Modeling Kinematic Dynamos in 2 and 3-D

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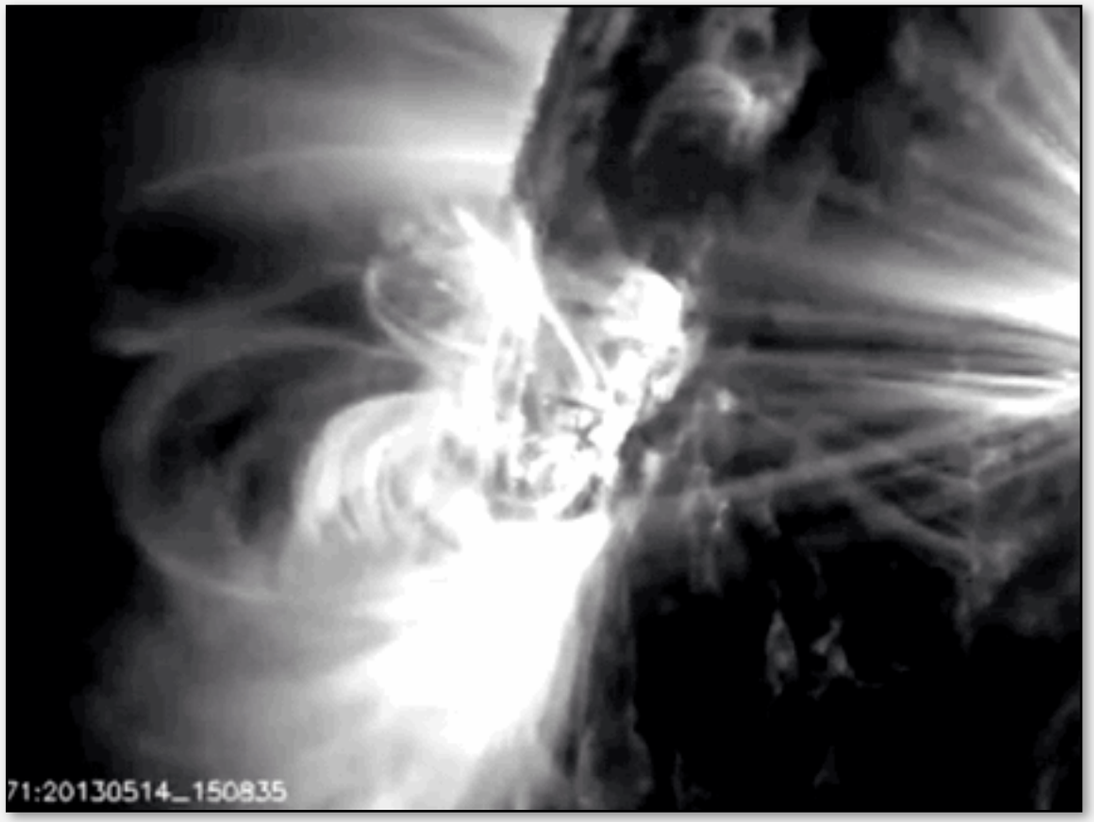
Motivation



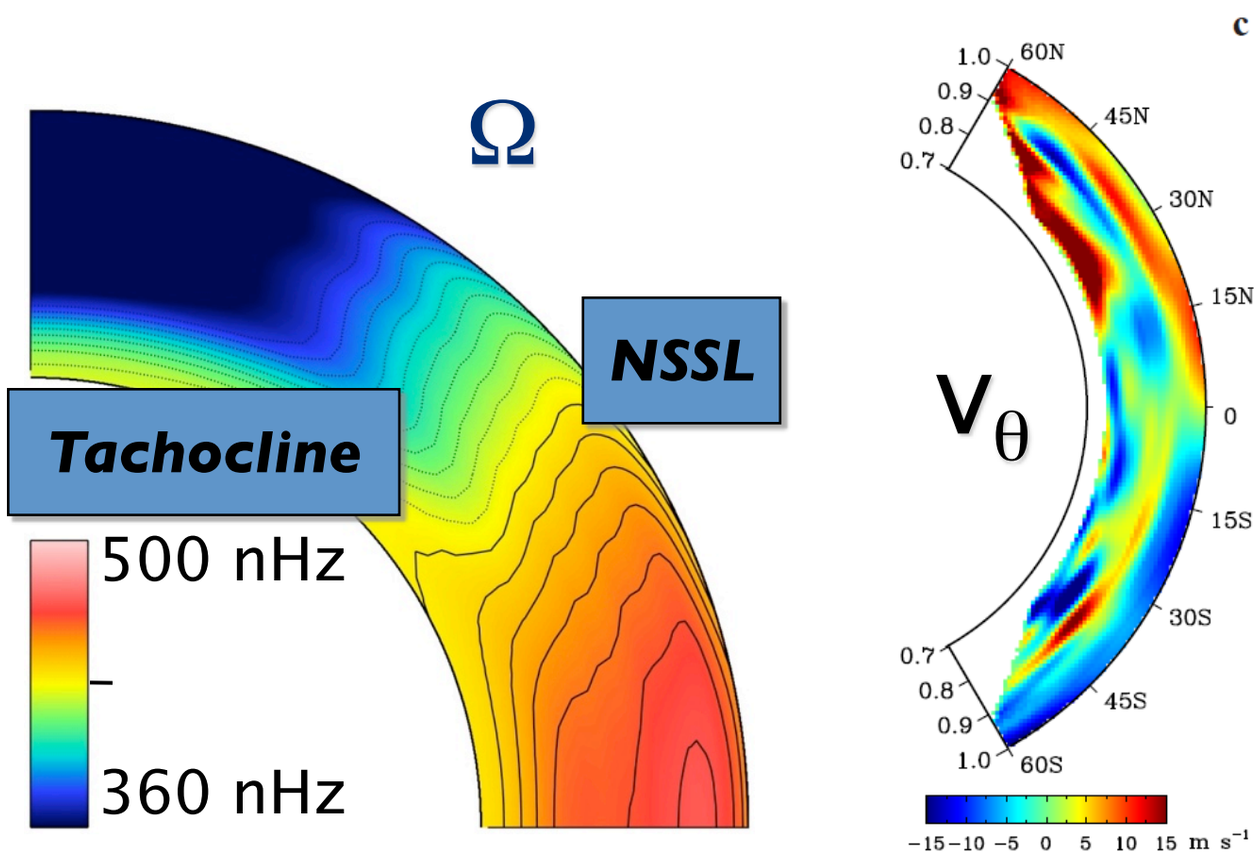
- Convection on many different scales
- Magnetic events on different scales
- Long (~22 year) solar cycle
- How do all these different scales fit together?
- How can we know how accurately our models are working?

The Dynamic Sun
Convection on Many Scales

Helioseismic Inference

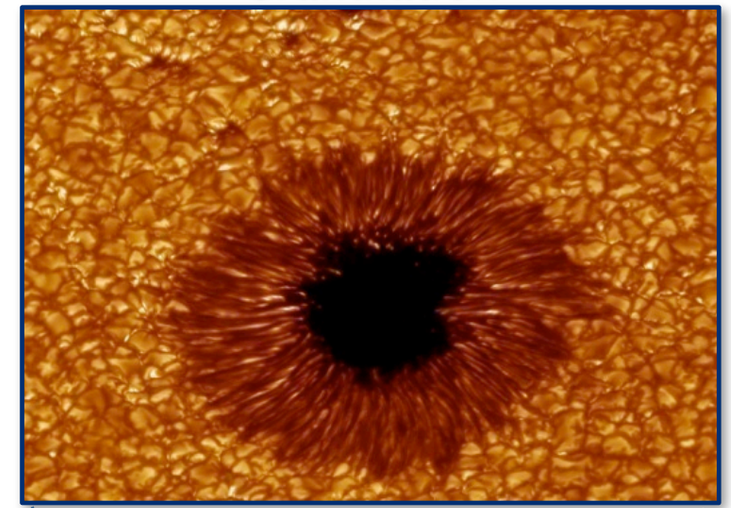
Differential Rotation

Meridional Circulation

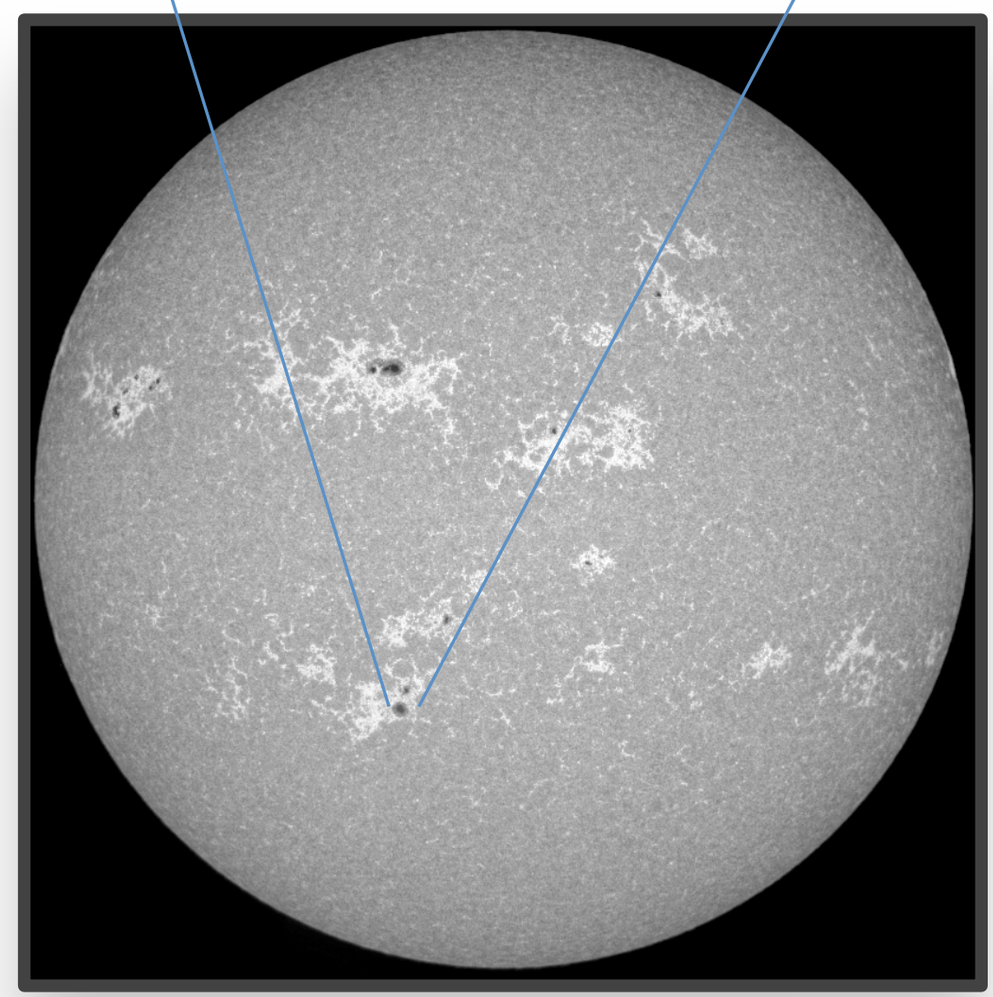


Howe et al. 2000; Schou et al. 2002 Zhao et al. 2012

Granulation



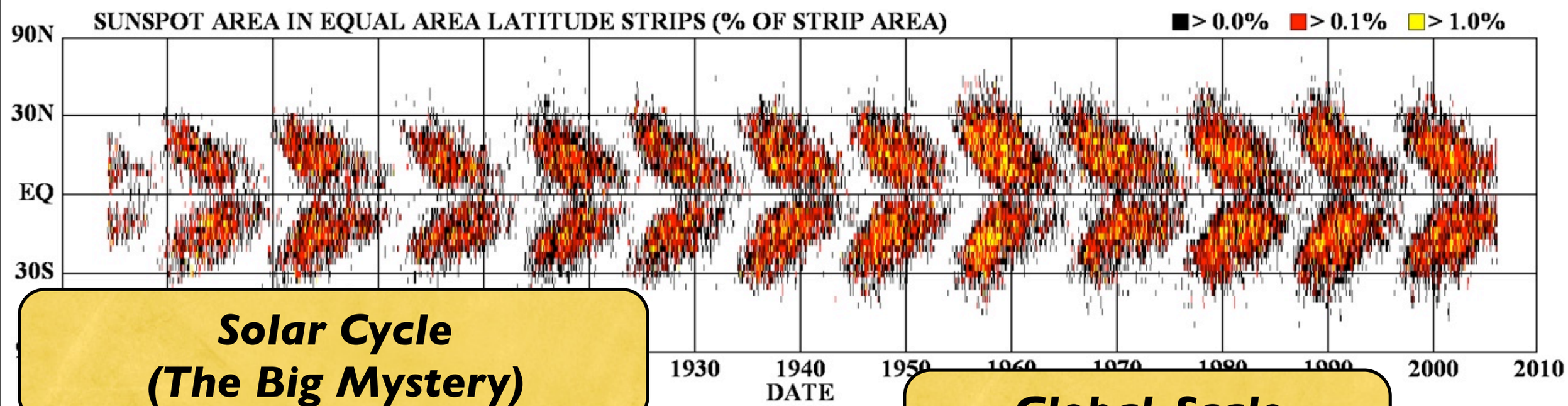
SST



SDO/AIA

Supergranulation

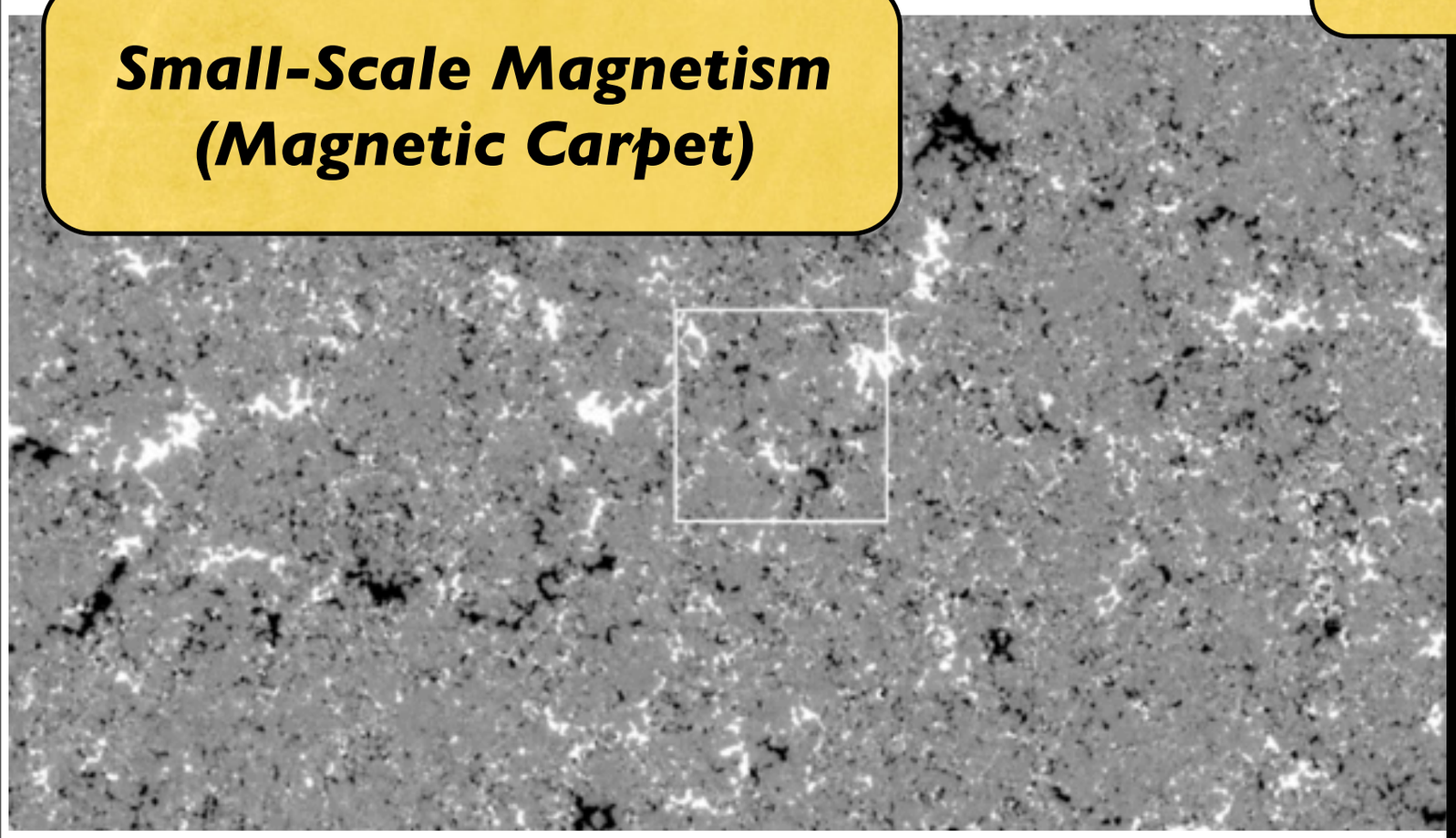
The Magnetic Sun

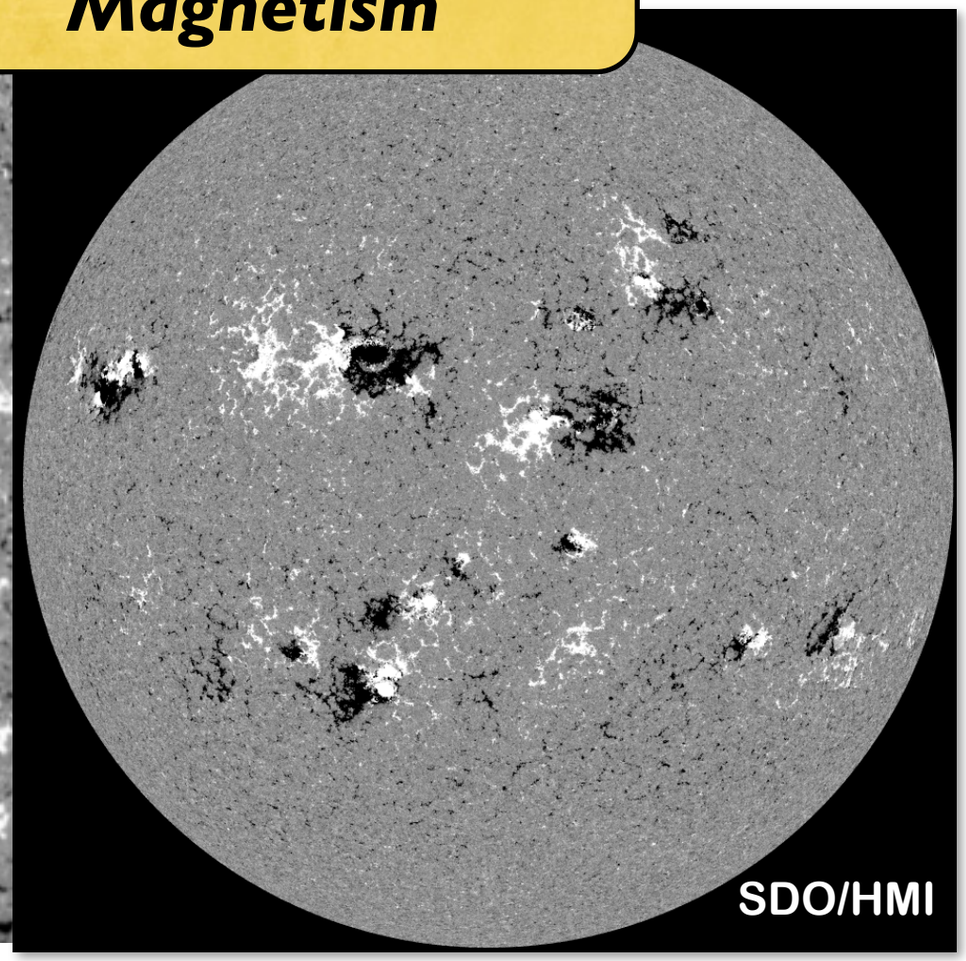


Solar Cycle (The Big Mystery)

Global-Scale Magnetism

Small-Scale Magnetism (Magnetic Carpet)





The Induction Equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}$$

Generation

Dissipation

$$\nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = \boldsymbol{v}(\nabla \cdot \boldsymbol{B}) - \boldsymbol{B}(\nabla \cdot \boldsymbol{v}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{v} - \boldsymbol{v} \cdot \nabla \boldsymbol{B}$$

Always zero

Compression and Expansion

Stretching and Shear

Advection (Movement)

Magnetic Reynolds Number

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}$$

Generation

Dissipation

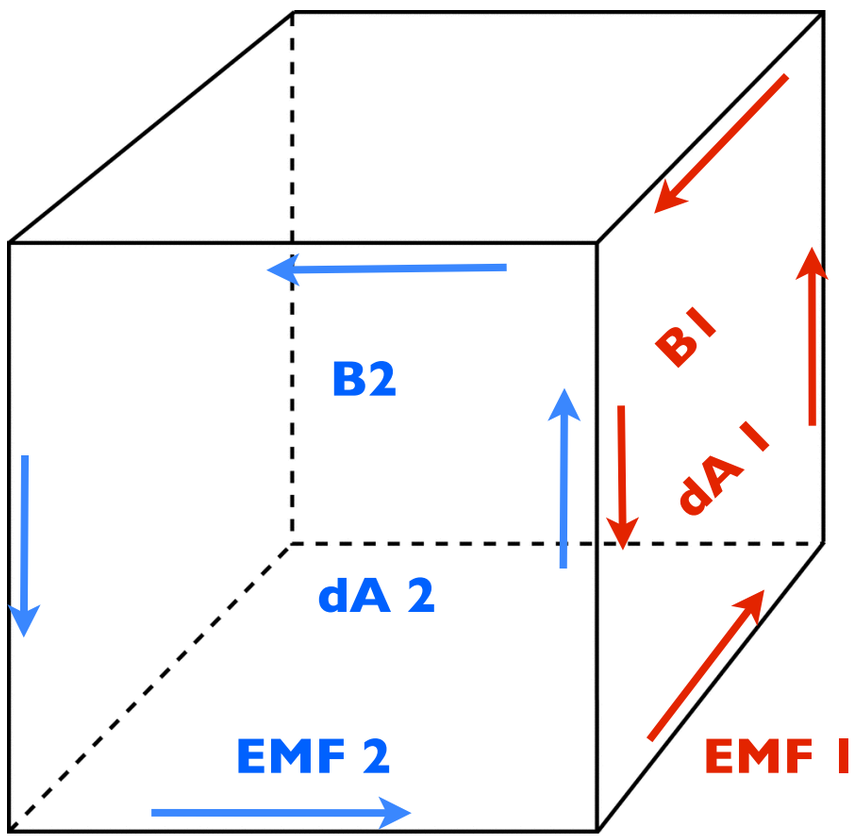
$$\mathrm{R_m} = \frac{UL}{\eta}$$

- Ratio of generation to dissipation
- Essentially the reciprocal of the diffusivity

Methods

Magneato

- Numerical solution to induction equation
- Periodic 3-D domain



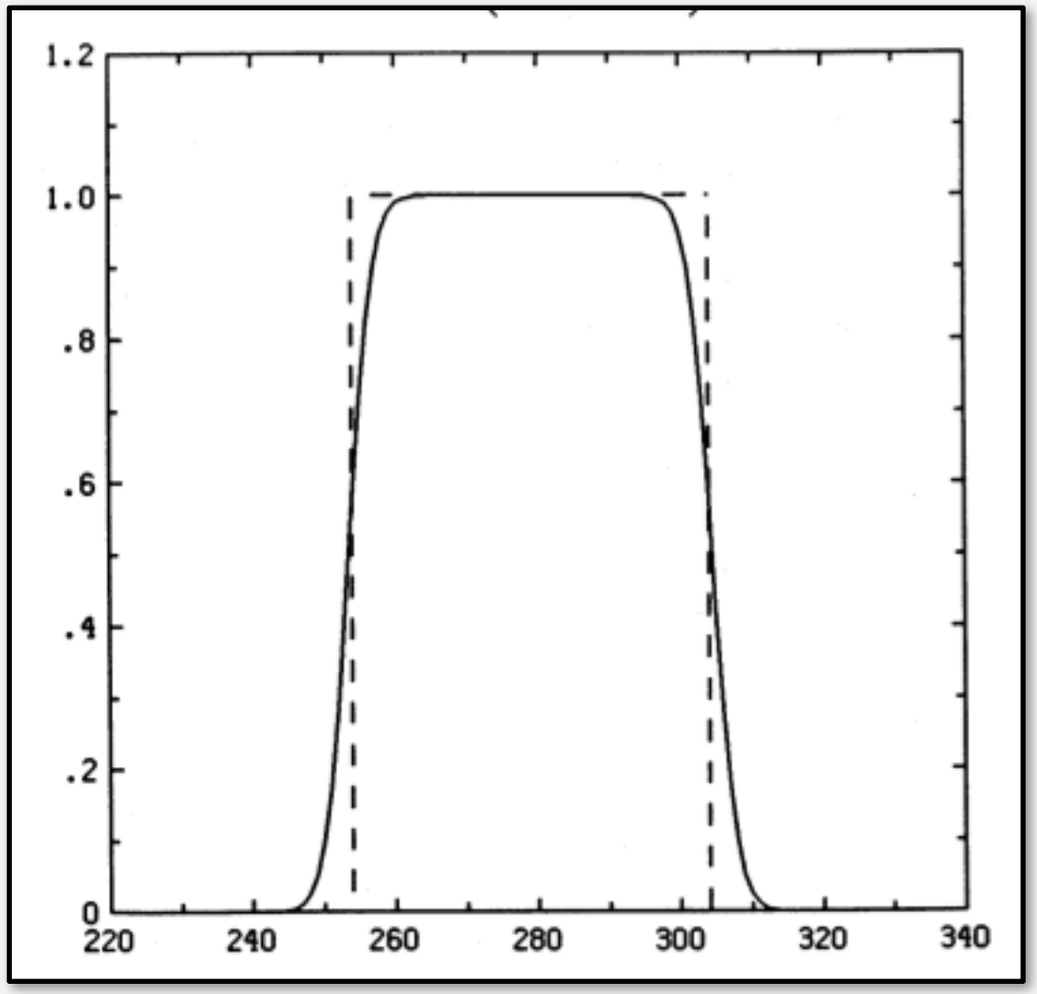
$$\Phi_i = B_i dA_i \qquad \frac{\partial \Phi_i}{\partial t} = \oint \boldsymbol{\mathcal{E}} \cdot \boldsymbol{dl}$$

- Computational domain broken up into many small cubes
- Constrained transport (Evans & Hawley 1988)
- Preserves divergence free magnetic field by integrating EMF around cell faces

Objectives

Assessing Dynamo Properties of Numerical Diffusion

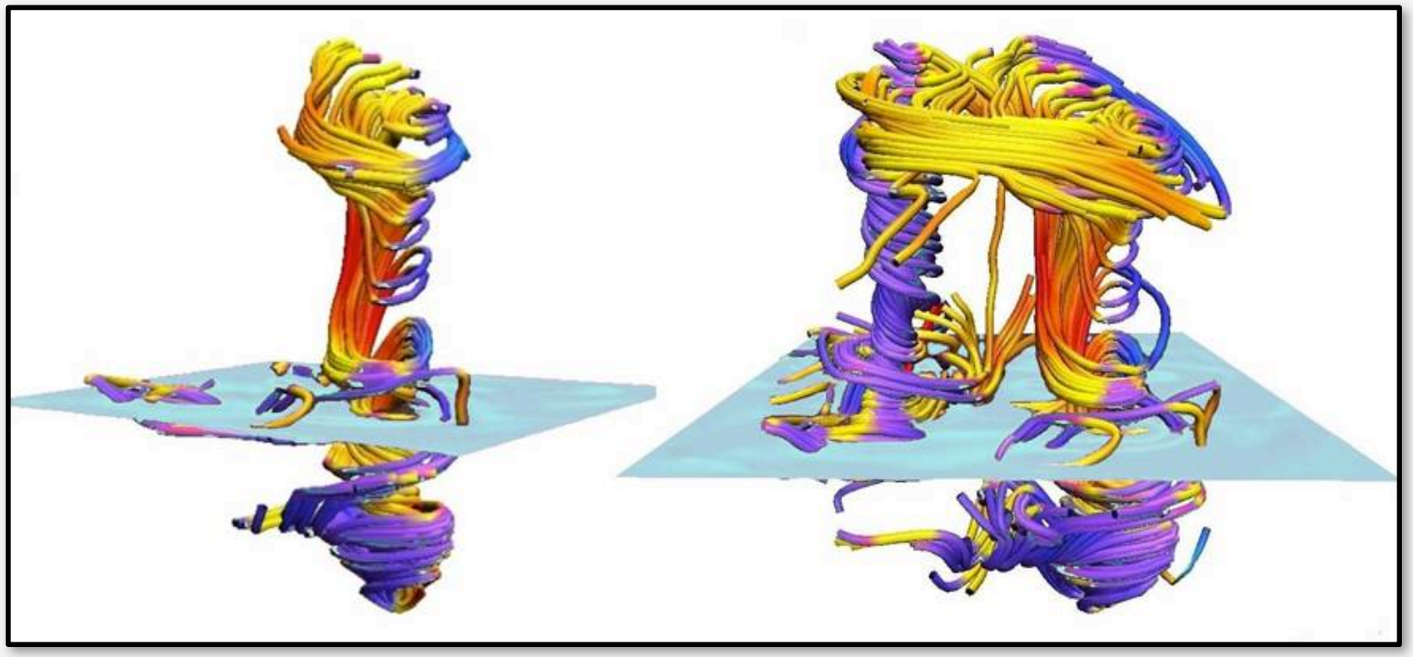
There is <u>always</u> diffusion present



Evans and Hawley 1988

Examine the dynamo properties of “solar-like” convective flows

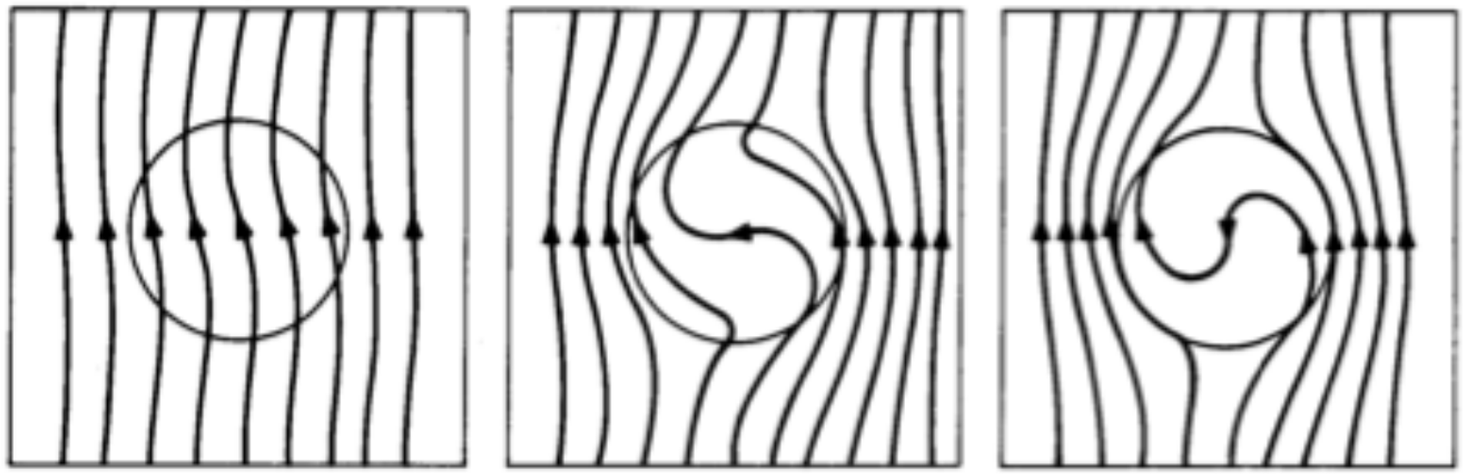
Some examples of flows in current solar models



Featherstone et al. 2009

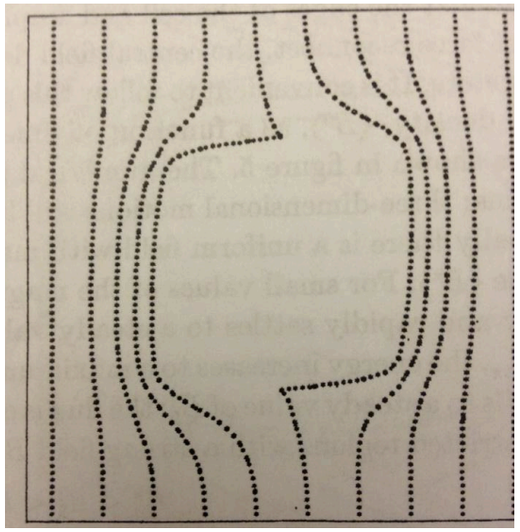
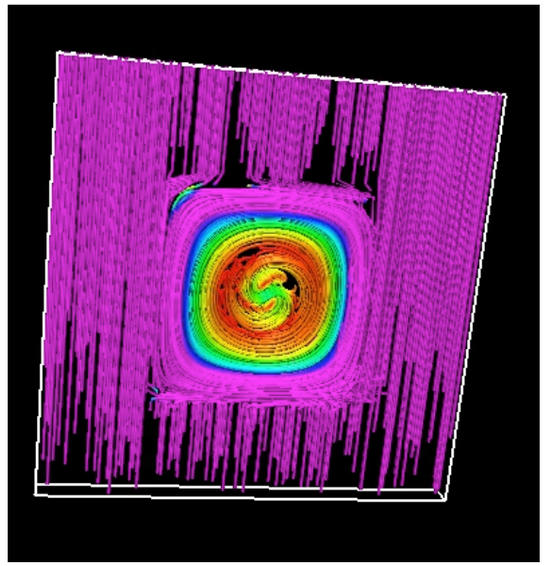
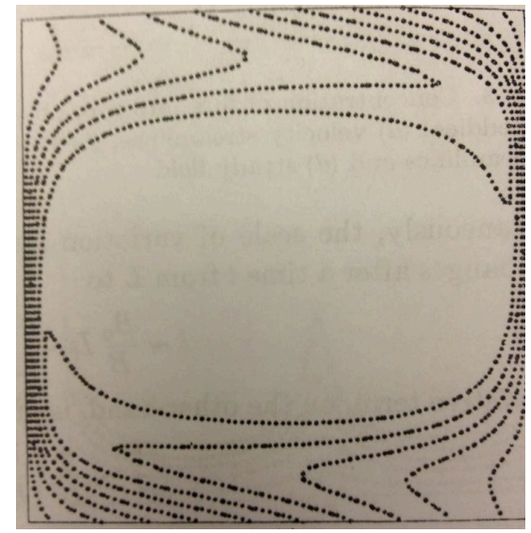
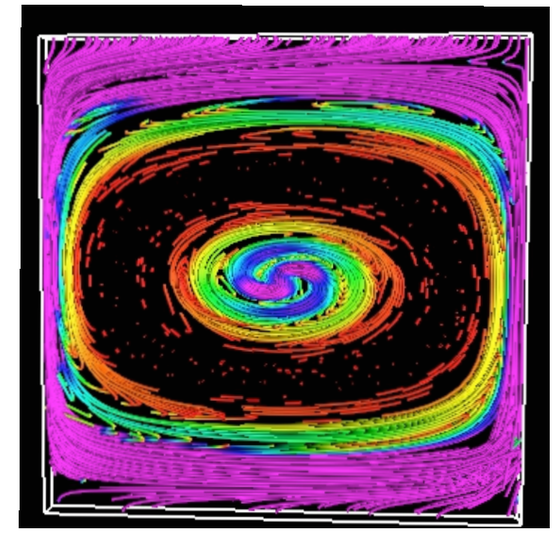
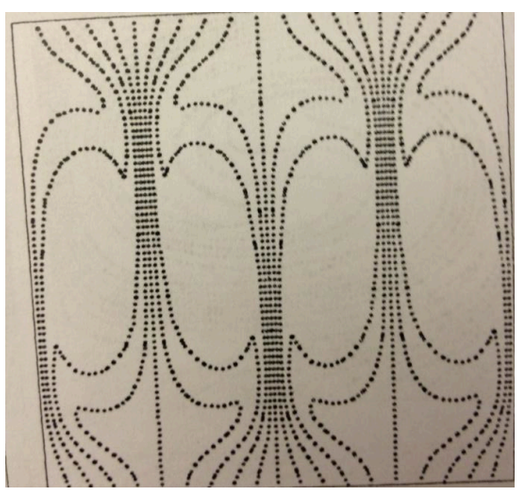
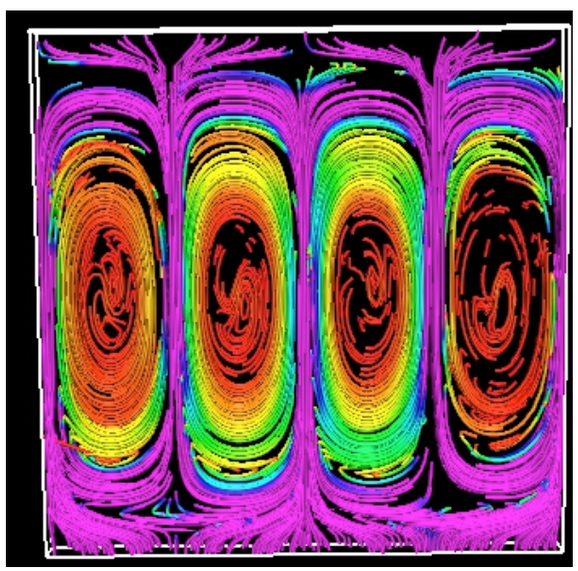
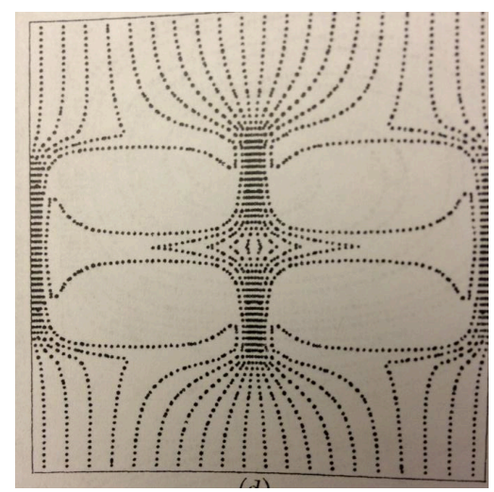
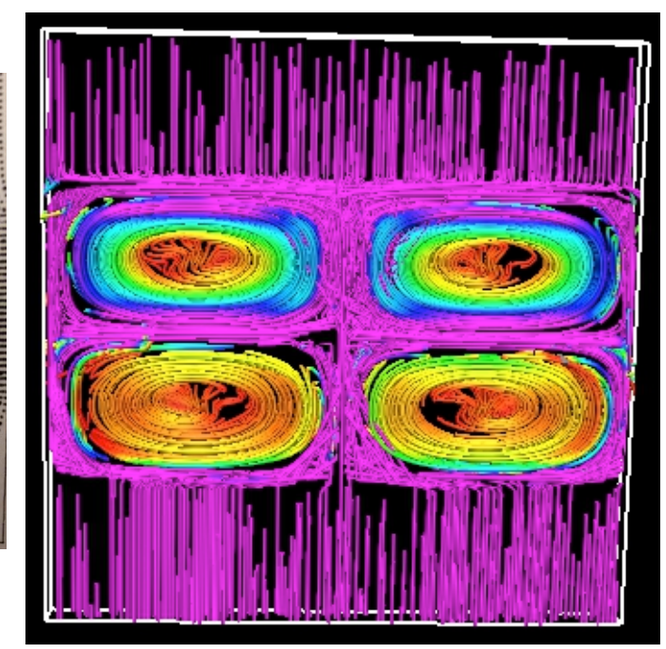
Effects of Vortical Flows in 2-D

- Flow “winds up” the field
- Takes field in one direction and generates field in another direction
- Eventually dissipation always wins as fields in opposing directions come together



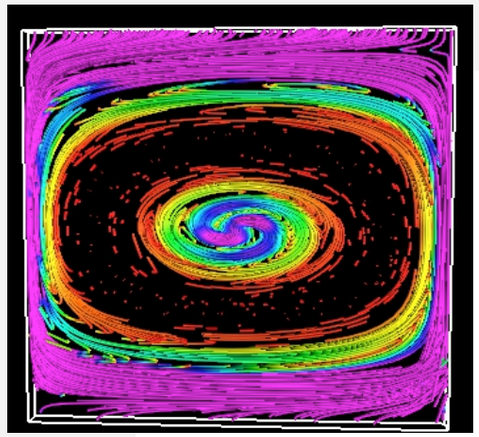
Moffatt 1978

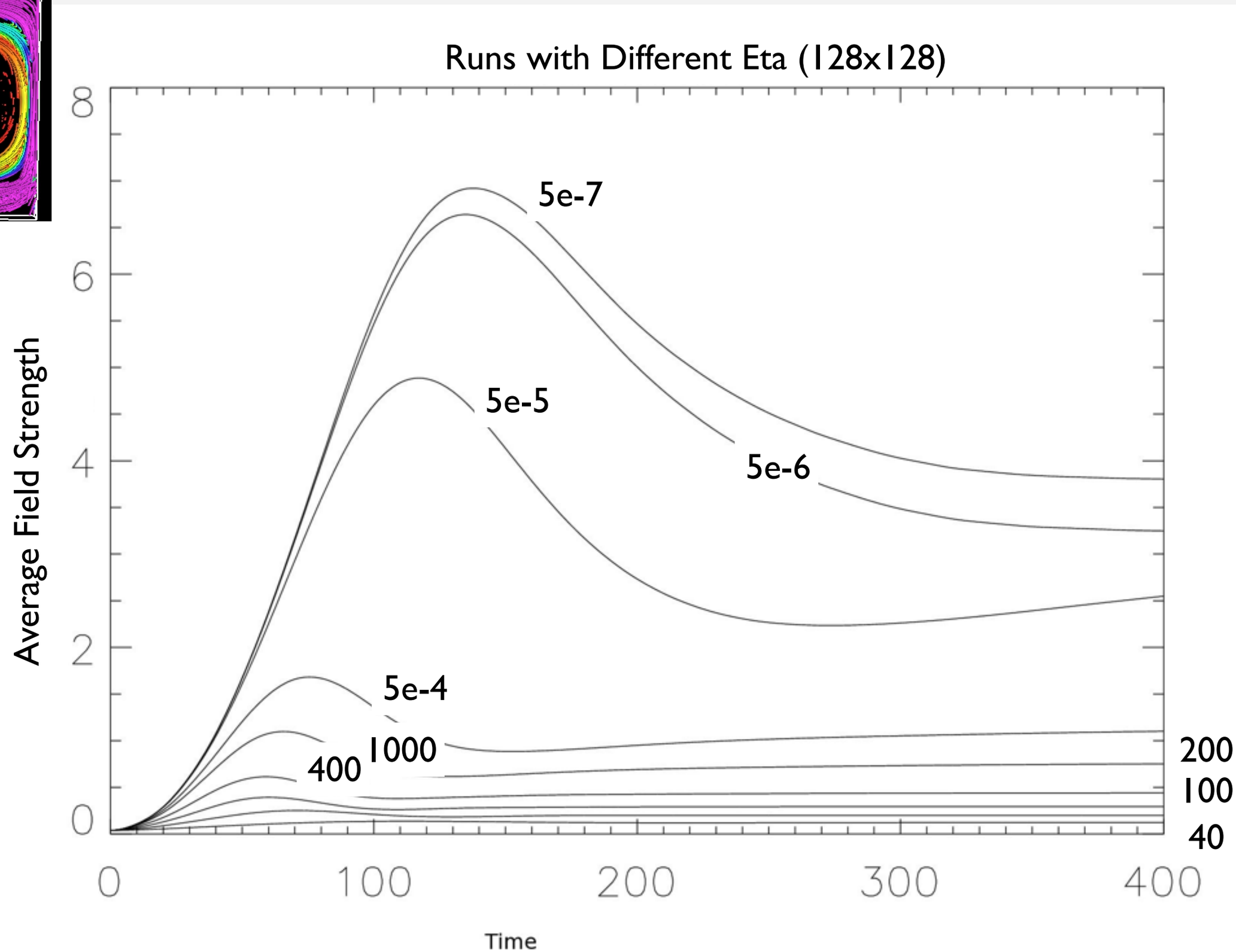
Code Validation

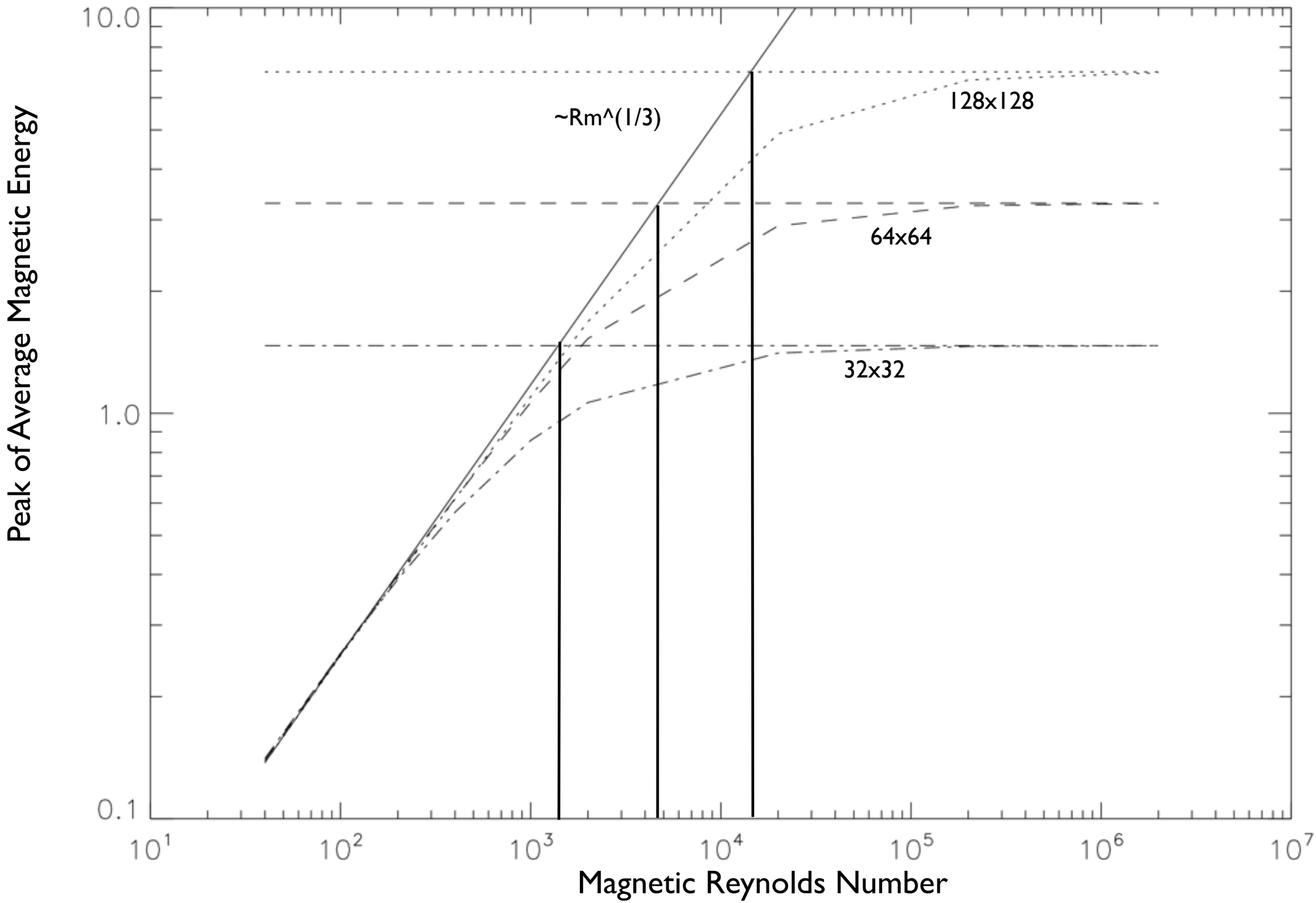
The Expulsion of Magnetic Fields by Eddies, Weiss 1966

Assessing Numerical Diffusion





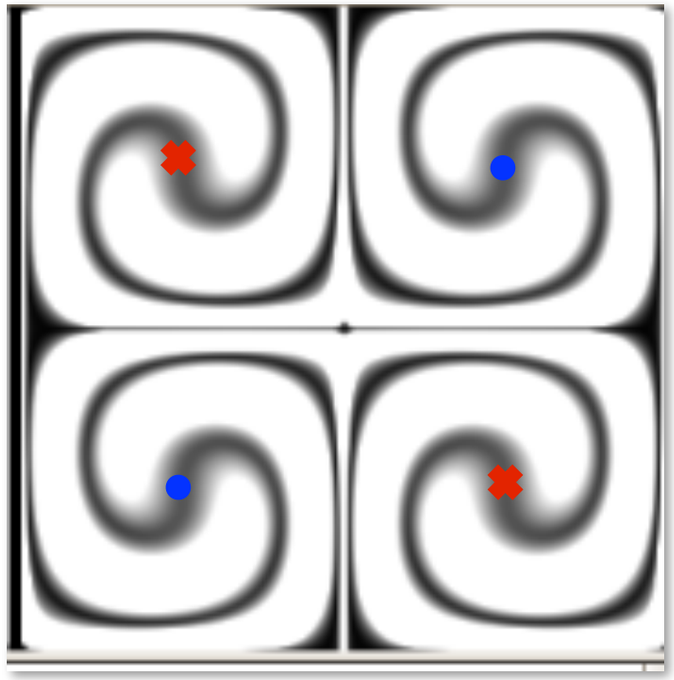
Magnetic Reynolds Number vs Peak of Magnetic Field for Different Grid Spacings



“2.5-D” Flow

$$
\begin{aligned}
v_x^a &= A\sin\left(\frac{2\pi x}{L_x}\right)\cos\left(\frac{2\pi y}{L_y}\right) \\
v_y^a &= -A\cos\left(\frac{2\pi x}{L_x}\right)\sin\left(\frac{2\pi y}{L_y}\right) \\
v_z^a &= \sin\left(\frac{2\pi x}{L_x}\right)\sin\left(\frac{2\pi y}{L_y}\right),
\end{aligned}
$$

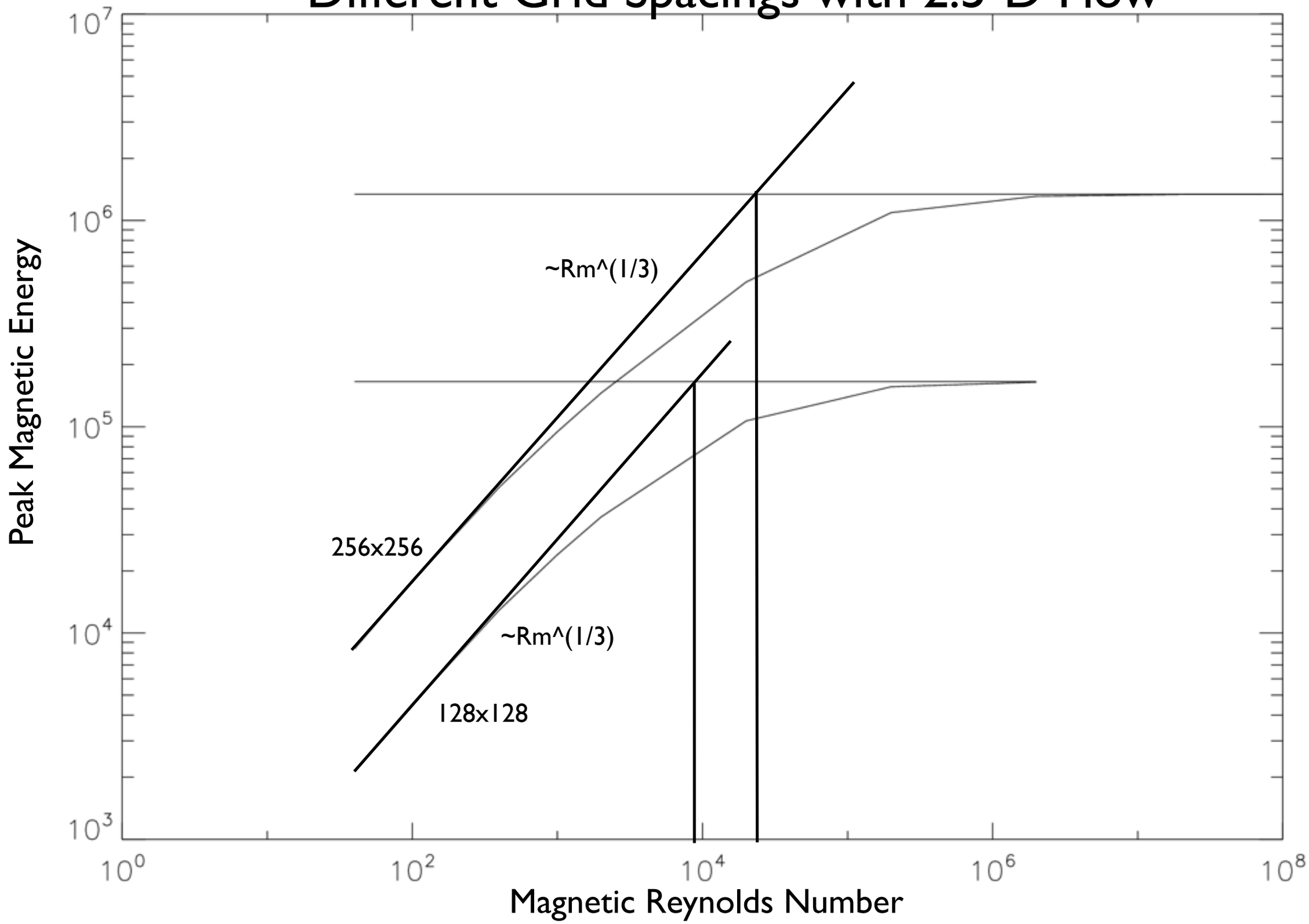
- Invariant in z-direction
- Vortices do not connect



Into Slide

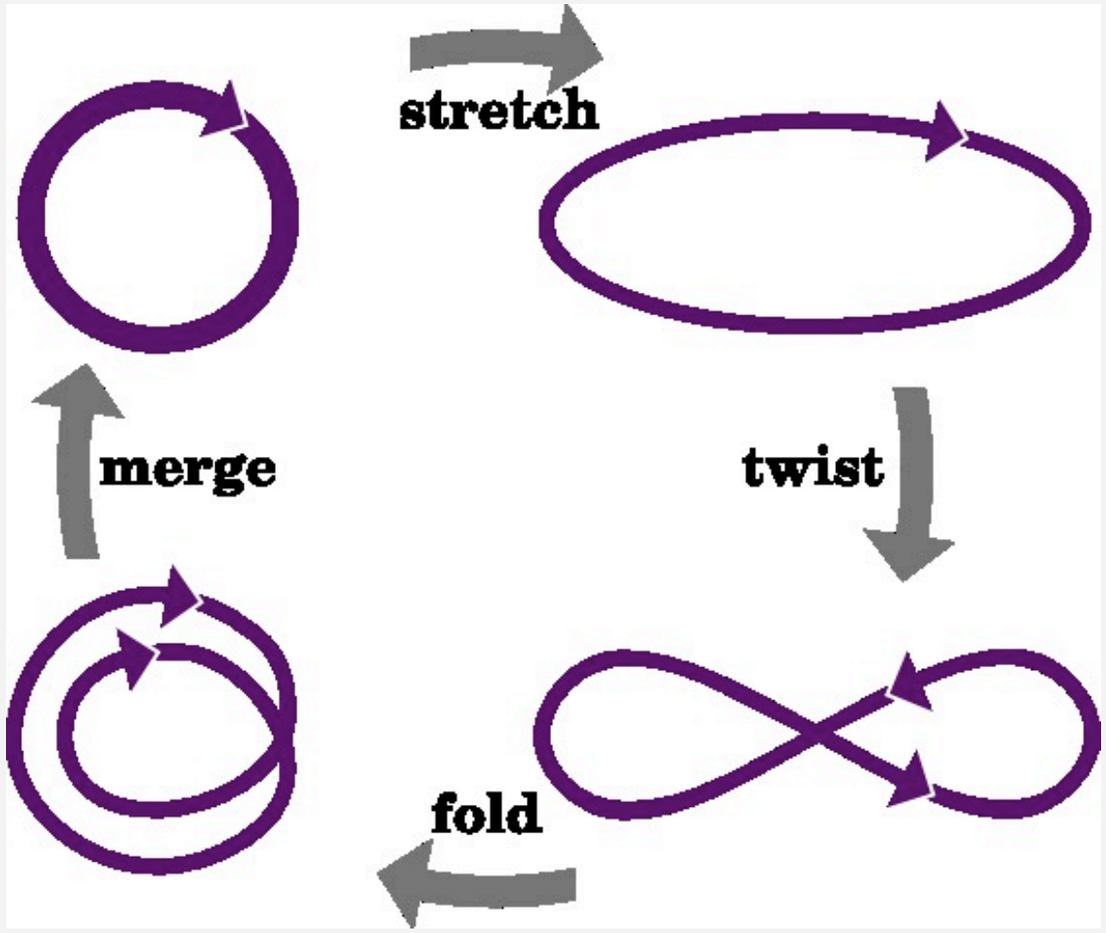
Out of Slide

Different Grid Spacings with 2.5-D Flow



Creating a Dynamo

- Impossible in 2-D (diffusion always wins)
- Field lines will wrap
- Opposing field comes together and cancels
- Possible in 3-D
- "stretch, twist, fold"



Fully 3-D Flow

A-flow

$$
\begin{aligned}
v_x^a &= A\sin\left(\frac{2\pi x}{L_x}\right)\cos\left(\frac{2\pi y}{L_y}\right) \\
v_y^a &= -A\cos\left(\frac{2\pi x}{L_x}\right)\sin\left(\frac{2\pi y}{L_y}\right) \\
v_z^a &= \sin\left(\frac{2\pi x}{L_x}\right)\sin\left(\frac{2\pi y}{L_y}\right),
\end{aligned}
$$

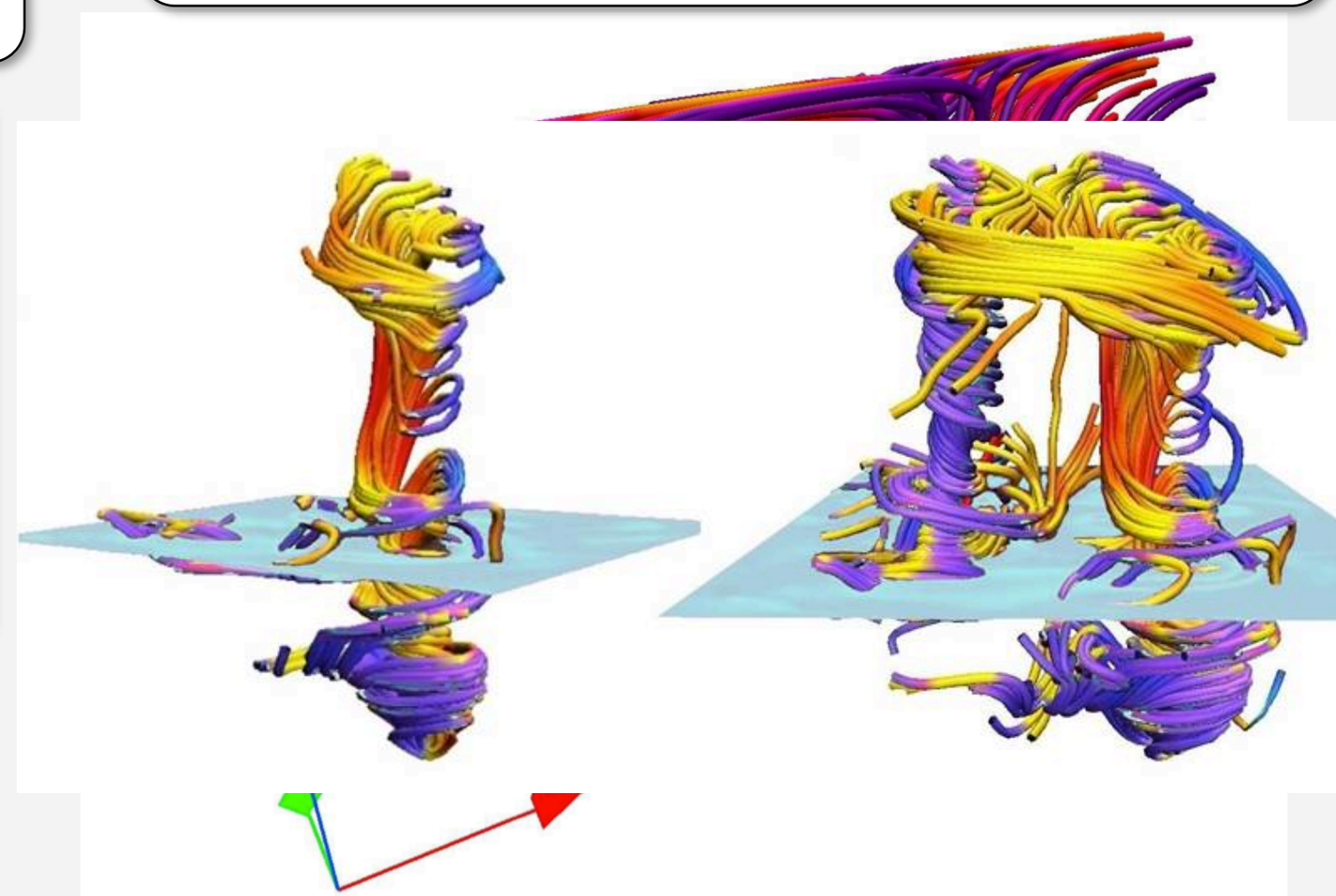
B-flow

$$
\begin{aligned}
v_x^b &= \cos\left(\frac{2\pi x}{L_x}\right)\sin\left(\frac{2\pi y}{L_y}\right)\cos\left(\frac{2\pi z}{L_z}\right) \\
v_y^b &= 0 \\
v_z^b &= \sin\left(\frac{2\pi x}{L_x}\right)\sin\left(\frac{2\pi y}{L_y}\right)\sin\left(\frac{2\pi z}{L_z}\right).
\end{aligned}
$$

Total flow is a weighted sum of the two

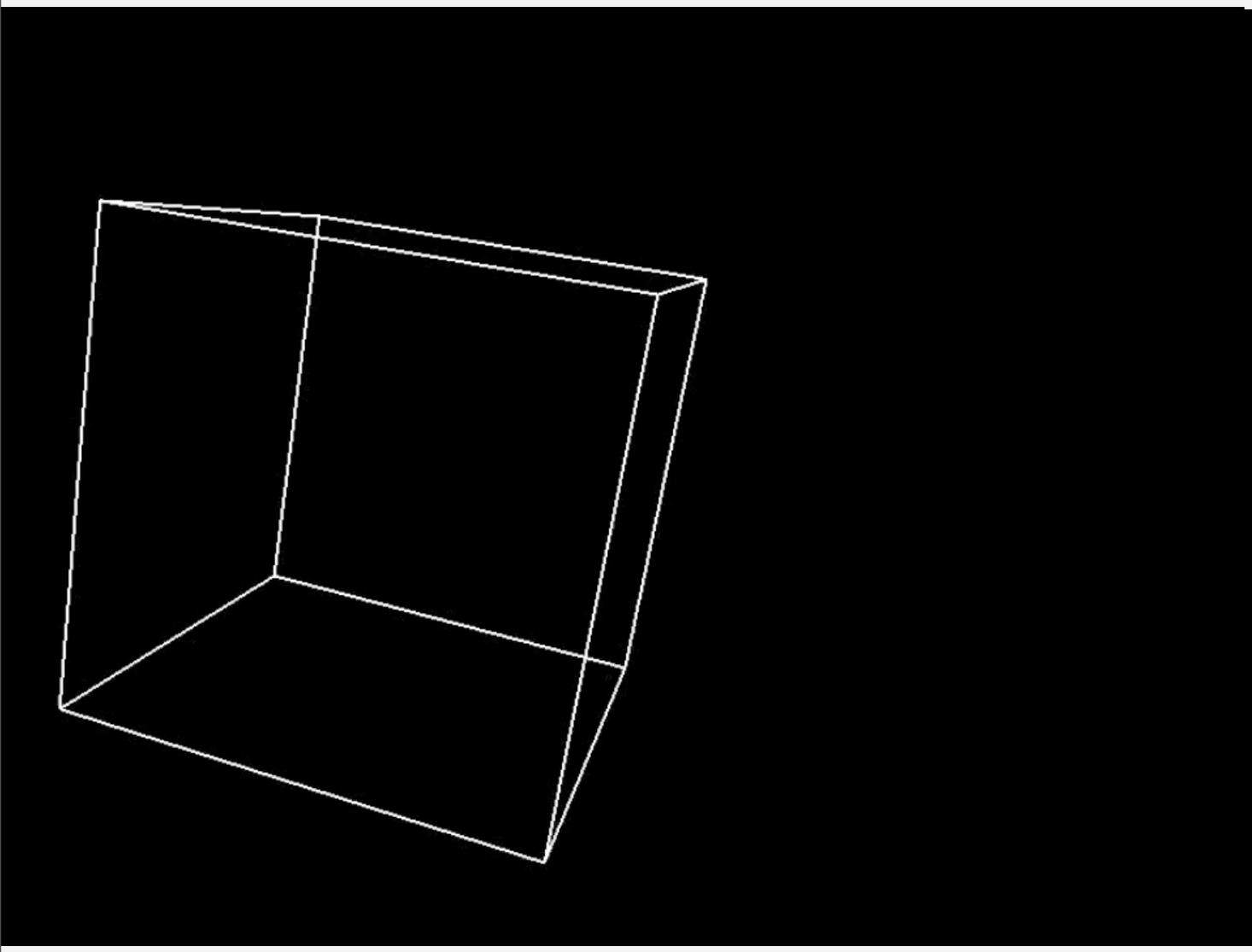
$$\boldsymbol{v} = (1 - \mathrm{w}(z))\boldsymbol{v}^a + \mathrm{w}(z)\boldsymbol{v}^b,$$

$$\mathrm{w}(z) = \frac{1}{2}\left(1 + \cos\left(\frac{4\pi z}{L_z}\right)\right).$$

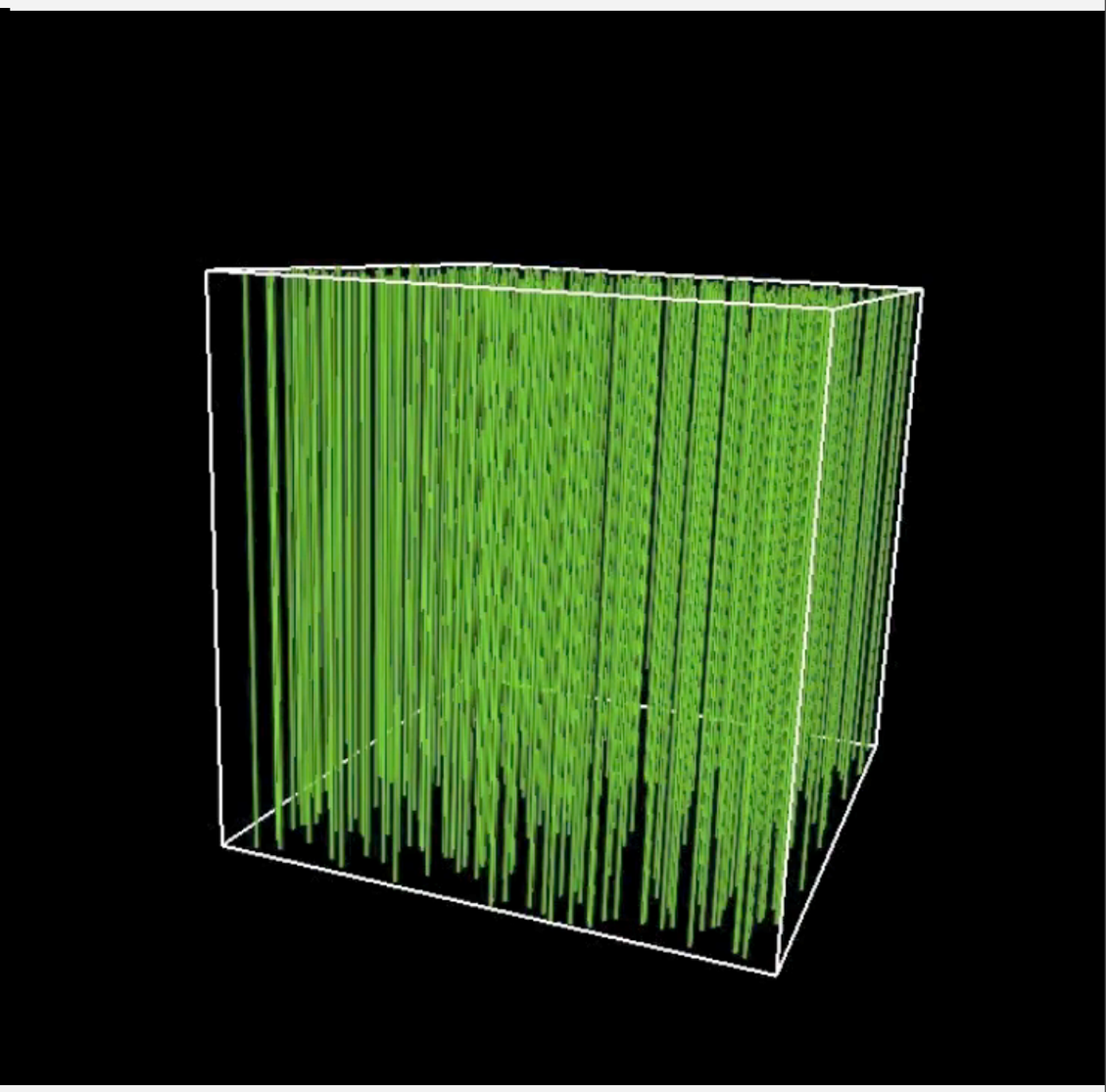


128 Cubed 3-D Flow

Magnetic Energy

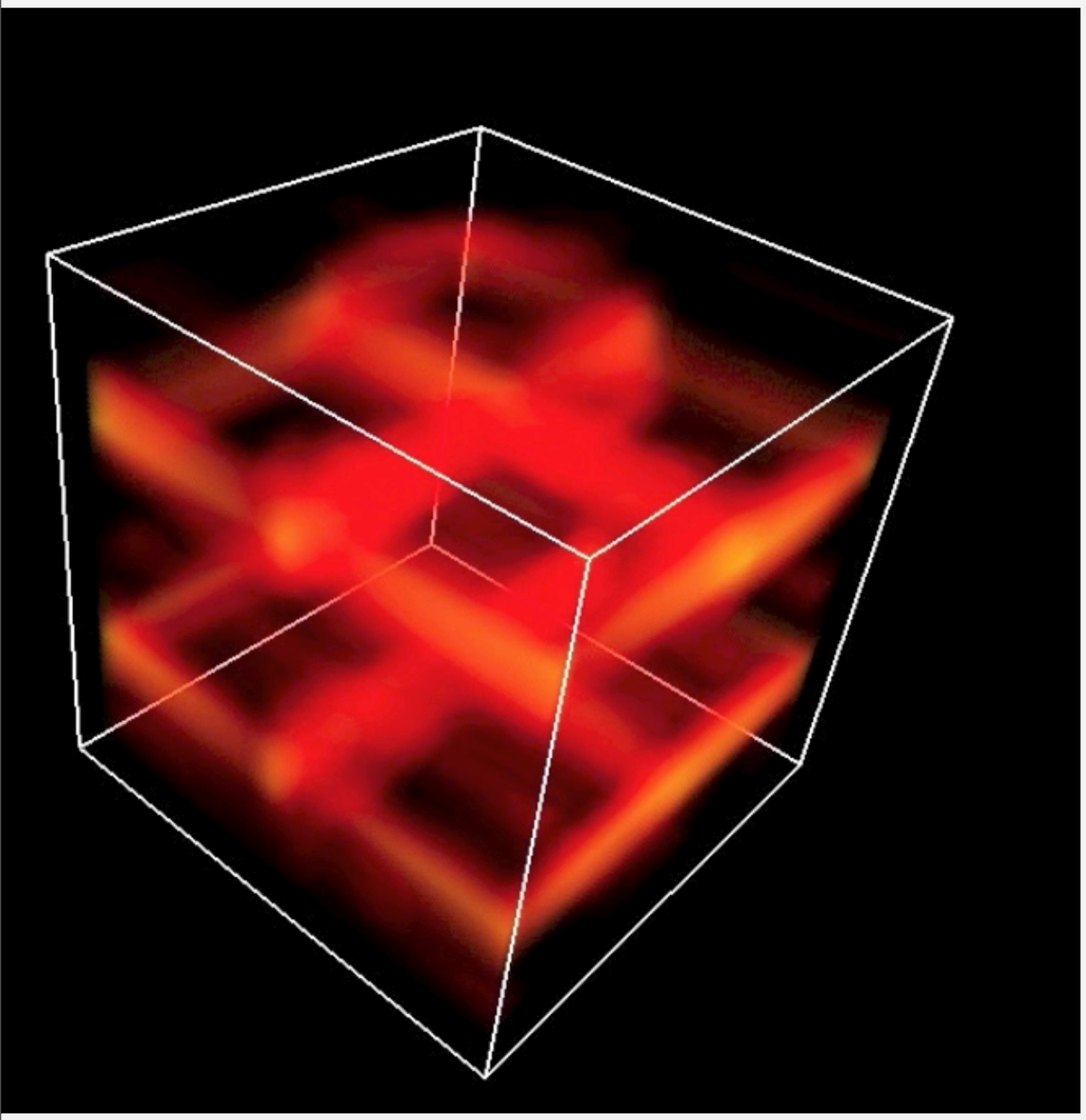


Field Lines

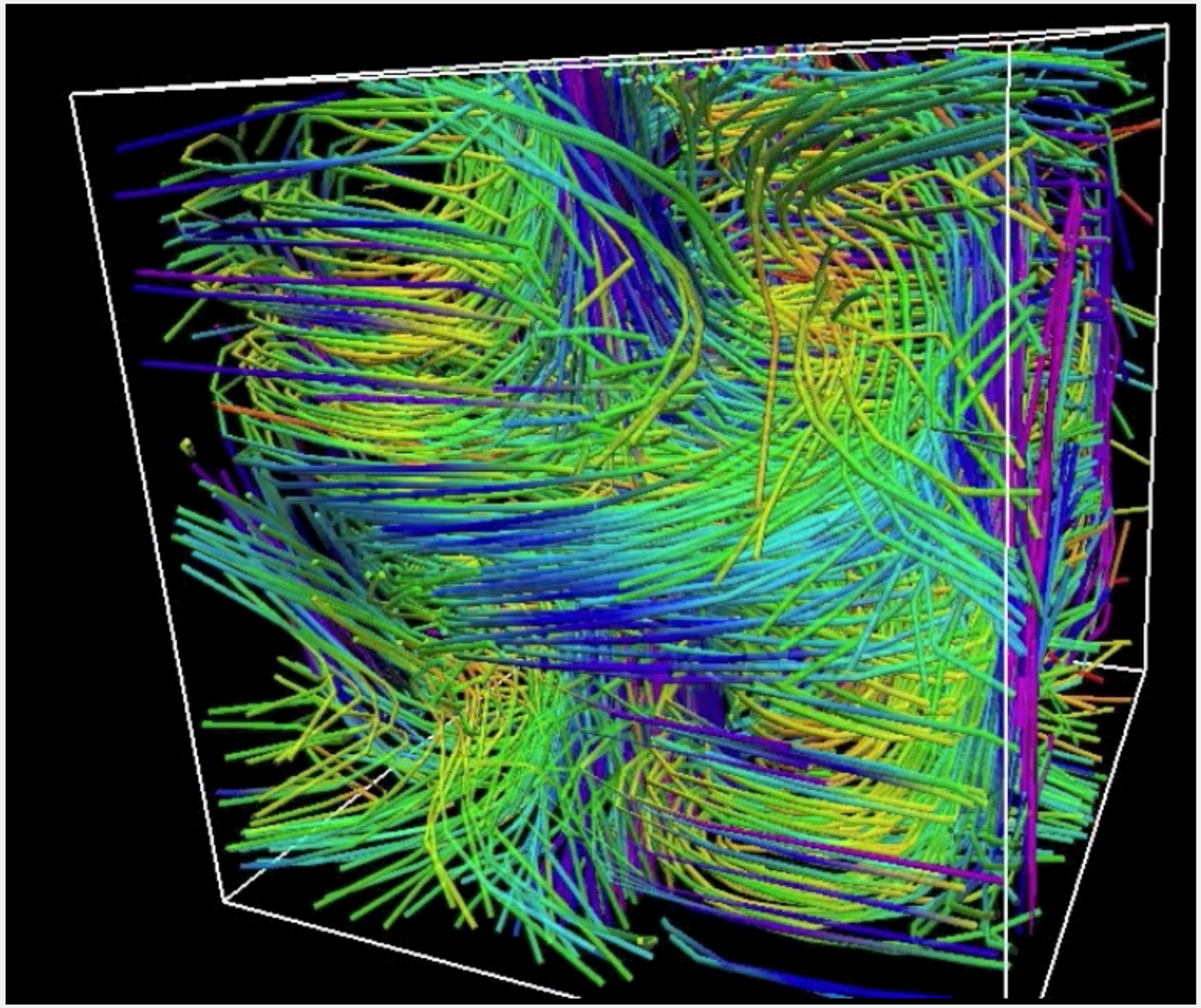


After Many Iterations...

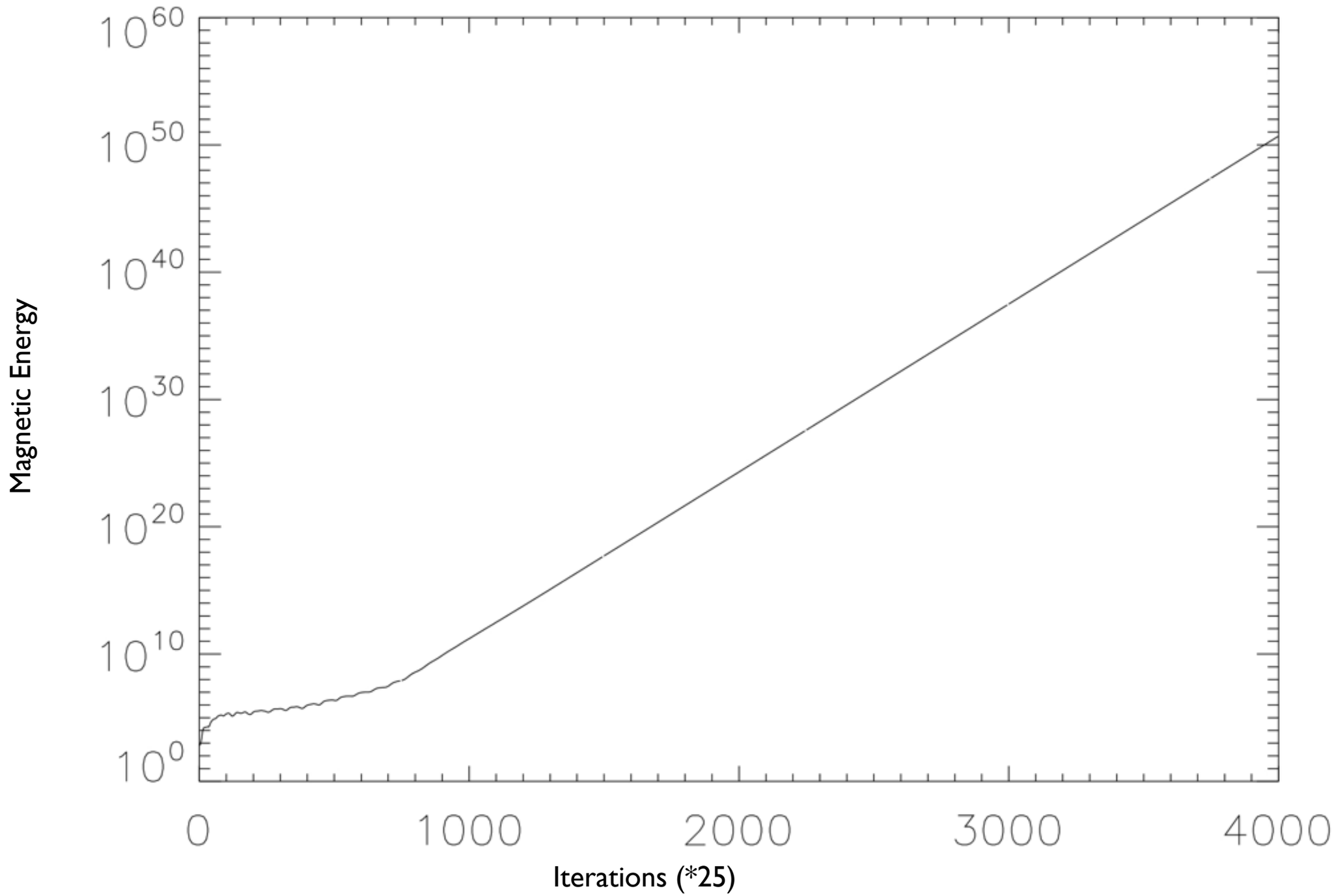
Magnetic Energy



Field Lines



Fully 3-D Dynamo



Overview

- Characterized numerical diffusion in two and three dimensional flows
- Showed that the numerical diffusion depended on grid size
- Showed the numerical diffusion varied with flow
- Creation of a 3-D dynamo motivated by those flows present in solar models