Polynomial reconstruction of the magnetic field with inferred structure velocity

– Richard Denton, Yi-Hsin Liu, Hiroshi Hasegawa & others

The original idea

- MMS Jp_{sc} from FPI is very accurate
- Find quadratic B_{model} from fits to B_{sc} and Jp_{sc} (assuming $Jp = \nabla \times B_{model}$)
- Conditions to keep $\nabla \cdot B_{model} = 0$
- Usually use a 17 parameter (RQ-3D) model motivated by the ordering $\partial/\partial n \gg \partial/\partial l \gg \partial/\partial m$ where n, l, and m are the directions of maximum, intermediate, and minimum gradient
- Find best least-squares fit for model coefficients

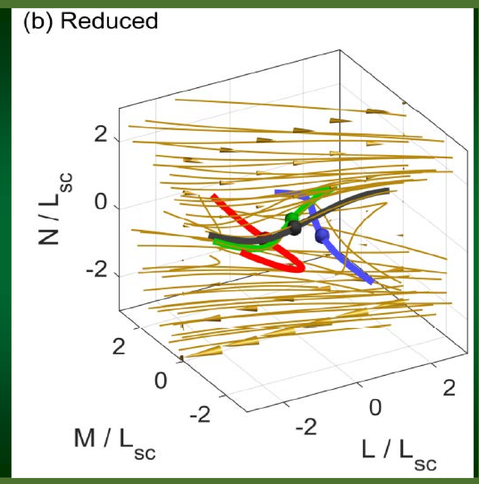
Equations – RQ-3D model

$$
\begin{aligned}
B_l &= B_{l,0} + \frac{\partial B_l}{\partial n} n + \frac{\partial B_l}{\partial l} l + \frac{\partial B_l}{\partial m} m + \frac{\partial^2 B_l}{\partial n^2} \frac{n^2}{2} \\
B_m &= B_{m,0} + \frac{\partial B_m}{\partial n} n + \frac{\partial B_m}{\partial l} l + \frac{\partial B_m}{\partial m} m \\
&\quad + \frac{\partial^2 B_m}{\partial n^2} \frac{n^2}{2} + \frac{\partial^2 B_m}{\partial n \partial l} nl + \frac{\partial^2 B_m}{\partial l^2} \frac{l^2}{2} \\
B_n &= B_{n,0} + \frac{\partial B_n}{\partial n} n + \frac{\partial B_n}{\partial l} l + \frac{\partial B_n}{\partial m} m + \frac{\partial^2 B_n}{\partial l^2} \frac{l^2}{2}
\end{aligned}
$$

$$
\frac{\partial B_n}{\partial n} + \frac{\partial B_l}{\partial l} + \frac{\partial B_m}{\partial m} = 0
$$

$$
\begin{aligned}
\mu_0 J_l &= \frac{\partial B_n}{\partial m} - \left(\frac{\partial B_m}{\partial n} + \frac{\partial^2 B_m}{\partial n^2} n + \frac{\partial^2 B_m}{\partial n \partial l} l \right) \\
\mu_0 J_m &= \frac{\partial B_l}{\partial n} + \frac{\partial^2 B_l}{\partial n^2} n - \left(\frac{\partial B_n}{\partial l} + \frac{\partial^2 B_n}{\partial l^2} l \right) \\
\mu_0 J_n &= \frac{\partial B_m}{\partial l} + \frac{\partial^2 B_m}{\partial n \partial l} n + \frac{\partial^2 B_m}{\partial l^2} l - \frac{\partial B_l}{\partial m}
\end{aligned}
$$

3D View at t = 31.92 s

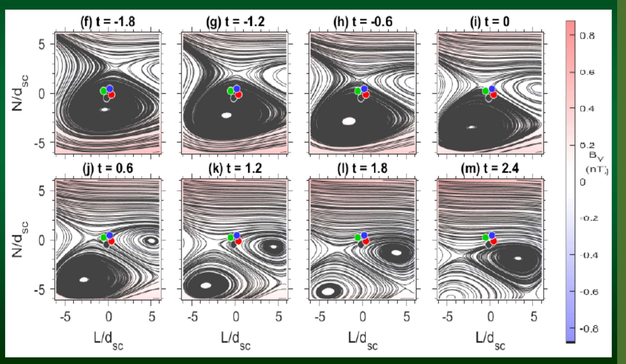


[Denton et al., JGR, 2020]

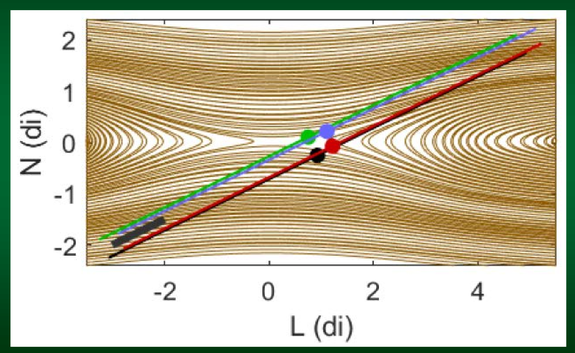
Extend to multiple times

- In the original scheme, the reconstruction is done at each particular time, yielding the reconstructed magnetic field at that time
- Here, we use a sequence of times assuming that a stationary magnetic structure is convecting past the spacecraft with a constant velocity
- We vary the structure velocity and polynomial coefficients of the reconstruction in order to get a best least-squares fit to the magnetic field and particle current density observed by the spacecraft
- By this means, we find not only the reconstructed magnetic field, but also the optimal structure velocity relative to the spacecraft

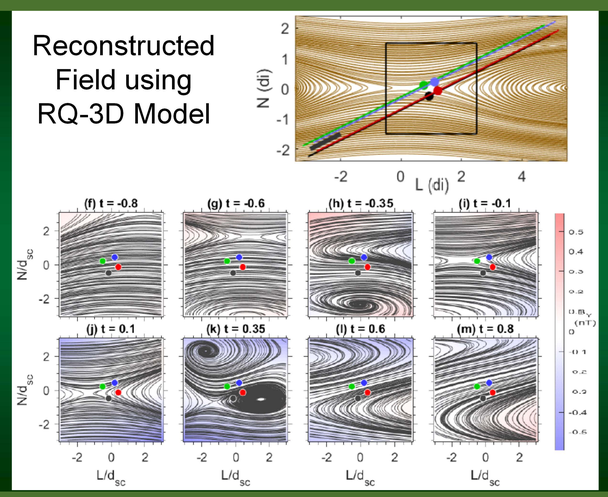
Analytical quadratic model exactly reconstructed with correct velocity



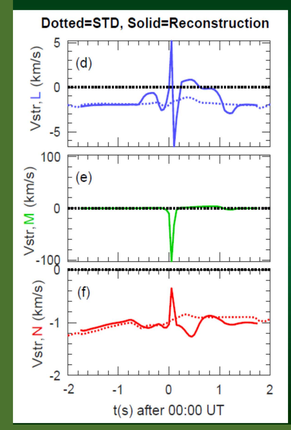
One of Yi-Hsin Liu's 3D anti-parallel simulations with virtual spacecraft trajectories



Reconstructed Field using RQ-3D Model

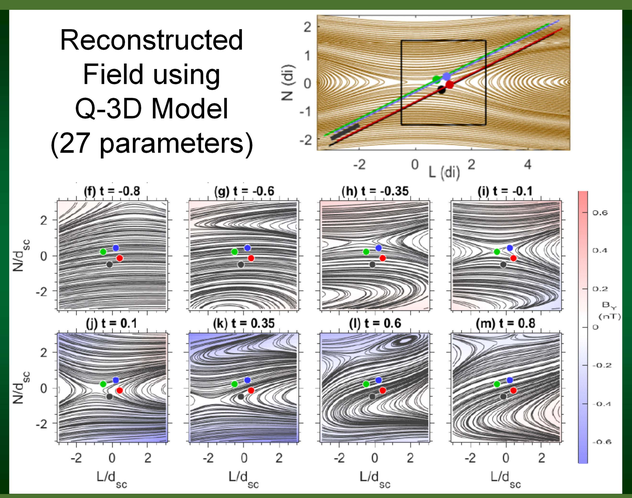


Inferred structure velocity

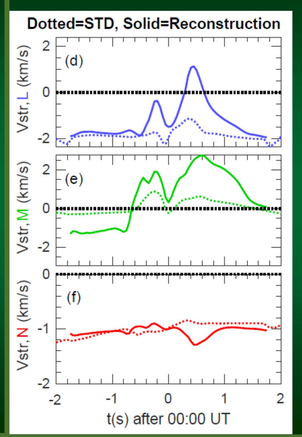


Compo-nent	Exact Velocity	Median Inferred Velocity	Standard Deviation of Inferred Velocity
L	-2	-1.97	± 1.4
M	-1	-0.2	± 13
N	-1	-1.03	± 0.12

Reconstructed Field using Q-3D Model (27 parameters)



Inferred structure velocity – Q3D model



Compo-nent	Exact Velocity	Median Inferred Velocity	Standard Deviation of Inferred Velocity
L	-2	- 1.63	± 0.79
M	-1	0.85	± 1.38
N	-1	-1.03	± 0.08

Conclusions

- The goal is to reconstruct the magnetic field and get the structure velocity at the same time
- We get exactly the right reconstruction fields and velocity for a test case with a quadratic field
- Results using simulation data yield reasonable reconstructions, though with some unrealistic features. The velocity in the maximum and intermediate gradient direction can be determined, though with some fluctuations

References

- Torbert, R. B., Dors, I., Argall, M. R., Genestreti, K. J., Burch, J. L., Farrugia, C. J., et al. (2020). A New Method of 3D Magnetic Field Reconstruction. *Geophysical Research Letters*, *47*, e2019GL085542. https://doi.org/10.1029/2019GL085542
- Denton, R. E., R. B. Torbert, H. Hasegawa,,I.Dors, K. J. Genestreti, et al. (2020), Polynomial reconstruction of the reconnection magnetic field observed by multiple spacecraft, J. Geophys. Res. Space Physics, doi:10.1029/2019JA027481.
- Denton, R. E., R. B. Torbert, H. Hasegawa, K. J. Genestreti, R. Manuzzo, et al. (2021), Two-dimensional velocity of the magnetic structure observed on 11 July 2017 by the Magnetospheric Multiscale spacecraft, J. Geophys. Res. Space Physics, doi:10.1029/2020JA028705.