

The Hydromagnetic Interior of a Solar Quiescent Prominence: Two Calculations on the Effects of Power-Law Radiative Loss and Viscosity on a Kippenhahn-Schlüter Slab

BARNARD

Andrea Egan^{1,2}, B.C. Low¹, & Yuhong Fan¹

¹High Altitude Observatory, National Center for Atmospheric Research

²Barnard College, New York, NY



ABSTRACT

This study is a part of an ongoing investigation conducted by BC Low and colleagues to understand the small-scale behaviors of quiescent prominences in recent Hinode/SOT SDO/AIA high-resolution observations. Two particular calculations are presented here, utilizing the one dimensional Kippenhahn-Schlüter infinite-slab prominence model in a steady state, to simplify the system without losing the nonlinear coupling between force-balance and energy-transport. The first calculation focuses on the thermodynamic balance in a non-resistive and non-viscous plasma subject to radiative loss that is nonlinear in temperature and heating that is directly proportional to the density of the plasma in the presence of magnetic-field aligned thermal conduction. This calculation generalizes the result found in Low et al. (2012, ApJ 755, 34) in which linear-temperature radiation is studied. Possible explanations of the observed dynamic interior of prominences are further investigated in the second calculation focusing on the effect of viscosity on the resultant resistive flow of the slab across the supporting field. Isothermal condition is assumed to remove the complication of energy-transport in order to isolate the effects of viscosity. The nonlinear ordinary differential equations of these calculations have interesting mathematical properties studied with a standard Runge-Kutta numerical solve

THE MODEL: KIPPENHAHN-SCHLÜTER INFINITE SLAB

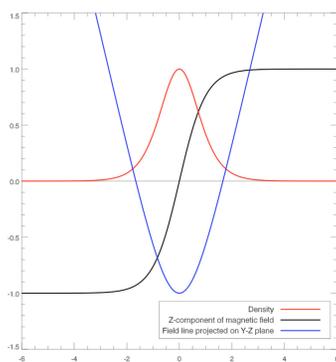


Fig. 1 –The original isothermal static Kippenhahn-Schlüter (1957) prominence slab. Displayed as a function of Y are the density (red), $B_z=B_0H$ (Black), and bowed field-line (blue) projected on the Y-Z plane.

$$\vec{v} = [0, 0, v(y)] \quad \vec{B} = B_0[0, 1, H(y)]$$

The Kippenhahn-Schlüter (KS) infinite slab is a 1D prominence model in which the weight of the plasma is supported against gravity by a horizontally-oriented bowed magnetic field, taking the electrical conductivity to be infinite. This unique model is generalizable to allow for a non-isothermal interplay between force-balance and energy-transport and the presence of a vertical resistive flow, without the complexity of 3D equations.

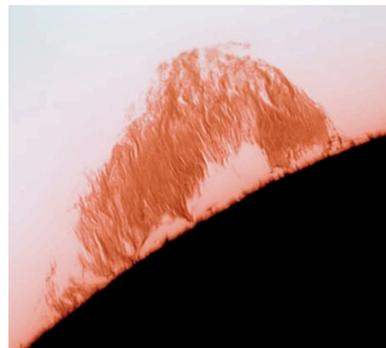
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MOTIVATION



Although the prominence is in a state of global equilibrium, on a smaller scale it is composed of dense well-defined vertical threads of material descending at less than free-fall speeds that add up to $10^{15}g$ falling over the course of a day—a mass over ten times greater than that of the total stable mass of the prominence. This investigation proposes that the observed drainage is a resistive flow resulting from force-balance and the nonlinear couplings among heating, cooling, resistivity, thermal conduction, and viscosity.

Properties of a viscous slab

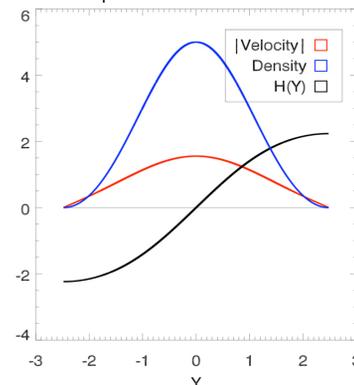


Fig 4. – A resistive viscous KS slab with density ρ and downward vertical velocity $|v|$ vanishing at the finite-Y boundaries, and $B_z=B_0H(Y)$; see force balance equation (5).

POWER-LAW RADIATIVE LOSS

$$r = \alpha_0 \rho^2 T^n \quad (1) \quad h = \gamma_0 \rho \quad (2)$$

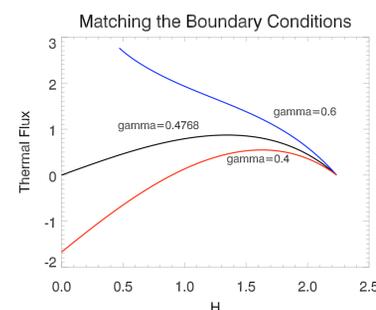


Fig 2 – Thermal conductive fluxes for $n=2$ as functions of $0 < B_z = B_0 H(Y) < 2.236$ that spans the physical space $0 < Y < \infty$ for different heating rates γ_0 in a KS slab symmetric about $Y=0$, all other parameters held fixed. Thermal conduction must vanish at $H=0$ and $H=2.236$ as natural boundary conditions. Only for $\gamma_0=0.4768$ in normalized unit are these conditions made. If $\gamma_0 > 0.4768$ equilibrium is impossible with temperature everywhere increasing in time. If $\gamma_0 < 0.4768$, thermal conduction vanishes at $H=2.236$ and at an intermediate point $0 < H = H_c < 2.236$, such that in physical space, the total mass in $-H_c < H < H_c$ becomes a cold, collapsed, equilibrium mass-sheet with δ -functions for its density and electric current density. The essential point is that at finite electrical conductivity, the δ -functions in electric current density must dissipate via a resistive flow.

This calculation solves equation (3) for the non-isothermal balance among radiative loss (1) with index $n \neq 1$, simple heating (2), and field-aligned thermal conduction in a static K-S slab with infinite electrical conductivity. Its results generalize the limited $n=1$ result of Low et al. (2012, ApJ 755, 34) that for most parametric regimes, this energy balance produces a cold, collapsed core in the slab that will resistively flow across the supporting field at the high but finite coronal electrical conductivity.

$$\frac{1}{10} \beta^2 \frac{d}{dH} \left[\frac{\kappa_0}{1+H^2} \frac{dT^{5/2}}{dH} \right] = (\beta - H^2) T^{n-1} - \beta \gamma \quad (3)$$

Density and Temperature Distributions for Eigenvalue Case

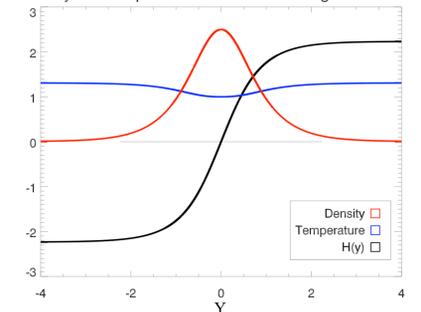


Fig 3 –Normalized density and temperature distributions are plotted for the eigenvalue solution (see Fig. 2) for which the density does not produce a delta function

ISOTHERMAL VISCOUS CASE

$$\mu_0 \frac{d^2 v}{dy^2} - \rho g + \frac{dH}{dY} = 0 \quad (5)$$

The second calculation treats the resistive flow with viscosity, simplifying the problem by taking the coefficients of resistivity and viscosity to be constants, with (5) describing the vertical balance of viscous, gravitational and magnetic-tension forces, where μ_0 is the product of the two coefficients, and g a uniform gravity. To avoid the complication of energy transport, the isothermal condition is assumed. The solution presented shows that the combined forces can compact the isothermal slab into a finite-width suspended in vacuum by an external potential field. The viscous force is upward in the central part of the slab, lightening the load on the supporting field, but downward near the boundaries, to make up for the total weight taken up by the supporting field. This viscous distribution of the forces in the resistive flow may be important for understanding prominence thread.

Density Profile Comparison

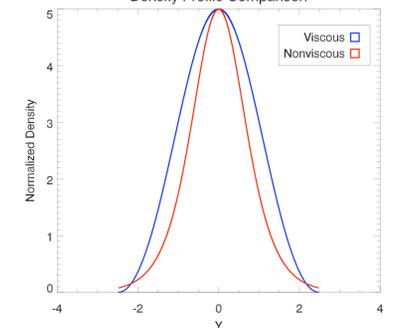


Fig 5. –A comparison of the density profiles of viscous-resistive and the original isothermal KS-slabs, showing the contraction of the latter into a finite width in the former.