10

Fundamentals of planetary magnetospheres

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10.1 Introduction

The comparative study of the magnetospheres associated with various planets (and with some other objects within the solar system) aims to develop a unified general description of magnetospheric phenomenology and physics that is applicable to a variety of different systems. In this way concepts and theories, often developed in the first instance to fit specific phenomena in a particular magnetosphere, can be tested for correctness and applicability in a more general context.

The subject matter of magnetospheric physics, in general terms, deals with the configuration and dynamics of systems that result from the magnetic field interaction between isolated objects and their environments. Such systems exhibit a variety of fascinating physical phenomena, both visible (e.g. auroral light emissions) and invisible (e.g. charged-particle radiation belts), some of which have significant aspects of practical application (e.g. space weather effects at Earth, radiation dosage problems for spacecraft near Jupiter or Saturn). They also present a special challenge for physical understanding: as regions of transition between an object and its environment, magnetospheres are by their very nature spatially inhomogeneous systems, characterized by the inclusion of very different physical regimes, large ranges of parameter values, and the overwhelmingly important role of gradients.

The aim of this chapter is to present magnetospheric physics, at least in outline, as a self-contained discipline – a branch of physics described in logical sequence, as distinct from a study of individual objects (often presented in historical sequence). The detailed observational and theoretical study of the magnetospheres at individual planets (Chapter 13) remains, of course, the foundation stone of the general discipline, along with the detailed examination and modeling of the most accessible and best known of all magnetospheres, that of the Earth (see also Chapter 11).

10.2 Definitions and classifications

The term "magnetosphere" was introduced by Gold (1959) as the name for "the region above the ionosphere in which the magnetic field of the [Earth] has a dominant control over the motions of gas and fast charged particles". It quickly acquired a broader connotation as the region of space dominated, in a not always precisely defined sense, by the magnetic field of the Earth (replacing the term "geomagnetic cavity" introduced earlier by Chapman and Ferraro) and was soon being applied to analogous regions at other planets as well.

In the most general context, we consider a *central object*: a distinct well-defined body held together (in most cases) by its own gravity. It is immersed in a tenuous *external medium*, assumed to be sufficiently ionized that it behaves as a plasma. The *magnetosphere* is then the region of space around the central object within which the object's magnetic field has a dominant influence on the dynamics of the local medium. An alternative and in some ways more precise view is to regard the magnetosphere as the region enclosed by its bounding surface, the *magnetopause*, the latter being defined as the discontinuity of the magnetic field, where its direction changes; inside the magnetopause the controlling field is that of the central object, while outside it primarily the magnetic field of the distant external medium. This definition is particularly useful for the magnetospheres of planets in the solar wind: the continual variability of the interplanetary magnetic field direction in contrast with the relative constancy of the planetary magnetic dipole allows in most cases an easy observational identification of the magnetopause.

For a magnetosphere to exist, the central object must have a magnetic field of sufficient strength; if this magnetic field is dipolar, as in most cases of interest, the magnetic dipole moment must exceed a minimum value (discussed in Section 10.3.1 below). Even if the magnetic field of the central object is too weak to produce a true magnetosphere, the interaction of the central object with the magnetic field of the external medium may nevertheless create structures similar to those found in magnetospheres; in such cases the designation *magnetosphere-like system* is often used.

Within the solar system, magnetospheres have been observed at the planets Mercury, Earth, Jupiter, Saturn, Uranus, Neptune (external medium, the solar wind) and at Jupiter's moon Ganymede (external medium, the plasma of Jupiter's magnetosphere), magnetosphere-like systems at the planets Venus, Mars (external medium, the solar wind), at Jupiter's moon Io (external medium, the plasma of Jupiter's magnetosphere), and at Saturn's moon Titan (external medium, the plasma of Saturn's magnetosphere); all these systems are discussed in Chapter 13. The interaction of a comet with the solar wind may also be viewed as a magnetosphere-like system. On a much larger scale, the entire heliosphere may be viewed as being produced by the interaction of the Sun (the central object) with the local interstellar medium (the external medium) and described either as a magnetosphere or as a magnetosphere-like system, depending on what one chooses to emphasize; see Section 10.6 for further discussion. Finally, the concepts of magnetospheric physics have also been applied, albeit speculatively in most cases, to astrophysical objects such as pulsars (e.g. Michel, 1982), X-ray sources in binary systems (e.g. Vasyliūnas, 1979) and even to systems as large as radio galaxies (e.g. Miley, 1980).

In this chapter, as noted already, magnetospheric physics will be discussed as a general discipline, abstracted from the consideration of individual cases. Since the development of the discipline to date has been based to a large extent on studies of planetary magnetospheres, it is convenient to use the terminology "planet" and "solar wind" in place of the more general but somewhat colorless "central object" and "external medium". The presentation proceeds in three steps of increasing complexity. In Section 10.3 the interaction of the solar wind with just the dipolar magnetic field of the planet is discussed. In Section 10.4 the flow of plasma within the magnetosphere is added, which immediately brings in the coupling between the magnetosphere and the ionosphere. In Section 10.5 the plasma sources and the processes of plasma transport are considered. Finally, in Section 10.6 dimensional analysis is applied to determine the scaling relations of magnetospheric properties, by the use of which the various individual objects can be compared or contrasted.

10.3 Interaction of solar wind with a planetary magnetic field

10.3.1 A closed magnetosphere

The basic configuration of a prototypical planetary magnetosphere is shown in Fig. 10.1. Many of its characteristic structures can be understood on the basis of a simple model that takes into account only the two ingredients indispensable for the formation of a magnetosphere: the solar wind, with mass density ρ_{sw} and bulk velocity $\mathbf{v}_{sw}$, and the planetary magnetic field, with dipole moment $\mu = B_p R_p{}^3$ where B_p is the surface magnetic field strength at the equator and R_p is the radius of the planet. As a consequence of the constraints imposed by the magnetohydrodynamic (MHD) approximation (discussed under various aspects in Chapters 3–7 and briefly in Section 10.4 below), the boundary surface between the solar wind and the planetary magnetic field – the magnetopause – is nearly impermeable both to plasma and to magnetic field, resulting in a clear separation between two distinct regions of space: the magnetosphere itself, within which the magnetic field lines from the planet are confined and from which the solar wind plasma is excluded, and

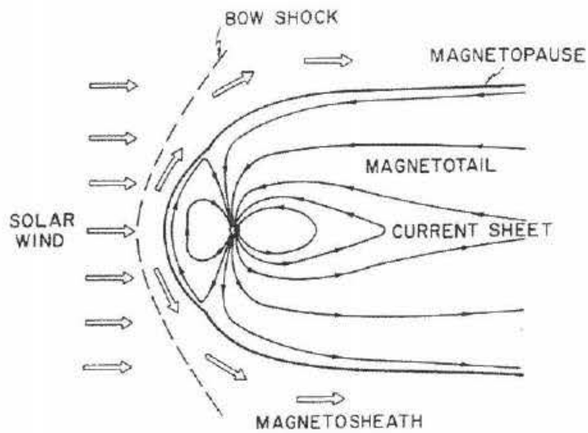

Fig. 10.1. Schematic view of a magnetically closed magnetosphere, cut in the noon–midnight meridian plane. Open arrows, solar wind bulk flow; solid lines within magnetosphere, magnetic field lines (directions appropriate for Earth).

the exterior region beyond the magnetopause, to which the plasma that comes from the solar wind is confined. This simple *closed magnetosphere* is only a first-order approximation (in reality the magnetopause is not completely impermeable but allows, under certain conditions, some penetration of plasma and of magnetic field to produce the *open magnetosphere* described in Section 10.3.3); it does, however, describe fairly accurately the size and shape of the main structures.

The solar wind flow, initially directed away from the Sun, is diverted around the magnetosphere, as indicated in Fig. 10.1. Since the initial flow speed is supersonic and super-Alfvénic (faster than both the speed of sound and the Alfvén speed v_A), the solar wind is first slowed down, deflected, and heated at a detached *bow shock* situated upstream of the magnetopause (the bow shock is analogous to the sonic boom in supersonic aerodynamic flow past an obstacle). The region between the bow shock and the magnetopause, within which the plasma from the solar wind is flowing around the magnetosphere, gradually speeding up and cooling, is called the *magnetosheath*, a term introduced by Dessler and Fejer (1963) replacing the term *transition region* used in the older literature.

The location of the magnetopause is determined primarily by the requirement of pressure balance: the total pressure (plasma plus magnetic) must have the same value on each side of the discontinuity. In the simple closed magnetosphere considered here, the plasma pressure inside the magnetopause and the magnetic pressure outside are both neglected. The exterior pressure then scales as the linear momentum flux density in the undisturbed solar wind, $\rho_{sw} v_{sw}{}^2$ (often called the *dynamic pressure* of the solar wind), and is a maximum in the sub-solar region, where the plasma near the magnetopause is almost stagnant. The interior pressure scales as the magnetic pressure of the dipole field, $(1/8\pi)(\mu/r^3)^2$ with μ the magnetic dipole moment of the planet, and thus varies strongly with distance from the planet. Equating the two gives an estimate for the distance R_{MP} of the sub-solar magnetopause:

$$R_{MP} = (\xi\mu)^{1/3}\,(8\pi\rho_{sw}v_{sw}{}^2)^{-1/6} \tag{10.1}$$

where ξ is a numerical factor that corrects for the added field from magnetopause currents ($\xi \simeq 2$ to first approximation).

The distance given in Eq. (10.1) (with various choices of ξ) is often called the Chapman–Ferraro distance. In this chapter, we will consistently use the symbol R_{CF} for the distance *defined* by Eq. (10.1) with $\xi = 2$, i.e. for the nominal distance of the sub-solar magnetopause predicted by pressure balance; we will reserve the symbol R_{MP} for the *actual* distance of the sub-solar magnetopause in any particular context. Thus, $R_{MP} \simeq R_{CF}$ in the present case of a simple closed magnetosphere but this is not necessarily so in the case of more general models.

The pressure balance condition, combined with assumptions about the sources of the magnetic field within the magnetosphere, may be used to calculate not only the distance to the sub-solar point but also the complete shape of the magnetopause surface; for a discussion of such models at Earth, see Chapter 11 and also reviews by e.g. Siscoe (1988). Typically the magnetopause is roughly spherical on the dayside of the planet, facing into the solar wind flow (the effective center of the sphere being located behind the planet, very roughly at a distance 0.5 R_{MP}), and is elongated in the anti-sunward direction.

The magnetopause distance R_{MP} may be regarded as the characteristic scale for the size of a magnetosphere. Being equal to R_{CF} in the case of negligible plasma pressure and no magnetic field sources other than the planetary dipole inside the magnetosphere, R_{MP} can be readily calculated from Eq. (10.1) given only a few basic parameters of the system. In the case where the plasma pressure or a non-dipolar field in the outer regions of the magnetosphere is not negligible, the qualitative effects on R_{MP} can still be estimated from the pressure balance, as illustrated in Fig. 10.2: (a) the actual distance R_{MP} is larger than the nominal distance R_{CF} (the value $\xi = 2$ instead of $\xi = 1$ is in fact a consequence of the non-dipolar field from the magnetopause currents); (b) a change in the solar wind dynamic pressure produces a larger change in the magnetopause distance – the magnetosphere is less "stiff" if the plasma pressure in the interior is significant.

10.3.2 Stress considerations

From a more general physical viewpoint, determining the magnetopause from the pressure balance is an instance of a recurrent theme in magnetospheric physics: the shaping of the magnetic field configuration by the mechanical stresses of the

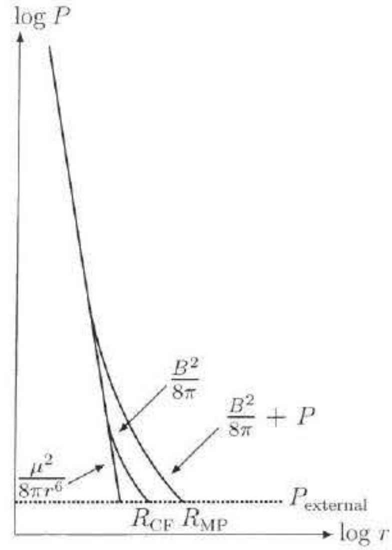

Fig. 10.2. Variation in the total pressure (magnetic plus plasma) with distance from the planet and its relation to the radial distance of the sub-solar magnetopause. The relationship for R_{MP} in Eq. (10.1) may be compared with the schematic representation of a more realistic plasma-filled, non-dipolar planetary magnetic field.

plasma. The dipole magnetic field as such exerts no stresses outside the planet, its magnetic pressure being exactly balanced by its magnetic tension; the pressure balance at the magnetopause arises solely because the planetary dipole magnetic field, which by itself would extend to infinity, has been pushed back by the solar wind and confined to a finite space. Since the magnetosphere presents an obstacle to the solar wind flow, it is subject to forces as the flow is deflected. Locally, at any point of the magnetopause, the pressure exerted by the exterior plasma is balanced by the pressure of the interior magnetic field. Globally, these questions arise: what is the total force on the magnetosphere from the exterior medium, and on what is it really exerted?

A powerful tool for investigating such questions is the well-known momentum equation of the plasma, expressed in the standard conservation form (the partial time derivative of the density of a conserved quantity plus the divergence of the flux density of that quantity):

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v} + \mathbf{P} - \mathbf{T}) = 0 \tag{10.2}$$

where ρ, $\mathbf{v}$, and $\mathbf{P}$ are, respectively, the mass density, bulk flow velocity, and pressure tensor of the medium; $\mathbf{T}$ is the Maxwell stress tensor $\mathbf{T}_{\mathrm{m}}$ plus the stress tensors representing any other forces, if significant:

$$\mathbf{T}_{\mathrm{m}} = \frac{\mathbf{BB}}{4\pi} - \frac{\mathbf{I}B^2}{8\pi} \tag{10.3}$$

($\mathbf{I}$ is the unit dyad, $I_{ij} = \delta_{ij}$; the units are Gaussian). The divergence of $\mathbf{T}_{\mathrm{m}}$ is equal to the Lorentz force per unit volume:

$$\nabla \cdot \mathbf{T}_{\mathrm{m}} = (\nabla \times \mathbf{B}) \times \frac{\mathbf{B}}{4\pi} = \frac{1}{c}\mathbf{J} \times \mathbf{B}. \tag{10.4}$$

The total force $\mathbf{F}$ acting on any given volume is given by the volume integral of the divergence terms in Eq. (10.2) and therefore can be written as the surface integral over the boundary of the volume:

$$\mathbf{F} = \int \left(\frac{\mathbf{BB}}{4\pi} - \frac{\mathbf{I}B^2}{8\pi} - \rho\mathbf{vv} - \mathbf{P} \right) \cdot \mathrm{d}\mathbf{S}. \tag{10.5}$$

By Newton's second law, this force must equal the net acceleration of the total mass enclosed within the volume, i.e. the integral of the time-derivative terms in Eq. (10.2); quantitatively, for a planetary magnetosphere (Vasyliūnas, 2007),

$$\mathbf{F} \simeq M \left(\frac{\mathrm{d}^2\mathbf{R}}{\mathrm{d}t^2} - \mathbf{g} \right) \tag{10.6}$$

where $\mathbf{g}$ is the gravitational acceleration due to the Sun, M is the total mass and $\mathbf{R}$ is the center of mass of the entire enclosed system, magnetosphere plus planet. Siscoe (1966) first recognized (in a terrestrial context) that, because the mass contained in the magnetosphere is smaller by many orders of magnitude than the mass of the planet, the entire force from the solar wind interaction must be exerted on the massive planet, with a negligible net force on the magnetosphere itself (for further discussion of the forces on the magnetosphere and the planet, see Siscoe (2006) and Vasyliūnas (2007) and references therein).

While the pressure from the external medium thus accounts for the formation and shape of the magnetosphere on the dayside of the planet, it cannot by itself explain the formation of the *magnetotail* on the nightside. This structure, shown also in Fig. 10.1, is a region of magnetic field lines pulled out into an elongated tail in the anti-sunward direction, with the magnetic field reversing direction between the two sides of a *current sheet* or *plasma sheet* in the equatorial region. To form this structure one needs an appropriate stress: a tension force pulling away from the planet. If we choose a closed volume bounded by a surface just outside the magnetopause plus a cross section of the magnetotail (a vertical cut at, say, the right-hand edge of Fig. 10.1) and evaluate the force by applying Eq. (10.5), the total tension force F_{MT} is given by the integral over the cross section and the total pressure force F_{MP} by the integral over the magnetopause:

$$F_{\mathrm{MT}} \simeq \frac{B_{\mathrm{T}}^2}{8\pi} A_{\mathrm{T}}, \qquad F_{\mathrm{MP}} \simeq \rho_{\mathrm{sw}} v_{\mathrm{sw}}^2 A_{\mathrm{T}} \tag{10.7}$$

where B_{T} is the mean magnetic field strength and A_{T} the cross-sectional area of the magnetotail (typically, A_{T} exceeds πR_{MP}^2 by a factor 3–4). Both F_{MP} and F_{MT} are directed away from the Sun and, as pointed out by Siscoe (1966), are exerted ultimately on the planet.

There are several possible physical mechanisms that could produce this tension force, and which of them is the most important may well differ between magnetospheres. The most straightforward mechanism (and, at least at Earth, the most widely assumed) is that for an open magnetosphere (discussed below in Section 10.3.3): magnetic field lines connect directly from the magnetotail into the solar wind flow and are dragged along with it. Alternatively, and less specifically, the solar wind flowing along the magnetopause is assumed to exert a *tangential drag* on the plasma adjacent to the interior of the magnetopause. Other mechanisms, as discussed in Section 10.5, rely on internal sources of magnetospheric plasma: a plasma pressure within the magnetosphere sufficiently strong to inflate the magnetic field lines, or a pronounced outflow from interior sources that pulls out the magnetic field, somewhat analogously to the action of the solar wind on the interplanetary magnetic field. In both cases the day–night asymmetry of the pressure outside the magnetosphere tends to confine the extended fields to the night-side of the planet.

10.3.3 The open magnetosphere

At the locations of the planets, the interplanetary magnetic field is weak in the sense that its energy density is very small in comparison to the kinetic energy density of the solar wind bulk flow: $v_{\mathrm{A}}^2 \ll v_{\mathrm{sw}}^2$. The flow of solar wind plasma past the magnetospheric obstacle deforms the magnetic field lines within the magnetosheath and drapes them around the magnetopause, a process well modeled at Earth (e.g. Spreiter *et al.*, 1968; see Section 11.4). The magnetic field is amplified and may become dynamically no longer negligible as the magnetopause is approached, but the total pressure is in general not greatly modified, an increase in magnetic pressure often being offset by a decrease in plasma pressure. One might therefore anticipate that the effect of the interplanetary magnetic field on planetary magnetospheres should be minimal.

What is overlooked in the above discussion is the possibility that, through the process of *magnetic reconnection* (the physics of which is discussed in detail in Chapter 5), the magnetic field lines from the planet may become connected with

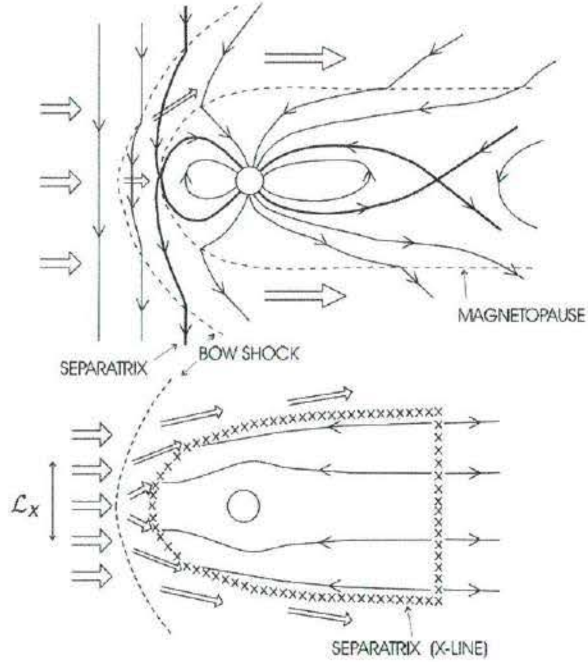

Fig. 10.3. Schematic representation of a magnetically open magnetosphere. The upper panel shows a cut in the noon–midnight meridian plane; the bold lines are magnetic field lines within the separatrix surfaces separating open lines from closed lines or open lines from interplanetary field lines; the other conventions are the same as in Fig. 10.1. The lower panel shows a cut in the equatorial plan; the perimeter line of $\times$ symbols represents the intersection with the two branches of the separatrix (compare with Fig. 4.3 for the hypothetical case with zero wind speed); the solid lines are streamlines of magnetospheric plasma flow, and $\mathcal{L}_X$ represents the projection of the dayside magnetic reconnection region along the streamlines into the solar wind (see Section 10.4.3).

those of the interplanetary magnetic field, to produce a magnetically *open magnetosphere*, shown Fig. 10.3 for the simplest case in which the interplanetary magnetic field is parallel to the planetary dipole moment. The magnetopause is now no longer impermeable to the magnetic field and, as a consequence, it need no longer be impermeable to plasma either.

The modifications of the magnetospheric system implied by an open character of the magnetosphere are in some ways minor and in other ways very far-reaching. The location and shape of the dayside magnetopause is for the most part not greatly modified (in agreement with our expectations above). The component B_n of the magnetic field normal to the magnetopause is in general small compared with the magnitude of the field, $|B_n| \ll B$ (so much so that it is often difficult to establish by direct observation that $B_n \neq 0$, and much of the evidence for an open magnetosphere has been indirect). However, the total amount of open magnetic flux Φ_M of one polarity can (at least at Earth) become comparable with the maximum amount that could reasonably be expected to be open, estimated as $\sim \mu/R_{\mathrm{MP}}$, the dipole flux beyond the distance of the sub-solar magnetopause; despite the fact that $|B_n| \ll B$, this is possible if the effective length of the magnetotail $\mathcal{L}_{\mathrm{T}} \gg R_{\mathrm{MP}}$. As noted in Section 10.3.2, that the magnetosphere is open provides immediately an explanation of the tension force needed to maintain the magnetotail, as well as a mechanism for magnetospheric convection (discussed below in Section 10.4.3). Finally, the efficiency of the reconnection process depends greatly on the relative orientation of the magnetic fields on the two sides of the magnetopause, one result of which is that the open character of the magnetosphere is most pronounced when the interplanetary magnetic field is parallel to the planetary dipole moment (i.e. antiparallel to the dipole magnetic field in the equatorial plane), $\mathbf{B}_{\mathrm{sw}} \cdot \hat{\boldsymbol{\mu}} > 0$. Since the direction of the interplanetary magnetic field is highly variable on all time scales, this can lead to pronounced time-varying changes of magnetospheric configuration as well as energy input and dissipation; such processes have been extensively observed and modeled at Earth.

From the point of view of comparing magnetospheres (see Chapter 13) an open magnetosphere embodies primarily the effects of the solar wind acting on the planetary magnetic field, and one major question is the relative importance of these effects in comparison with the effects of internal processes arising from rotation and interior sources of plasma (Sections 10.4 and 10.5.1).

10.4 Plasma flow and magnetosphere–ionosphere interaction

10.4.1 Fundamental principles

A well-known consequence of the MHD approximation is a constraining relation between the plasma bulk flow and the magnetic field: plasma elements that are initially on a common field line remain on a common field line as they are carried by the bulk flow. Since magnetic field lines in the magnetosphere of a planet connect to the ionosphere of the planet, any discussion of plasma flow in the magnetosphere immediately involves questions of magnetosphere–ionosphere interaction. The field lines extend in fact into the interior of the planet, which in many cases is highly conducting electrically; hence it might seem that magnetospheric flow is constrained by the planet itself. This does not happen, however, because, as pointed out by Gold (1959) in the same paper in which he introduced the term "magnetosphere", most planets possess an electrically neutral (and effectively non-conducting) atmosphere sandwiched between the ionosphere and the planetary interior. Although very thin in comparison with the radius of the planet, this layer suffices to break the MHD constraints and thus allow the plasma in the ionosphere and the magnetosphere to move without being necessarily attached to the planet; without such an insulating layer, much magnetospheric dynamics as we know it would not be possible.

While the plasma in the ionosphere can thus move relative to the planet, it remains constrained to move more or less together with the plasma in the magnetosphere. The conventional formulation, however, describes the plasma flow rather differently in the two regions. On the one hand the magnetosphere is treated, to a first approximation at least, as an MHD medium, with the electric field $\mathbf{E}$ related to the plasma bulk flow $\mathbf{v}$ by the MHD approximation and with the electric current $\mathbf{J}$ related to the plasma pressure by stress balance. On the other hand the ionosphere is treated as a moving conductor (the conductivity results primarily from collisions between the ions and the neutral particles, planetary ionospheres being for the most part weakly ionized), with $\mathbf{J}$ related by a conductivity tensor to $\mathbf{E} + \mathbf{v}_n \times \mathbf{B}/c$, where $\mathbf{v}_n$ is the bulk velocity of the neutral medium. The magnetosphere and the ionosphere are coupled by current continuity,

$$\nabla \cdot \mathbf{J} = 0, \tag{10.8}$$

which implies a connection between the currents in the two regions, and by Faraday's law

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}, \tag{10.9}$$

which implies, in this context, the continuity of the tangential electric field.

The resulting scheme of calculation is illustrated in Fig. 10.4 in the form in which it was first proposed by Vasyliūnas (1970) and Wolf (1970), generalizing earlier work by Fejer (1964). Since then it has been elaborated considerably in detail, but neither the logic nor the equations have changed appreciably (for a review see e.g. Wolf (1983) and references therein). What has changed to some extent in recent years is the physical understanding of the equations: stress and flow are emphasized for the ionosphere as well as for the magnetosphere, and the limitation of the theory to quasi-steady equilibrium situations is better appreciated. Motivated in part by discussions on the relative role of $\mathbf{B}$ and $\mathbf{v}$ versus $\mathbf{J}$ and $\mathbf{E}$ in plasmas (Parker, 1996, 2000, 2007; Vasyliūnas, 2001, 2005a,b; and references therein), this new understanding is particularly relevant when discussing the magnetospheres of the giant planets (see Chapter 13): within these, rotational stresses are of obvious importance, and the long wave-travel times imply correspondingly long time scales for establishing equilibrium. Accordingly, before describing corotation and other plasma flows of the various magnetospheres, we will summarize the new aspects in the understanding of magnetosphere–ionosphere coupling; a more conventional treatment of the theory, as applied at Earth, is given in Chapter 11.

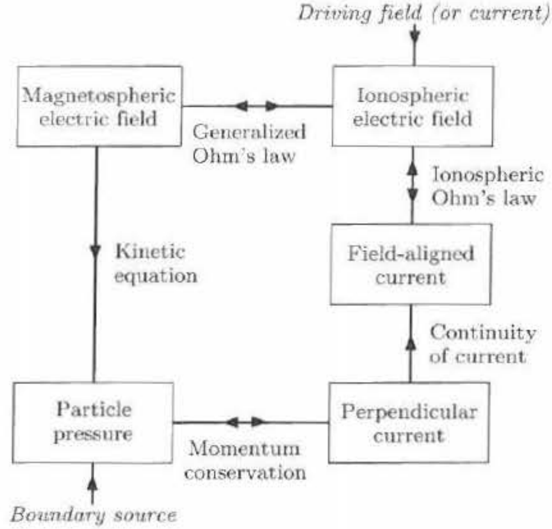

Fig. 10.4. Schematic diagram of magnetosphere–ionosphere coupling calculations (after Vasyliūnas, 1970).

The principal insights are the following.

(1) As long as $v_A{}^2/c^2 \ll 1$ (i.e. the inertia of the plasma is dominated by the rest mass of the plasma particles, not by the relativistic energy-equivalent mass of the magnetic field), $\mathbf{v}$ produces $\mathbf{E}$ but $\mathbf{E}$ does not produce $\mathbf{v}$ (Buneman, 1992; Vasyliūnas, 2001). The primary quantity physically is thus the plasma bulk flow, established by appropriate stresses. The electric field is the result of the flow, not the cause; its widespread use in calculations is primarily for mathematical convenience (one example is mentioned below in the discussion of Eq. (10.14)).

(2) The electric current in the ionosphere is not an Ohmic current in the physical sense, and its conventional expression by the "ionospheric Ohm's law"

$$\mathbf{J}_\perp = \sigma_P \mathbf{E}_\perp^* + \sigma_H \hat{\mathbf{B}} \times \mathbf{E}^*, \qquad \text{where} \qquad \mathbf{E}^* \equiv \mathbf{E} + \frac{1}{c}\mathbf{v}_n \times \mathbf{B} \qquad (10.10)$$

has a merely mathematical significance. Here σ_P and σ_H are the Pedersen and the Hall conductivities (e.g. Matsushita, 1967), $\mathbf{E}^*$ is the electric field in the frame of reference of the neutral atmosphere, and $\hat{\mathbf{B}}$ is a unit vector.

Physically, the current is determined by the requirement that the Lorentz force should balance the collisional drag between the plasma and the neutral atmosphere when their bulk flow velocities differ (Song *et al.*, 2001; Vasyliūnas and Song, 2005). The governing equations (giving horizontal components only) are the momentum equation of the ionospheric plasma

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \cdots = \frac{1}{c}\mathbf{J} \times \mathbf{B} - \nu_{in}\rho(\mathbf{v} - \mathbf{v}_n) \qquad (10.11)$$

(ν_{in} is the ion–neutral-particle collision frequency), in which the time derivative term and the inertial and pressure terms (indicated by the ellipses) on the left-hand side are neglected, and the generalized Ohm's law in the form

$$0 = \mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} - \frac{\mathbf{J} \times \mathbf{B}}{n_e e c} + \cdots. \tag{10.12}$$

Eliminating **v** between Eqs. (10.11) and (10.12) then yields Eq. (10.10). The current in the ionosphere is thus governed by stress balance in the same way as the current in the magnetosphere, the latter being determined by the momentum equation for the magnetospheric plasma (cf. Eq. (10.2)),

$$\frac{\partial \rho \mathbf{v}}{\partial t} = -\nabla \cdot (\rho \mathbf{v}\mathbf{v} + \mathbf{P}) + \frac{1}{c}\mathbf{J} \times \mathbf{B}, \tag{10.13}$$

with the time derivative term on the left-hand side neglected.

(3) Underlying this neglect of the time derivative (i.e. acceleration) terms in the momentum equations is the implicit assumption that any imbalance between the mechanical and the magnetic stresses (which, fundamentally, is what determines the acceleration of the plasma) produces a bulk flow that acts to reduce the imbalance, which then becomes negligible over a characteristic time scale that is easily shown to be of the order of the Alfvén wave travel time across a typical spatial scale, e.g. along a field line. The theory is thus applicable only to systems that are stable and evolve slowly, on time scales $\tau \gg \mathcal{L}/v_A$.

(4) For phenomena that are large-scale in the sense that their time and length scales are long in comparison with those of plasma oscillations ($\tau \gg 1/\omega_p$ and $\mathcal{L} \gg \lambda_e \equiv c/\omega_p$ where ω_p is the electron plasma frequency and λ_e is the electron inertial length, also known as the collisionless skin depth), **J** adjusts itself to become equal to $(c/4\pi)\nabla \times \mathbf{B}$, not the other way around (Vasyliūnas, 2005a, b). Although **B** is in principle determined from a given **J** by Maxwell's equations (on a time scale of the light travel time $\sim \mathcal{L}/c$), in a large-scale plasma any $\mathbf{J} \neq (c/4\pi)\nabla \times \mathbf{B}$ is immediately (on a time scale $\sim 1/\omega_p$) changed by the action of the displacement-current electric field on the free electrons in the plasma. The current continuity condition $\nabla \cdot \mathbf{J} = 0$ is thus satisfied automatically; there is no separate requirement of current closure – what is often discussed under that rubric is in reality the coupling of the Maxwell stresses along different portions of a field line.

10.4.2 Corotation

Corotation with the planet is the simplest pattern of plasma flow in a planetary magnetosphere and one that plays a major role particularly in the magnetospheres

of the giant planets (see Chapter 13). Understanding the conditions that are needed for corotation to exist as well as those that result in significant departures from corotation is an essential aspect of magnetospheric studies.

If the planet possesses an insulating atmosphere, the rotation of the planet itself has no *direct* effect on plasma flow in the magnetosphere, as discussed in Section 10.4.1. What does affect plasma flow is the motion of the neutral upper atmosphere (the thermosphere) at altitudes of the ionosphere where the neutral and the ionized components coexist and interact. "Corotation with the planet" is therefore not quite an accurate description. What really is meant is co-motion with the upper atmosphere, which in turn is then assumed to corotate with the planet, for reasons unrelated to the magnetic field, i.e. the vertical transport of horizontal linear momentum from the planet to the neutral atmosphere (e.g. by collisional or eddy viscosity or similar processes) together with an assumed small relative amplitude of the neutral winds.

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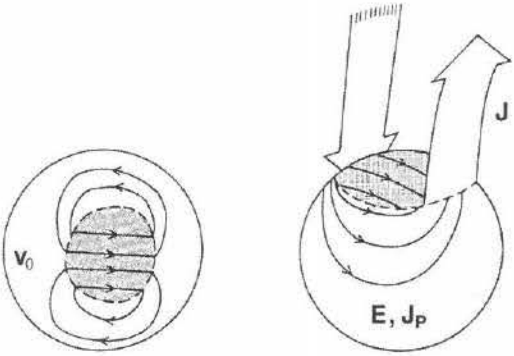

Fig. 10.5. Schematic diagram of magnetospheric convection over the Earth's north polar region (after Vasyliūnas, 1975a). (Left) Streamlines of the plasma bulk flow; the Sun is on the left. (Right) Electric field lines and associated Pedersen currents, and the magnetic-field-aligned current (also known as the Birkeland current; see the large arrows). Section 10.4.3 gives a detailed description and Section 11.6 and Fig. 11.9 give corresponding MHD model results for the electric potential. For a schematic representation of magnetospheric convection throughout the magnetosphere, see Fig. 13.4.

10.4.3 Magnetospheric convection

Magnetospheric convection may be considered as the other canonical pattern, besides corotation, of plasma flow in a planetary magnetosphere, one that plays an overwhelmingly important role in the magnetosphere of Earth (see Chapter 11 for a detailed discussion). The basic concept is that the flow of solar wind plasma past the magnetosphere imparts some of its motion to plasma in the outermost regions of the magnetosphere, either directly by MHD coupling along open field lines (Dungey, 1961) or through an unspecified tangential drag near the magnetopause (Axford and Hines, 1961).

By continuity of mass and magnetic flux transport, the flow then extends into the region of closed field lines or the interior of the magnetosphere, setting up a large-scale circulation pattern (which has some superficial resemblance to, but no real physical commonality with, what is called convection in ordinary fluid dynamics). Figure 10.5 illustrates the pattern, projected onto the topside ionosphere of the planet: shown on the left-hand side are the streamlines of the plasma bulk flow $\mathbf{v}_0$, which are also the equipotentials of the electric field according to Eq. (10.15). The electric field lines and the associated Pedersen currents are shown on the right-hand side, along with a sketch of the implied Birkeland (i.e. magnetic-field-aligned) currents. The shaded region is the polar cap, identified with the region of open field lines in the open magnetosphere; it represents the mapping (along field lines) of the boundary region where the solar wind motion is being imparted to magnetospheric plasma. The equatorial-plane counterpart of the flow outside the polar cap was shown in the bottom panel of Fig. 10.3.

A quantitative global measure of the strength of magnetospheric convection is the emf (which equals the maximum line integral of the electric field) across the polar cap, Φ_{PC}. Its physical meaning is the rate of magnetic flux transport (advection) through the polar cap. In an open magnetosphere, $c\Phi_{PC}$ equals the rate of reconnection of magnetic flux between the interplanetary and the planetary magnetic fields. Numerous empirical studies at Earth (e.g. Boyle *et al.*, 1997; Burke *et al.*, 1999; and references therein) have shown that for a southward interplanetary magnetic field (i.e. $\mathbf{B}_{sw} \cdot \hat{\boldsymbol{\mu}} > 0$), Φ_{PC} can be related to solar wind parameters approximately as follows:

$$c\Phi_{PC} \simeq v_{sw}(\mathbf{B}_{sw} \cdot \hat{\boldsymbol{\mu}})\mathcal{L}_X \tag{10.16}$$

where $\mathcal{L}_X$ is a length that typically is a fraction (~ 0.2 to ~ 0.5) of the magnetopause radius R_{MP}. When comparing different magnetospheres, one often supposes that the ratio $\mathcal{L}_X/R_{MP}$ is a more or less universal constant. Physically, $\mathcal{L}_X$ may be looked at as the length of the reconnection X-line on the magnetopause at which the magnetic field lines from the solar wind and from the planet first become interconnected, projected along the streamlines of the magnetosheath flow back into the undisturbed solar wind, as illustrated in Fig. 10.3 (see Section 13.1.4 for applications to other planets).

Although magnetospheric convection is often talked about as if it were simply a mapping-plus-continuation of the solar wind or boundary layer flow, it does involve mechanical stresses in an essential way. The Birkeland currents shown in Fig. 10.5 indicate that the magnetic field is deformed both at the near-planet end and also at the distant end, with Lorentz forces that must be balanced by mechanical stresses at both ends. The Lorentz force in the ionosphere is balanced by the collisional drag of the neutral atmosphere; the total force integrated over the entire ionosphere points approximately anti-sunward (to the right in Fig. 10.5), in agreement with the preponderance of anti-sunward plasma flow and hence sunward drag force. The Lorentz force in the distant region is sunward and acts to slow down the anti-sunward flow of the solar wind plasma. In the presence of the strongly converging field lines of the dipole, all these forces interact, also in a complicated way, with the interior of the planet, as recently elucidated by Siscoe (2006) and Vasyliūnas (2007).

An alternative and in some ways physically more accurate view is to consider magnetospheric convection as the result not directly of an imposed flow but of an imposed stress on the magnetic field, in this case an anti-sunward pull on the field lines by the solar wind flow. Once this point of view is adopted, mechanisms for producing flows of the magnetospheric convection pattern other than by the solar wind are immediately apparent. A mechanical stress in the magnetosphere will deform the magnetic field to reach local equilibrium; the deformation of the field extends necessarily into the ionosphere, where it must be balanced in turn by a mechanical stress; however, the only one available is collisional drag and hence plasma flow relative to the neutrals. Thus any quasi-localized unidirectional mechanical force in the magnetosphere can produce a flow pattern similar to that of Fig. 10.5, the anti-sunward direction being replaced by the direction of the force and the polar cap being replaced by the region mapped along field lines from the location of the force in the magnetosphere.

The most significant mechanisms for such internally driven convection flows in planetary magnetospheres are one-sided (azimuthally non-symmetric) enhancements of plasma pressure or, in the case of corotating plasmas subject to centripetal acceleration, of mass density; these play an important role in transport processes, as discussed below in Section 10.5. If these enhancements are fixed relative to the rotation, e.g. if they are associated with asymmetries or anomalies of the planetary magnetic field (Dessler and Hill, 1975), the convection pattern remains fixed in the corotating frame of reference and is known as *corotating convection*; see Hill *et al.* (1981) and references therein.

10.4.4 Limits to corotation

The conditions (1)–(4) for corotation discussed in Section 10.4.2 are all in essence local conditions at a given magnetic flux tube. Deviations from corotational flow when one or other of these conditions is no longer satisfied need not, therefore, be global but can be confined to limited regions. Typically, plasma flow in any particular magnetosphere may follow corotation in the inner regions, out to a critical radial distance in the equatorial plane, and then deviate significantly from corotation at larger distances. The critical distance depends on which of conditions (1)–(4) given in Section 10.4.2 is violated and by which process.

Violation of (1): Deviations of $\mathbf{v}_n$ from the corotation velocity $\mathbf{\Omega} \times \mathbf{r}$ may occur for two reasons: (a) the presence of a neutral wind velocity field, which in most cases is not directly associated with the magnetosphere and is not connected with any particular distance (but which can have effects in the magnetosphere since the plasma tends to follow the combined flow field of corotation plus winds); (b) when the plasma flow is already strongly non-corotational because of stresses applied from the magnetosphere, the collisional drag on the neutrals may become non-negligible in comparison with the stresses from the planet, modifying the flow of the neutrals to reduce the difference from the plasma.

Violation of (2): The mechanical stresses associated with magnetospheric convection (Section 10.4.3) may act to produce non-corotational flows. The importance

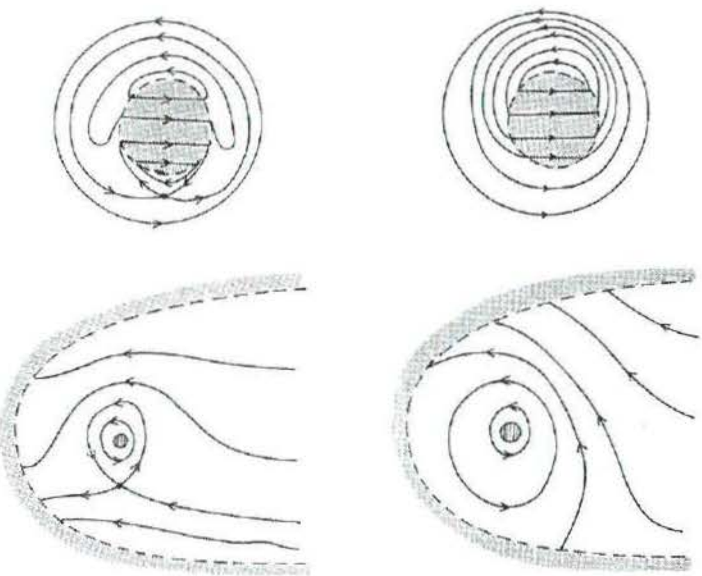

Fig. 10.6. Streamlines of plasma flow in a magnetosphere: (upper panels) looking down on the topside ionosphere, and (lower panels) projected along magnetic field lines to the equatorial plane of the magnetosphere. On the left magnetospheric convection is dominant and on the right corotation is dominant (after Vasyliūnas, 1975); compare with the observation-based model for Jupiter's magnetospheric configuration in Fig. 13.6).

of solar-wind-driven magnetospheric convection in comparison with corotation may be gauged by comparing the polar cap potential given by Eq. (10.16) with the corresponding corotational potential between the colatitude θ_{pc} of the polar cap boundary and the pole:

$$c\Phi_{CR} \simeq \Omega R_P \theta_{pc}{}^2 B_P. \tag{10.17}$$

By conservation of magnetic flux,

$$\Phi_M = \pi R_P \theta_{pc}{}^2 (2B_P) \approx \mathcal{L}_X \mathcal{L}_T (\mathbf{B}_{sw} \cdot \hat{\boldsymbol{\mu}}) \tag{10.18}$$

(although perhaps confusing, the use of Φ both for potential and for magnetic flux is traditional; for $\mathcal{L}_X$ and $\mathcal{L}_T$ see Section 10.3.3). From Eqs. (10.16)–(10.18) we obtain

$$\frac{\Phi_{CR}}{\Phi_{PC}} \approx \frac{\Omega \mathcal{L}_T}{2\pi v_{sw}}. \tag{10.19}$$

The plasma flow patterns for the two extreme cases of small and large values for the ratios in Eq. (10.19) are shown in Fig. 10.6. When $\Phi_{CR}/\Phi_{PC} \gg 1$ (right-hand panels), the magnetosphere is said to be corotation-dominated; the corotation extends almost to the magnetopause. When $\Phi_{CR}/\Phi_{PC} \ll 1$ (left-hand panels), the magnetosphere is convection-dominated; corotation is confined to the inner region, which extends to a distance R_{cr} given approximately by

$$\left(\frac{R_{cr}}{R_{MP}}\right)^{\nu} \sim \frac{\Omega \mathcal{L}_T}{v_{sw}} \tag{10.20}$$

where the exponent ν varies between 1.5 and 2 depending on the details of the model for the convection flow. The two extreme cases, convection-dominated and corotation-dominated, were first distinguished and applied to the magnetospheres of Earth and Jupiter by Brice and Ioannidis (1970) (see also Vasyliūnas, 1975a).

Nevertheless, even when $\Phi_{CR}/\Phi_{PC} \gg 1$ the corotation of plasma does not necessarily extend to the magnetopause but may be self-limited by another mechanical stress that arises when the magnetosphere has sources of plasma deep within its interior (see Section 10.5.1 below). The plasma supplied from such a source must be transported outward and, as long as corotation is maintained, the angular momentum per unit mass of the plasma increases as Ωr^2. This requires an appropriate torque to be applied to the plasma by the azimuthal component of the Lorentz force, which is then balanced in the usual way by an azimuthal collisional drag in the ionosphere, constituting a departure from corotation. Physically, an outward-moving plasma element initially lags behind corotation (by the conservation of angular momentum), pulling the magnetic field line with it; the resulting deformation of the magnetic field in the ionosphere reduces the corotation of the plasma until the drag force needed to balance the torque in the magnetosphere is reached. This process of *partial corotation* was first described and quantitatively modeled by Hill (1979), who found that the departure from corotation is very slight at small equatorial radial distances and increases gradually with increasing required torque; the flow becomes strongly sub-corotational beyond a distance R_H (often called the Hill radius), given for a dipole field by the simple expression

$$R_H{}^4 = \left(\frac{\pi \Sigma_P}{c^2}\right)\frac{\mu^2}{S} \tag{10.21}$$

where S is the net outward mass flux from the internal source (recall that μ is the magnetic dipole moment of the planet).

Violation of (3): By the very nature of the MHD approximation, departures from it occur for the most part only for small-scale structures. Hence one does not expect a direct influence on the mapping of large-scale flows from non-MHD effects (which can, however, have a very significant influence on related phenomena such as the aurora or particle acceleration).

Violation of (4): The radial stress balance needed to maintain the centripetal acceleration of corotating flow can exist only as long as the Lorentz force, or equivalently the tension of the stretched-out field lines, is sufficiently strong to match or exceed $\rho\Omega^2 r$ in the equatorial region of the magnetosphere. This is no longer possible beyond a critical distance R_0, often estimated by the rough argument of setting the Alfvén speed equal to the corotation speed. A more precise estimate can easily be made if the plasma source is located at a radial distance r_s, beyond which the plasma is assumed to be confined to a planar sheet of thickness $h_s \ll r$, with magnetic flux tube content (the mass per unit magnetic flux)

$$\eta \equiv \int \frac{\mathrm{d}l\rho}{B} \approx \frac{\rho h_s}{B_z} \tag{10.22}$$

being assumed approximately constant for $r > r_s$; then R_0 is given by (Vasyliūnas, 1983, and references therein)

$$R_0{}^4 \simeq \frac{\mu^2}{\pi \rho_s h_s r_s{}^3 \Omega^2} \tag{10.23}$$

where ρ_s is the plasma mass density at the source location. Under the same assumptions about the geometry of the plasma distribution, the total mass M from the internal source can be estimated as

$$M \approx 2\pi \rho_s h_s r_s{}^2; \tag{10.24}$$

hence R_0 can also be written as

$$R_0{}^4 \simeq \frac{2\mu^2}{M r_s \Omega^2}. \tag{10.25}$$

Obviously the above estimate of R_0 and indeed the entire concept of the breakdown of radial stress balance is applicable only if the plasma is still corotating at distances as far out as R_0, which requires that $R_0 \ll R_H$; when comparing the two distances, one may relate M and S by $M = S\tau_{tr}$, where τ_{tr} is the global transport time to be discussed below in Section 10.5.2. If this is the case, the plasma flow beyond R_0 is expected to change from corotation to a more nearly outward motion and ultimately to a general quasi-radial outflow, which then also stretches out the magnetic field lines until they break, via the reconnection process. Such a rotationally driven radial outflow of a corotating plasma from an internal source (possibly channeled into the magnetotail by the external pressure on the magnetopause) was proposed for the magnetosphere of Jupiter by Hill *et al.* (1974) and Michel and Sturrock (1974), who named it the *planetary wind*; the term *magnetospheric wind* is also used, sometimes in the more general sense of a radial outflow without reference to a specific physical origin. A widely cited but rather speculative diagram of the associated changes in the magnetic field topology (Vasyliūnas, 1983) is shown in Fig. 10.7.

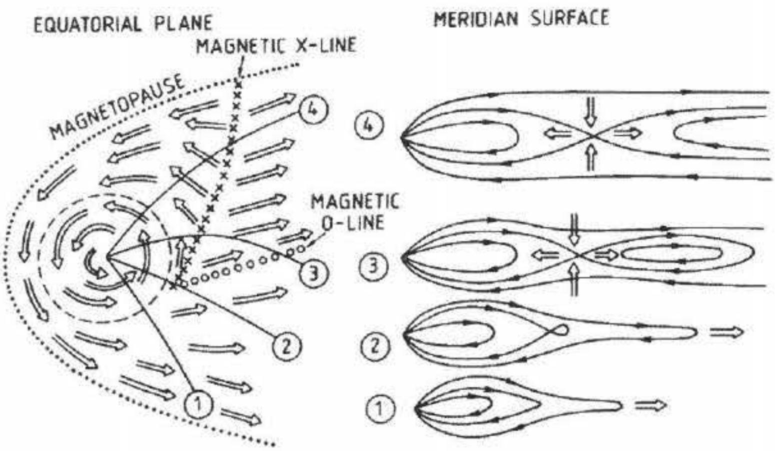

Fig. 10.7. Schematic of planetary wind flow and magnetic topology (after Vasyliūnas, 1983).

10.5 Plasma sources and transport processes

Planetary magnetospheres exhibit an enormous variety; almost every observed magnetosphere differs significantly and qualitatively from all the others (see Section 13.1.3). A primary reason for such diversity, in the face of the few structures of more or less universal character predicted by the interaction of the solar wind with the planetary magnetic field (Section 10.3), lies in the existence of several different types of source for the plasma within the magnetosphere, as well as several different types of transport process that determine the motion and distribution of the plasma.

10.5.1 Sources of magnetospheric plasma

Three types of plasma source can be distinguished on the basis of their location: (1) at the outer boundary of the magnetosphere, i.e. the interface with the solar wind; (2) at the inner boundary of the magnetosphere, i.e. the interface with the ionosphere and atmosphere of the planet; (3) within the interior volume of the magnetosphere, at interfaces with objects within the magnetosphere, e.g. the moons and the rings of the planet. Figure 10.8 shows schematically the various locations and the associated plasma source regions.

(1) **Outer-boundary source: solar wind.** In a magnetically open magnetosphere (Section 10.3.3), plasma from the solar wind can flow along open magnetic field lines directly into the magnetotail, forming a *plasma mantle*. Although much of this plasma remains on open field lines and continues to flow down the magnetotail, some plasma may, as a result of magnetic reconnection in the magnetotail,

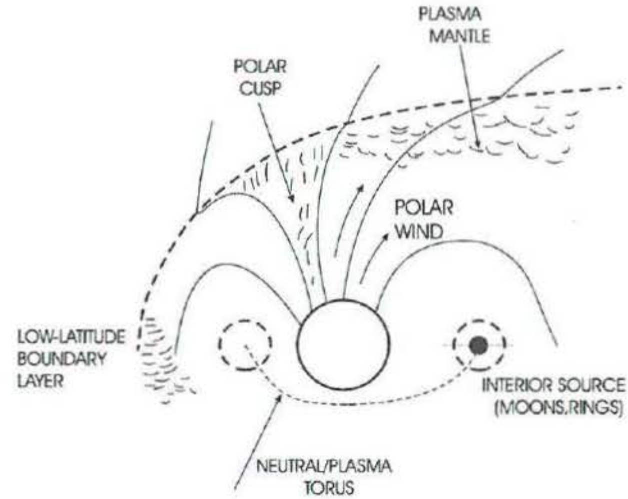

Fig. 10.8. Schematic of magnetospheric plasma source locations (not to scale, schematic, only northern hemisphere shown). See Fig. 13.3 for a diagram for Earth's magnetosphere in particular.

be transported into the closed-field-line region of the magnetosphere. In addition, even when the magnetopause ideally should be impenetrable to plasma, as in a closed magnetosphere (Section 10.3.1) or in those parts of the open magnetosphere boundary where the normal component of the magnetic field $B_n = 0$, some solar-wind plasma crosses as the result of non-ideal MHD processes; this is evidenced by the existence of the *low-latitude boundary layer*, where plasma with properties typical of the magnetosheath is found on closed field lines just inward of the magnetopause. The *polar cusp* (see e.g. Fig. 1.3) is another prominent region of plasma entry, but whether it represents flow along open or transport across closed field lines (or a combination of both) is still disputed.

(2) **Inner boundary source: ionosphere**. Plasma from the ionosphere can flow directly along the magnetic field lines upward into the magnetosphere, provided only that there is sufficient energy to overcome the gravitational binding to the planet. The process may be simply a plasma analog of Jeans escape, involving those particles with thermal speeds in excess of the gravitational escape speed (but with the added complication that, since electrons are much lighter than ions, plasma escape also involves electrostatic effects). There may be a systematic upward bulk flow, the *polar wind*, formed by processes similar to those that lead to the outflow of the solar wind from the Sun. Finally, magnetospheric processes, especially those connected with the aurora, may lead to acceleration of both electrons and ions in the ionosphere to speeds well above the gravitational escape speed.

(3) **Interior source: moons and planetary rings.** Solid bodies traveling on orbital paths that lie wholly or partly within the volume of the magnetosphere may act as sources of plasma, in most cases by an indirect process: the release of neutral gas, which is subsequently ionized. Such a source is significant particularly in the case of the giant planets. Jupiter and Saturn both possess a number of moons, some of size comparable with or larger than the Earth's moon, located deep within the magnetosphere; Saturn has in addition the well-known rings, the constituents of which may range from dust to boulder size. The neutral gas may be released from the surface of the body or from its atmosphere (in the case of moons large enough to keep one), by evaporation, or by the sputtering produced by impacting plasma particles from the magnetosphere; it may be ionized by photoionization or by electron-impact ionization. For a more detailed discussion of specific objects, see Chapter 13.

The strength of any particular plasma source is generally parameterized by the rate S, i.e. the mass added per unit time; the role of the mass added from an interior source in limiting corotation was discussed in Section 10.4.4. For the solar wind source, S is given roughly by the incident solar wind mass flux (which is reasonably well known) multiplied by the efficiency of entry into the magnetosphere (which is poorly known, but probably small). For an S from the ionospheric source the efficiency may be taken as effectively 1, but the upward mass flux is very uncertain in most cases. For an interior source, S depends on the rate of injection of neutral particles and on the rate of ionization, both of which (if sputtering and electron impact ionization are important) depend on the parameters of magnetospheric plasma, of which S itself is a source; thus a complex nonlinear interaction determines S. As a result of all these uncertainties, at present the values of S for the different sources in various magnetospheres can, for the most part, only be estimated empirically (see Section 10.6 for further discussion).

An important marker for identifying the plasma from the different sources is the composition of the plasma ions, which reflects the ionic composition of the source. The solar wind contributes primarily H^+ (protons), with a small admixture of He^{++} (alpha particles). The contribution of the ionosphere depends on the atmosphere of the planet. For the giant planets, the atmospheres consist primarily of hydrogen, hence the contribution of their ionospheres is not easily distinguishable from that of the solar wind, except for some subtle effects, e.g. the absence of He^{++} and the possible presence of molecular hydrogen ions; for Earth, the presence of O^+ is an indicator of the ionospheric contribution. The contribution of planetary moons and rings varies with the object but in almost all cases is dominated by heavy ions – e.g. sulfur and oxygen ions from the moon Io at Jupiter and water-group ions from most of the other (ice-covered) moons or rings at Jupiter and Saturn.

10.5.2 What moves the plasma?

Plasma in the magnetosphere from the various sources described in Section 10.5.1 is located initially on magnetic field lines that thread the respective source regions. For a solar wind source, they are field lines near the magnetopause: open (plasma mantle, possibly a part of polar cusp) or closed (low-latitude boundary layer, possibly the other part of polar cusp). For a interior source, they are the field lines crossing the orbits of the moons or rings. An ionospheric source does, in principle, populate all the field lines, but its contribution is, at least initially, heavily concentrated toward low altitudes. If the plasma is to be redistributed over wide regions of the magnetosphere, as observed in many magnetospheres for plasma from different sources, transport processes that move the plasma away from the source regions must be present.

Plasma from an ionospheric source may be redistributed by direct flow along the magnetic field lines (its density, however, becomes greatly reduced as the cross-sectional area of a flux tube increases with increasing altitude). This is the simplest transport process, and one not impeded by any MHD constraints. To redistribute the plasma from other types of source, transport processes are required that necessarily involve flow across magnetic field lines, including in particular flow in the radial direction.

Plasma bulk flows that are transverse to the magnetic field are subject to the generalized Ohm's law, e.g. Eq. (10.12), and hence are predominantly $\mathbf{E} \times \mathbf{B}$ drifts, with some contribution from gradient and curvature drifts in the azimuthal direction only; thus they transport magnetic flux along with the plasma. The plasma sources described in Section 10.5.1, however, are sources only of plasma and not of magnetic flux: they add a net amount of plasma to the magnetosphere, while the total amount of magnetic flux threading the magnetosphere remains fixed (it is set by the number of field lines of one polarity emerging from the planet). The added plasma must ultimately be removed from the magnetosphere, predominantly by transport out of it. Of the other loss processes, recombination is negligible in most cases; precipitation of plasma onto the planet, although potentially of appreciable local impact (on e.g. the formation of the aurora or the heating and ionization of the atmosphere), is of very little significance for the global mass budget simply because the surface area of the planet is usually very small in relation to the magnetosphere.

The quintessential problem of plasma transport in planetary magnetospheres, therefore, is how to achieve a flow that carries both plasma and magnetic flux yet removes plasma but not magnetic flux. The almost universally accepted solution is to consider circulating flow patterns that do not appreciably change the magnetic field (they are often referred to as *interchange motions* (Gold, 1959), at least in a qualitative sense – the precise denotation of that term is sometimes disputed) but in which the outflowing segments have a larger plasma content than the inflowing segments. The prototype of such circulating interchange motions is magnetospheric convection, in the generalized sense discussed at the end of Section 10.4.3; the various models of the transport process may be viewed as applications of magnetospheric convection, driven by different physical processes and with different assumed spatial and temporal scales. There are two basic dichotomies of the adopted model: (I) magnetospheric convection is either (a) driven by the solar wind or (b) driven by the internal dynamics of the magnetosphere; (II) magnetospheric convection is either (1) quasi-steady (systematic) or (2) fluctuating (random or chaotic). Each of the four resulting possibilities has found application in some magnetosphere or other.

(1a) **Quasi-steady magnetospheric convection driven by the solar wind**. This is expected to be the primary transport process in magnetospheres that have no significant interior sources and that are convection-dominated in the sense defined by Eq. (10.19), e.g. the magnetospheres of Mercury and Earth. In magnetospheres that are corotation-dominated, however, magnetospheric convection may still be present but produces no net radial transport; the latter is effectively suppressed since each plasma element is carried by the rotation between inbound and outbound segments of the convection pattern.

(1b) **Quasi-steady magnetospheric convection driven by internal dynamics**. As pointed out in Section 10.4.3, for the internal dynamics to give rise to magnetospheric convection, a necessary condition is an azimuthally asymmetric configuration of the stresses in the magnetosphere; for the convection to be quasi-steady, the asymmetry should be permanent for a given location. An example of transport by this process is corotating convection, proposed as a possibility for the magnetosphere of Jupiter (Hill *et al.*, 1981, and references therein), the permanent asymmetry being provided by an observed asymmetry (the "magnetic anomaly") of Jupiter's internal magnetic field. Very recently, corotating convection has also been proposed as a transport process for the magnetosphere of Saturn (Goldreich and Farmer, 2007; Gurnett *et al.*, 2007); since no azimuthal asymmetry of Saturn's internal magnetic field has been observed to date, however, it is not clear what could hold an asymmetric stress configuration in place.

(2a) **Fluctuating magnetospheric convection driven by the solar wind**. Since the strength of solar-wind-driven magnetospheric convection depends on the direction of the interplanetary magnetic field (Eq. 10.16), which is highly variable, a fluctuating state may be viewed as the normal condition. As far as the plasma transport process is concerned, the main difference from the quasi-steady state is that the suppression of radial transport by the effects of rotation, discussed under (1a) above, can be overcome by fluctuations that have time scales appropriately related to the rotation period. The principal application of this concept has been to the inner regions of the magnetosphere of Earth, roughly inside the distance given by Eq. (10.20), where corotation is the dominant flow and also the azimuthal flows associated with gradient and curvature drifts may be large. Fluctuating magnetospheric convection has been invoked, sometimes under different designations, to explain how radial transport (needed to account for e.g. buildup of the ring current, erosion of the plasmasphere, and formation of the radiation belts; see Chapter 11) can be maintained despite the large azimuthal motions.

(2b) **Fluctuating magnetospheric convection driven by internal dynamics.** Almost universally, this is assumed to be the primary transport process in the corotation-dominated magnetospheres of the giant planets, Jupiter and Saturn. If the plasma comes predominantly from interior sources deep within a corotating magnetosphere, its distribution is unstable to the formation of localized mass concentrations that move outward, alternating with adjacent mass dispersions that move inward; this is the *rotational* (or centrifugal) *interchange instability*, which is somewhat analogous to the Rayleigh–Taylor instability of a heavy fluid on top of a light one. As the primary mechanism of radial transport, it was first proposed for Jupiter by Ioannidis and Brice (1971) and has been extensively developed, e.g. by Siscoe and Summers (1981), Southwood and Kivelson (1989), Pontius and Hill (1989), Vasyliūnas (1989b), Hill (1994), Ferrière *et al.* (2001), Vasyliūnas and Pontius (2007), and others.

To describe quantitatively the mean transport resulting from random fluctuations, a diffusion equation for the mass per unit magnetic flux (i.e. the flux tube content) η, defined in Eq. (10.22), is commonly employed. It is most simply derived by a method (adapted from work by G. I. Taylor in 1921 on fluid turbulence) first applied in the magnetosphere of Earth by Cole (1964) and independently in an astrophysical context by Jokipii and Parker (1968). If each quantity Q is represented as a mean $\langle Q \rangle$ plus a fluctuation δQ, the mean transport of flux tube content is then given by

$$\langle \eta \mathbf{v} \rangle = \langle \eta \rangle \langle \mathbf{v} \rangle + \langle \delta\eta \delta\mathbf{v} \rangle. \tag{10.26}$$

The first term on the right-hand side, giving the plasma transport by the mean flow, is *advection*; the second term, giving the transport by the correlated density and velocity fluctuations without any mean flow, is *diffusion*. In the absence of sources and sinks, the flux tube content is conserved by the flow:

$$\frac{\partial \eta}{\partial t} + \mathbf{v} \cdot \nabla \eta = \text{sources} - \text{sinks}. \tag{10.27}$$

Now, if $\delta\eta$ is calculated in the quasi-linear approximation by integrating Eq. (10.27) this allows the second term in Eq. (10.26) to be rewritten in standard diffusion form:

$$\langle \delta\eta \delta\mathbf{v} \rangle \simeq -\mathbf{D} \cdot \nabla \langle \eta \rangle \tag{10.28}$$

with diffusion coefficient

$$\mathbf{D} = \int_{-\infty}^{t} \mathrm{d}t' \langle \delta\mathbf{V}(t')\delta\mathbf{V}(t)\rangle \tag{10.29}$$

(it has been assumed that the autocorrelation of the velocity field $\langle\delta\mathbf{V}\delta\mathbf{V}\rangle$ is non-negligible only for separations short in comparison with the evolution time scale of the mean configuration). Note that the concept of diffusive transport is not restricted to flux tube content but can also be derived for any quantity that is conserved by the flow, i.e. obeys an equation of the same form as Eq. (10.27); it has been extensively applied in studies of trapped particles and radiation belts (e.g. Schulz and Lanzerotti, 1974, and references therein).

The diffusion coefficient (when not simply taken as an empirical parameter to be determined by fitting to observations) can be expressed in various ways: as the mean square amplitude of the velocity fluctuations multiplied by a correlation time (Cole, 1964) or as the power spectrum at zero frequency (Jokipii and Parker, 1968). In the present context of fluctuating magnetospheric convection, it is most conveniently represented in terms of the characteristic linear dimension λ of the circulating flow pattern and of the circulation time τ_c as

$$D \simeq \lambda^2/\tau_c. \tag{10.30}$$

The net outward mass flux S is given by

$$S = r \oint \mathrm{d}\phi\, B_e \langle\delta\eta\delta\mathbf{v}\rangle \simeq -r \oint \mathrm{d}\phi\, B_e D \frac{\partial\eta}{\partial r} \tag{10.31}$$

where B_e is the equatorial magnetic field at radial distance r and the integration is over all azimuthal angles around the planet. With the total mass M given by

$$M = \int r\mathrm{d}r \oint \mathrm{d}\phi\, B_e\eta, \tag{10.32}$$

the global transport time defined by $\tau_{tr} = M/S$ can be estimated to an order of magnitude as

$$\tau_{tr} \sim \tau_c \left(\frac{r_s}{\lambda}\right)^2 \tag{10.33}$$

where r_s is the radial distance of the source region.

The circulation time τ_c can be estimated relatively reliably from considerations of magnetosphere–ionosphere coupling (Siscoe and Summers, 1981), but the detailed spatial geometry of the transport process is still largely unknown. Not only the values of λ and their variation with distance but even the basic flow patterns – nearly circular, or radially elongated? – remain uncertain, with no firm, generally accepted, conclusions to date from the extensive observational, theoretical, and computational studies that have been made (e.g. Krupp *et al.*, 2004; Vasyliūnas and Pontius, 2007; Yang *et al.*, 1994; and references therein).

10.5.3 Plasma structures

The great variety of magnetospheric structures that can be formed by the interplay of plasma source and transport processes are surveyed in Chapter 13 and (for Earth) in Chapter 11. Here we simply make a few remarks on some generic relationships.

(1) *Neutral* and *plasma tori* are the direct result of an interior source from a moon of the planet. A neutral torus around the orbit of the moon forms if the neutral particles leave the moon with speed high enough to escape its gravitational field but not the gravitational field of the planet (as first suggested by McDonough and Brice, 1973); the corresponding plasma torus then forms by ionization of the neutral torus. If the ionization time is appreciably shorter than the time for the neutrals to spread over the orbit of the moon, the neutral torus will not be complete but will cover only a segment of the orbit near the moon and will move with its Keplerian speed; the plasma torus, however, will still be complete (except in the case of a moon at synchronous orbit) as long as it is carried around by corotation.

(2) *Plasma sheets* (or, equivalently, *current sheets*) can be formed by two different processes. With an interior source, plasma is transported outward, stretching out the magnetic field; the resulting plasma or current sheet is also called the *magnetodisk*, particularly if the stretching is by rotational stresses. With a solar wind source, plasma is transported inward, preventing the field lines stretched by the solar wind from collapsing back to a quasi-dipolar form.

(3) *Ring currents* in the traditional terrestrial sense of plasma populations with energy content high compared with that supplied by local sources (as distinct from merely a synonym for the magnetodisk) and *radiation belts* are structures that arise from strong adiabatic compression of the plasma and suprathermal particles, respectively. They therefore presuppose inward transport over appreciable distances and are most readily produced by an exterior boundary source. They can, however, be produced by an interior source if the plasma or particles are first transported outward to the boundary regions, heated or accelerated, and then transported (recirculated) back inward. A unique feature of radial diffusion is that it transports always in the direction of decreasing content; thus it is capable of such bidirectional transport if the radial gradients at low and high energies are opposite.

(4) The *plasmasphere* is usually understood as the region filled with the plasma coming primarily from the ionosphere. Because of the unfavorable area to volume ratio of the ionospheric source, high densities in the magnetosphere can be reached only if the source acts for a long time; hence the outward transport from the region

must be negligible (if this is the case, the reason is usually the local predominance of corotation).

10.6 Scaling relations for magnetospheres

The comparative study of magnetospheres is a very broad topic, both qualitatively (regarding the diversity of phenomena) and quantitatively (regarding the wide range of the numerical values of parameters). For a systematic comparison and classification, one would like to identify a few key parameters characterizing each magnetosphere and its environment; from these, one would like to be able to decide which differences among magnetospheres are essential, in the sense that they arise from different physical processes, and which differences result merely from the same process acting at different scales. Also, if such distinctions are to be made on the basis that some particular parameter is large or small, the parameter in question must be expressed in a dimensionless form, for the designation "large" or "small" is meaningful only when applied to a dimensionless quantity (whenever something is described as large or small, the question must always be asked: large or small compared to what?).

The parameters may be separated into two classes. The *input* or given parameters define the system; in principle they may be specified arbitrarily. The *derived* parameters describe the properties of the given system; they are determined by the relevant physical processes.

10.6.1 Input parameters

The physical parameters that may be assumed given for a particular magnetosphere fall naturally into three groups: those describing the solar wind or, more generally, the external medium, those describing the planet or central object, and those describing the given – as distinct from derived – properties of the magnetosphere itself.

(1) The **solar wind parameters** important for the interaction are the following (their dependence on radial distance r from the Sun is shown, with values at $r = 1$ AU designated by the subscript a):

mass density, $\rho_{sw} \simeq \rho_a (1\ \mathrm{AU}/r)^2$,

bulk flow speed, $v_{sw} \simeq v_a$,

magnetic field transverse to the radius vector from the Sun, $B_{sw-t} \simeq B_a (1\ \mathrm{AU}/r)$,

radial magnetic field, $B_{sw-r} \simeq B_a (1\ \mathrm{AU}/r)^2$

(the radial magnetic field is generally of minor significance for studies of magnetospheres, because $B_{sw-r} \ll B_{sw-t}$ for magnetospheres located beyond $r = 1$ AU and, more importantly, because the radial field is aligned with the bulk flow and thus does not contribute to the $-\mathbf{v} \times \mathbf{B}$ electric field of the MHD approximation).

Not included in the above list is the temperature of the solar wind, which has been largely "forgotten" at the bow shock of the magnetosphere; the post-shock temperature, equal to some fraction of the solar wind bulk-flow kinetic energy per particle, is essentially independent of the pre-shock temperature value, and the latter therefore plays no significant role in the interaction (except possibly in the distant downstream regions). Note that an important dimensionless parameter, the Alfvén Mach number based on the transverse magnetic field,

$$M_{\mathrm{A}} = \left(\frac{4\pi \rho_{\mathrm{sw}} v_{\mathrm{sw}}^{2}}{B_{\mathrm{sw-t}}^{2}} \right)^{1/2} \approx \left(\frac{4\pi \rho_{\mathrm{a}} v_{\mathrm{a}}^{2}}{B_{\mathrm{a}}^{2}} \right)^{1/2} \tag{10.34}$$

is independent of the distance from the Sun and hence has the same value, on average, at all the planets.

(2) The **planetary parameters** relevant to the magnetosphere and the interaction with the solar wind include

- radius of the planet, R_{p},
- rotation frequency, Ω,
- surface magnetic field strength at the equator, B_{p},
- magnetic dipole moment, $\mu = B_{\mathrm{p}} R_{\mathrm{p}}^{3}$,
- typical value of Pedersen conductance in the ionosphere, Σ_{P},
- mass of the planet, M_{p} (important only for gravitational effects).

For describing most aspects of the magnetosphere, the parameters B_{p} and R_{p} appear only in the combination $B_{\mathrm{p}} R_{\mathrm{p}}^{3} = \mu$; the values of B_{p} and R_{p} taken separately are of little or no significance (e.g. Vasyliūnas, 1989a, 2004). Also, M_{p} hardly ever appears explicitly in any magnetospheric calculation.

(3) **Magnetospheric parameters** are not always easily classified as given or derived. Most of the simple parameters one can construct to represent the characteristic or global properties of a magnetosphere are to a large extent determined by the interaction with the solar wind and can hardly be considered as specifiable a priori. If the magnetosphere has an interior source of plasma associated with a single moon, e.g. Io at Jupiter, the

- radial distance of the interior source, r_{s},

is one obvious given parameter. Aside from that, almost the only other magnetospheric parameter that is generally treated as given is the

- strength of the interior source of plasma, S

(Section 10.5.1). The theoretical arguments for this are rather inconclusive: on the one hand, it may be argued that the interior source is governed primarily by the injection of neutral particles and is independent of the magnetosphere to first order, while on the other hand one may point to plasma effects on the ionization rate and even (via sputtering) on the injection rate. The real motivation for treating S as given appears to be primarily the availability of reasonably robust empirical estimates of S for the magnetospheres of Jupiter and Saturn.

In addition to the magnitudes of the various parameters listed above, one must also include the directions of the vector quantities: the solar wind velocity $\mathbf{v}_{sw}$, the interplanetary magnetic field $\mathbf{B}_{sw}$, the planetary rotation axis $\mathbf{\Omega}$, and the magnetic moment $\boldsymbol{\mu}$.

10.6.2 Derived parameters

Despite the almost boundless complexity in detail of magnetospheres, several simple parameters that prove useful in characterizing their essential gross properties can be defined; some have been mentioned in earlier sections. Specifically, we now consider the following *derived parameters*:

- distance to the sub-solar magnetopause, R_{MP} (Section 10.3.1),
- effective length of the magnetotail, $\mathcal{L}_T$ (Section 10.3.3),
- amount of open magnetic flux, Φ_M (Section 10.3.3),
- EMF across the polar cap, $c\Phi_{PC}$ (Section 10.4.3),
- rate of energy dissipation in the magnetosphere, $\mathcal{P}$,
- mass input from the solar wind, S_{sw} (Section 10.5.1),
- limiting distance of corotation, R_{cr} (Section 10.4.4).

The question now is whether scaling relations can be found that allow the derived parameters to be expressed in terms of the input parameters.

10.6.3 Constraints from dimensional analysis

Two guiding principles constrain the search for scaling relations. The first is the principle of dimensional similitude: if a derived (i.e. dependent, calculated) parameter is to be expressed as a combination of input (independent, given) parameters, that combination must have the same physical dimensions as the dependent parameter. The second is the principle of physical relevance: parameters associated with a particular physical process should appear only if that process plays a significant role.

The first principle imposes a specific functional form for each derived parameter: it must be proportional to a dimensionally correct combination of the given parameters, multiplied by a function of any or all dimensionless quantities formed from the given parameters, including the relative angles between the vectors (the form of the function is not constrained by dimensional analysis). The second principle, less specific than the first, is of help in deciding which of the many dimensionless quantities that can be formed are to be included.

The basic length scale defined by a combination of solar wind and planetary parameters is the Chapman–Ferraro distance R_{CF} (see Eq. (10.1) and the following discussion). The dependences of the derived parameters of Section 10.6.2 can then be expressed as follows:

$$R_{\mathrm{MP}} \sim R_{\mathrm{CF}} \Psi_{\mathrm{MP}}, \tag{10.35}$$

$$\mathcal{L}_{\mathrm{T}} \sim R_{\mathrm{CF}} \Psi_{\mathrm{T}}, \tag{10.36}$$

$$\Phi_M \sim B_{\mathrm{sw}} R_{\mathrm{CF}}{}^2 \Psi_M, \tag{10.37}$$

$$S_{\mathrm{sw}} \sim \rho_{\mathrm{sw}} v_{\mathrm{sw}} R_{\mathrm{CF}}{}^2 \Psi_{\mathrm{S}}, \tag{10.38}$$

$$c\Phi_{\mathrm{PC}} \sim v_{\mathrm{sw}} B_{\mathrm{sw}} R_{\mathrm{CF}} \Psi_{\mathrm{PC}}, \tag{10.39}$$

$$\mathcal{P} \sim \rho_{\mathrm{sw}} v_{\mathrm{sw}}{}^3 R_{\mathrm{CF}}{}^2 \Psi_{\mathcal{P}}, \tag{10.40}$$

$$R_{\mathrm{cr}} \sim R_{\mathrm{CF}} \Psi_{\mathrm{cr}}, \tag{10.41}$$

where the Ψs are (dimensionless) functions of the appropriate dimensionless quantities. Not all the Ψs are independent: Eqs. (10.16) and (10.18) imply that

$$c\Phi_{\mathrm{PC}} \sim \frac{v_{\mathrm{sw}}}{\mathcal{L}_{\mathrm{T}}} \Phi_M, \quad \text{which is equivalent to} \quad \Psi_{\mathrm{PC}} \Psi_{\mathrm{T}} \sim \Psi_M . \tag{10.42}$$

Equations (10.35)–(10.41) represent the derived parameters as dimensioned scale factors multiplied by dimensionless proportionality constants; the form of the equations has been chosen so that the dimensioned scale factors depend only on the given solar wind parameters and on the Chapman–Ferraro distance R_{CF}, which has been taken as the scale factor for the parameters with the dimension of length. For all the other derived parameters, the scale factors are simply the corresponding solar wind values: for $c\Phi_{\mathrm{PC}}$, the solar wind electric potential over a distance R_{CF}; for Φ_M, S_{sw}, and $\mathcal{P}$, the interplanetary magnetic flux, the solar wind mass flux, and the solar wind bulk-flow kinetic energy flux, respectively, through an area $R_{\mathrm{CF}}{}^2$. Everything to do with the magnetosphere is thus contained in the dimensionless proportionality constants, the Ψs, which are functions of the relevant dimensionless quantities.

With the exception of the angles and the Alfvén Mach number M_{A}, most of the dimensionless quantities have no universally used symbols; we will designate them simply as $Q_1, Q_2, \ldots$ and choose between a quantity and its inverse in such a way as to make the dimensionless quantity small when the associated physical process is not important.

Of the angles between the vectors, the most important is the angle θ between $\mathbf{B}_{\mathrm{sw}}$ and $\boldsymbol{\mu}$; empirically, θ is found to be the dominant factor for determining the values of Ψ_M, Ψ_{PC}, and $\Psi_{\mathcal{P}}$ in the magnetosphere of Earth. Aside from the angles, the only dimensionless quantity that can be formed from the solar wind parameters alone is the Alfvén Mach number, Eq. (10.34); for magnetospheric purposes, the inverse quantity

$$M_A{}^{-1} = \left(\frac{B_{sw}^2}{4\pi\rho_{sw}v_{sw}{}^2}\right)^{1/2} \equiv Q_1 \tag{10.43}$$

is more informative: $M_A{}^{-1}$ represents the influence of the interplanetary magnetic field and is thus, despite its typically small numerical value, $\sim O(10^{-1})$ (see Table 9.1), always important if the magnetosphere is open.

Of the planetary parameters, the only one that appears explicitly in Eqs. (10.35)–(10.41) is the magnetic moment μ, through its role in determining the Chapman–Ferraro distance R_{CF}. Any other planetary parameters that have magnetospheric effects must therefore appear as factors in the dimensionless arguments of the Ψs. No dimensionless quantity that is relevant to the magnetosphere, however, can be formed from the planetary parameters alone: they must be taken together with solar wind parameters. The electrodynamic influence of the ionosphere can be represented by the dimensionless quantity

$$\frac{4\pi\Sigma_P v_{sw}}{c^2} \equiv Q_2 \tag{10.44}$$

and that of the planetary rotation by

$$\frac{\Omega R_{CF}}{v_{sw}} \equiv Q_3\,. \tag{10.45}$$

Finally, a dimensionless quantity that involves, of the planetary parameters, only the dipole moment is

$$\left(\frac{e}{mc}\frac{B_{sw}R_{CF}}{v_{sw}}\right)^{-1} \equiv Q_4 \tag{10.46}$$

where e and m are the charge and mass of the proton. In essence, this is the ratio of the kinetic energy per unit charge of the solar wind bulk flow and the solar wind electric potential across the size of the magnetosphere, but it can also be related (Vasyliūnas *et al.*, 1982) to the comparative importance of the gradient and curvature as against $\mathbf{E} \times \mathbf{B}$ drifts, as well as to other manifestations of finite gyroradius effects, in particular an effective viscosity. The essentially new aspect of the quantity Q_4 is the explicit appearance of the particle mass and charge, implying that Q_4 is important only if non-MHD processes play a significant role (the MHD equations do not contain any reference to individual particle properties). Its numerical value is generally quite small ($\sim O(10^{-2})$ for Earth and much smaller for Jupiter and Saturn); its possible importance for Mercury was suggested by Ogilvie *et al.* (1977).

From the magnetospheric given parameters, three dimensionless quantities can be formed. The first, somewhat trivial, is the ratio of r_s and R_{CF} (or some other length scale),

$$\frac{r_s}{R_{CF}} \equiv Q_5. \tag{10.47}$$

The second is the ratio

$$\frac{S}{\rho_{sw} v_{sw} {R_{CF}}^2} \equiv Q_6 \tag{10.48}$$

of the interior source strength and the solar wind mass flux through an area roughly comparable with the cross section of the magnetosphere; it has proved useful in describing the relative importance of interior sources (e.g. Vasyliūnas, 1989a). The third (little known, recently introduced in Vasyliūnas, 2008) is

$$\frac{S\Omega {r_s}^5}{\mu^2} \equiv Q_7 \tag{10.49}$$

(in this form it may look rather incomprehensible, but writing it as

$$S\left\{\left[\left(\mu/{r_s}^3\right)^2/(\Omega r_s)^2\right](\Omega r_s)\left({r_s}^2\right)\right\}^{-1} \tag{10.50}$$

should make its derivation apparent); it contains magnetospheric and planetary but no solar wind parameters and, as discussed in Section 10.6.4 below, is closely related to the limits on corotation.

10.6.4 Interrelationships of parameters

We have now six independent dimensionless quantities, Q_1–Q_6, defined by Eqs. (10.43)–(10.48); Q_7, defined by Eq. (10.49), can be shown to satisfy $Q_7 \sim Q_6 Q_3 (Q_5)^5$. Obviously, if the proportionality constants were to depend on all six quantities at comparable levels of importance, dimensional analysis would not impose any significant constraints on the derived parameters. Some dimensionless quantities may be neglected, however, if the associated physical processes play only a minor role.

In the simplest case (at the level of approximation discussed in Section 10.3) of an MHD interaction between the solar wind and the planetary magnetic dipole, the only dimensionless quantity (aside from angles) that is relevant is $Q_1 = {M_A}^{-1}$. The set of all magnetospheres can be approximated in this limit as a two-parameter family: the proportionality constants in the scaling relations (10.35)–(10.41) depend only on ${M_A}^{-1}$ and on $\hat{\mathbf{B}}_{sw} \cdot \hat{\boldsymbol{\mu}}$ (for simplicity, any dipole tilt away from perpendicular to the solar wind flow is ignored here). Plausible rough dependences are suggested by the empirical results for Earth's magnetosphere (see e.g. Chapter 11); for the derived parameters that can be discussed at this level of approximation, they are

$$\Psi_{\mathrm{MP}} \sim 1 - O(10^{-1})F, \tag{10.51}$$

$$\Psi_{\mathrm{T}} \sim O(M_{\mathrm{A}}), \tag{10.52}$$

$$\Psi_{\mathrm{M}} \sim O(M_{\mathrm{A}})F, \tag{10.53}$$

$$\Psi_{\mathrm{S}} \sim O({M_{\mathrm{A}}}^{-3}) \tag{10.54}$$

where F (here and in the following equations) represents $F(\hat{\mathbf{B}}_{\mathrm{sw}} \cdot \hat{\boldsymbol{\mu}})$, the function for the angle dependence, which is frequently approximated as follows:

$$F(x) = \begin{cases} x, & x > 0, \\ 0, & x < 0. \end{cases} \tag{10.55}$$

The remaining derived parameters, $c\Phi_{\mathrm{PC}}$, $\mathcal{P}$, and R_{cr}, are meaningful only if plasma flow and ionospheric effects are included (at the level discussed in Section 10.4), which brings in Q_2 and Q_3 as additional dimensionless arguments on which the proportionality constants depend; the set of magnetospheres is now a four-parameter family. The derived parameter primarily affected by this extension is the emf across the polar cap, $c\Phi_{\mathrm{PC}}$, which is observed to approach a limiting value as $v_{\mathrm{sw}}\mathrm{B}_{\mathrm{sw}}$ increases (Siscoe *et al.*, 2002c, and references therein), a saturation effect predicted as a result of ionospheric currents by Hill *et al.* (1976); the model developed by Siscoe *et al.* (2002c) is equivalent to setting

$$\Psi_{\mathrm{PC}} \sim O(1)\frac{F}{1 + 1/(Q_1 Q_2 F)}; \tag{10.56}$$

note that

$$Q_1 Q_2 = \frac{4\pi\, \Sigma_{\mathrm{P}} v_{\mathrm{A}}}{c^2}. \tag{10.57}$$

For the energy dissipation rate $\mathcal{P}$, the two most widely used empirically based models are the Burton–McPherron–Russell equation (Burton *et al.*, 1975) and the so-called ϵ parameter (Perrault and Akasofu, 1978), both first put into a dimensionally correct form by Vasyliūnas *et al.* (1982). The equivalent values of $\Psi_{\mathcal{P}}$ are given by Eqs. (10.58) and (10.59), respectively:

$$\Psi_{\mathcal{P}} \sim O\left({M_{\mathrm{A}}}^{-1}\right) F \tag{10.58}$$

$$\sim O\left({M_{\mathrm{A}}}^{-2}\right) F', \tag{10.59}$$

F being taken from Eq. (10.55) for the first form and a somewhat modified, smoothed, function F' for the second. Both values of $\Psi_{\mathcal{P}}$ depend (in slightly different ways) on ${M_{\mathrm{A}}}^{-1}$ and on angles only, with no explicit reference to ionospheric parameters.

As the final step of increasing complexity, one may add the interior plasma sources (at the level of approximation discussed in Section 10.5), bringing in Q_5 and Q_6 as additional dimensionless quantities. With the large number of dimensionless quantities now included, dimensional analysis as a predictive tool may be of limited power; however, it is also useful as a classifying tool, for encompassing the great variety of observed magnetospheres and magnetosphere-like systems that result from the diversity of plasma sources and transport processes.

One significant result of a strong interior source is to increase the size of the magnetosphere, scaled by the R_{MP}, to well above the value R_{CF} set by the pressure balance between the solar wind and the magnetospheric magnetic field, as the pressure of plasma inside the magnetosphere becomes larger; this effect is discussed in Section 10.3.1 and illustrated there in Figure 10.2. The increase in the ratio $R_{\mathrm{MP}}/R_{\mathrm{CF}}$ as the result of internal plasma pressure can be considerably larger ($R_{\mathrm{MP}}/R_{\mathrm{CF}} \sim 2$–3 is observed at Jupiter and Saturn) than the decrease associated with an open magnetosphere ($\sim O(10^{-1})$ is observed at Earth). To represent the effect quantitatively in dimensionless form, one may therefore neglect the influence of M_{A}^{-1} and replace Eq. (10.51) by

$$\frac{R_{\mathrm{MP}}}{R_{\mathrm{CF}}} \sim \Xi\left(\frac{S}{\rho_{\mathrm{sw}} v_{\mathrm{sw}} {R_{\mathrm{CF}}}^2}\right) \tag{10.60}$$

where the function $\Xi(x)$ increases monotonically with increasing x from $\Xi(0) \sim 1$. (In this context, S need not be limited to the interior source from moons and planetary rings but may be taken as including any sources from the ionosphere.)

One may envisage a situation in which S becomes so large that the plasma pressure inside the magnetosphere is much larger than the magnetic pressure almost everywhere, to the point that the planetary magnetic field no longer plays any significant role in balancing the external pressure from the solar wind; according to the definitions of Section 10.2 we then have a magnetosphere-like system rather than a true magnetosphere. We may still designate the distance to the interface with the solar wind as R_{MP}, but its value as given by Eq. (10.60) obviously must be independent of R_{CF} (which in this limit plays no role). This requires that

$$\Xi(x) \to \sqrt{\frac{x}{\zeta}} \quad \text{as} \quad x \to \infty \tag{10.61}$$

where ζ is a constant, equal to the limiting value of $S/(\rho_{\mathrm{sw}} v_{\mathrm{sw}} {R_{\mathrm{MP}}}^2)$. It is easily shown that, for a pressure balance between the solar wind and plasma flowing outward with speed v from a localized source, $\zeta \sim v_{\mathrm{sw}}/v$.

It is convenient to rewrite Eq. (10.60) by normalizing S to the solar wind mass flux through the observed distance ${R_{\mathrm{MP}}}^2$ instead of the theoretical distance ${R_{\mathrm{CF}}}^2$

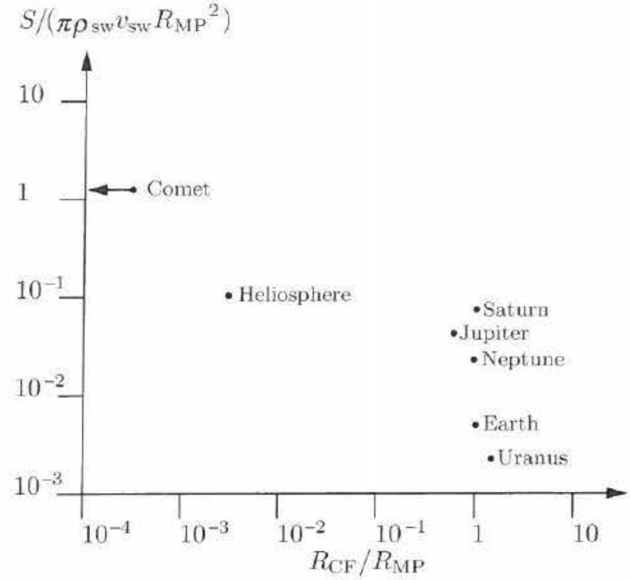

Fig. 10.9. Normalized interior mass source vs. R_{CF}/R_{MP}. The parameter values are mostly taken from Chapter 13. The values for a comet and for the heliosphere are only very rough estimates. The distance R_{CF} for the Sun (admittedly a somewhat artificial concept) was calculated by balancing a dipole field of 1 G at the solar surface against an interstellar pressure that stops the solar wind at $\sim$ 100 AU.

and by inverting to give S as a function of $1/R_{MP}$:

$$\frac{S}{\rho_{sw}v_{sw}R_{MP}{}^2} \sim \Upsilon\left(\frac{R_{MP}}{R_{CF}}\right) \tag{10.62}$$

where the function $\Upsilon(x)$ has the following limiting values:

$$\Upsilon(x) \to \zeta \quad \text{as} \quad x \to 0, \qquad \Upsilon(x) \to 0 \quad \text{as} \quad x \to 1\,. \tag{10.63}$$

A plot of the two variables of Eq. (10.62) for various observed or inferred magnetospheres and magnetosphere-like systems is shown in Fig. 10.9. The true magnetospheres are clustered near $R_{CF} \simeq R_{MP}$ and spread upward and to the left ($R_{CF} < R_{MP}$ as the interior source strength increases). On the far left ($R_{CF} \ll R_{MP}$) are magnetosphere-like systems of the type exemplified by a cometary interaction with the solar wind. (Magnetosphere-like systems of the other type, exemplified by Venus and Mars, in which the solar wind interacts directly with the ionosphere or atmosphere, may be included in this diagram at the far right, where $R_{CF} \gg R_{MP}$.)

The heliosphere may be viewed as a magnetosphere-like system in which a central object (the Sun) interacts with an external medium (the interstellar gas and plasma); the solar wind now plays the role of the internal plasma of the system. The interaction is in some ways similar to that of a comet with the solar wind:

the pressure to hold off the external medium is provided primarily by the outflow of plasma from the central object. The Sun differs from a comet in possessing a magnetic field, but the stresses of the interplanetary magnetic field are negligible in comparison with the dynamic pressure of the solar wind. From Fig. 10.9, the heliosphere is in fact intermediate between Jupiter and a comet: the interaction with the interstellar medium may be viewed either as *Jupiter-like* but with the interior source greatly enhanced (to the point of dominating dynamically over the magnetic field) or as *comet-like* but with the addition of a magnetic field from the central object (a field not strong enough, however, to have a significant dynamical effect).

The final topic in which the interior plasma source plays a major role is that of providing a limit to corotation. This was extensively discussed in Section 10.4.4, and the result obtained was that the limiting distance of corotation R_{cr} is the smallest of the distances R_{c} (Eq. 10.20), R_{H} (Eq. 10.21), and R_0 (Eq. 10.24); here we simply recast these distances in the appropriate dimensionless form. In terms of the dimensionless quantities $Q_1, Q_2, \ldots$ we have,

$$\frac{R_{\mathrm{c}}}{R_{\mathrm{CF}}} \sim \left(\frac{Q_3}{Q_1}\right)^{1/\nu}, \tag{10.64}$$

$$\frac{R_{\mathrm{H}}}{R_{\mathrm{CF}}} \sim \left(\frac{Q_2}{Q_6}\right)^{1/4}, \tag{10.65}$$

$$\frac{R_0}{R_{\mathrm{CF}}} \sim (Q_3 Q_5 Q_6 \Omega \tau_{\mathrm{tr}})^{-1/4} \tag{10.66}$$

where ν is the exponent introduced in Eq. (10.20) and τ_{tr} is the global transport time (Section 10.5.2). The distances R_{H} and R_0 can also be expressed in dimensionless form in another way, with the use of only the planetary and magnetospheric given parameters:

$$\frac{R_{\mathrm{H}}}{r_{\mathrm{s}}} \sim \left[\left(\frac{4\pi \Sigma_{\mathrm{P}} \Omega r_{\mathrm{s}}}{c^2}\right)\left(\frac{1}{Q_7}\right)\right]^{1/4}, \tag{10.67}$$

$$\frac{R_0}{r_{\mathrm{s}}} \sim (Q_7 \Omega \tau_{\mathrm{tr}})^{-1/4}. \tag{10.68}$$

This was to be expected, since the physical processes associated with these distances are purely internal to the magnetosphere and do not involve the solar wind.