Kinetic Entropy-Based Measures of Distribution Function Non-Maxwellianity: Theory and Simulations

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Abstract and Conclusions (Liang et al., in press)

- We investigate kinetic entropy-based measures of the non-Maxwellianity of distribution functions in plasmas
- We assess the properties of a form previously employed by Kaufmann and Paterson and derive analytical expressions for three common non-Maxwellian plasma distribution functions
- We show that there are undesirable features of this measure and elucidate the reason
- We introduce a new non-Maxwellianity measure based on the velocity-space kinetic entropy density
- We use collisionless particle-in-cell simulations of two-dimensional anti-parallel magnetic reconnection to assess the kinetic entropy-based non-Maxwellianity measures
- We show that regions of non-zero non-Maxwellianity are linked to kinetic processes occurring during magnetic reconnection
- The simulated non-Maxwellianity agrees reasonably well with predictions for distributions resembling those calculated

Kaufmann and Paterson non-Maxwellianity

- At a given position and time, consider a local distribution function f
- The Maxwellianized distribution is f_M defined as

$$f_M(\vec{v}) = n\left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-m(\vec{v}-\vec{u})^2/2k_B T}$$

where n, u, and T are the density, bulk flow velocity, and scalar temperature derived from f

- The (continuous) kinetic entropy density s is

$$s = -k_B \int d^3v f(\vec{v}) \ln f(\vec{v})$$

- Then, the Kaufmann and Paterson non-Maxwellianity is

$$\bar{M}_{KP} = \frac{s_M - s}{(3/2)k_B n}$$

- Properties: dimensionless, zero only for Maxwellians, non-negative, in collisional systems, measure of irreversible entropy production

Analytic Values of $\bar{M}_{KP}$

- We calculate $\bar{M}_{KP}$ for some common non-Maxwellian distributions
  - Two beams separated in velocity space

$$\bar{M}_{KP,beam} \simeq \ln\left(\frac{T_{beam}/n^{2/3}}{(T_1/n_1^{2/3})^{n_1/n}(T_2/n_2^{2/3})^{n_2/n}}\right)$$

$$T_{beam} = \frac{m n_1 n_2}{3k_B(n_1+n_2)^2}(u_{z1}-u_{z2})^2 + \frac{n_1T_1 + n_2T_2}{n_1+n_2}$$

  - Bi-Maxwellian distributions

$$\bar{M}_{KP,biM} = \ln\left(\frac{T}{T_\perp^{2/3}T_\parallel^{1/3}}\right) = \ln\left[\frac{2}{3}\left(\frac{T_\perp}{T_\parallel}\right)^{1/3} + \frac{1}{3}\left(\frac{T_\parallel}{T_\perp}\right)^{2/3}\right]$$

  - "Egedal" distributions (e.g., Egedal et al., 2013)

$$s_{Eg} = \frac{3}{2}k_B n\left[\frac{n_\infty G}{n} + \ln\left(\frac{2\pi k_B T_\infty}{m n_\infty^{2/3}}\right)\right]$$

$$G = 2b\sqrt{\frac{\Phi}{\pi}} + \left(1-\frac{2\Phi}{3}\right)\mathrm{erfcx}\left(\sqrt{\Phi}\right) - \sqrt{1-b}\left(1-b-\frac{2\Phi}{3}\right)\mathrm{erfcx}\left(\sqrt{\frac{\Phi}{1-b}}\right)$$

A Problem and a Resolution

- The problem: One can see from the above forms that the non-Maxwellianity can diverge in various limits, including the temperature going to 0 or ∞
  - This is not only an issue of non-Maxwellianity, but of the kinetic entropy density itself!
  - This makes interpreting non-Maxwellianity challenging
- The cause of the problem: a fixed velocity space grid scale is no longer appropriate when the temperature goes to 0 or ∞
- A resolution: instead of using the kinetic entropy density, use the velocity space kinetic entropy density (Liang et al., 2019)

$$s_{\mathrm{velocity}} = k_B\left[n\ln\left(\frac{n}{\Delta^3 v}\right) - \int d^3v f(\vec{v})\ln f(\vec{v})\right] = k_B n \ln\left(\frac{n}{\Delta^3 v}\right) + s.$$

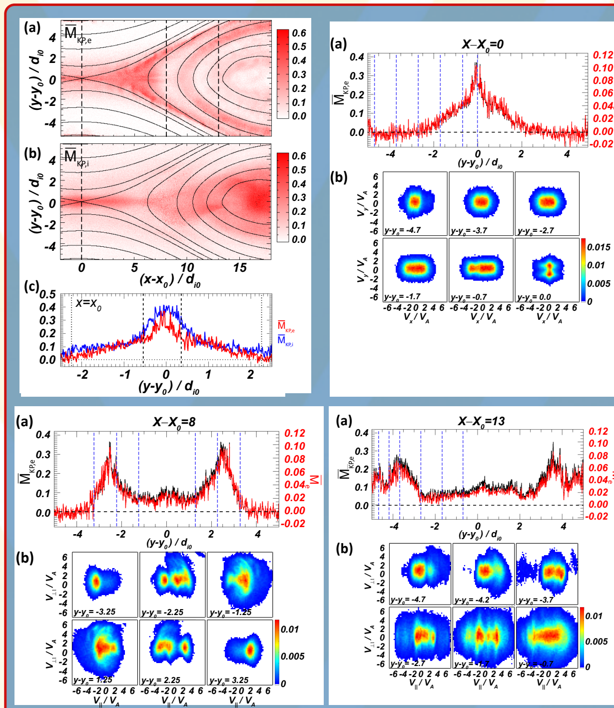
  - This quantity only considers permutations of particles in a single phase space cell rather than the whole domain; it has an explicit dependence on velocity space grid scale, as entropy should
- We define a new non-Maxwellianity measure as

$$\bar{M} = \frac{s_{\mathrm{velocity},M} - s_{\mathrm{velocity}}}{s_{\mathrm{velocity},M}} = \frac{\bar{M}_{KP}}{1+\ln\left(\frac{2\pi k_B T}{m(\Delta^3 v)^{2/3}}\right)}$$

  - This has the additional property that it is between 0 and 1 for properly defined velocity space grid scales

Comparison to PIC Simulations

- We investigate non-Maxwellianity in 2D PIC simulations using the P3D code (Zeiler et al., 2002)
- We validate the implementation: (top left plot) the electron and ion non-Maxwellianity go to zero upstream as they should, and are non-zero in the EDR and IDR and in the magnetic island
- In a vertical cut through the X-line (top right), the electrons are Maxwellian far from the current sheet, become Egedal-like distributions in the IDR, and undergo bouncing orbits in the EDR



- Downstream (lower left), the electrons display two beams near the separatrix
- Further downstream (lower right), electrons in the island are nearly bi-Maxwellian as a result of Fermi acceleration in the contracting island
- We compare these three distributions to the analytical predictions - each are within about 20%, even though non-Maxwellianities themselves differ by a factor of 4

References

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