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THE NON-MAXWELLIAN ENERGY DISTRIBUTION OF IONS IN THE WARM IO TORUS

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Abstract. Observations of Io's torus indicate that the majority of ions have energies of 55-75 eV, with a high-energy tail extending up to the corotation energy. We have found that such a distribution can be established via the Coulomb cooling of ions heated at the corotation energy onto the cold 5-eV electrons. The energy E^* of the main body of the ions and the shape of the energy distribution are functions of the transport loss time. Matching E^* with the data ($E^* = 55$-75 eV) requires transport loss times in the range 25-100 days.

Introduction

The bulk of the plasma in Io's warm torus ($L \approx 6 - 7.5$) was observed by the Voyager 1 plasma experiment to have energies of 55-75 eV [Bagenal and Sullivan, 1981]. The probable origins of this plasma are neutral clouds of sulfur and oxygen which have escaped from Io and are ionized by charge exchange and electron impact ionization. The acceleration of these newly created ions by the corotation electric field imparts an initial energy of 540 eV to sulfur ions and 270 eV to oxygen ions.

The question therefore raised is why do the bulk of the ions have a much lower energy than their creation energy? Goertz [1980] proposed that mass loading near Io locally decreases the corotation speed, enabling ions to form at a lower energy. However, the UV emissions which should result from a locally concentrated source region are not observed [Shemansky, 1980]. Since it does not seem possible to create the ions at the observed energy, some cooling mechanism must be found which can provide the observed ion energy distribution. Loss of energy from ions to the cold electrons is an obvious possibility. However, in order to assess the likelihood of the possibility, one must determine whether the time required to cool to the observed temperature is consistent with estimated residence times of the ions.

The exchange of energy between ions and electrons has been studied in the context of trying to provide enough energy to the electrons to power the UV torus emissions. Plasma waves are not of sufficient intensity to provide the ion-electron energy exchange necessary to power the torus [Thorne, 1981; Barbosa et al., 1982]. If ions provide the energy for the torus emissions, this leaves Coulomb interactions as the probable energy transfer mechanism.

Barbosa et al. [1983] have modeled the exchange of energy in the torus via Coulomb collisions. Their model consists of a plasma with three Maxwellian components: a hot ion component at the corotation energy, a cooler ion component at which the bulk of the plasma resides, and a cold electron component. By balancing the energy flow between these components and the amount of energy lost to radiation, they obtained temperatures of 50-eV for the cool ion component and 5-eV for the electrons, in good agreement with observations. This result requires that the hot component contain 20% of the ions and that the entire torus be doubly ionized.

It has been suggested that electron-electron heating may also be an important source of energy for the torus emissions. Thorne [1981] has shown that a large flux of electrons can be backscattered from the auroral zones. Shemansky and Sandel [1982] argue that the local time asymmetry in the electron temperature inferred from the UV emissions requires electron-electron heating, since ion-electron interactions are too slow to maintain a local time asymmetry. However, it has been shown that the convection electric field set up by outward moving plasma in the tail can create this observed asymmetry [Barbosa and Kivelson, 1983]. Thus neither electron-electron heating nor ion-electron heating can be ruled out.

Whichever of these two mechanisms provides the predominant amount of energy for powering torus emissions, the ions will be cooled by their interactions with electrons. It is expected therefore that the average ion temperature should be a function of the transport time in the torus.

If transport is fast, this implies a larger source of hot ions to maintain the torus density along with shorter cooling times, and thus a plasma with a higher mean energy. The opposite is true for slow transport. Many methods have been used to obtain estimates of the transport time. They include calculating the ion creation rate needed to cause the observed corotation lag outside of Io's torus [Hill, 1980], modeling the ion partitioning in the hot torus [Shemansky, 1980], and computing the ionization rate of the neutral clouds [Smyth and Shemansky, 1983]. These methods yield quite different results, giving a range for the torus transit time of 15-200 days.

Our objective in this paper is to find the ion energy distribution F(E) for the Io torus that results when a continuous source of hot ions is cooled by cold electrons. We also include the effect of the transport time for the loss of the ions in our model and will obtain an estimate of this radial transit time based on the observed energy of the torus.

The Model

We assume a steady state situation in which a constant influx of ions is added to the torus at the corotation energy. For mathematical convenience we will not include individual ion species but will instead treat an ion having average torus properties: a mass of 24, a charge state of

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1.5, a density of 2000 ions/cm^3, and a corotation energy of 400 eV. It is also assumed that energy exchange in the torus plasma is a smooth and continuous process. This should be the case, since in Coulomb collisions most energy transfer is a result of small angle interactions rather than direct collisions.

The governing equation for our model is the equation for particle flux conservation along the energy axis.

$$-\frac{d}{dE}\left[m(E)\frac{dE}{dt}\right] = S - L \qquad (1)$$

where m(E) is the number of ions between E and E + ΔE and S and L are the sources and losses of ions with energy E. There are two sources of ions, charge exchange and electron impact ionization (the loss of ions by charge exchange enters into the loss term below), so that the source term can be written

$$S = (n^o n^+ \alpha_{cx} + n^o n^- \alpha_i)\ \ \delta(E_c - E) \qquad (2)$$

where n^o, n^+, and n^- are the neutral, ion, and electron number densities, α_{cx} and α_i are the charge exchange and ionization rates, and δ is the Dirac delta function indicating that all new ions are created at the corotation energy E_c. Ions are lost from the system by charge exchange and transport out of the torus:

$$L = n^o\, m(E)\, \alpha_{cx} + \frac{m(E)}{\tau} \qquad (3)$$

where τ is the transport time.

Using the conservation equation for ions,

$$\frac{dn^+}{dt} = n^o n^- \alpha_i - \frac{n^+}{\tau} \qquad (4)$$

together with the assumption of steady state ($dn^+/dt \doteq 0$) and integrating (1) over energy give

$$m(E)\frac{dE}{dt} = K\int_0^E m(E)\, dE \qquad (5)$$

where

$$K = \left(1 + \frac{\alpha_{cx}}{z\alpha_i}\right)\frac{1}{\tau} \qquad (6)$$

and z is the average ion charge state ($n^- = zn^+$).

Note that this is the only place where the charge exchange and ionization rates, and thus implicitly the roll of the neutrals, enter our model. The important quantity is the ratio of these two rates, α_{cx}/α_i, which varies throughout the torus. We use a value of 1.5 for this ratio, consistent with the results of Johnson and Strobel [1982], which should be accurate to within a factor of two in most of the torus.

We now need expressions for $(dE/dt)_e$ and $(dE/dt)_i$, the rates of interchange of energy between ions and electrons and between ions of different energy. Spitzer [1953] has derived an expression for the interactions of two Maxwellian populations,

$$\frac{dT_2}{dt} = \frac{T_1 - T_2}{\tau_{eq}} \qquad \tau_{eq} = \frac{3\text{x}10^5 A_1 A_2}{n_1 Z_1^2 Z_2^2}\left(\frac{T_1}{A_1} + \frac{T_2}{A_2}\right)^{3/2} \qquad (7)$$

where A is the atomic weight and the Coulomb logarithm in the numerical factor in τ_{eq} was evaluated for Io torus conditions. Using the relation $T_e/A_e >> T_i/A_i$ and rewriting (7) in terms of energies rather than temperatures give

$$\left(\frac{dE}{dt}\right)_i = \int_0^{E_c} C^+\, m(E')\,\frac{E'-E}{(E'-E)^{3/2}}\, dE' \qquad C^+ = \frac{Z^4}{2.5\text{x}10^5 A_i^{\frac{1}{2}}}$$

$$\left(\frac{dE}{dt}\right)_e = C^-\ \frac{E_e - E}{E_e^{3/2}} \qquad C^- = \frac{Z^2 n^- A_e^{\frac{1}{2}}}{2.5\text{x}10^5 A_i} \qquad (8)$$

These expressions are for interactions between Maxwellian populations. The electron distribution is probably Maxwellian; the ions, however, are not expected to have a Maxwellian distribution. Nevertheless, we will use these quations now and return to the question of their applicability later in this paper.

Protons have been suggested as a possible intermediary in the transfer of energy from ions to electrons [Thorne, 1982]. Calculations based on whistler dispersion in the torus region indicate the proton number density is about 0.1 the heavy ion density [Tokar et al., 1982]. The proton temperature should result from an equilibrium between energy gained by the protons from heavy ions and that lost by the protons to electrons. Using (8), taking $\bar{T}$ = 60 eV, and using the "average ion" parameters given earlier, we find $T_p \approx$ 17 eV. Again by use of (8) it can be shown that in order for protons to provide as much energy to electrons as the heavy ions do, a proton density equal to one third the ion density would be required. A proton component this large is not supported by observations. We conclude that protons do not contribute significantly to the transfer of energy in the torus.

Substitution of (8) into (5) yields an integral equation for the ion distribution m(E). To find the form of the solution of this equation, the ion interaction term (8), which governs the exchange of energy between ions, can be initially neglected. For ions interacting only with electrons, the solution is

$$m(E) = m(E_c)\left(\frac{E - E_e}{E_c - E_e}\right)^{-Q} \qquad (9)$$

where

$$Q = 1 - \frac{KE_e^{3/2}}{C^-}$$

and $m(E_c)$ is the density of ions at the corotation energy. To give an example based on representative values, a transport time of 50 days and a charge exchange rate 1.5 times the ionization rate yield a value of about 0.8 for Q.

Next consider what happens when the ion interaction term is included. We do not attempt to find an exact analytic solution to the governing integral equation in this case. However, general considerations show that there will be some energy E^* at which the energy an ion loses to electrons is balanced by the energy an ion gains from higher energy ions. The ions, instead of cooling to the electron energy, cool to this intermediate energy. The solution of the equation should then have approximately the same general form for the distribution function as (4) but with E_c replaced by E^*:

$$m(E) = m(E_c) \left(\frac{E - E^*}{E_c - E^*}\right)^{-\gamma} \qquad (10)$$

Here we regard E^*, the energy of the bulk of the plasma, and γ; the exponent, as parameters to be determined to give (10) the best fit to the actual solution. Note for future reference that an expression for $m(E_c)$ is obtained by integrating (10) over energy, giving

$$m(E_c) = \frac{n^+(1 - \gamma)}{E_c - E^*} \qquad (11)$$

The necessity for a distribution function of the form given by (10) is seen by evaluating (1) at E^*. If the density $m(E^*)$ does not equal infinity, (1) yields a restriction on the transport time τ:

$$\tau < E_c^{3/2}\left(1 + \frac{\alpha_{cx}}{z\alpha_i}\right)/C^- n^- \approx 8 \text{ days} \qquad (12)$$

This condition is clearly not physically valid, as the transport and cooling mechanisms are independent of each other; thus $m(E^*)$ must equal infinity.

Since there are two unknowns, E^* and γ, two equations are needed to solve for them. The first is the condition of energy flow balance at E^*,

$$\left(\frac{dE}{dt}\right)_e = \left(\frac{dE}{dt}\right)_i \qquad (13)$$

The second is obtained by integrating (5) over energy. The ion interaction term drops out, as this governs the exchange of energy between ions and causes no net change in the total ion energy. This leaves the equation

$$\int_{E^*}^{E_c} m(E) \frac{C^- n^-}{E_e^{3/2}} (E_e - E)dE = \int_{E^*}^{E_c} K \int_{E^*}^{E} m(E')dE' \, dE \qquad (14)$$

Using (13) and (14), it is possible to solve for E^* and γ.

Before showing any results let us return to the question of the expressions used for $(dE/dt)_i$ and $(dE/dt)_e$. The expressions (8) used above are appropriate for interactions between Maxwellian populations. The situation our equations govern is one in which there is a superposition of many Maxwellians, as shown in Figure 1b, one at each temperature between the corotation energy and E^*, with the total density of each separate population given by $m(E)$. This representation will overestimate the number of ions at the highest and lowest energies.

The other extreme is a distribution shown in Figure 1a, in which there are no ions with energies greater than E_c or less than E^*. This is clearly not a realistic distribution, since the lower energy ions will approach a Maxwellian distribution on a time scale of a few days, much shorter than the transport time. The lower energy ions probably have a distribution close to that shown by Figure 1a, while the higher energy ions have a distribution similar to Figure 1b. The correct solution should be bracketed by these two extremes.

We now need expressions for $(dE/dt)_i$ and $(dE/dt)_e$ for the distribution shown in Figure 1a. Butler and Buckingham [1962] have derived an expression for the loss of energy by an ion of speed V to a Maxwellian electron population with thermal speed ω_e;

$$\frac{dE}{dt} \approx -\frac{8\sqrt{\pi}\, z^2 e^4 n^- \ln \Lambda}{m_e \omega_e} \left(-\frac{A_e}{A_i} + \left(\frac{2}{3} + \frac{A_e}{A_i}\right)\left(\frac{V}{\omega_e}\right)^2\right) \qquad (15)$$

where m_e is the electron mass and $\ln \Lambda$ is the Coulomb logarithms. The exchange of energy between an ion with speed v and the distribution function $F(\omega)$ is

$$\frac{dE}{dt} = \frac{4\pi}{m_i} z^4 e^4 n^+ \ln \Lambda \, F(\omega^*, \gamma)$$

$$F(\omega^*, \gamma) = \left[(\omega_c^2 - \omega^{*2})^{1-\gamma} + \frac{1}{v}\int_{\omega^*}^{v} (\omega^2 - \omega^{*2})^{1-\gamma} d\omega - \left(1 + \frac{m}{M}\right)(v^2 - \omega^{*2})^{1-\gamma}\right] \cdot \left[\omega_c^2(\omega_c^2 - \omega^{*2})^{1-\gamma} - \int_{\omega^*}^{\omega_c} (\omega^2 - \omega^{*2})^{1-\gamma} d\omega\right]^{-1} \qquad (16)$$

where m_i is the ion mass and ω_c and ω^* are the velocities corresponding to E_c and E^*. Using these equations in (5), we again want to solve for E^* and γ. The integral over (5) can again be used as one of the equations

$$\int_{E^*}^{E_c} m(E) \left(\frac{dE}{dt}\right)_e dE = \int_{E^*}^{E_c} K \int_{E^*}^{E} m(E') \, dE' \, dE \qquad (17)$$

The second equation in this case is obtained by evaluating (5) at E_c,

$$m(E_c) \left[\left(\frac{dE}{dt}\right)_e + \left(\frac{dE}{dt}\right)_i\right]E_c = Kn^+ \qquad (18)$$

Results

Table 1 shows the solutions for E^* and γ for our two cases as a function of the transport time τ. Observations indicate the bulk of the plasma has energies between 55 and 75 eV. For

TABLE 1. Solution for E^* and γ for Two Cases as a Function of τ

τ, days	E_c	T_e	γ	E^*	$\bar{E}$	Power, 10^{12} w
			Non-Maxwellian Case			
25	400	5	0.80	180	216	0.92
50	400	5	0.83	110	151	0.62
100	400	5	0.87	61	100	0.38
200	400	5	0.91	33	63	0.21
400	400	5	0.94	20	41	0.11
			Maxwellian Case			
10	400	5	0.38	105	218	2.24
25	400	5	0.79	77	132	1.33
50	400	5	0.91	54	83	0.80
100	400	5	0.97	38	48	0.43
200	400	5	0.985	24	30	0.23
400	400	5	0.995	17	18	0.12

the Maxwellian case (Figure 1b) this corresponds to transport times of 25-50 days, and for the non-Maxwellian case (Figure 1a) to transport times of about 100 days. Thus the range of values for the transport time bracketed by our two solutions is 25 to just over 100 days, with the actual time probably lying closer to the shorter values, as the Maxwellian case should be much closer to the real distribution than the non-Mawellian case.

These values of the transit time correspond to values of γ ranging from 0.79 - 0.91. Figure 2 shows the distribution function for the case where $E^* = 61$ eV and $\gamma = 0.87$, which is appropriate for the non-Maxwellian case with a 100-day transport time. The distribution is heavily skewed in favor of ions with energies near E^*. Half of the ions have energies between 61 and 75 eV, 20% have energies greater than 145 eV, and 10% have energies greater than 225 eV. Thus ions lose most of their energy in a small fraction of the tranport time.

The distribution function shown in Figure 2 should be quite accurate in the upper two thirds of the energy range shown. At lower energies (50 eV) the self collision time is short enough ($\sim$ 4 days) compared to the transport loss time that a more Maxwellian profile should result.

Also shown in Table 1 is the amount of power transferred from the ions to the electrons for each value of the transport time. This calculation is model dependent in that a volume must be chosen for the torus. Here we have assumed a volume of 2×10^{31} cm^3 for the emitting region. It also depends on our assumption of 5 eV for the electron temperature. The results can be compared with a UV output of 1.6×10^{12} watts based on the same values for the volume and electron temperature (D. E. Shemansky, private communication, 1983). For the Maxwellian case, transit times which give good agreement with observations of E^*, 25-50 days, give power outputs

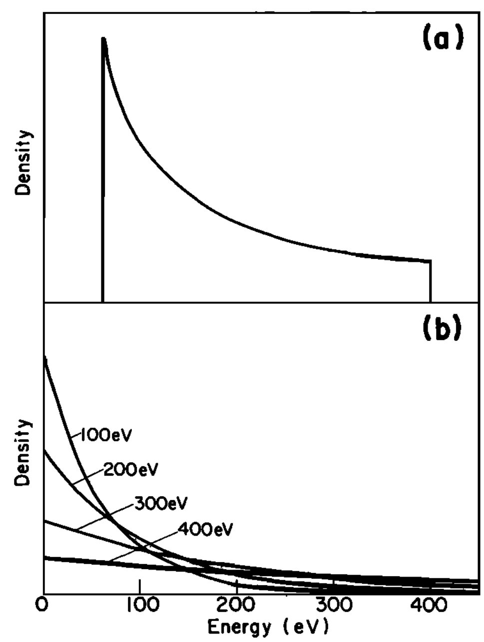

Figure 1. Shown here are the two types of energy distribution used in calculating energy transfer rates. The non-Maxwellian case (a) has no ions of energy levels below E^* or above E_c. The Maxwellian case (b) is the distribution arrived at by summing up the assumed Maxwellian at each energy, four of which are shown here. The density scales used in (a) and (b) are not the same.

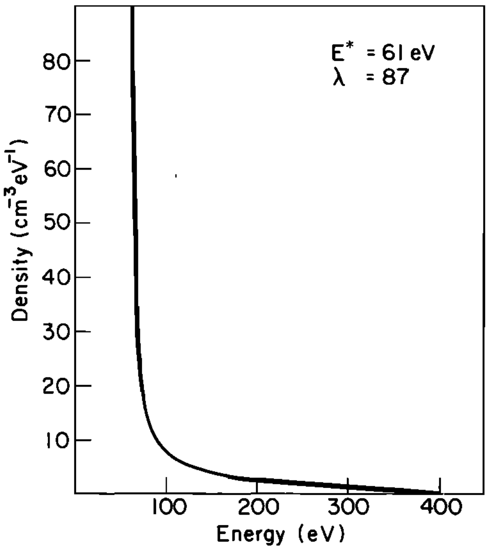

Figure 2. This shows an example of the distribution function we have derived. This case is appropriate for the Maxwellian case with a 100 day torus transport time.

1-2 times less than observed. The non-Maxwellian case yields even less power.

We do not consider the stated differences between the observed and calculated powers necessarily to be a major problem. Factors of 2 or 3 are not outside the uncertainties of these parameters. Also, as mentioned earlier, it is not necessarily a requirement that ion-electron energy transfer power the entire torus emission, as electron-electron heating may also be important.

Finally, we note that the average energy of the torus plasma has been reported to be about 60 eV for SII and 90 eV for SIII [Brown, 1981, 1982]. These numbers are consistent with the values we obtain for the average energy shown in Table 1.

It should be noted that we have not determined that value of the electron temperature that gives equal rates of energy transfer from ions to electrons and energy output as UV radiation. Instead we have chosen a value of 5 eV as typifying the electron temperature found by other means [e.g., Shemansky and Smith, 1981; Barbosa et al., 1983]. These results are quite sensitive to this parameter, as the exchange rate of energy between ions and electrons is proportional to $T_e^{-3/2}$. For example, considering just the Maxwellian case, raising the electron temperature to 7.5 eV increases the transport time needed to match the observations from 25-50 days to 75-150 days. It also, however, reduces the power provided to the electrons by a fqctor of ∿ 3 to 3 x 10^{11} watts, a factor of 5 less than observed. Reducing the electron temperature in our model to 3 eV allows us to provide enough energy to the electrons to power the torus. However, an electron temperature below 4 eV would not be consistent with the UV observations. Thus small variations in the electron temperature allow us to either increase the torus residence time but run into difficulty in powering the torus, or to provide enough energy to power the torus but require small residence times.

Summary

We have obtained a distribution function for ions in the hot Io torus of the form

$$m(e) = m(E_c) \left(\frac{E-E^*}{E_c-E^*}\right)^{-\gamma}$$

where γ is between 0.8 and 0.9. This is a highly non-Maxwellian distribution with ions bunched at energies near E^*, which, based on observations, is in the range 55-75 eV. An E^* in this range was found to require a transit time through the torus in the range 25 to 100 days. This, in turn, implies an ion creation rate of 6 x 10^{27} - 2 x 10^{28} ions/s, with shorter transit times and higher source strengths being preferred.

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References

Bagenal, F., and J. D. Sullivan, Direct plasma measurements in the Io torus and inner magnetosphere of Jupiter, J. Geophys. Res., 86, 8447-8466, 1981.

Barbosa, D. D., and M. G. Kivelson, Dawn dusk electric field asymmetry of the Io plasma torus, Geophys. Res. Lett., 10, 210-213, 1983.

Barbosa, D. D., F. V. Coroniti, W. S. Kurth, and F. L. Scarf, Detection of lower hybrid emissions in the Io plasma torus: bounds on anomalous electron heating rates, paper presented at the 1982 Fall Meeting of the American Geophysical Union, San Francisco, Calif., Dec. 7-15, 1982.

Barbosa, D. D., F. V. Coroniti, and A. Eviatar, Coulomb thermal properties and stability of the Io plasma torus, in press, Astrophys. J., 1983.

Brown, R. A., The Jupiter hot plasma torus: Observed electron temperature and energy flow, Astrophys. J., 244, 1072-1080, 1981.

Brown, R. A., The probability distribution of S^+ gyrospeeds in the Io torus, J. Geophys. Res., 87, 230-234, 1982.

Butler, S. T., and M. J. Buckingham, Energy loss of a fast ion in a plasma, Phys. Rev., 126, 1-4, 1962.

Goertz, C. K., Io's interaction with the plasma torus, J. Geophys. Res., 85, 2949-2956, 1980.

Hill, T. W., Corotation lag in Jupiter's magnetosphere: Comparison of observation and theory, Science, 207, 301-302, 1980.

Johnson, R. E., and D. F. Strobel, Charge exchange in the Io torus and exosphere, J. Geophys. Res., 87, 10385-10394, 1982.

Shemansky, D. E., Mass loss and diffusive loss rate of the Io plasma torus, Astrophys. J., 242, 1266-1277, 1980.

Shemansky, D. E., and G. R. Smith, The Voyager 1 EUV spectrum of the Io plasma torus, J. Geophys. Res., 86, 9179-9192, 1981.

Shemansky, D. E., and B. R. Sandel, The injection of energy into the Io plasma torus, J. Geophys. Res., 87, 219-229, 1982.

Smyth, W. H., and D. E. Shemansky, Escape and ionization of atomic oxygen from Io, Astrophys. J., in press, 1983.

Spitzer, L., Jr., Physics of Fully Ionized Gases, Interscience, New York, 1956.

Thorne, R. M., Jovian auroral secondary electrons and their influence on the Io plasma torus, Geophys. Res. Lett., 8, 508-512, 1981.

Thorne, R. M., Injection and loss mechanism for energetic ions in the inner Jovian magnetosphere, J. Geophys. Res., 87, 8105-8110, 1982.

Tokar, R. L., D. A. Gurnett, and F. Bagenal, The proton concentration in the vicinity of the Io plasma torus, J. Geophys. Res., 87, 10395-10400, 1982.

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