VI. Summary

The dynamical processes that govern the earth's radiation belts have a phenomenological representation in the form of a modified Fokker-Planck equation. The operand of this partial differential equation is the drift-averaged phase-space distribution function $\bar{f}=\bar{J}_{\perp}/p^2$, where $J_{\perp}$ is the differential unidirectional particle flux at pitch angle $\pi/2$ and p is the scalar momentum. The transport coefficients that enter the modified Fokker-Planck equation correspond to the violation of one or more of the three adiabatic invariants of charged-particle motion. Adiabatic invariants of radiation-belt particles can be violated either by collisions with atmospheric constituents or by interactions with magnetospheric waves and other disturbances. For processes not involving collisions of radiation-belt particles with the atmosphere, the Fokker-Planck equation typically reduces to a diffusion equation. The basic objective of radiation-belt physics is to account for the observational data on energetic particles within the context of the Fokker-Planck equation by inserting realistic transport coefficients for the operative dynamical processes.

Non-Diffusive Phenomena. The principal effect of the atmosphere on radiation-belt protons and heavier ions is to cause charge exchange and deceleration through Coulomb collisions. Charge-exchange collisions of protons can be represented by a simple loss term of the form $-\bar{f}/\tau_q$. However, for multiply-charged ions (mainly helium in the case of the earth's magnetosphere), charge exchange serves to couple the transport equations governing the distribution functions of variously charged ions derived from identical nuclei. Charge exchange and the deceleration of ions by Coulomb collisions are accompanied by very little pitch-angle diffusion or energy diffusion (range straggling).

Pitch-Angle Diffusion. The atmospheric scattering of radiation-belt electrons involves diffusion in both pitch angle and energy, as well as deceleration. The Coulomb collisions that are important in the context of deceleration and range straggling involve the individual electrons that surround the nucleus. Pitch-angle diffusion involves the nuclear charge, as shielded by the electron cloud. Only at $L \lesssim 1.3$ do atmospheric

collisions importantly affect electrons that mirror near the geomagnetic equator. Atmospheric scattering of electrons at $L \gtrsim 1.3$ is important only in that it defines a loss cone in velocity space for electrons acted upon by other dynamical processes.

Beyond $L \approx 1.3$, electron pitch angles diffuse into the atmospheric loss cone primarily by virtue of interactions between the electrons and magnetospheric plasma waves. Except for electrons that mirror very near the magnetic equator, the interaction that produces pitch-angle diffusion is probably a Doppler-shifted cyclotron resonance with electromagnetic cyclotron waves (whistler mode). The pitch-angle diffusion of electrons mirroring near the equator can be produced by cyclotron resonance with electrostatic cyclotron waves (Bernstein modes), by bounce resonance with MHD waves, or by Landau resonance ($\omega = k_{\parallel} v_{\parallel}$) with whistler-mode waves propagating at oblique angles to the geomagnetic field.

The empirical result of wave-particle interactions is a pitch-angle-diffusion lifetime that varies strongly with L, but only moderately with energy, for radiation-belt electrons. The lifetime at fixed energy ($E \sim 0.5$ MeV) typically decreases by a factor ~ 60 (from ~ 300 days to ~ 5 days) between $L=1.5$ and $L=5$ (*cf.* Figs. 39—41, Section IV.3). Electrons having $E \sim 1$ MeV are more long-lived ($\tau \sim 10$ days at $L \sim 5$). It is plausible that the "slot" between the inner and outer electron belts (at $L \sim 3$) may correspond to a region of anomalously intense pitch-angle diffusion[57]. Quantitative evaluation of pitch-angle diffusion for electrons inside the plasmasphere, resulting from resonant interactions with obliquely-propagating whistler-mode waves, has provided reasonable agreement with observed electron lifetimes [121].

Proton pitch-angle diffusion, presumably caused by electromagnetic ion-cyclotron waves, is believed to play an important role in the dynamics of radiation-belt protons having $E \lesssim 400$ keV. Isolated instances of pitch-angle diffusion have been detected also for protons with $E \sim 5$—70 MeV near synchronous altitude. However, pitch-angle diffusion is not known to play an important role in establishing the observed flux profile of outer-zone protons, and currently successful models of the earth's proton radiation environment seldom include pitch-angle diffusion for protons having $E \gtrsim 1$ MeV.

Radial Diffusion. The processes that are known to produce radial diffusion of geomagnetically trapped particles generally involve disturbances

[57] However, formation of the slot could also be related to the diminishing importance of radial diffusion at $L \lesssim 3$ and/or to the presence of an internal (CRAND?) source [20] at $L \lesssim 2$. The flux profile of outer-zone electrons (terminating at the slot) can be understood in terms of a balance between pitch-angle diffusion and radial diffusion [70] from a source at $L \gtrsim 6$.

of magnetospheric extent. Sudden magnetic and electrostatic impulses violate only the third invariant of adiabatic motion for radiation-belt particles. The radial-diffusion coefficients D_{LL} produced by these disturbances (whose power spectra vary approximately as ω^{-2}) are found to be strong functions of L. Magnetic sudden impulses typically yield for D_{LL} a magnitude that is independent of particle species and energy, but which is much larger for particles mirroring near the equator than for those mirroring at high magnetic latitudes. Electrostatic sudden impulses produce a D_{LL} whose magnitude varies as the inverse square of the particle drift frequency $\Omega_3/2\pi$. Thus, the value of D_{LL} resulting from electrostatic impulses depends upon particle species and energy, but the dependence of this D_{LL} on equatorial pitch angle is quite weak.

Several empirical estimates for D_{LL} at constant M and $J=0$ are collected in Fig. 83 [122]. The numbered curves correspond to the reference list that follows this chapter. These estimates have been obtained by investigators using a variety of analytical methods, as indicated in the caption. The results reported to apply beyond $L\sim 2$ vary by an order of magnitude up and down from a "compromise" D_{LL} of the form $D_{LL}\approx 1.3\times 10^{-9}\ L^{10}\ \mathrm{day}^{-1}$. Thus, there is considerable room for disagreement concerning the "best" numerical value[58] for D_{LL}.

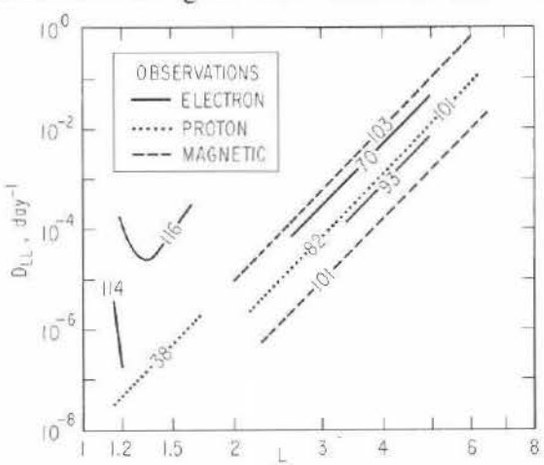

Fig. 83. Compilation [122] of radial diffusion coefficients obtained by various empirical methods, assuming constant M and J: counting of magnetic impulses [101, 103]; spatial quadrature [114, 116]; variational technique [93]; temporal integration [70, 82]; spatial integration [38, 101].

[58]In working with algebraic expressions for D_{LL}, it is convenient to remember certain approximate numerical relationships, *e.g.*, $2^{10}=1024\sim 10^3$, $4^{10}\sim 10^6$, $8^{10}\sim 10^9$; $3^{10}\sim 6\times 10^4$, $6^{10}\sim 6\times 10^7$; $5^{10}\sim 10^7$; $7^{10}\sim 3\times 10^8$; $\pi^2\sim 10$, $\pi^3\approx 31$; 1 year $\sim \pi\times 10^7$ sec; $e^3\approx 20$, $e^{0.7}\approx 2$, $e^{2.3}\approx 10$.

It is important to realize, when considering the spread of results in Fig. 83, that these values of D_{LL} were obtained for particles having a considerable range of M values. Some of the spread could well result from a genuine variation of D_{LL} with M and/or particle species among the various observations, *e.g.*, from the contribution of electrostatic impulses to radial diffusion. Moreover, the magnitude of D_{LL} surely varies with geomagnetic activity, and hence with time. Finally, discrepancies among several of the reported values of D_{LL} may originate from an incompatibility among the assumptions that underlie the various methods of empirical analysis (in particular [70], [93], and [116]).

The inverse variation of D_{LL} with L for electrons at low L values (see Fig. 83) is apparently a real effect; *i.e.*, not an artifact of the analytical procedure. The enhanced radial diffusion at $L\lesssim 1.2$ may possibly result from pitch-angle diffusion in the presence of drift-shell splitting (see Section III.7). No similar enhancement is evident in D_{LL} for protons [38], which do not suffer significant pitch-angle diffusion from atmospheric collisions. If pitch-angle diffusion is indeed responsible for enhanced electron radial diffusion below $L\approx 1.2$, then the hypothesis of constant M and J in the analysis should be replaced by a hypothesis of constant energy; this would approximately double the required magnitude of D_{LL} in this region. Elsewhere in the radiation belts, the magnitudes of D_{LL} that could result from constant-energy processes are typically found to be too small (by factors ~ 4—100) to account for the particle observations. Moreover, the direction of stochastic flow in L at constant energy is often opposite to the flow direction at constant M and J.

Quo Vadimus? Radial diffusion and pitch-angle diffusion are essential processes in the dynamics of geomagnetically trapped radiation. The determination of numerical values for the various diffusion coefficients will surely continue to be a subject of major scientific interest, since it is important to be able to understand and, ultimately, to predict the evolution of the radiation belts.

Progressively sophisticated methods of data analysis are being brought to bear on the problem of extracting diffusion coefficients from particle observations. One of the next major steps will probably involve the fitting of observational data within the framework of a multi-dimensional Fokker-Planck equation through the use of least-squares techniques. In order for such procedures to be utilized most profitably, the particle distribution functions must be specified with as much detail as is technologically feasible. This objective requires extensive spatial and spectral coverage of the radiation belts by coherently instrumented satellites. In many analyses of this type it will be necessary to augment the basic diffusion equation with terms representing non-diffusive phenomena, *e.g.*, distributed particle sources, particle acceleration

in situ, particle "injection" associated with magnetic storms, and inelastic collisions with the atmosphere.

Much more effort will be devoted in the future to spectral analysis of the electromagnetic and electrostatic fields present in the radiation belts. The spectra that result from such analyses can be utilized directly to calculate the magnitudes expected of the various diffusion coefficients. Much more effort will probably be devoted as well to relating these *in situ* spectral measurements with similar measurements made at ground stations. Eventually it may become possible to utilize the extensive geographical coverage of ground measurements to infer the condition of the magnetospheric environment. This would make it possible to monitor disturbances from the ground while observing the response of trapped particles in space, with the ultimate goal of gaining predictive insight into the consequences of magnetospheric dynamical processes.

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Frequently used Symbols

A, A_j	atomic weight (number of nucleons in ion)
$\mathbf{A}$	magnetic vector potential
a	earth radius (6371.2 km)
$\mathbf{B}$	magnetic field
B	magnitude of $\mathbf{B}$
$\hat{\mathbf{B}}$	unit vector $\mathbf{B}/B$
B_0, B_1, B_2	coefficients of Mead field
B_e, B_m	equatorial field, mirror field
B_r, B_θ, B_φ	components of $\mathbf{B}$ (spherical coordinates)
B_t	tail-field intensity
$\mathbf{b}$	magnetic-field perturbation
b	stand-off distance; magnitude of $\mathbf{b}$
$b_\perp, b_\parallel$	components of $\mathbf{b}$ (relative to $\hat{\mathbf{B}}$)
b_x, b_y	components of $b_\perp$
$\mathscr{B}_\parallel, \mathscr{B}_\perp, \mathscr{B}_z$	magnetic spectral densities
c	speed of light (3×10^5 km/sec)
c_A	Alfvén speed
$D, D(y)$	dipole drift function, $=(1/2)\,T(y)-(1/12)\,Y(y)$
$\mathbb{D}, \tilde{\mathbb{D}}$	diffusion tensors
D_{xx}	pitch-angle (-cosine) diffusion coefficient
D_{LL}	radial diffusion coefficient
D_{LL}^*	Bohm diffusion coefficient
D_t'	Fokker-Planck coefficient
D_{st}	ring-current index
$\mathbf{E}$	electric field
E	kinetic energy
E_c	convection electric field (equatorial magnitude)
E_0	e-folding energy
E_r, E_θ, E_φ	components of $\mathbf{E}$ (spherical coordinates)
$\mathbf{e}$	electric-field perturbation
$\mathscr{E}_c, \mathscr{E}_m$	electrostatic spectral densities
$\mathscr{E}_\perp, \mathscr{E}_\parallel$	electrostatic spectral densities
$\mathbf{F}$	force; guiding-center force
F_j	unit-normalized distribution function

f, f_j	phase-space distribution function
$\bar{f}, \bar{f}_j$	phase average of f (j identifies particle species)
$\tilde{f}, \tilde{f}_j$	perturbation of f, *i.e.*, $\tilde{f}=f-\bar{f}$
$f_{\|}, f_{\|}(s,t)$	perturbing force (parallel to **B**)
f_n	Fourier amplitude of $f_{\|}(s,t)$
$\mathscr{F}_{\|}$	spectral density of $f_{\|}(s,t)$
$G(J_i;Q_j)$	Jacobian, $=\det(\partial J_i/\partial Q_j)$
$G_1(y), G_2(y)$	shell-tracing functions
$g_n(x)$	pitch-angle eigenfunction ($n=0,1,2,\ldots$)
H	Hamiltonian function
$\hbar$	Planck's constant ($\div 2\pi$)
I	$J/2p$, geometrical "invariant"; electrical current
I_i	mean excitation energy
$I_\alpha, I_\perp$	integral unidirectional flux ($I_\perp \equiv I_{\pi/2}$)
$I_{4\pi}$	integral omnidirectional flux
$I^*_{4\pi}, I^*$	maximum integral omnidirectional flux
Im	imaginary part
J	second adiabatic invariant
J_i	fundamental action integral ($i=1,2,3$)
$J_\alpha, J_\perp$	differential unidirectional flux ($J_\perp = J_{\pi/2}$)
$\bar{J}_\alpha, \bar{J}_\perp$	phase averages of J_α and $J_\perp$
$J_{4\pi}$	differential omnidirectional flux
J_0, J_l	Bessel function (order zero, order l)
J	electrical current density
K	derived invariant, $=J/(8m_0M)^{1/2}$
K_p	index of geomagnetic activity
k	wave-propagation vector
L	invariant drift-shell parameter, $=2\pi B_0 a^2 \|\Phi\|^{-1}$
L_d	field-line label
L_m	McIlwain shell parameter
M	first adiabatic invariant
m	relativistic mass; harmonic number
m_0, m_j	rest mass (j identifies particle species)
N_e, N_p	electron and proton densities (particles/cm^3)
$\bar{N}_e, \bar{N}_i$	atmospheric densities (drift averaged)
n	refractive index, $=ck_{\|}/\omega$; harmonic number
P_l^m	associated Legendre function (Schmidt-normalized)
$P_\perp, P_{\|}$	plasma pressures (relative to $\hat{\mathbf{B}}$)
p	particle momentum
$Q(y)$	shell-tracing function
Q_i	new coordinate (generalized)
$\mathbf{q}, q_i$	canonical position ($i=1,2,3$)
q	particle charge; charge-exchange (subscript)

Re	real part
r	radial coordinate
S	arc length of field line; source for $\partial\bar{f}/\partial t$
$T, T(y)$	dipole bounce function
$\tilde{T}(y;L_d,\varphi)$	bounce function in distorted field
t	time
u	solar-wind velocity
V_e, V_m	electric and magnetic scalar potentials
$\mathbf{v}_d$	guiding-center drift velocity
v_g, v_p	group velocity ($d\omega/dk_{\|}$), phase velocity ($\omega/k_{\|}$)
v	particle velocity
$\mathscr{V}, \mathscr{V}_m$	spectral density of electrostatic potential
W	energy (kinetic plus potential), $=E+qV_e(\mathbf{r})$
w	energy-like variable, $p^2/2m_0$
X	field-line coordinate, $X^2 \equiv 1-(B_e/B)$
x	cosine of equatorial pitch angle
x_b, x_c	cosine of (bounce-, drift-) loss-cone half-angle
$Y, Y(y)$	dipole bounce function
$\tilde{Y}(y;L_d,\varphi)$	bounce function in distorted field
y	sine of equatorial pitch angle
Z, Z_i	ionic charge number, nuclear charge number
α	local pitch angle; Euler potential
β	Euler potential
$\beta_\perp, \beta_{\|}$	plasma indices ($8\pi P_\perp/B^2$ and $8\pi P_{\|}/B^2$)
$\Gamma(s+1)$	gamma function, $s!$
γ, γ_j	ratio of relativistic mass to rest mass
γ	unit of magnetic intensity, $=10^{-5}$ gauss
δ_{ij}	Kronecker symbol ($=1$ if $i=j$; $=0$ otherwise)
ε	adiabaticity index, $=\langle v/\Omega_1 S\rangle$
$\varepsilon_1, \varepsilon_2$	field-expansion indices, $\varepsilon_l \equiv (B_l/B_0)(L_d a/b)^{l+2}$
ζ	M/y^2, approximate invariant
θ	colatitude; angle between $\hat{\mathbf{k}}$ and $\hat{\mathbf{B}}$
θ_m	colatitude of mirror point
Λ	"invariant" magnetic latitude ($L=\sec^2\Lambda$)
λ_D	Debye length
$\mu_{\|}, \mu_\perp, \mu_l$	electrical mobilities (Ohm, Pedersen, Hall)
$\boldsymbol{\pi}, \pi_i$	canonical momentum ($i=1,2,3$)
ρ_s	solar-wind mass density
τ	interaction time; pitch-angle-diffusion lifetime
Φ	third adiabatic invariant
φ, φ_a	magnetic longitude, longitude of "anomaly"
φ_i	phase conjugate to $J_i/2\pi$ ($i=1,2,3$)

ψ_n	Fourier phase of wave or field
ψ_s	angle of deflection (solar wind)
$\Omega_i/2\pi$	frequency of gyration, bounce, or drift ($i=1,2,3$)
Ω_j	nonrelativistic gyrofrequency ($\times 2\pi$), $-q_jB/m_jc$
$\mathbf{\Omega}_0$	angular velocity of earth's rotation
ω	frequency ($\times 2\pi$)
ω_j	ion or electron plasma frequency ($\times 2\pi$)
$\omega_n/2\pi$	Fourier frequency of wave or field ($n=1,2,3,\ldots$)

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