JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 86, NO. A10, PAGES 8485–8490, SEPTEMBER 30, 1981

Factors Governing the Ratio of Inward to Outward Diffusing Flux of Satellite Ions

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The primary loss process for the ionized component of satellite debris in Jupiter's magnetosphere appears to be radial transport, which carries the satellite-derived ions away from the source satellite. Cross L (interchange) diffusion is a likely candidate for the radial transport mechanism. Many consequences are predicted to result from satellite ions that reach regions close to and remote from Jupiter. The ratio of the inward to outward flux of diffusing ions from a given satellite is therefore important as an input to the estimates of the magnitudes of expected consequences. In the formal analysis of cross L diffusion, the flux ratio depends on poorly known boundary conditions and on a poorly determined diffusion coefficient. We present here an analysis of the sensitivity of the flux ratio to the uncertainties in the mentioned parameters. The main result is that if the diffusion is driven externally, for example by winds in Jupiter's ionosphere, the inward flux exceeds or is of the same order of magnitude as the outward flux. On the other hand, if the diffusion is driven locally by the centrifugal interchange instability, as one interpretation of Voyager plasma data suggests, the outward flux can be one or two orders of magnitude greater than the inward flux. Fits to the Voyager in situ plasma torus data obtained from solutions to the transport equations appropriate to wind driven and centrifugally driven diffusion are compared. In both cases, good fits are obtained. Both require a substantial discontinuity in the diffusion coefficient at Io's orbit, and both require an onset of ion injection a finite time prior to Voyager encounter. The values of the source strength, the duration of injection, and the percentage of inward flux transport, determined thereby differ by a factor of four or less between the two types of diffusion.

INTRODUCTION

Io is known to be a source of heavy ions, principally various charge states of sulfur and oxygen. These are dispersed through the combined action of corotation and radial transport to create a distinctive plasma formation within the Jovian magnetosphere. The Io-derived plasma feature has been variously described as a sulfur-ion nebula, a plasma torus and a plasma disk. Each designation reflects the particular way in which this spatially distributed, non-homogeneous structure is sensed by different detectors, namely, ground-based optical telescopes, the Voyager ultraviolet spectrometer and the Voyager in situ plasma detector.

The other Galilean satellites also might be sources of ions that disperse to form satellite-derived magnetospheric plasma formations. While the spread of satellite ions in azimuth is believed to be a simple kinematical consequence of enforced corotation, and the spread in latitude to be controlled and confined by the centrifugal force, the spread in radius is generally assumed to result from radial 'cross L' diffusion, for which the operative equation is [*Fälthammar*, 1968]

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial L} \frac{D_{LL}}{L^2} \frac{\partial (NL^2)}{\partial L} + S - R \tag{1}$$

Here N is the total number of ions in a magnetic L shell per unit L, L is the equatorial crossing distance of the flux-shell in units of Jovian radii, D_{LL} is the diffusion coefficient, and S and R are the appropriate source and loss terms.

Equation (1) governs the type of cross L diffusion referred to as interchange diffusion. It is expected to dominate over single particle diffusion in the same way that eddy diffusion in an atmosphere dominates over molecular diffusion. In this process two radially adjacent flux tubes containing equal magnetic flux are envisioned to rotate about the axis forming their common border until they have exchanged places. If the number of particles per unit magnetic flux in the ambient plasma decreases outward, the exchange of flux tubes just described results in a net transport of mass outward. Diffusive transport occurs if there are many pairs of interchanging flux tubes, which continually form and disappear, and if they are dispersed randomly throughout the medium.

Alternatively, the interchange motion could be organized into one or two large, permanent cells. The resulting radial mass transport would then be described as convective. Convective transport in Jupiter's magnetosphere driven by the solar wind, such as occurs in earth's magnetosphere, was considered by *Brice and Ionnidis* [1970]; and convective transport driven by the centrifugal force acting on a non-axially symmetric injection of mass from Io was considered by *Chen* [1977]. Both investigations lead to the conclusion that diffusion should dominate organized convection.

However, *Vasyliunas and Dessler* [this issue] argue that a number of magnetospheric phenomena are explained if the existence of a substantial deviation from axial symmetry in magnetospheric mass, corotating with Jupiter, is accepted. One possible consequence of such an 'anomaly model' is the existence of organized convection as just described. In the absence of a definitive answer to questions of whether diffusion or convection dominates in radial mass transport, it is desirable that the consequences of both mechanisms be explored separately in terms of mathematical models, in order eventually to determine which is better able to account for the observations. This paper is concerned with exploring options that exist solely within the framework of diffusive transport. All of the assumptions appropriate to that hypothesis will naturally therefore be adopted.

Applicability of (1) requires an axially symmetric distribution of ions. This is compatible with the expected ordering of the three relevant transport time scales. Namely, the time scale for latitudinal dispersion (a particle bounce time) is less than the time for rotational dispersion (the planetary rotation period in the frame of reference of the source), which in turn

Paper number 80A1587.
0148-0227/81/080A-1587$01.00

is less than the time for radial dispersion (the diffusion time scale).

The source term S is assumed to be nonzero only at the L value of the source satellite. It may be a function of time. As an additional prerequisite for axial symmetry, time variations in the source must be slower than the time scale of rotational dispersion.

In the region $L > 2$, the component of centrifugal force acting parallel to the magnetic field very effectively inhibits loss of ions into Jupiter's atmosphere by means of precipitation. The magnetic mirror force restricts precipitation loss at lesser distances. Recombination is therefore the only distributed loss mechanism that might in principle act. However, all transport time scales appear to be much less than the time scale for recombination. The loss term R in (1) is, therefore, neglected.

Given a particular source function, the solution to (1) depends on two boundary conditions and the explicit form for the diffusion coefficient. Although many studies have attempted to specify these factors, and as a result the a priori possibilities have been greatly narrowed, there still remain significant uncertainties in both the applicable boundary conditions and the applicable diffusion coefficient.

It is the purpose of this article to explore the sensitivity of the solutions to (1) to variations in the uncertain factors over a reasonable range as defined below. We direct the analysis especially to a determination of the uncertainty in the percentages of the total ions that diffuse inward and outward from the source.

The result of this study provides a simple yet significant measure of effects of variations in model parameters. Furthermore, the ratio of inward to outward fluxes is important to studies concerned with interactions involving satellite ions. For example, inward diffusing ions and electrons may interact with particles in Jupiter's ring, and with Amalthea, and they might contribute to chemical processes in the Jovian upper atmosphere [*Strobel and Yung,* 1979]. The determination of the interaction between the inward diffusing electrons and the Io neutral sodium cloud remains a major unsolved problem, (W. H. Smyth, private communication, 1980). Its solution requires knowledge of the inward flux of electrons. Outward diffusing ions are important to the dynamics of the outer magnetosphere [*Hill and Michel,* 1976; *Eviatar et al.,* 1978; *Hill,* 1980], and they can contribute to surface physics and chemistry at other satellites [*Eviatar et al.,* 1981].

NATURE AND RANGE OF UNCERTAINTIES IN THE DIFFUSION PARAMETERS

Two boundary conditions are needed to fix the integration constants in the solution of (1). It seems apparent that if distributed recombination is ignored, all inward diffusing ions are lost somewhere in the region inside of $L = 2$. Complete absorption at the ring is one possibility ($N(L = 1.8) = 0$). Another is complete absorption in Jupiter's upper atmosphere ($N(L = 1) = 0$), which would ignore ion losses to the ring.

The outer boundary condition is physically less well defined. Again we ignore distributed recombination, field-aligned precipitation, and also for the present satellite sweep-up. Then the outward moving ions are obliged to diffuse until they reach a distance where they can be carried off by some other, more direct transport process. The boundary condition has not been identified that properly joins the diffusion region to the remote domain of direct transport. In the absence of an adequate treatment of this problem, we simply terminate the region at a perfectly absorbing boundary located near the expected transition. The location of the boundary is considered to be a model parameter that can be varied to test the sensitivity of the solutions to the uncertainty in the outer boundary condition.

The indeterminancy in the diffusion coefficient can be expressed by means of uncertainties in the parameters that enter into its mathematical representation. Theoretical treatments of specific mechanisms that produce the kind of diffusion envisioned to be operative here lead to the following generic form for the diffusion coefficient (see, for example, *Goertz et al.* [1979])

$$D_{LL} = kL^m \tag{2}$$

where k and m are constants that depend respectively upon the strength and type of the mechanism considered. Diffusion driven by fluctuations in the solar wind and in magnetospheric convection are characterized by large values of m, namely in the range 6–10. However, these processes have been found to be inadequate to account for indirectly inferred diffusion rates. Diffusion driven by fluctuating winds in the upper atmosphere of Jupiter is characterized by a low value for m, which is often taken to be 3 as predicted for an idealized model. In actuality it could be smaller or larger. This process appears capable of satisfying some estimates of the required diffusion rates [*Goertz et al.,* 1979].

Attempts to infer the value of m empirically from the intensity of the Jovian decimetric radio emission and the magnitude of absorption suffered by energetic particles as they cross Io's orbit have returned values of m ranging from −0.3 to 4. In a recent review of the particle absorption studies, *Goertz et al.* [1979] give the uncertainty in m as 2.5 ± .5. They also present a new determination that gives added preference to the value $m = 3$. These determinations refer to a region extending toward Jupiter from about the orbit of Io at $L = 6$ and primarily to the time of the Jupiter encounter of Pioneer 10. On the other hand, the density gradient in the region $L > 6$, as obtained from the Voyager 1 ultraviolet spectrometer experiment and the planetary radio experiment, are consistent with a steady state solution of (1) with $m = 4$ to 5 [*Froidevaux,* 1980].

Detailed analysis of the Voyager 1 low-energy plasma data also revealed a marked transition in the character of the plasma across the orbit of Io [*Bagenal et al.,* 1980; *Richardson et al.,* 1980]. *Richardson et al.* [1980] concluded on the basis of a comparison of the radial gradients of the plasma density inside and outside of Io's orbit that the source of the plasma was time dependent and that at the time of Voyager 1 encounter the diffusion coefficient operating to transport material outward ($L > 6$) was much greater than the one transporting material inward ($L < 6$).

A discontinuity in D_{LL} at Io's orbit is consistent with diffusion driven by the centrifugal force acting on the torus material itself [*Richardson et al.,* 1980]. This type of diffusion acts to transport plasma primarily outward from Jupiter. In the case of centrifugally driven diffusion, the diffusion coefficient in the domain of outward transport takes the form

$$(D_{LL})_\Omega = -K_\Omega L^{4+p} \frac{d(NL^2)}{dL} \tag{3}$$

where k_Ω is a constant and the parameter p was found to fall in the range $2 \leq p \leq 4$ at the time of the Voyager 1 encounter

[*Siscoe and Summers*, this issue]. In the domain of inward transport, inside of Io's orbit, diffusion must be driven indirectly as previously considered to be true at all distances, and for which the diffusion coefficient is given by equation (2).

While the situation at the time of the Voyager encounters was probably characterized by a discontinuity in diffusion properties as just described, the situation as regards the plasma torus at the time of the Pioneer encounter was evidently quite different [e.g., *Broadfoot et al.*, 1979]. We therefore have explored the sensitivity of the inward to outward flux ratio for a wide variety of conditions, and the results are given in the next section.

$$\frac{F_I}{F_O} = \frac{(L_O^{3-m_O} - L_S^{3-m_O})/[(3-m_O)k_O]}{(L_{Io}^{3-m_I} - L_I^{3-m_I})/[(3-m_I)k_I] + (L_S^{3-m_O} - L_{Io}^{3-m_O})/[(3-m_O)k_O]} \qquad (m_{I,O} \neq 3) \qquad (8a)$$

FLUX RATIOS FOR TIME INDEPENDENT, LINEAR DIFFUSION

Consider first the simplest case in which there is no time dependence and the diffusion coefficient has the linear form (2) at all distances. We will, however, allow D_{LL} to be discontinuous at Io's orbit. Equation (1) is then solved analytically in the two domains of inward and outward transport, in both of which $\partial N/\partial t = 0$, $S = 0$, and $R = 0$. The results contain four constants of integration. The boundary conditions are $N_I(L_I) = 0$, $N_I(L_S) = N_O(L_S)$, and $N_O(L_O) = 0$, where the subscripts I and O identify parameters belonging to the intervals of inward and outward diffusion. L_I and L_O are the locations of the limits to the two domains. The limits are assumed to be fully absorbing, non-reflecting boundaries, in the sense that is used in diffusive transport problems. The two domains have a common boundary at the distance of the source satellite ($L = L_S$), where the ion density must be continuous.

The flux either inward or outward is given by

$$F_{I,O} = \frac{(D_{LL})_{I,O}}{L^2} \left| \frac{d(N_{I,O}L^2)}{dL} \right| \qquad (4)$$

The fourth condition needed to fix the integration constants is

$$F_I + F_O = F_T \qquad (5)$$

where F_T is the total flux of ions away from the source satellite. F_T is the net production rate of ions by the satellite, which we regard as a given parameter.

The problem is now completely specified; the algebra is readily carried out, and the results are as follows. If k and m are the same in regions I and O, we find

$$\frac{F_I}{F_O} = \frac{L_O^{3-m} - L_S^{3-m}}{L_S^{3-m} - L_I^{3-m}} \qquad (m \neq 3) \qquad (6a)$$

$$\frac{F_I}{F_O} = \frac{\ln(L_O/L_S)}{\ln(L_S/L_I)} \qquad (m = 3) \qquad (6b)$$

If k and m are discontinuous across $L = L_S$, then

$$\frac{F_I}{F_O} = \frac{3-m_I}{3-m_O}\frac{k_I}{k_O}\frac{L_O^{3-m_O} - L_S^{3-m_O}}{L_S^{3-m_I} - L_I^{3-m_I}} \qquad (m_{I,O} \neq 3) \qquad (7a)$$

$$\frac{F_I}{F_O} = \frac{k_I}{(3-m_O)k_O}\frac{L_O^{3-m_O} - L_S^{3-m_O}}{\ln(L_S/L_I)} \qquad \begin{pmatrix} m_I = 3 \\ m_O \neq 3 \end{pmatrix} \qquad (7b)$$

We consider also the possibility that D_{LL} is discontinuous at Io's orbit, $L = L_{Io}$, but that the source satellite of interest is one of the other Galilean satellites, then

$$\frac{F_I}{F_O} = \frac{(L_O^{3-m_O} - L_S^{3-m_O})/[(3-m_O)k_O]}{[\ln(L_{Io}/L_I)]/k_I + (L_S^{3-m_O} - L_{Io}^{3-m_O})/[(3-m_O)k_O]} \qquad \begin{pmatrix} m_I = 3 \\ m_O \neq 3 \end{pmatrix} \qquad (8b)$$

The sensitivity of the flux ratio to the uncertainty in the locations of the inner and outer boundaries may be determined with the use of (6*a*) and (6*b*). Table 1 displays an indicative range of results. The dependent parameter in the table is the inward flux expressed as a percentage of the total production rate of ions, F_T. The table consists of two parts. In one the outer boundary is held fixed at $L_O = 45$ and the inner boundary is put at three different distances; namely, $L_I = 1$, corresponding to loss in Jupiter's upper atmosphere, $L_I = 1.8$, corresponding to loss to the outer edge of Jupiter's ring, and $L_I = 4.5$, corresponding to loss by recombination relatively close to Io's orbit. In the other part of the table, the inner boundary is held fixed at $L_I = 1.8$, and the outer boundary is put successively at $L_O = 30$, 40, and 50. In this case where the diffusion coefficient is presumed to be continuous across $L = 6$, the flux ratio is independent of the parameter k. The value of m is taken to be 3.

By comparison with order of magnitude differences, the sensitivity of the percentage of inward flux to the locations of the boundaries is seen to be relatively little. With only one exception, the flux ratio F_I/F_T expressed as a percentage falls in the range 10–90. The greatest imbalances of inward to outward flux occur when one of the boundaries is quite close to the satellite, as in the case of Io with $L_I = 4.5$ and of Callisto with $L_O = 30$. In most of the other cases, roughly equal portions of the created ions diffuse inward and outward. Another way to express the relative insensitivity of F_I/F_T to boundary locations is to compare the minimum and maximum values of

TABLE 1. Effect of Boundary Location on the Percentage of Flux Going Inward

			Percentage of Flux Going Inward							
				L_I Variable				L_O Variable		
Satellite	m	L_S	L_O, Fixed	1.0	1.8	4.5	L_I, Fixed	30	40	50
Io	3	5.9	45	53.4	63.1	88.2	1.8	57.8	61.7	64.3
Europa	3	9.4	45	41.1	48.7	68.0	1.8	41.2	46.7	50.3
Ganymede	3	15.0	45	28.9	34.1	47.7	1.8	24.6	31.6	36.2
Callisto	3	26.3	45	14.1	16.7	23.3	1.8	4.7	13.5	19.3

TABLE 2. Effect of the Form of the Diffusion Coefficient on the Percentage of Flux Going Inward

				Percentage of Flux Going Inward											
				m, k Constant						m, l change at Io so D_{LL} is Continuous			m, k change at Io so D_{LL} out = 100 × D_{LL} in		
Satellite	L_I	L_S	L_O	0	1	2	3	4	5	0.5	2.5	3.5	0.5	2.5	3.5
Io	1.8	5.9	45	99.8	98.4	90.5	63.1	27.6	9.2	63.6	43.4	30.5	1.6	0.74	0.43
Europa	1.8	9.4	45	99.1	95.8	82.4	48.6	15.8	3.5	23.9	16.4	11.5	0.62	0.28	0.16
Ganymede	1.8	15.0	45	96.3	89.0	69.4	34.1	8.3	1.3	8.7	6.0	4.2	0.22	0.10	0.06
Callisto	1.8	26.3	45	80.0	65.9	43.3	16.7	3.0	0.3	2.1	1.4	1.0	0.06	0.03	0.01

this parameter in the table for a given satellite. For example, in the case of Io, the ratio of $(F_I/F_T)_{max}$ to $(F_I/F_T)_{min}$ is 1.65, or less than a factor of 2. Also in the case of Io, the ratio of $(F_O/F_T)_{max}$ to $(F_O/F_T)_{min}$ is 3.95. These numbers again illustrate that the uncertainty in either the inward or outward flux that results from the uncertainty in the boundary locations is considerably less than an order of magnitude.

Table 2 exhibits the sensitivity of F_I/F_T to the uncertainty in the parameters that specify the diffusion coefficient. The inner and outer boundaries are held fixed at $L_I = 1.8$ and $L_O = 45$. The dependence on the value of m is given in the first part of the table. Over the given range, $0 \leq m \leq 5$, the sensitivity is seen to be great, by comparison with the variability in F_I/F_T found in Table 1, which resulted from varying L_I and L_O. Small values of m favor inward transport, and large values outward transport. For example, in the case of Io with $m = 0$, the inward flux exceeds the outward flux by a factor approaching 500. However, with $m = 5$, the outward flux dominates by nearly a factor of 10. Again for Io, the ratio of (F_I/F_T) max to (F_I/F_T) min is 10.85, and the ratio of (F_O/F_T) max to (F_O/F_T) min is 454. The sensitivity of both the inward and outward flux corresponding to the uncertainty in the parameter m is thus seen to be more than an order of magnitude.

The other two parts of the table will be discussed in the next section.

DISCONTINUITIES IN THE DIFFUSION COEFFICIENT

Equations (7) and (8) give the inward to outward flux ratio for steady state, linear diffusion with discontinuities in k and m at a satellite orbit, $L = L_S$. Examples in which the discontinuity occurs only at Io's orbit and in which the inner and outer boundaries are held fixed as before are given in the second and third parts of Table 2. In both parts the discontinuities are expressed in terms of paired choices of m representing the values in the regions interior and exterior to Io. The inner values, 0, 2, and 3, are taken from the low end of the range of m. These are appropriate to the inner region based on interpretations of the Pioneer 10 and 11 data [e.g., *Goertz et al.*, 1979]. In the region exterior to Io, m is put equal to 5, as determined by *Froidevaux* [1980] on the basis of an interpretation of the Voyager data in the region $6 \leq L \leq 8$.

In the second part of the table, the discontinuity in the parameter k is adjusted to compensate for the discontinuity in m so that the diffusion coefficient itself suffers no discontinuity across Io's orbit. There is thus only a discontinuity in the L dependence of D_{LL} at Io's orbit. Comparison of the first and second parts of the table shows that the stronger L dependence for $L > L_{Io}$ greatly increases that fraction of the total number of new ions that diffuse outward. The change relative to the situation with no discontinuity at Io is greatest for small values of m in the inner region. For example, in the case of Io with $m_I = 0$, 0.2 percent of new ions diffuse outward if there is no discontinuity compared to 36.4% if there is the specified discontinuity, an increase by a factor of 182. The size of the change between the two parts of the table decreases with larger values of m_I, and, of course, it would go to zero when $m_I = m_O = 5$. Compared with the situation with no discontinuity, the increase in the fraction of outward diffusing ions in this example results from the more rapid increase of D_{LL} with L in the region $L > L_{Io}$. In a sense, it is correct to say that the average diffusion coefficient in the region exterior to Io is greater than in the previous situation because of the discontinuity in the L dependence of D_{LL}, although D_{LL} is itself continuous at Io.

TABLE 3. Effective of the Mode of Diffusion on Transport Parameters

Mode of Diffusion	L_I	L_0	D_{LL} in	D_{LL}out	D_{LL} out/ D_{LL} in at Io	S,* ions S^{-1}	t,* days	F_I/F_T
Linear	1.8	4S	$k_{in}L^3$*	$k_{out}L^3$	20	$2\times10^{28\pm1}$	$21\times10^{\pm1}$	0.08
Nonlinear	1.8	4S	$k_{in}L^3$*	$k\,L^{12}\partial NL^2/\partial L$	16	$8\times10^{28\pm1}$	$16\times10^{\pm1}$	0.12

*k_{in} is assumed to lie in the range $2\times10^{-9\pm1}$. The uncertainties in S and t result directly from the assumed range in k.

The third part of the table corresponds more closely to the condition inferred from the Voyager data. The numbers in this part of the table refer to a factor of 100 discontinuous increase in D_{LL} across the orbit of Io. Although a factor of 100 is used to provide a concrete example, the values corresponding to an arbitrary discontinuity in D_{LL} are readily calculated for the Io source. Equation (7), which gives the inward to outward flux ratio at Io, can be generalized to accommodate a discontinuity in the diffusion at Io, namely

$$\frac{F_I}{F_O}=\frac{3-m_I}{3-m_O}\left.\frac{(D_{LL})_I}{(D_{LL})_O}\right|_{L_S} L_S^{m_O-m_I}\frac{L_O^{3-m_O}-L_S^{3-m_O}}{L_S^{3-m_I}-L_I^{3-m_I}} \qquad (9)$$

$$(m_{I,O}\neq3)$$

and the corresponding expression for $m_I = 3$. Equation (9) shows that the inward to outward flux ratio is proportional to the amplitude of the discontinuity in the diffusion coefficient, expressed as a ratio. The situation in which $m_I \neq m_O$ but $(D_{LL})_I = (D_{LL})_O$ is illustrated in part two of Table 2. Other situations can then be determined by multiplying the F_I/F_O ratio corresponding to this continuous diffusion coefficient case by the amplitude of the discontinuity in D_{LL}, expressed as a ratio.

From the above discussion, it follows that a discontinuous increase in D_{LL} as one moves away from Jupiter across Io's orbit will favor outward diffusion of newly created ions. This general conclusion is illustrated by comparing the third part of Table 2 with the first two parts. In the chosen example, in which $(D_{LL})_O = 100\,(D_{LL})_I$, all evaluated instances diffuse less than 2% of new ions inward. Compared with the situation for externally driven, continuous diffusion, which is illustrated in the first part of the table, inward flux is reduced by a factor ranging between 50 and 1500. Thus, in all instances virtually the total flux of new ions is outward.

As was discussed earlier and in more detail by *Siscoe and Summers* [this issue], the existence of a discontinuity in the diffusion coefficient at Io's orbit carries with it the strong implication that outward diffusion is driven by the centrifugal interchange instability, for which the structure of the diffusion coefficient is given by (3). In this situation, it is useful to write the inward to outward flux ratio in the most general form from (4)

$$\frac{F_I}{F_O}=\frac{(D_{LL})_I}{(D_{LL})_O}\frac{[d(NL^2)/dL]_I}{[d(NL^2)/dL]_O} \qquad (10)$$

It is readily shown that if $(D_{LL})_0$ is given by (3), then the flux ratio (10) is the same as would be found if $(D_{LL})_0$ were given by (2) provided that the parameters m and p in these equations are related by $m = 3 + p/2$. That is, we can replace the steady state nonlinear problem by an equivalent steady state linear problem of the type we have already considered. The replacement follows immediately from the observation that the solution for N is identical in the linear and non-linear problems if m and p are related as indicated above [*Siscoe and Summers*, this issue]. Thus, the radial gradients of N that enter into (10) would also be identical in the two types of problems.

As the above argument implies, and as can be shown directly, because of the coincidence in the linear and nonlinear solutions, the inward to outward ratio of fluxes from Io in the nonlinear case can be found directly from (9) with m_0 replaced by $3 + p/2$. For example, if we take $p = 4$, which is the upper limit on p inferred from the Voyager data, then we must use $m_0 = 5$ in (9). This is the value of m_0 that was used to construct the second and third parts of Table 2. The table, therefore, illustrates the behavior of the flux ratio for both the linear and nonlinear situations. Thus the results attained earlier for the linear problem with discontinuity apply unchanged to the nonlinear problem.

TIME DEPENDENCE

In the linear problem, the flux ratio resulting from a time dependent production rate of ions is constant in time and equal to the ratio resulting from a steady state source with the same boundary conditions and the same values of k and m that specify the linear diffusion coefficient. This assertion follows simply from the superposition principle that applies to linear problems. A steady source can be considered to be composed of a time series of equal amplitude, contiguous, differential square-wave source pulses. Since the pulses are identical, and each contributes equally to the resulting flux ratio, the flux ratio for each pulse must be the same as that for the sum of all pulses, in other words, the same as that for the steady source. In a linear problem, ratios are independent of amplitudes. Thus, for a problem fully specified except for the source behavior, all elemental source pulses have the same flux ratio. Since any time dependent source can be decomposed into a time series of contiguious source pulses with variable amplitudes, it follows that the flux ratio is independent of time variations of the source, and equal to the steady state value. Of course, by contrast, in the nonlinear problem, a variable source will result in a variable flux ratio.

It is instructive to compare the fits of the linear and non-linear diffusion equations to the Voyager I in situ plasma data. The linear fit was given previously by *Richardson et al.* [1980]. Both a nonlinear fit and a new linear fit to a radially more extended data set than used previously are shown in Figure 1. In both cases the fits are seen to be relatively good, although there are significant small-scale features that are not reproduced in either case. An immediate and important conclusion that can therefore be drawn from the figure is that the issue of whether the diffusion coefficient for the outer domain should take the linear or nonlinear form can not be resolved on the basis of their relative abilities to fit the data.

In both fits, the linear form of the diffusion coefficient was used for the inward domain, wherein we would not expect the influence of the centrifugally driven interchange instability to be felt [see *Siscoe and Summers*, this issue]. As was formerly found to be the case for the linear fit to both domains, for the nonlinear fit it was necessary to assume that the injection of ions into the torus began a finite time before the arrival of Voyager.

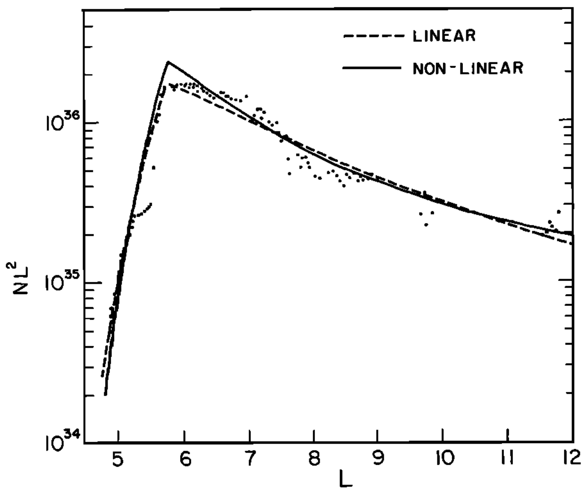

Fig. 1. Fits of the linear and nonlinear forms of the diffusion equation to the radial variation of NL^2 as determined from in situ Voyager plasma data taken in the Io torus. We note that the power to which L is raised in the nonlinear expression for D_{LL} out is larger than that assumed by *Siscoe and Summers* [this issue]. The larger power is needed to fit the large dropoff in density at $L = 7.5$. This dropoff may have an explanation that lies outside of the range of considerations treated in this paper. [*Siscoe et al.*, this issue].

The main parameters of the system that one derives from the fits are the strength of the ion-injection source S, the time that elapsed between the start of the injection event and the Voyager encounter t, and the percentage of the newly created ions that diffuse inwards.

Table 3 gives the parameters used to specify the diffusion coefficients in each case and compares the parameters derived from the fits. The remarkable result is that they differ by a factor of four or less in all cases. We see here in the fitting of the Voyager data a final illustration of the conclusion drawn earlier from theoretical case studies and also arrived at by *Siscoe and Summers* [this issue]. Namely, the observable consequences of nonlinear diffusion driven by the centrifugal interchange instability are remarkably similar to those of linear, externally driven diffusion. The primary observable that can determine which of the two is operative remains the discontinuity at Io's orbit, for which a natural interpretation has been given only in terms of a scenario in which nonlinear outward diffusion is an integral part. In order to achieve an acceptable linear fit, a discontinuity must be imposed at Io's orbit, for which there is no natural reason within the conceptual framework that gives rise to a fully linear diffusion coefficient.

Acknowledgments. We thank J. D. Sullivan and F. Bagenal for providing the extended data set of NL^2 versus L that was used in Figure 1. This work was supported in part by the National Aeronautics and Space Administration under contract NAS7-100 to Jet Propulsion Laboratory, subcontract 953733 (JPL to MIT) and subcontract 26834 (MIT to UCLA).

REFERENCES

Bagenal, F., J. D. Sullivan, and G. L. Siscoe, Spatial distribution of plasma in the Io torus, *Geophys. Res. Lett., 7,* 41–44, 1980.

Brice, N. M., and G. A. Ioannidis, The magnetospheres of Jupiter and earth, *Icarus, 13,* 173–183, 1970.

Broadfoot, A. L., et al., Extreme ultraviolet observations from Voyager I encounter with Jupiter, *Science, 204,* 979–982, 1979.

Chen, C.-K., Topics in Planetary Plasmaspheres, Ph.D. dissertation, Department of Atmospheric Sciences, Univ. of Calif., Los Angeles, 1977.

Eviatar, A., G. L. Siscoe, T. V. Johnson, and D. L. Matson, Effects of Io ejecta on Europa, submitted to *Icarus,* 1980.

Eviatar, A., C. F. Kennel, and M. Neugebauer, Possible origins of time variability in Jupiter's outer magnetosphere, 3, Variations in the heavy ion plasma, *Geophys. Res. Lett., 5,* 287–290, 1978.

Fälthammar, C.-G., Radial diffusion by violation of the third adiabatic invariant, in *Earth's Particles and Fields,* edited by B. M. McCormac, pp. 157–169, Reinhold, New York, 1968.

Froidevaux, L., Radial diffusion in Io's torus: Some implications from Voyager 1, *Geophys. Res. Lett., 7,* 33–35, 1980.

Goertz, C. K., J. A. Van Allen, and M. F. Thomsen, Further observational support for the lossy radial diffusion model of the inner Jovian magnetosphere, *J. Geophys. Res., 84,* 87–92, 1979.

Hill, T. W., Corotation lag in Jupiter's magnetosphere: Comparison of observation and theory, *Science, 207,* 301–302, 1980.

Hill, T. W., and F. C. Michel, Heavy ions from the Galilean satellites and the centrifugal distortion of the Jovian magnetosphere, *J. Geophys. Res., 81,* 4561–4565, 1976.

Richardson, J. D., G. L. Siscoe, F. Bagenal, and J. D. Sullivan, Time dependent plasma injection by Io, *Geophys. Res. Lett., 7,* 37–40, 1980.

Siscoe, G. L., and D. Summers, Centrifugally driven diffusion of iogenic plasma, *J. Geophys. Res.,* this issue.

Siscoe, G. L., A. Eviatar, R. M. Thorne, J. D. Richardson, F. Bagenal, and J. D. Sullivan, Ring current impoundment of the Io plasma torus, *J. Geophys. Res.,* this issue.

Strobel, D. F., and Y. L. Yung, The Galilean satellites as a source of CO in the Jovian upper atmosphere, *Icarus, 37,* 256–263, 1979.

Vasyliunas, V. M., and A. J. Dessler, The magnetic anomaly model of the Jovian magnetosphere: A post-Voyager assessment, *J. Geophys. Res.,* this issue.

(Received July 3, 1980;
accepted November 7, 1980.)