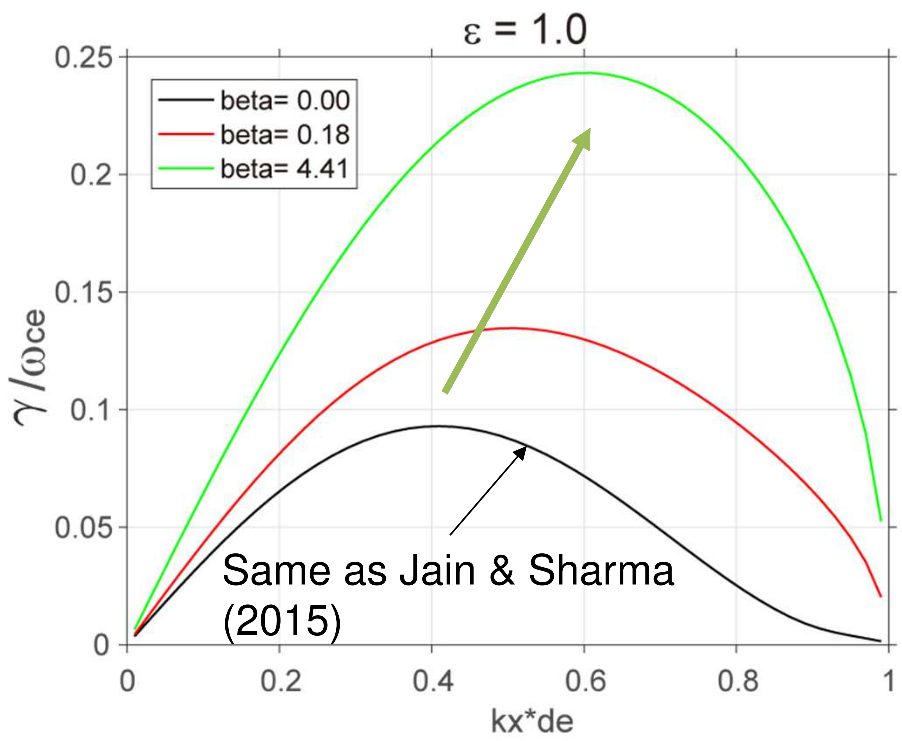
Half-thickness of CS [electron inertial length d_e]

Electron beta dependence (ε = 1 case)

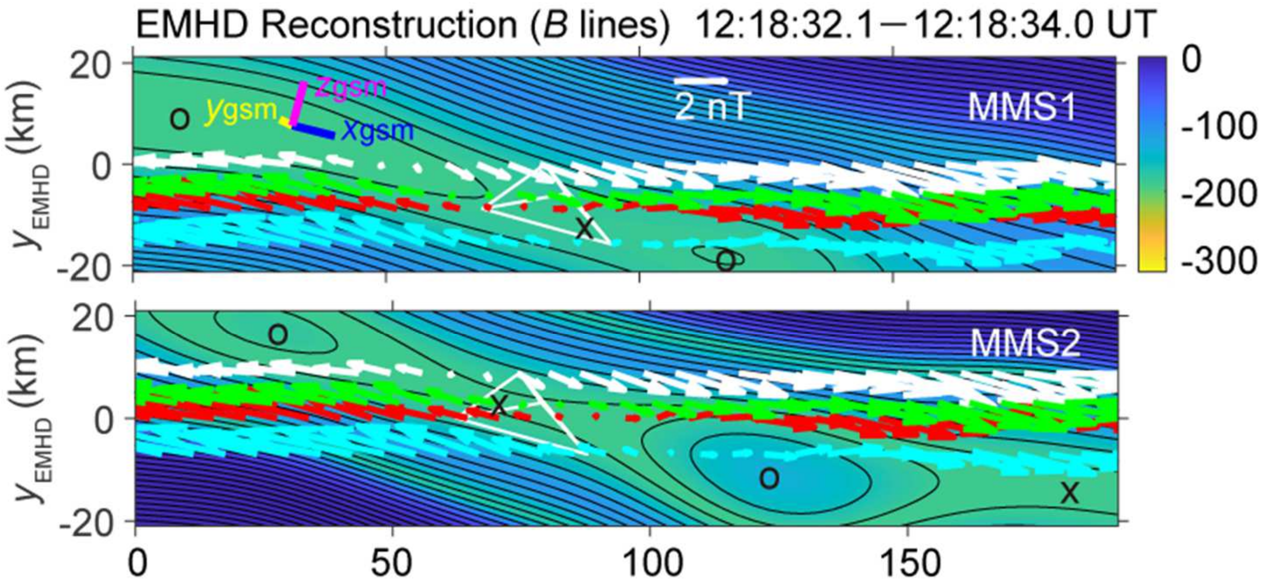


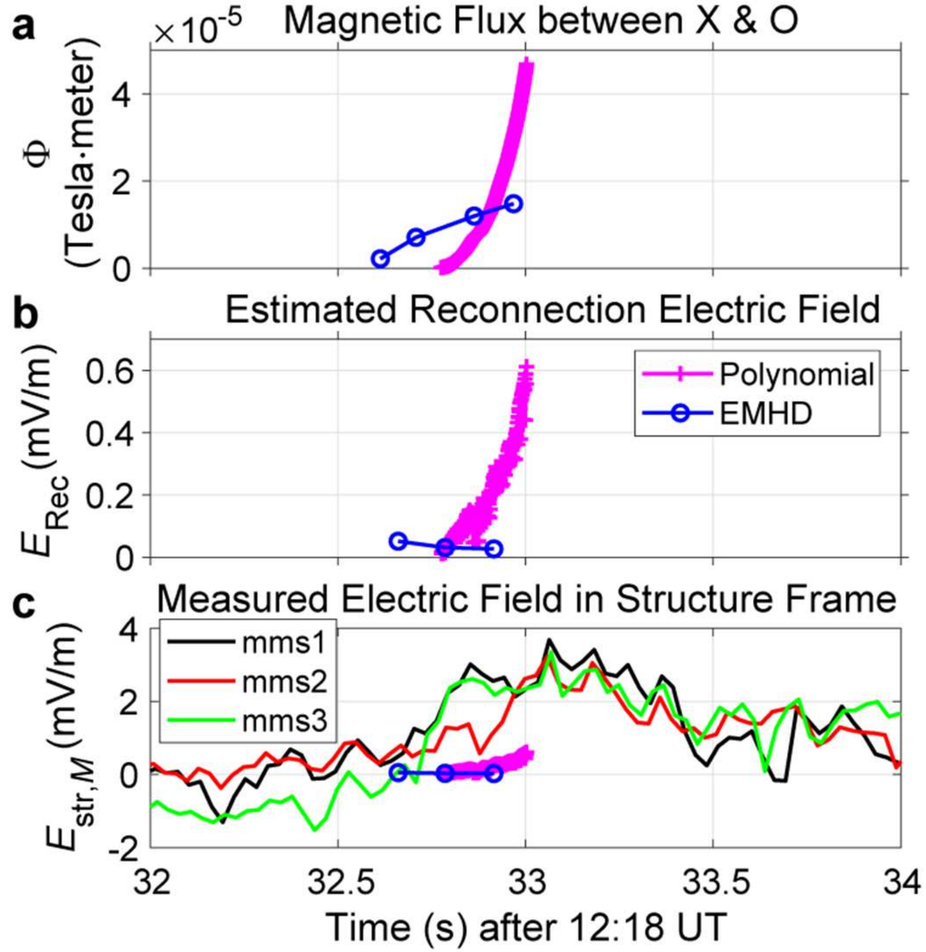
- The growth rate increases with electron β.
- The wavelength for the fastest-growing mode becomes shorter with increasing electron beta.

Half-thickness = d_e case



Growth time of the observed magnetic island





- In-plane magnetic flux increases with time, consistent with the island growth.
- However, the growth time ~0.1 s is far longer than the electron cyclotron time scale ($2\pi/\omega_{ce}$ ~0.0002 s), i.e.,
- The growth time is inconsistent with the tearing instability.
- The magnetic field in the observed EDR was mostly annihilated, rather than reconnected.

Magnetic field annihilation in elongated EDR

Ohm's law:

$$\boldsymbol{E} = -\boldsymbol{v}_{\rm e} \times \boldsymbol{B} + \frac{1}{e}\frac{\partial v_{{\rm e}L}}{\partial L}\sqrt{2m_{\rm e}k_{\rm B}T_{\rm e}}\boldsymbol{M}$$

Faraday's law:

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} = \nabla \times (\boldsymbol{v}_{\rm e} \times \boldsymbol{B}) - \frac{\sqrt{2m_{\rm e}k_{\rm B}T_{\rm e}}}{e}\nabla\left(\frac{\partial v_{{\rm e}L}}{\partial L}\right) \times \boldsymbol{M}$$

L-component of Faraday's law: $B_N \approx 0$

$$\frac{\partial B_L}{\partial t} \approx \frac{\sqrt{2m_{\rm e}k_{\rm B}T_{\rm e}}}{e}\frac{\partial}{\partial N}\left(\frac{\partial v_{{\rm e}L}}{\partial L}\right)$$

Incompressible flow in/around EDR $\nabla \cdot \boldsymbol{v}_{\rm e} = \frac{\partial v_{{\rm e}L}}{\partial L} + \frac{\partial v_{{\rm e}N}}{\partial N} = 0$

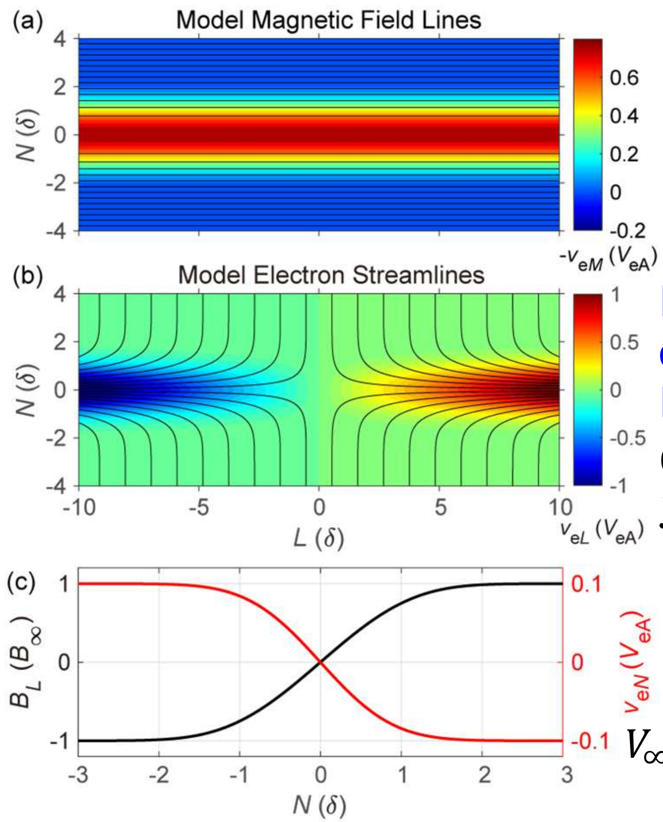
$$\frac{\partial B_L}{\partial t} \approx -\frac{\sqrt{2m_{\rm e}k_{\rm B}T_{\rm e}}}{e}\frac{\partial^2 v_{{\rm e}N}}{\partial N^2}$$

Using $-B_\infty v_{{\rm e}N} \approx B_L V_\infty$ (right figure), we derive a diffusion equation for B_L

$$\frac{\partial B_L}{\partial t} \approx D_B \frac{\partial^2 B_L}{\partial N^2}$$

Diffusion coefficient: $D_B = r_{\rm ge} V_\infty$

- In an elongated EDR, the electron pressure tensor term can act for magnetic field annihilation.
- The diffusion coefficient D_B is proportional to the inflow speed V_∞ and electron gyroradius $r_{\rm ge}$.



Exact solution of dissipative EMHD equations (Sonnerup+, JGR16)

$V_\infty \approx 0.1 V_{\rm eA}$

Summary & Discussion

- The linear eigenmode analysis of the 2-D tearing instability with non-gyrotropic electron pressure effect (Hesse-Kuznetsova term) suggests
  - The growth rate increases with increasing electron beta β_e.
  - The wavelength for the fastest-growing mode is shorter for larger β_e.
- However, the rate of magnetic-field annihilation in an extended electron-scale current sheet may also be enhanced with increasing β_e.
- The tearing instability and magnetic-field annihilation might be competitive in electron-scale current sheets.
- Is the EDR width scaled by the electron inertial length or gyroradius?
  - If scaled by gyroradius (β_e), the tearing would be faster for lower β_e while the annihilation may not depend much on β_e (larger EDR width for larger β_e).
  - Any β_e dependence of either or both processes in simulation & observations?