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Best regards,

Mike

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M. Schulz · L. J. Lanzerotti

Particle Diffusion in the Radiation Belts

With 83 Figures



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Michael Schulz

The Aerospace Corporation, Los Angeles, CA 90009/USA

Louis J. Lanzerotti

Bell Telephone Laboratories, Murray Hill, NJ 07974/USA

The illustration on the cover is adapted from Figure 58, which shows the evolution of an inner-zone electron-flux profile ($E > 1.9$ MeV, omnidirectional) observed on Explorer 15 following the high-altitude nuclear explosion of 1 November 1962. The data demonstrate the simultaneous effects of radial diffusion and pitch-angle diffusion.

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Preface

The advent of artificial earth satellites in 1957–58 opened a new dimension in the field of geophysical exploration. Discovery of the earth's radiation belts, consisting of energetic electrons and ions (chiefly protons) trapped by the geomagnetic field, followed almost immediately [1, 2]. This largely unexpected development spurred a continuing interest in magnetospheric exploration, which so far has led to the launching of several hundred carefully instrumented spacecraft.

Since their discovery, the radiation belts have been a subject of intensive theoretical analysis also. Over the years, a semiquantitative understanding of the governing dynamical processes has gradually evolved. The underlying kinematical framework of radiation-belt theory is given by the adiabatic theory of charged-particle motion [3], and the interesting dynamical phenomena are associated with the violation of one or more of the kinematical invariants of adiabatic motion.

Among the most important of the operative dynamical processes are those that act in a stochastic manner upon the radiation-belt particles. Such stochastic processes lead to the diffusion of particle distributions with respect to the adiabatic invariants. The observational data indicate that some form of particle diffusion plays an essential role in virtually every aspect of the radiation belts.

With radiation-belt physics now in its second decade, it seems desirable that the existing observational and theoretical knowledge of radiation-belt dynamics be consolidated into a unified presentation. This indeed is one purpose of the present series of monographs. Two of the earlier volumes of *Physics and Chemistry in Space* have treated the subjects of geomagnetic micropulsations [4] and adiabatic charged-particle motion in the magnetosphere [5], respectively. The present volume is concerned principally with the diffusion processes that affect radiation-belt particles. Some of these diffusion processes can cause energetic particles to precipitate into the atmosphere, thereby creating the aurora, as described in the fourth volume of this series [6].

Readers may find previous volumes of this series useful references for a more elaborate discussion of geomagnetic-activity indices [4] and adiabatic motion [5] than can be given here. To the extent that adiabatic

invariants are discussed in the present volume (Chapter I), they are treated from the Hamiltonian viewpoint [7], since this formulation is especially suitable for subsequent application to the analysis of particle diffusion. In this and other respects, an effort has been made to present the existing state of knowledge in a manner that is instructively novel.

Radiation-belt physics is especially rewarding in that it requires a close collaboration of ideas between the observational and theoretical spheres. Perhaps the necessity for this collaboration arises from the fact that (as in any environmental science) there can be no hope of controlling the entire magnetosphere by experimental technique, or even of measuring *all* the parameters that may have a bearing on the dynamical phenomena. By the same token, the environment is sufficiently complex that there can be no hope of theoretically predicting radiation-belt behavior solely from a set of mathematical postulates. Instead, the theoretical groundwork only makes it possible to relate one set of observations to another. The governing diffusion equations may be known in analytical form, but the transport coefficients that enter them are essentially empirical.

The principal purposes of this book are (1) to convey a quantitative understanding of the fundamental ideas prevalent in radiation-belt theory, and (2) to instruct the reader in how to recognize the various diffusion processes in observational data, and how then to extract numerical values for the relevant transport coefficients. Since the subject is rather specialized, the book will be of value primarily to those who have a professional interest in the subject. This prime audience would include active researchers in the field of space physics, post-doctoral scientists redirecting their interests from another branch of physics, and graduate students seeking to do thesis research on the topic of radiation-belt dynamics. The book is probably unsuitable as a graduate-course textbook (unless used in conjunction with other volumes in the series), but should serve well as a reference work. Moreover, although the applications specifically cited refer to the earth's magnetosphere, many of the basic ideas are current in the understanding of laboratory-plasma devices and non-terrestrial magnetospheres.

While the present volume is essentially self-contained, some sections offer much more difficult reading than others. The reader should feel free to pass over those portions that least interest him, accepting on faith (where necessary) the validity of such intermediate results that find subsequent application. For the more critical reader, an attempt has been made to indicate the basic justification for the various analytical procedures, although little attention has been given to mathematical rigor in the formal sense.

References to the literature are numbered approximately in order of first citation. These numbers (in square brackets) are keyed to a list that appears at the end of the volume. The length of the reference list has been held to the minimum consistent with the avoidance of plagiarism. The list includes a few truly classic papers in the field, a few sources of supplementary information too complicated for inclusion here, and the sources of observational and theoretical data used in figures and tables, or otherwise invoked in support of an argument. The list is therefore not representative of the excellent contributions of many investigators, and is not intended to be used for scorekeeping purposes.

The authors are pleased to thank their many colleagues whose suggestions have helped to optimize the presentation of current ideas in this monograph, especially Dr. J. G. Roederer, Dr. J. M. Cornwall, Dr. T. A. Farley, Dr. M. Walt, Dr. G. A. Paulikas, Dr. J. B. Blake, Dr. W. L. Brown, Dr. H. C. Koons, Dr. A. Eviatar, Dr. Y. T. Chiu, Dr. A. L. Vampola, Dr. D. P. Stern, Dr. A. Hasegawa, and Ms. C. G. MacLennan. Finally, it is a pleasure to thank Miss Doreen Bracht for typing the final manuscript.

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Michael Schulz

L. J. Lanzerotti

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Introduction

The earth's radiation belts consist of energetic electrons and ions (mostly protons) whose motion is controlled by the geomagnetic field. Radiation-belt particles typically have energies ranging upward from 100 keV, and each is constrained by the field to execute a rather complicated orbital motion that encircles the earth with a characteristic *drift period* (typically tens of minutes). Thus, the particles are magnetically trapped by the earth's field.

Ideally, the radiation belts are mapped by means of satellite-borne sensors that distinguish between particle species and count the differential unidirectional particle flux at each energy and pitch angle of interest, and at all points in space. Practical considerations generally limit the available information quite severely, but a reasonable picture of the earth's radiation environment can be assembled from the available data. For example, Fig. 1 provides a contour plot of the observed *omnidirectional* electron flux (integrated over all directions of incidence relative

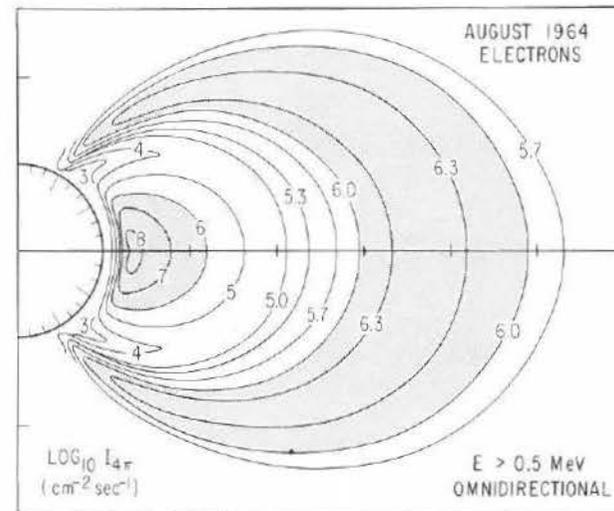


Fig. 1. Contours of constant integral electron flux $I_{4\pi}$ in the earth's radiation environment [8]. Shaded regions correspond to $I_{4\pi} > 10^6 \text{ cm}^{-2} \text{sec}^{-1}$.

neutral sheet is immersed in a *plasma sheet*, whose currents are in fact those responsible for maintaining the magnetic configuration described above. Various collective excitations of this plasma lead to a *nonvanishing* electrical resistivity, and so an electrostatic field appears across the nightside magnetosphere from east to west. In reality, this field extends across the dayside magnetosphere as well, and leads to a sunward convection of magnetospheric plasma [11] by virtue of the $\mathbf{E} \times \mathbf{B}$ drift (see below). This electrostatic field is therefore called the *convection electric field*; it extends across the magnetosphere from dawn to dusk. The sunward plasma flow caused by this electric field counterbalances an outward flow induced in a "viscous" boundary layer (~ 100 km thick) by a plasma-kinetic interaction between the solar wind and the magnetospheric plasma [12]. It is this "viscous" interaction (which probably involves several distinct plasma wave modes and instabilities) that is ultimately responsible (quite indirectly) for formation of the neutral sheet and geomagnetic tail.

As suggested above, the plasma sheet may be a repository for plasma of interplanetary origin. In addition, hydrogen plasma from each polar ionosphere is believed to flow outward along field lines that extend into the magnetotail. This flow, known as the *polar wind*, results from an ambipolar electric field established by electrons, protons, and oxygen ions (O^+) in the ionosphere. The ambipolar field is sufficiently strong to eject the lighter ions (H^+) against the restraint of gravity. It is presumed that some portion of the polar wind enters the plasma sheet, while the remainder escapes to interplanetary space.

Were it not for the rotation of the earth, all cold magnetospheric plasma of ionospheric origin would also escape. The sunward flow induced by the convection electric field would transport this plasma either directly to interplanetary space, or at least to the "viscous" boundary layer of the magnetosphere (see above). In fact, the earth's rotation creates an electrostatic field whose equipotential surfaces form closed shells. Cold plasma (by definition) follows trajectories that coincide with equipotential surfaces of the superimposed electric fields. Some of these surfaces form closed shells only weakly distorted from equipotentials of the *corotation electric field*. Others form open surfaces only weakly distorted from equipotentials of the convection electric field, and these intersect the magnetopause. The open equipotential surfaces intersect the ionosphere at high latitudes, but below the limit of open (to the tail) field lines that define the magnetic polar cap and carry the polar wind. Ionospheric plasma from these latitudes flows upward into the magnetosphere by convection across closed field lines to the boundary. The result is a high-latitude *trough* in ionospheric density, and

the associated flow is sometimes called the *trough wind*. The upward flow velocity here is generally smaller than in the polar wind.

Closed equipotential surfaces of the magnetospheric electric field (convection plus corotation) intersect the ionosphere at low and middle latitudes. Since cold plasma from the ionosphere cannot escape the magnetosphere either along open field lines or open equipotentials from this region, the associated magnetospheric plasma can build up an appreciable density and attain diffusive equilibrium with the ionosphere. In fact, the "last" closed equipotential surface is closely associated with a morphological feature known as the *plasmopause*, across which the observed plasma density can vary by three orders of magnitude over a distance ~ 1000 km in the equatorial plane. Beyond the plasmopause a cold-plasma density $\sim 1 \text{ cm}^{-3}$ is typical. Within the *plasmasphere*, which is bounded by the plasmopause, densities $\sim 10^3$ to 10^4 cm^{-3}

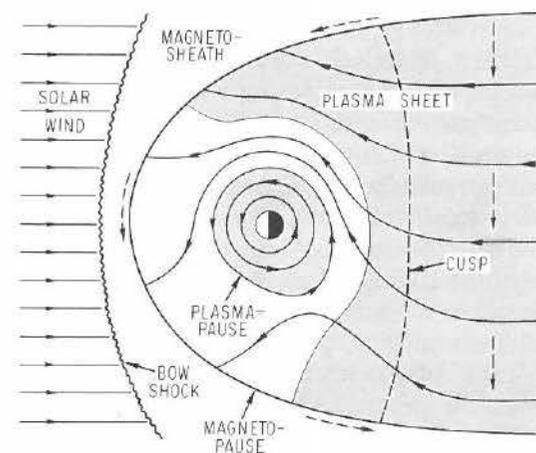


Fig. 3. Distribution (shaded areas) and flow pattern (solid arrows) of magnetospheric plasma ($E \leq 10$ keV) in the equatorial plane. Dashed arrows indicate current pattern. The plasmasphere is the shaded region inside the plasmopause.

are usual [13]. Figure 3 is a polar view of the magnetospheric equatorial plane in which the various electric equipotentials and regions of plasma concentration are indicated schematically³.

³While the plasmasphere contains truly cold plasma (temperature ~ 1 eV), the plasma sheet has a proton temperature ~ 6 keV.

As distinguished from cold plasma, there exists a class of particles (hot plasma) whose motion is governed not only by magnetospheric convection and corotation fields, but also by drifts related to the gradient and curvature of the local magnetic field \mathbf{B} . By contrast with the energy- and charge-independent drift caused by an electric field \mathbf{E} , plasma drifts related to $\nabla_{\perp} B$ and $\partial \hat{\mathbf{B}}/\partial s$ (gradient and curvature, where s is a coordinate measured along \mathbf{B}) increase monotonically with particle energy and proceed in a direction determined by the signature of the particle charge q . As a rather general rule, application of a force \mathbf{F} to a particle confined by a magnetic field results in a drift velocity $\mathbf{v}_d = (c/qB^2)\mathbf{F} \times \mathbf{B}$, where c is the speed of light (see Section III.6). In case $\mathbf{F} = q\mathbf{E}$, this formula is valid whenever it predicts $v_d < c$. Otherwise, the requirement is that $v_d \ll v$, where v is the total velocity of the particle. Curvature drift, for example, is driven by the centrifugal force $\mathbf{F} = mv_{\parallel}^2 (\partial \hat{\mathbf{B}}/\partial s)$, where m is the particle mass and $v_{\parallel} = \mathbf{v} \cdot \hat{\mathbf{B}}$.

A typical magnitude for the convection electric field is $5 \mu\text{V/cm}$ [14]. On an azimuthal-drift orbit having a diameter ~ 8 earth radii ($\sim 5 \times 10^9 \text{ cm}$), therefore, a typical particle energy $\sim 25 \text{ keV}$ might characterize the hot plasma, for which electric and gradient-curvature drifts are supposed to be of comparable importance. In fact, the majority of magnetospheric particle energy resides in protons of this type, the spatial distribution of which often exhibits a peak density $\gtrsim 1 \text{ cm}^{-3}$. The gyration of these particles in a field $\sim 100 \gamma$ can produce a significant diamagnetic effect locally, and the ring current associated with their drifts (and modified by diamagnetic effects) can produce a field depression observable even at the earth's surface [15]. Electrons $\sim 25 \text{ keV}$ in energy are found to produce a smaller, but still significant, contribution to the ring current (perhaps 25% of the total). Trajectories of ring-current particles (Fig. 4) can follow either open or closed surfaces, depending upon energy, particle species, and location in space [5]. This is true even though the field lines on which ring-current (hot-plasma) particles reside typically generate closed drift shells for particles of much larger energy (*e.g.*, $> 1 \text{ MeV}$).

In practice, the hot magnetospheric plasma is easily distinguished from the coexisting cold plasma, which has a temperature $\sim 1 \text{ eV}$ [16]. In view of the typical electric-field magnitude ($5 \mu\text{V/cm}$), gradient and curvature drifts must be negligible for particle energies $\lesssim 1 \text{ keV}$. The demarcation between ring-current (hot-plasma) and radiation-belt particles is somewhat more nebulous. Ideally, a radiation-belt particle should be sufficiently energetic that the steady magnetospheric electric field exerts no significant influence on the drift trajectory. In this limit the drift shell, generated by specifying an initial location in space and local pitch angle $\cos^{-1}(\hat{\mathbf{v}} \cdot \hat{\mathbf{B}})$, is independent of particle energy. Moreover,

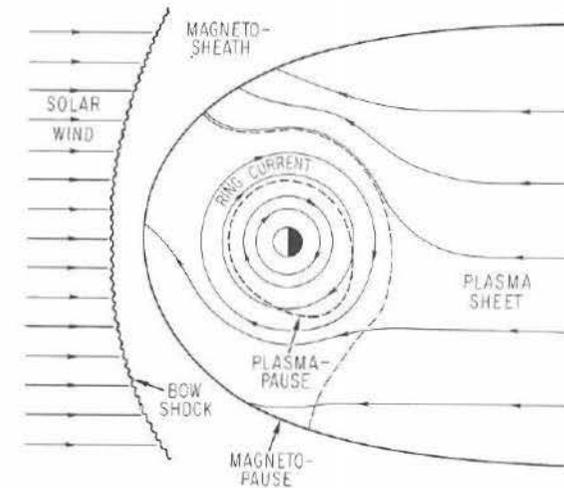


Fig. 4. Adiabatic flow pattern of magnetospheric protons ($E \sim 10-200 \text{ keV}$) in the equatorial plane. Dashed curves represent boundaries of plasma sheet and plasmasphere.

the mirror field (denoted B_m), *i.e.*, the magnitude of \mathbf{B} at which $\hat{\mathbf{v}} \cdot \hat{\mathbf{B}} = 0$, remains constant as the particle drifts in azimuth. Thus, a radiation-belt particle for which \mathbf{v} remains perpendicular to \mathbf{B} at all times will experience only gradient drift, and so its drift motion will trace a contour of constant magnetic-field intensity on the equatorial surface (Fig. 5). From this viewpoint, the magnetic equator is the set of all points at which the magnitude of \mathbf{B} attains a relative minimum with respect to displacement along the field line, *i.e.*, the set of points for which $\partial B/\partial s = 0$ and $\partial^2 B/\partial s^2 > 0$.

Strictly speaking, the convenient idealization of ignoring the steady \mathbf{E} field should be applicable only for particle energies $\gtrsim 200 \text{ keV}$. It is customary, however, to analyze charged-particle behavior not only in this limit, but also for energies as low as $\sim 40 \text{ keV}$, within the general framework of radiation-belt theory. For many purposes the steady magnetospheric electric field plays a complicating but nonessential role in the analysis at energies in the range $40-200 \text{ keV}$. In other situations the inclusion of \mathbf{E} is absolutely necessary. The degree of analytical sophistication required in the treatment of any given topic in radiation-belt dynamics is usually indicated by the quality of the available data. The question of whether to include \mathbf{E} falls within this context.

The dynamical behavior of radiation-belt particles is most conveniently expressed and most easily visualized within the kinematical framework of the adiabatic theory of charged-particle motion [3]. A

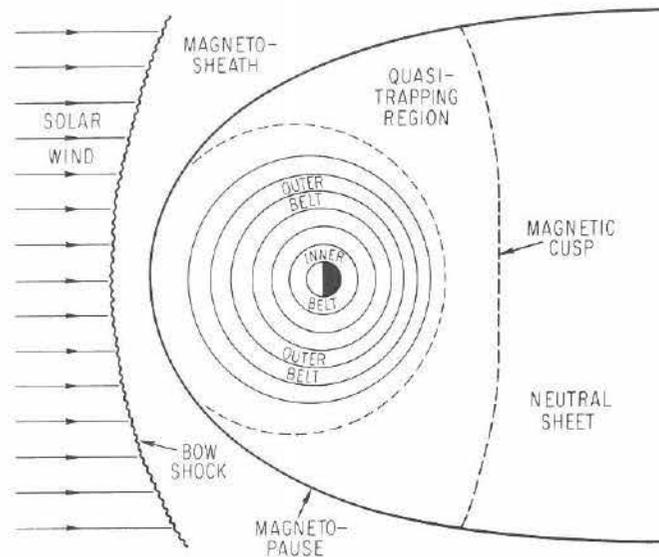


Fig. 5. Adiabatic drift paths of equatorially mirroring charged particles ($E \gtrsim 200$ keV). Protons drift westward (clockwise) and electrons eastward.

particle whose azimuthal drift motion defines a closed shell coincident with closed field lines at all longitudes can be assigned a set of three *adiabatic invariants*, which are (apart from dimensional constants) identifiable with the canonical momenta (action variables) of Hamilton-Jacobi theory [7]. As long as the particle remains undisturbed by forces varying suddenly on the drift time scale, the three adiabatic invariants are conserved quantities. The transformation to action-angle variables thus provides a kinematical framework in which the static and quasi-static characteristics of the magnetosphere are suppressed. Preservation of the adiabatic invariants is analogous to force-free rectilinear motion, in which the three components of linear momentum are separately conserved.

Geophysically interesting dynamical phenomena related to radiation-belt physics involve the violation of one or more adiabatic invariants. Such violation can be caused by particle-particle collisions, by wave-particle interactions, or by sufficiently sudden temporal changes in magnetospheric fields. Adiabatic invariants can be violated separately or in combination, depending upon the spatial structure and temporal character of the disturbing forces. Radiation-belt particles can be active conspirators in creating the disturbing forces, or they can be passive victims (test particles) to forces not of their own making. The various non-adiabatic perturbations that act upon geomagnetically trapped par-

ticles are unified, however, by an essential element of randomness, in that they distinguish among otherwise equivalent particles according to the instantaneous value ($\text{mod } 2\pi$) of a phase variable associated with the quasi-periodic unperturbed (adiabatic) motion.

Since the conditions of observation typically preclude the distinction of particle phases over time scales longer than a few drift periods, the day-to-day evolution of the earth's radiation belts cannot be traced with deterministic detail. Thus, for essentially all practical purposes, radiation-belt observations are phase-averaged. Moreover, the phase-averaged non-adiabatic evolution of the radiation belts proceeds via Brownian motion in canonical-momentum space, *i.e.*, by diffusion of the particle distribution among values of the adiabatic invariants.

Because particle diffusion in the radiation belts is best formulated in terms of the adiabatic invariants, a working knowledge of adiabatic theory is prerequisite to an understanding of particle diffusion. For this reason, the entire first chapter of the present volume is devoted to the adiabatic motion of charged particles, as applied to simple models of the earth's magnetosphere. The second and third chapters cover the theory of selected dynamical processes that violate adiabatic invariants and consequently lead to particle diffusion. The fourth and fifth chapters describe the interpretation and analysis of observational data having quantitative significance in terms of radial and pitch-angle diffusion, and the final (sixth) chapter briefly summarizes the present state of knowledge concerning particle diffusion in the radiation belts.