MMS SWT meeting 7 October 2020

Estimation of inhomogeneity factor for nonlinear wave particle interaction in an EMIC wave event

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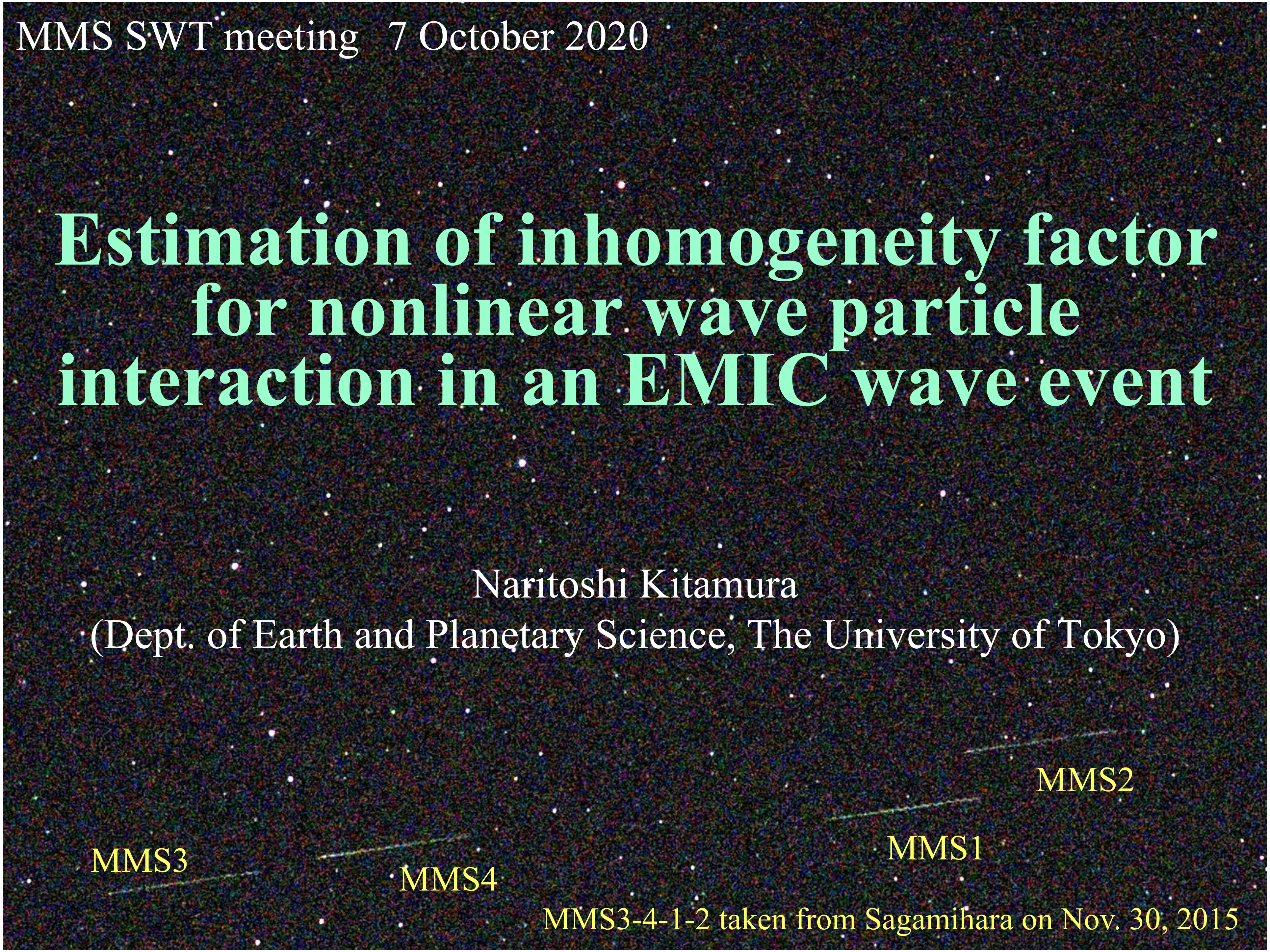


MMS3-4-1-2 taken from Sagamihara on Nov. 30, 2015

1. Introduction

◆ Electromagnetic ion cyclotron (EMIC) wave

- Free energy source: temperature anisotropy of hot ions ($T_{\mathrm{para}} < T_{\mathrm{perp}}$)
- Frequency: below H^+ cyclotron frequency
- Polarization at the source region: L-mode
- Location of maximum linear growth rate: near minimum-B

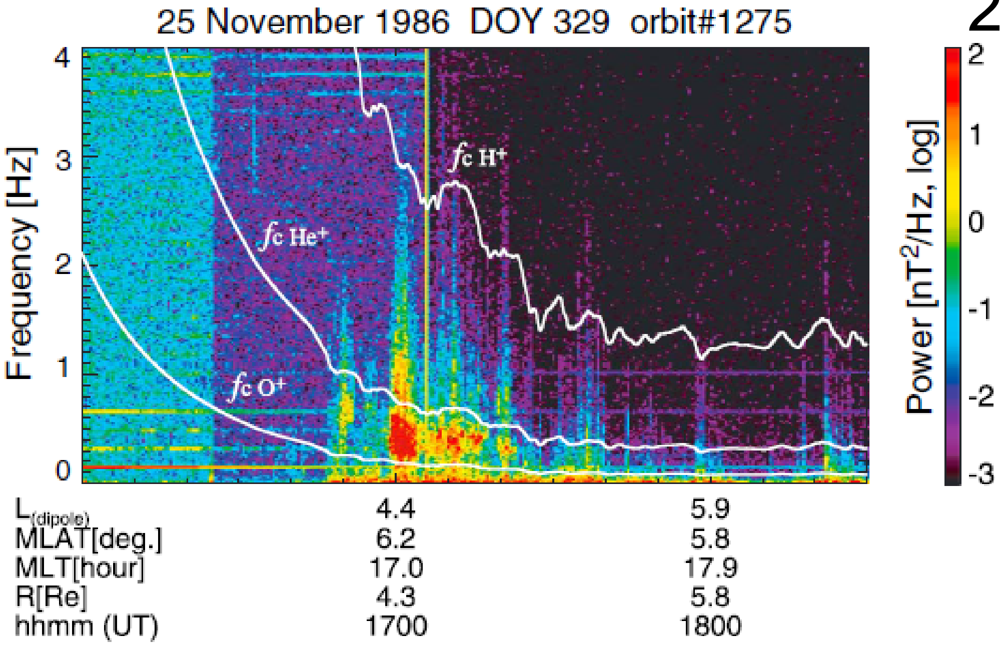


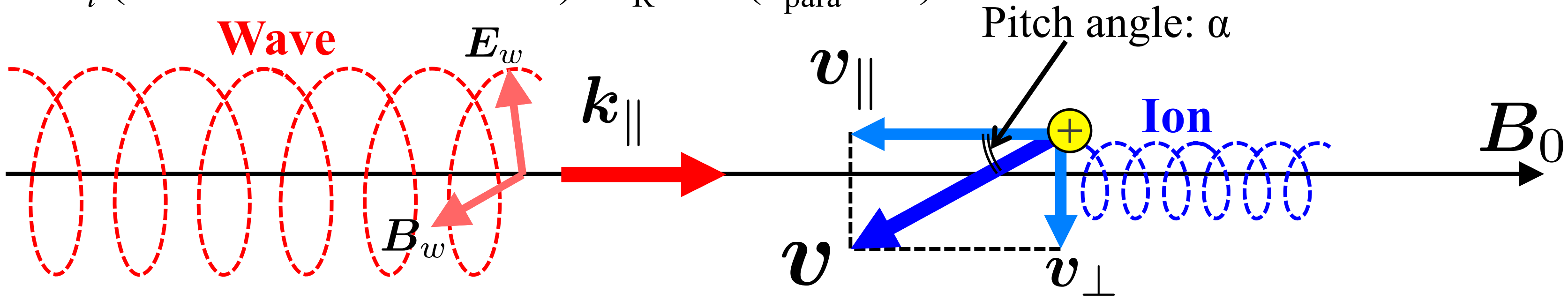
Fig. He band EMIC wave observed by the AMPTE/CCE spacecraft [Keika et al., *JGR*, 2013]

Resonance condition (1st order)

$$V_{\mathrm{R}} = \frac{\omega - \Omega_i}{k_{\mathrm{para}}}$$

V_{R}: Resonance velocity
ω: Wave angular frequency
Ω_i: Ion cyclotron angular frequency
k_{para}: Wavenumber (parallel)

$\omega < \Omega_i$ (L-mode EMIC waves): $V_{\mathrm{R}} < 0$ ($k_{\mathrm{para}} > 0$)



Resonance condition (2nd order)

For large amplitude electromagnetic cyclotron waves, the Lorentz force due to $\boldsymbol{v}_{\mathrm{perp}} \times \mathbf{B}_{\mathrm{w}}$ become important.

$$\frac{d\zeta}{dt} = -\theta \qquad \theta = k\left(v_{para} - V_R\right)$$

$$\frac{d^2\zeta}{dt^2} = \omega_{tr}^2(\sin\zeta + S)$$

$$\omega_{tr}^2 = k v_{perp} \Omega_w$$

$$\Omega_w = \frac{qB_w}{m}$$

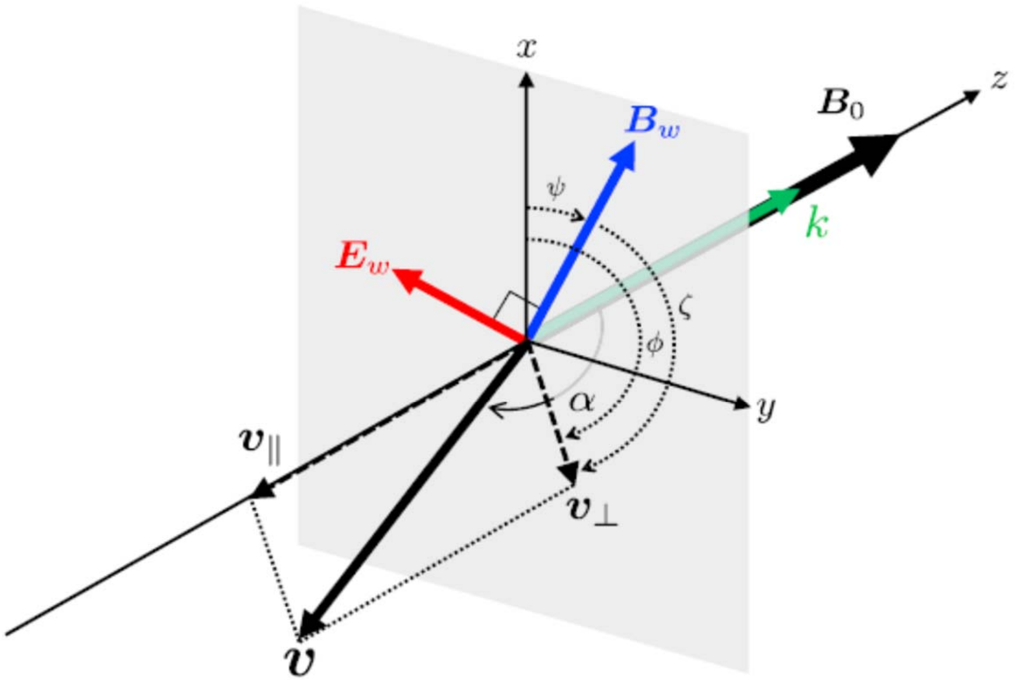


Figure 1. Configuration of the wave fields, the velocity vector of resonant electrons, and typical angles in a Cartesian coordinate system for the case of $k > 0$ and $v_\parallel < 0$.

[Kitahara and Katoh, 2019] (A case for whistler mode wave)

V_{R}: Resonance velocity
V_{P}: Phase velocity
ω: Wave angular frequency
Ω_H: Protom cyclotron angular frequency
k: Wavenumber
q: Charge
m: mass

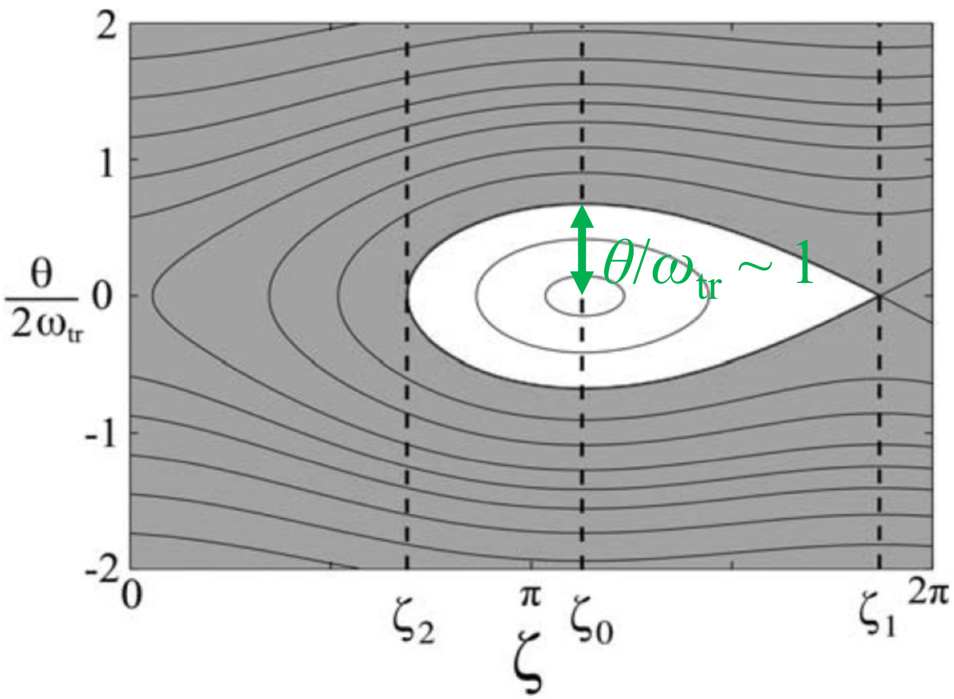


Figure 2. Trajectories of resonant protons in the $(\theta - \zeta)$ phase space for the inhomogeneity ratio S = 0.4. The phase angle ζ_0 is the center of trapping motion, while ζ_1 is the saddle point and ζ_2 is the boundary of the trapping region at $\theta = 0$.

[Omura et al., 2010]

$\theta/\omega_{\mathrm{tr}} \sim 1$ $(S \sim 0.4)$

$v_{\mathrm{para}} - V_{\mathrm{R}} \sim \omega_{\mathrm{tr}}/k$

Resonance condition (2nd order)

$$\frac{d\zeta}{dt} = -\theta \qquad \theta = k\left(v_{para} - V_R\right)$$

$$\frac{d^2\zeta}{dt^2} = \omega_{tr}^2(\sin\zeta + S)$$

$$S = \frac{1}{s_0\omega\Omega_w}\left(s_1\frac{\partial\omega}{\partial t} + V_p s_2\frac{\partial\Omega_H}{\partial h}\right)$$

$$s_0 = \frac{V_{perp0}}{V_P}$$

$$s_1 = \left(1 - \frac{V_R}{V_G}\right)^2$$

$$s_2 = \left(\frac{V_{perp0}^2}{2V_P^2} + \frac{V_R^2}{V_P V_G}\right)\frac{\omega}{\Omega_H} - \frac{V_R}{V_P}$$

V_{R}: Resonance velocity
V_{P}: Phase velocity
ω: Wave angular frequency
Ω_H: Protom cyclotron angular frequency
k: Wavenumber

Purpose

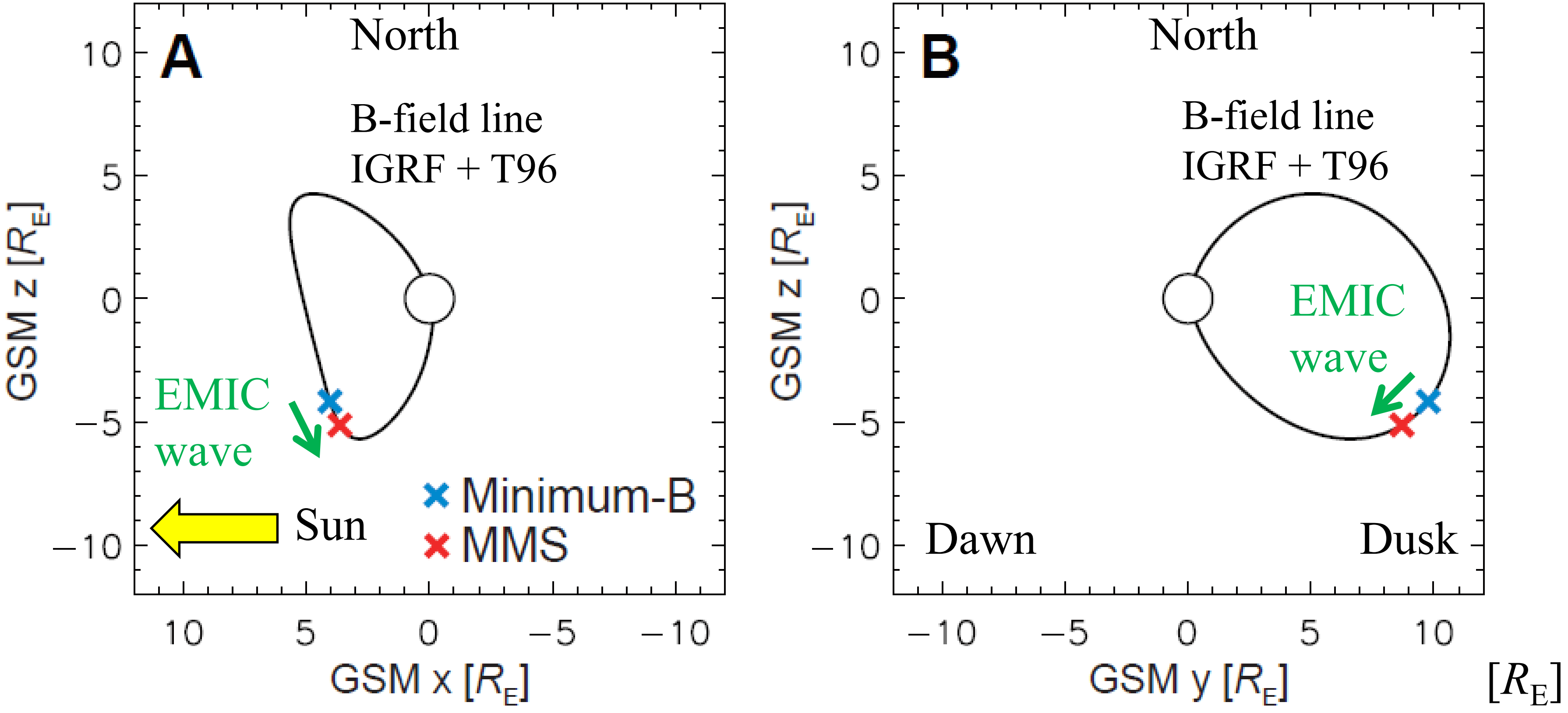
Direct identification of energy transfer from ions to EMIC waves (wave particle interaction analyzer (WPIA) method)

Measurements of grad *B* by four spacecraft

⇩

Is the feature of the energy exchange consistent with the prediction of the nonlinear theory [Omura et al., 2010]?

2. Data and Analysis Methods



2015/09/01 ~12:18 UT

Position of MMS spacecraft: near minimum-B (~1.5 R_E southward)
MLT 16.1 h, MLAT −24°, Dipole-L 12.7

2015/09/01 ~12:18 UT

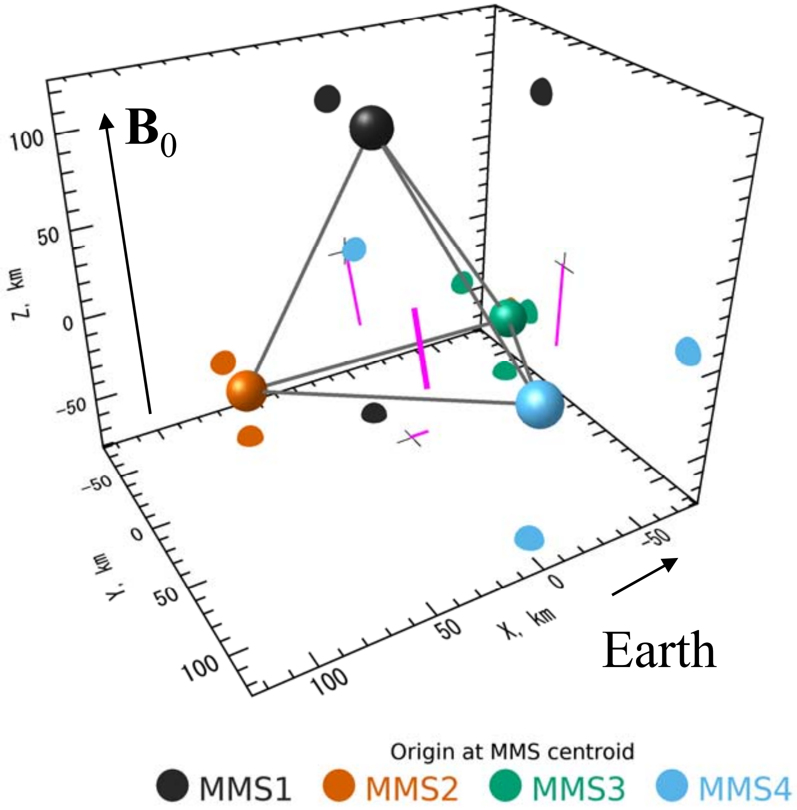
Spacecraft separation:
~160 km (~0.025 R_E)

Cyclotron-radius of H^+ (B: 22.5 nT)
10 keV: 640 km
30 keV: 1110 km

Wavelength of EMIC wave:
~3300–6300 km
(Dispersion relation calculated using KUPDAP [Sugiyama et al., *JGR*, 2015])

Spacecraft separation was much smaller than cyclotron-radius of hot H^+ (10–30 keV) and wavelength of EMIC wave.
→Average of data from 4 spacecraft

MMS Formation
2015-09-01/12:18:30.71 UTC



Field-aligned coordinates (FAC)
(Phigeo in SPEDAS)
+*x*: Radially outward ($\mathbf{e}_x = \mathbf{e}_\varphi \times \mathbf{e}_z$)
+*y*: Eastward ($\mathbf{e}_y = \mathbf{e}_z \times \mathbf{e}_x$)
+*z*: Direction of background magnetic field
($\mathbf{e}_\varphi$: Eastward basis vector (geographic coordinate))

◆FIELDS

Magnetic field

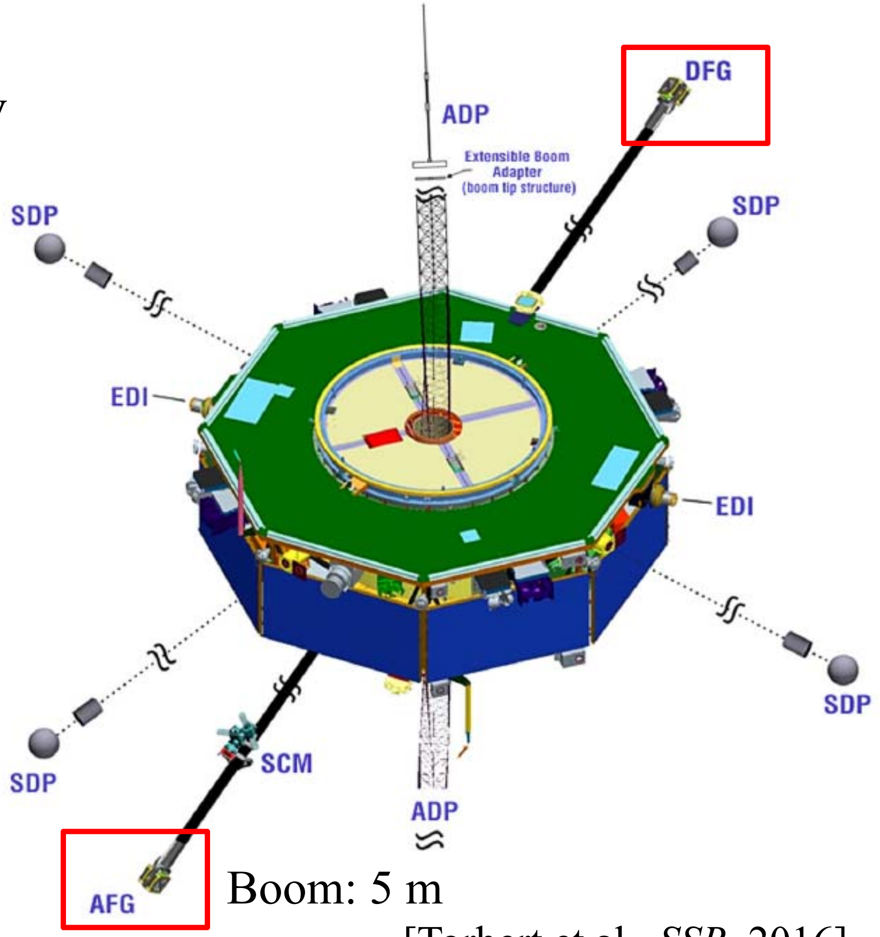
- 16 vectors/s (Fluxgate magnetometers, Fast Survey) [Russell et al., *SSR*, 2016]
- Background magnetic field: <0.05 Hz ($\mathbf{B}_{0\_EMIC}$)
- Wave magnetic field: 0.05–0.15 Hz ($\mathbf{B}_{w\_EMIC}$)
- Accuracy: 0.1 nT (0.1 nT/150 km ~ 0.67 pT/km)

Electric field

DC electric field data were not usable directly for the EMIC wave owing to the periodic fluctuation near the frequency of the EMIC waves due to the operation of ASPOC [Torkar et al., *SSR*, 2016].

Perpendicular electric fields for EMIC waves ($\mathbf{E}_{w\_EMIC}$) were estimated using cold ion (9.72–257 eV) bulk velocity, because of the significantly smaller (~1/5) frequency of EMIC waves than H^+ cyclotron frequency [Kitamura et al., *Science*, 2018].

- Wave electric field ($\mathbf{E}_{w\_EMIC}$): 0.05–0.15 Hz



Boom: 5 m

[Torbert et al., *SSR*, 2016]

◆Particle

Fast Plasma Investigation (FPI)

Dual Electron Spectrometers (DES), Dual Ion Spectrometers (DIS)

8 heads (4 × 2) per spacecraft (total 32 heads (each of DES and DIS))

Energy range (Phase-1A): 10 eV–30 keV (32 (64) bins)

Angular resolution: 11.25° (32 (Azimuth) × 16 (Elevation) (=512) pixels)

Temporal resolution (Burst data):

DIS: 0.15 sec (6.67 Hz >> EMIC wave frequency in the outer magnetosphere (~0.1 Hz))

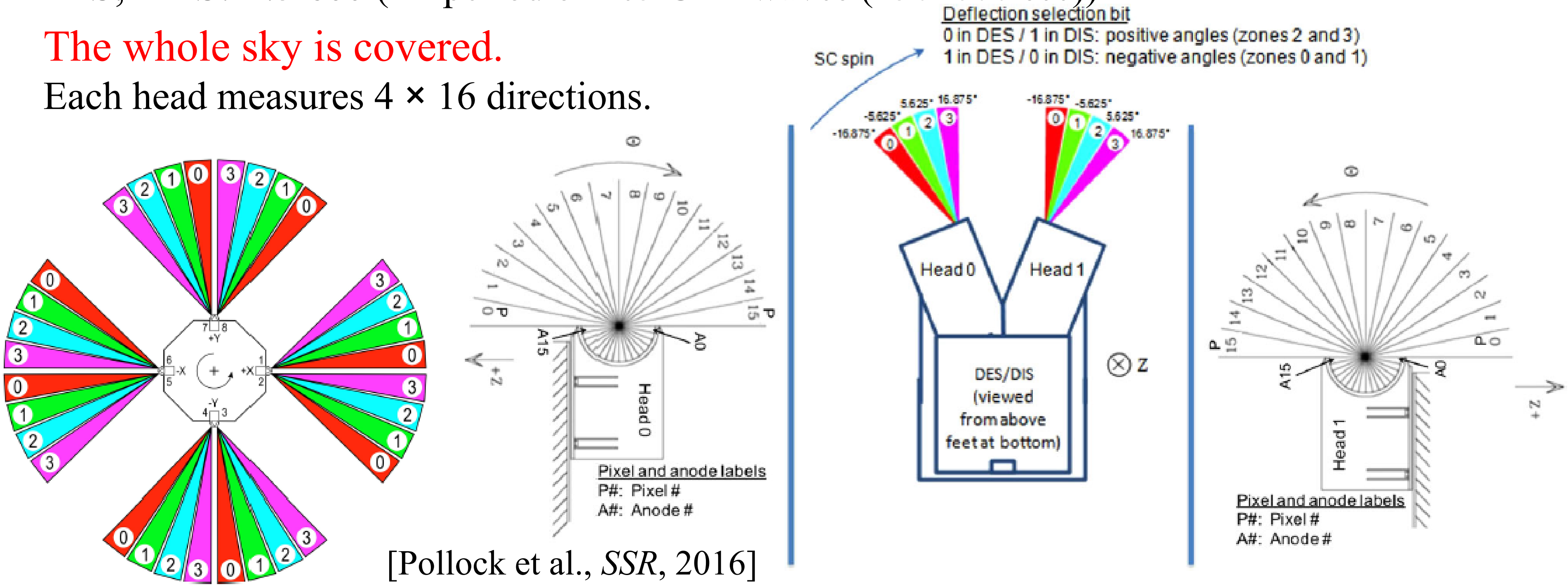
DES: 0.03 sec

Temporal resolution (Fast survey):

DIS, DES: 4.5 sec (<< period of Pc5 ULF waves (150–600 sec))

The whole sky is covered.

Each head measures 4 × 16 directions.



[Pollock et al., *SSR*, 2016]

◆Wave particle interaction analyzer (WPIA)

[e.g., Fukuhara et al., *EPS*, 2009; Katoh et al., *AnGeo*, 2013; Katoh et al., *EPS*, 2018; Shoji et al., *GRL*, 2017; Kitamura et al., *Science*, 2018]

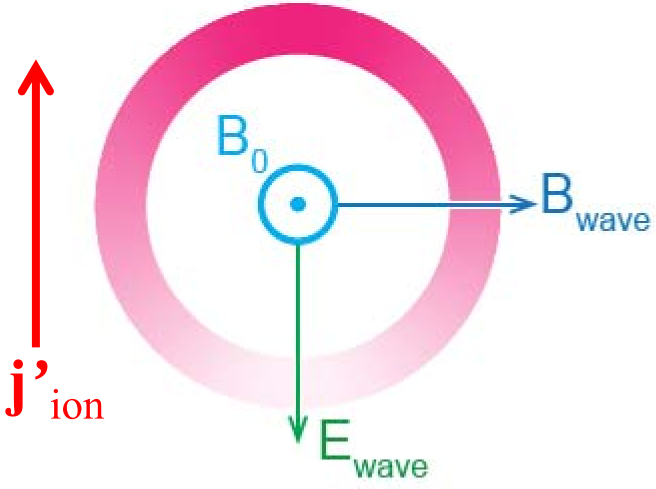
$\mathbf{j}\cdot\mathbf{E}$ indicates the energy transfer rate between particles and fields (waves).

$\mathbf{j}\cdot\mathbf{E} < 0$

Fields (wave) get energy from particles.
(wave growth)

$\mathbf{j}\cdot\mathbf{E} > 0$

Particles get energy from fields (wave).
(particle acceleration)



Denseness indicates ion fluxes.

$$\mathbf{j}(t,\varepsilon,\alpha)\cdot\mathbf{E}_{\mathrm{w}}(t)/n(t,\varepsilon,\alpha) = q\mathbf{E}_{\mathrm{w}}(t)\cdot\mathbf{v}_{\mathrm{average}}(t,\varepsilon,\alpha) = \frac{d}{dt}\left(\frac{1}{2}mv_{\mathrm{average}}^2\right)(t,\varepsilon,\alpha)$$

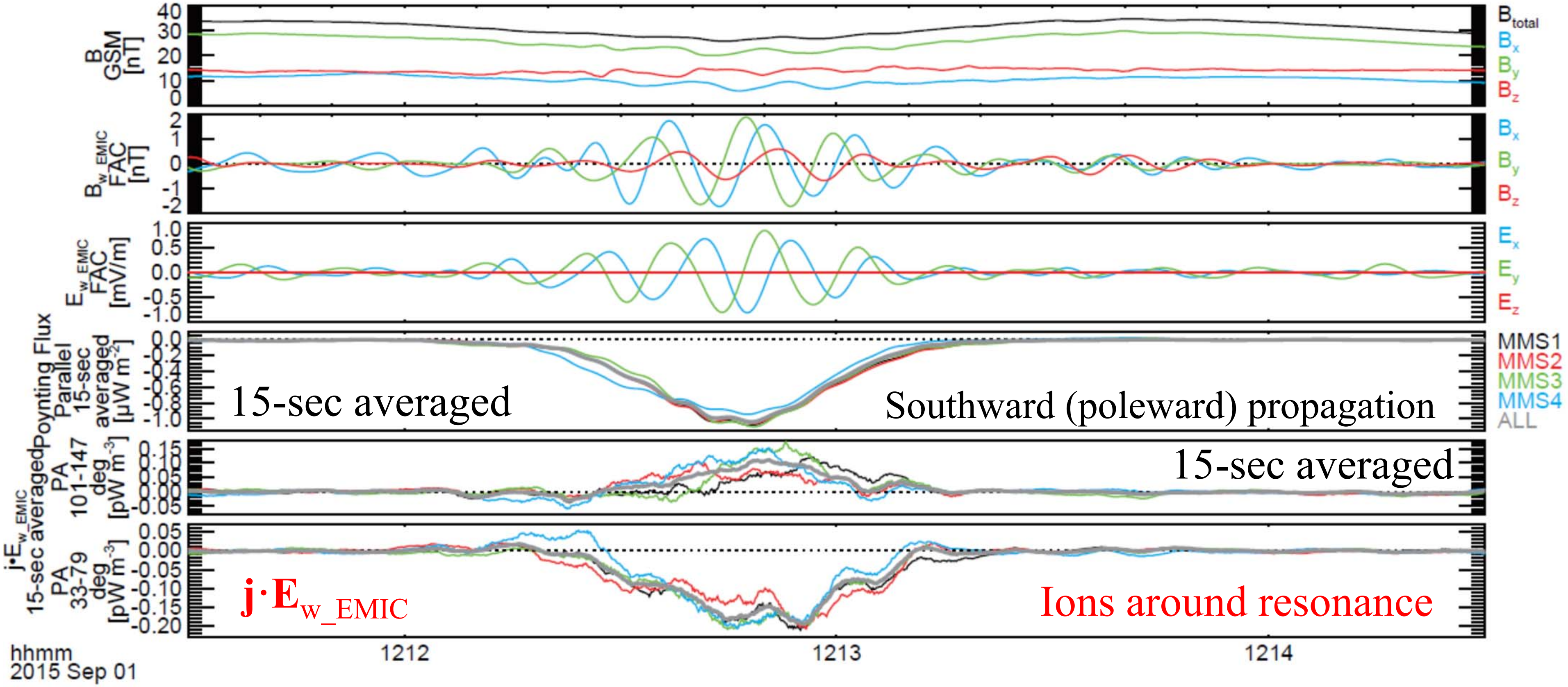
q: Electric charge
m: Mass
$\mathbf{v}$: Velocity
n: Number density (partial)
$\mathbf{j}$: Current density (partial)
$\mathbf{E}_{\mathrm{w}}$: Wave electric field

Frozen-in condition is assumed for electrons.

We derived $\mathbf{j}_{\mathrm{ion}}\cdot\mathbf{E}_{\mathrm{w}}$ using ion current density ($\mathbf{j} = q\mathbf{j'}$ ($\mathbf{j'}_{\mathrm{ion}}$: Number flux of ions)) in multiple ranges of energy (ε), and pitch angles (PA, α).

3. Result

Event1



15-sec averaged

Southward (poleward) propagation

15-sec averaged

$\mathbf{j} \cdot \mathbf{E}_{w\_EMIC}$

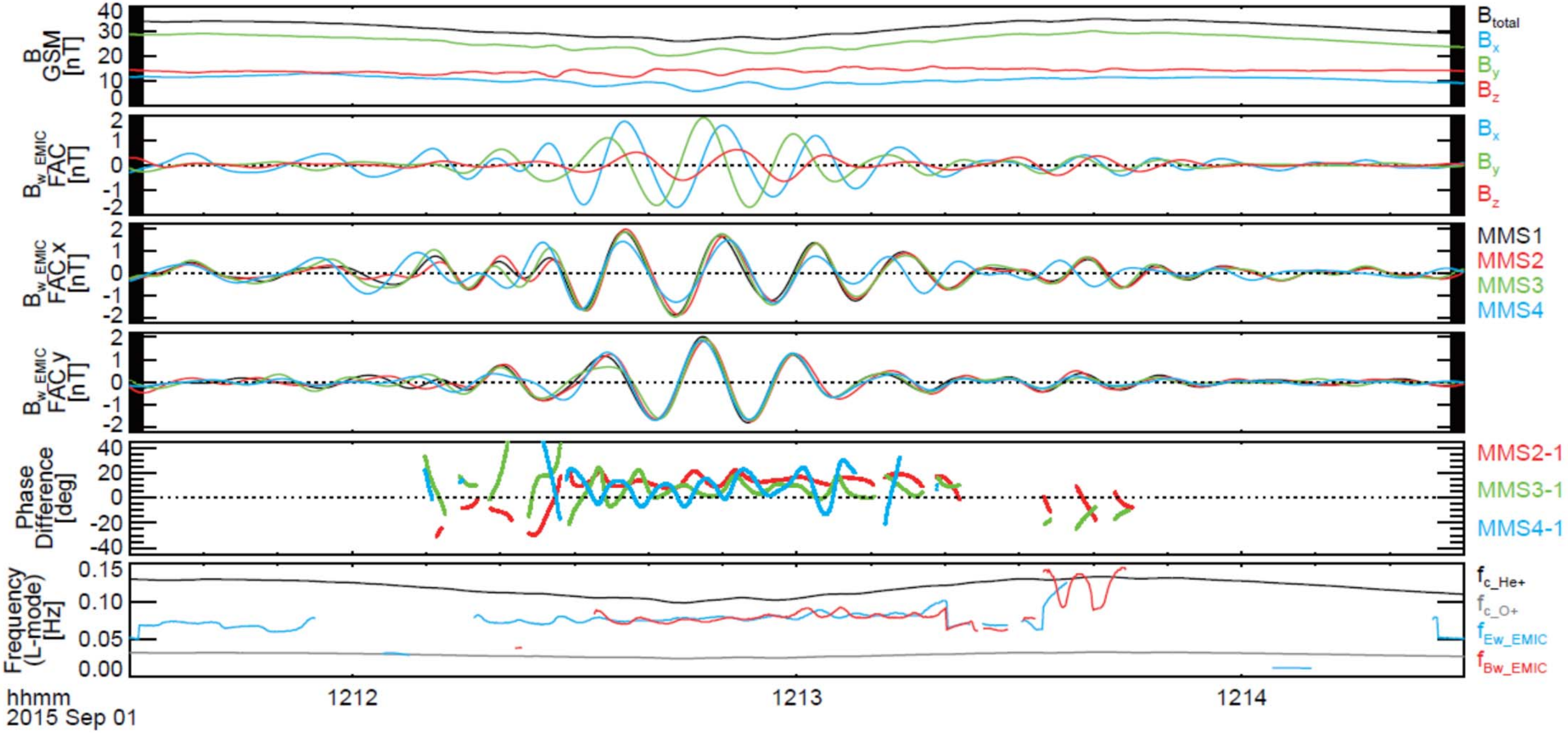
Ions around resonance

- Negatve Poynting flux parallel to $\mathbf{B}_0$

→Almost anti-parallel propagation

- Negatve $\mathbf{j} \cdot \mathbf{E}_{w\_EMIC}$ near the cyclotron resonance velocity

Event1



12:12:33.82–12:12:59.83 (2 rotation of $\mathbf{B}_{w\_EMIC}$)

Wave normal angle: ~22° (MVA)

Wavelength: ~4000 km (para: ~4300 km, perp: ~11,000 km)

(phase difference of MMS1-2)

Phase velocity (para): ~−330 km/s (PA: ~99° (18–30 keV))

Cyclotron resonance velocity: ~1370 km/s (PA: 42°–55° (18–30 keV))

A single plane wave approximation did not hold even at small spatial scales.

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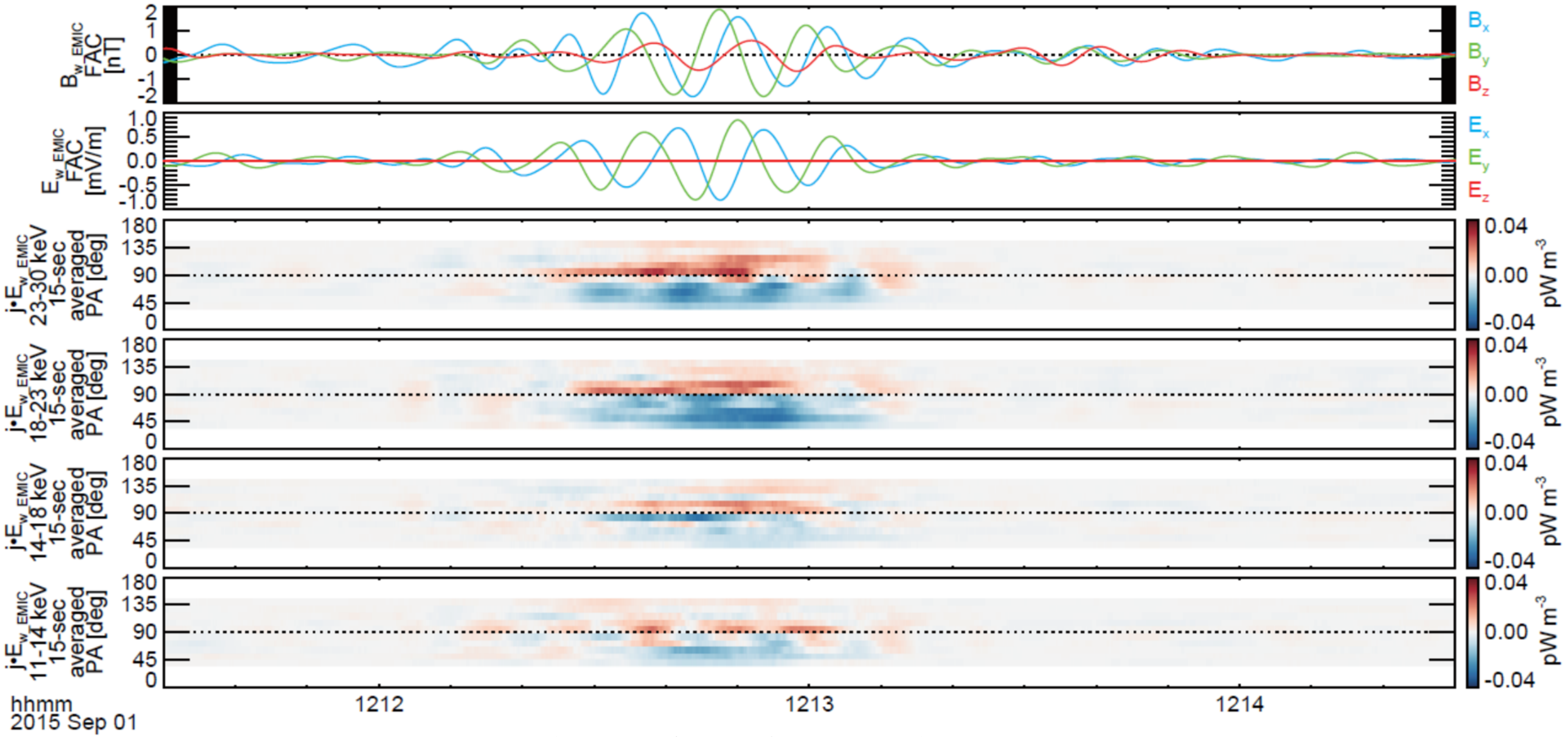

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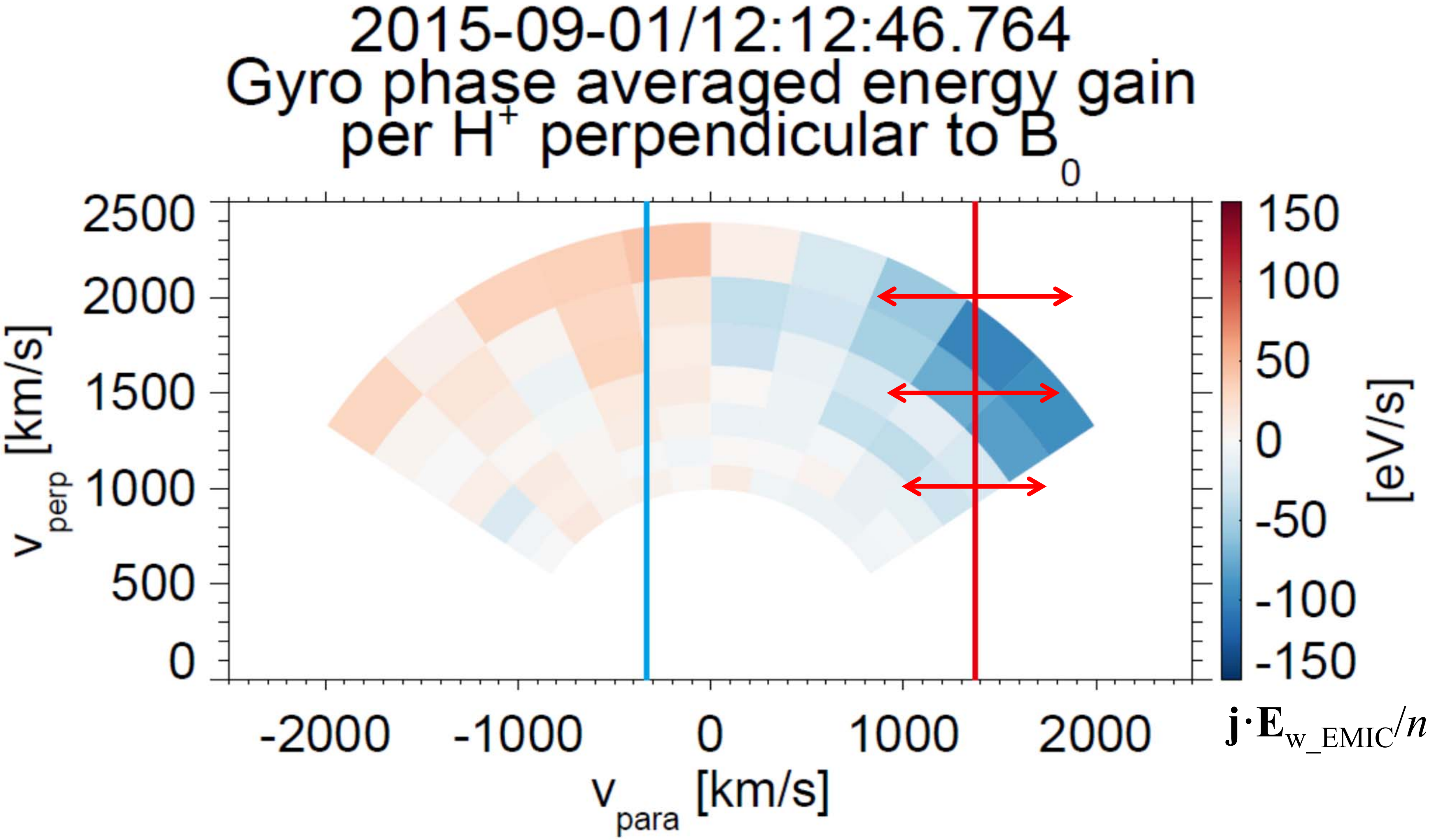
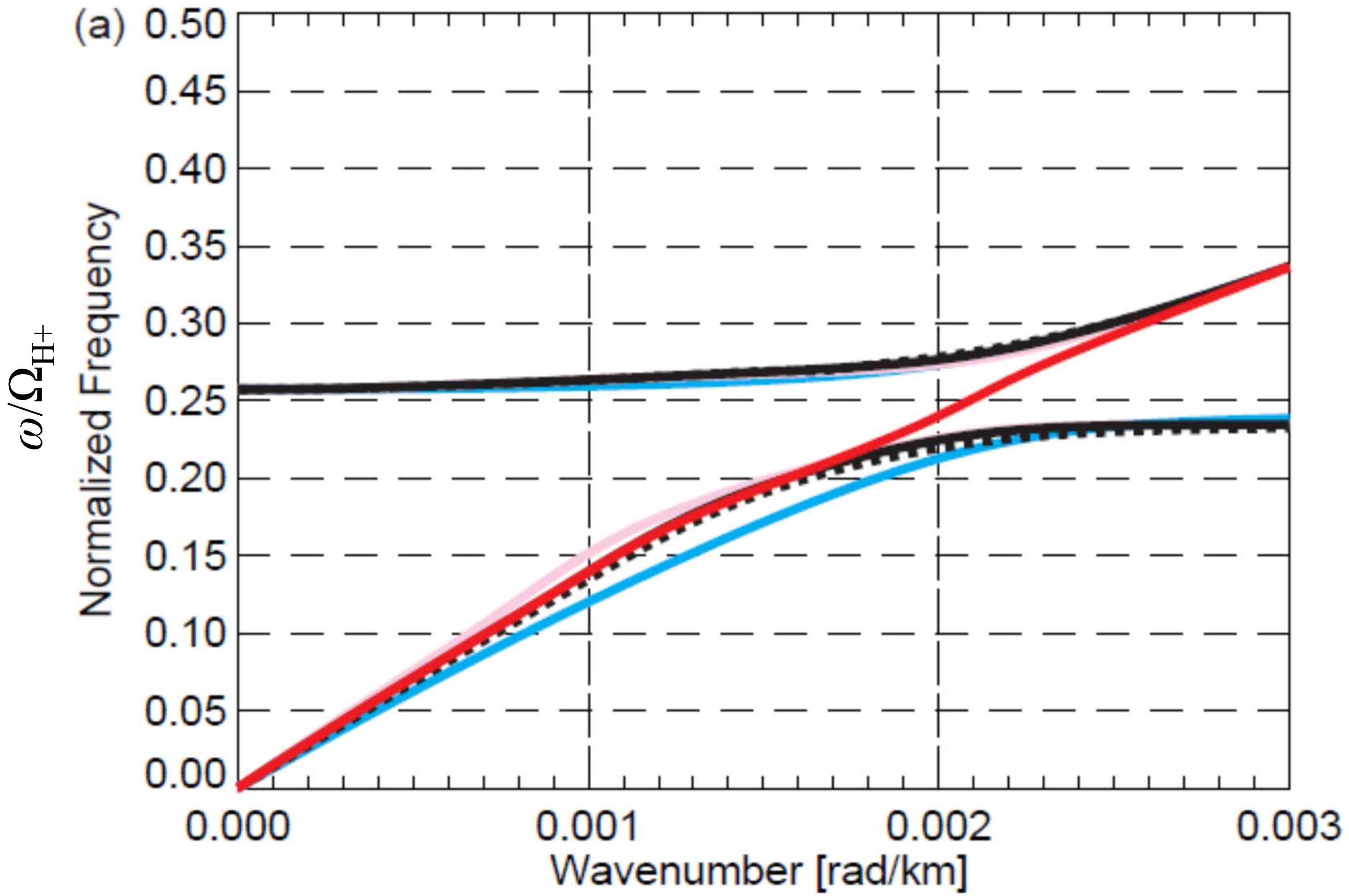


2015-09-01/12:12:46.764
Gyro phase averaged energy gain per H^+ perpendicular to B_0

$\mathbf{j} \cdot \mathbf{E}_{\mathrm{w\_EMIC}}/n$

Phase velocity (para): ~−330 km/s (PA: ~99° (18–30 keV))
Cyclotron resonance velocity: ~1370 km/s (PA: 42°–55° (18–30 keV))
$\omega_{tr}/k_{||}$ (v_{perp} = 1000, 1500, 2000 km/s): 360, 440, 510 km/s

◆ Dispersion relation (KUPDAP)



B: 26 nT
Plasma density: 3.0 /cm^3
Cold H$^+$: 81%, 99% (T = 1 eV)
He$^+$: 1% (T = 1 eV, 10 eV, 100 eV)
T_e = 1 eV

Hot H$^+$: 18%, 0% (from DIS (0.3–30 keV))
(T_{para} = 3.7 keV, T_{perp} = 5.6 keV)

The effect of hot H$^+$ reduces (~10%) the wave number around 0.2Ω_{H+}.

$$\frac{d^2\zeta}{dt^2} = \omega_{tr}^2(\sin\zeta + S)$$

$$S = \frac{1}{s_0\omega\Omega_w}\left(\cancel{s_1\frac{\partial\omega}{\partial t}} + V_P s_2\frac{\partial\Omega_H}{\partial h}\right)$$

$$s_0 = \frac{v_{perp}}{V_P}$$

$$s_2 = \left(\frac{v_{perp}^2}{2V_P^2} + \frac{V_R^2}{V_P V_G}\right)\frac{\omega}{\Omega_H} - \frac{V_R}{V_P}$$

Constant density along field line

$$s_2 = \left(\frac{v_{perp}^2}{2V_P^2} + \frac{V_R^2}{V_P V_G} - \frac{V_R^2}{2V_P^2}\right)\frac{\omega}{\Omega_H} - \frac{V_R}{V_P}$$

Density proportional to B

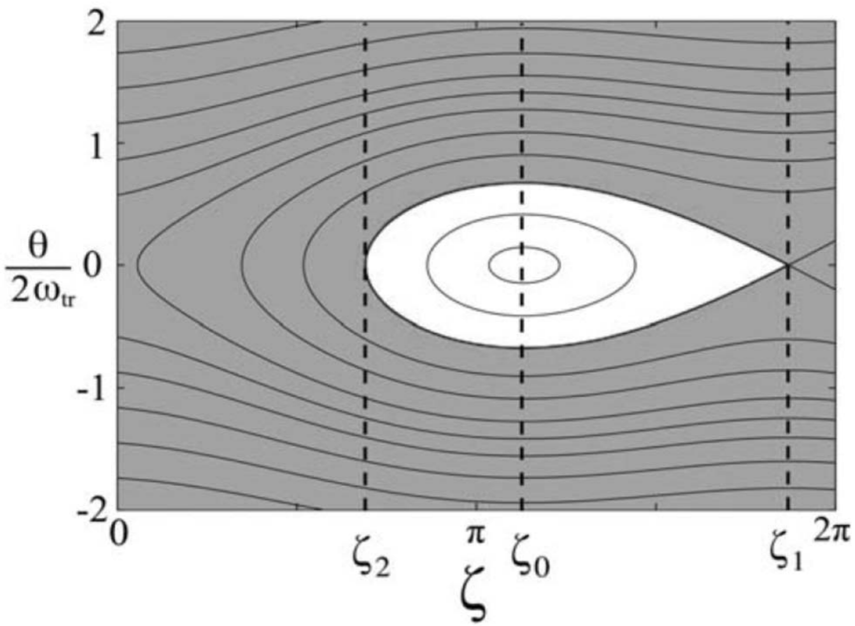


Figure 2. Trajectories of resonant protons in the $(\theta - \zeta)$ phase space for the inhomogeneity ratio S = 0.4. The phase angle ζ_0 is the center of trapping motion, while ζ_1 is the saddle point and ζ_2 is the boundary of the trapping region at $\theta = 0$.

Phase velocity (V_P): −330 km/s
Cyclotron resonance velocity (V_R): 1370 km/s
Group velocity (V_G): −173 km/s
B, B_{w_EMIC}: 26, 2 nT
dB/dh = 1.0±0.67 pT/km

The inhomogeneity factor S may be smaller than 1.

S

(v_{perp}: 1000 km/s): 0.60–3.02 (0.51–2.59)
(v_{perp}: 1500 km/s): 0.44–2.22 (0.38–1.92)
(v_{perp}: 2000 km/s): 0.37–1.87 (0.33–1.65)