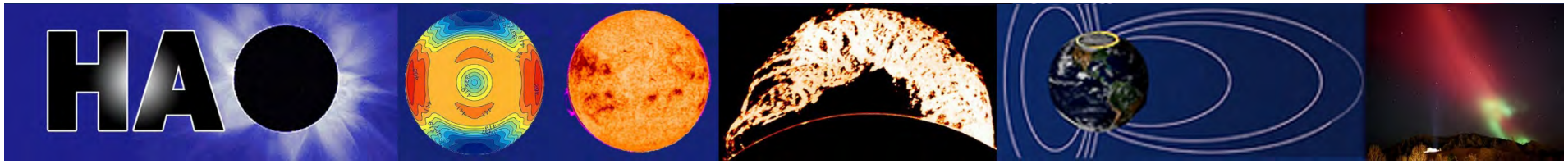

Using Modal Decomposition to Study Beating Patterns of Solar Cycle Data

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NCAR

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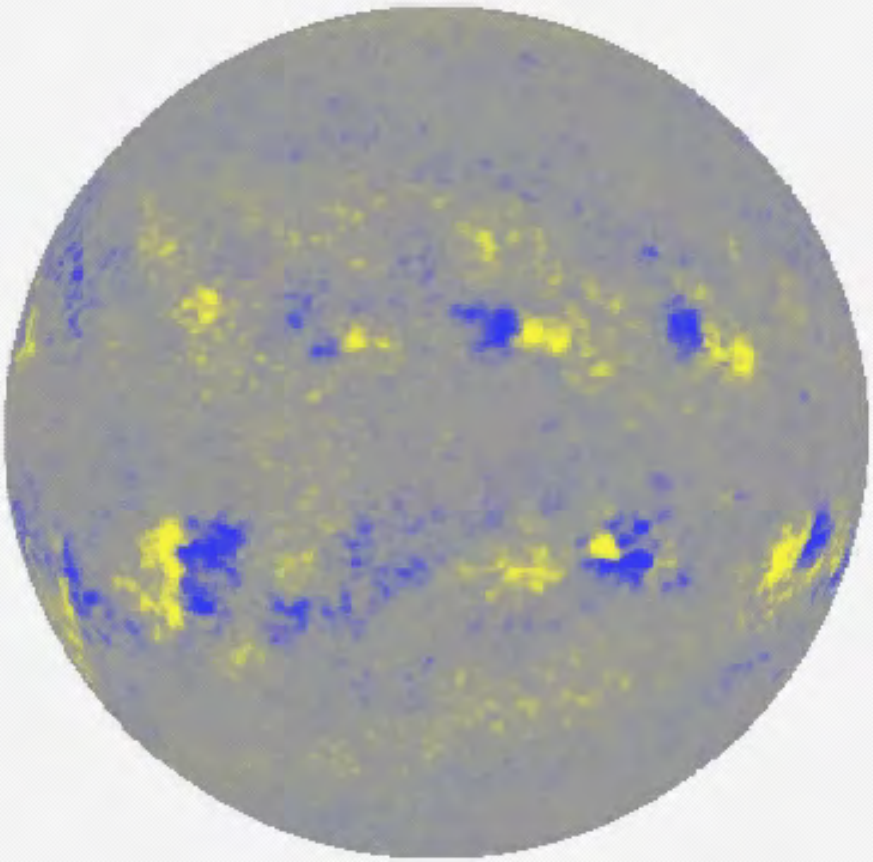
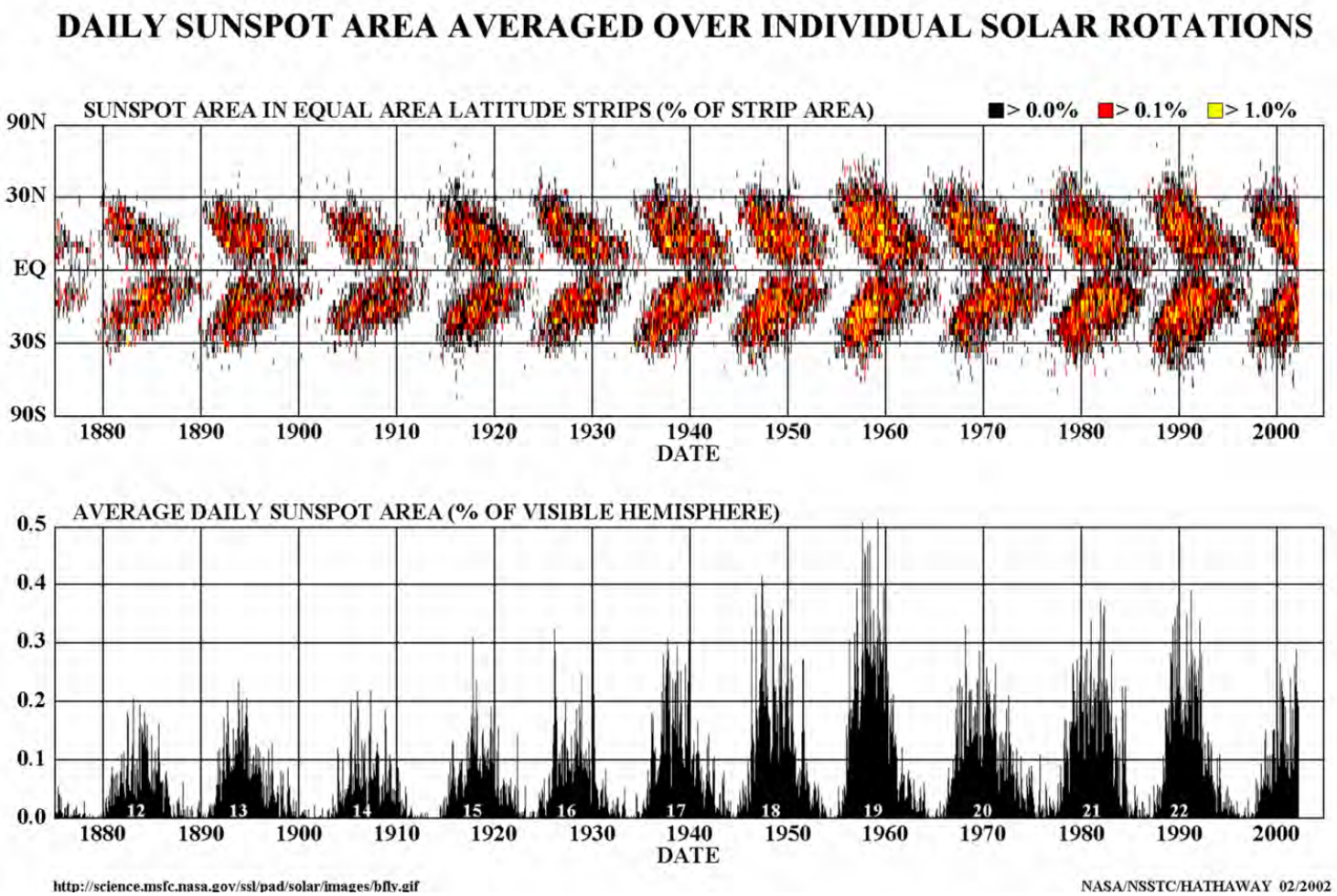
Solar activity

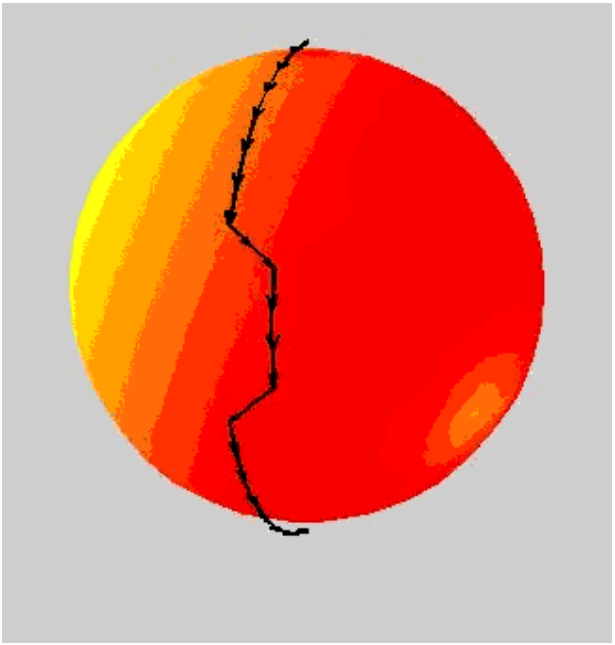


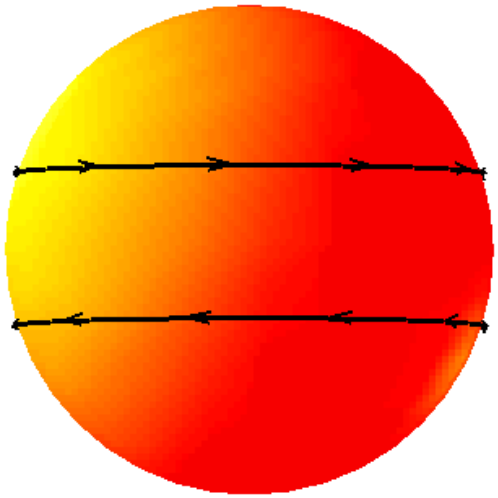
Solar butterfly diagram and spot-area cycle

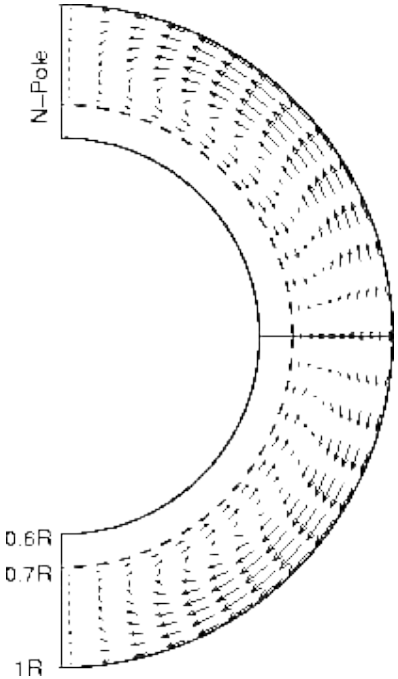


Most likely a hydromagnetic dynamo operating inside the Sun governs the solar activity cycle.

Solar dynamo mechanism







(i) Generation of toroidal field by shearing of a pre-existing poloidal field through differential rotation (Ω-effect)

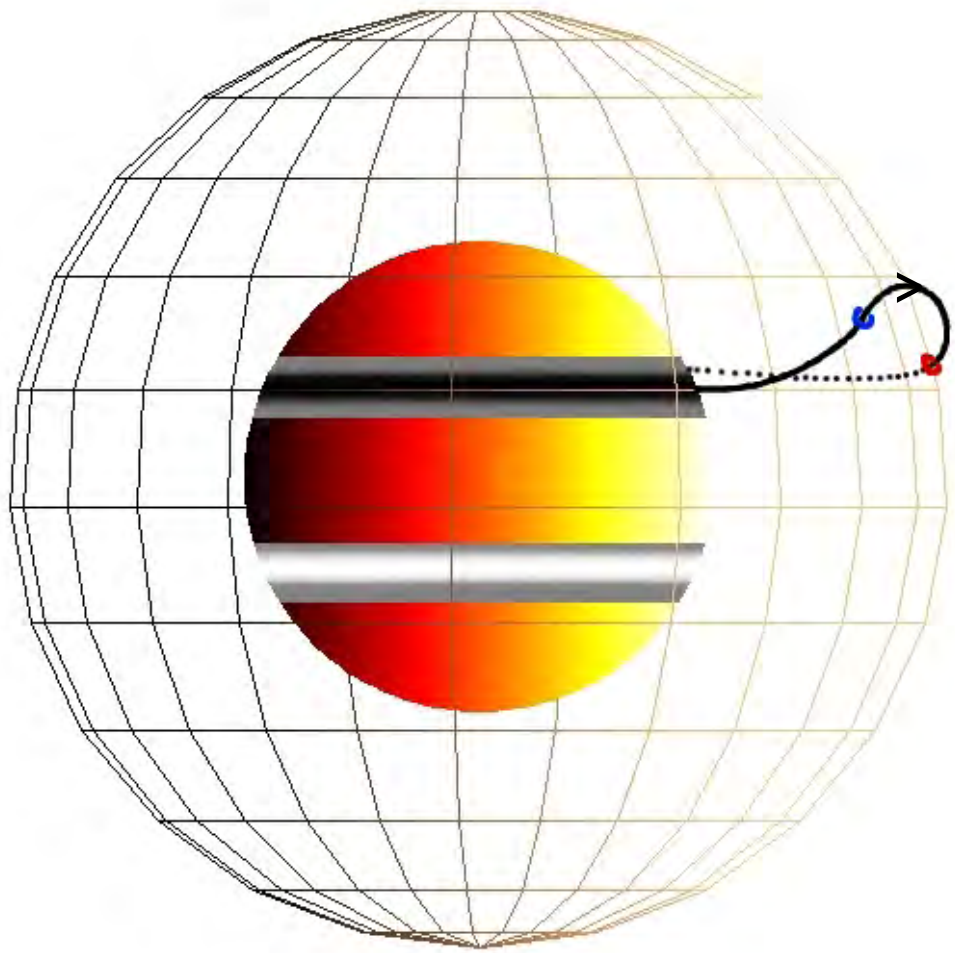
(ii) Re-generation of poloidal field by lifting and twisting a toroidal flux tube by helical turbulence (α-effect)

(iii) Flux-transport by meridional circulation

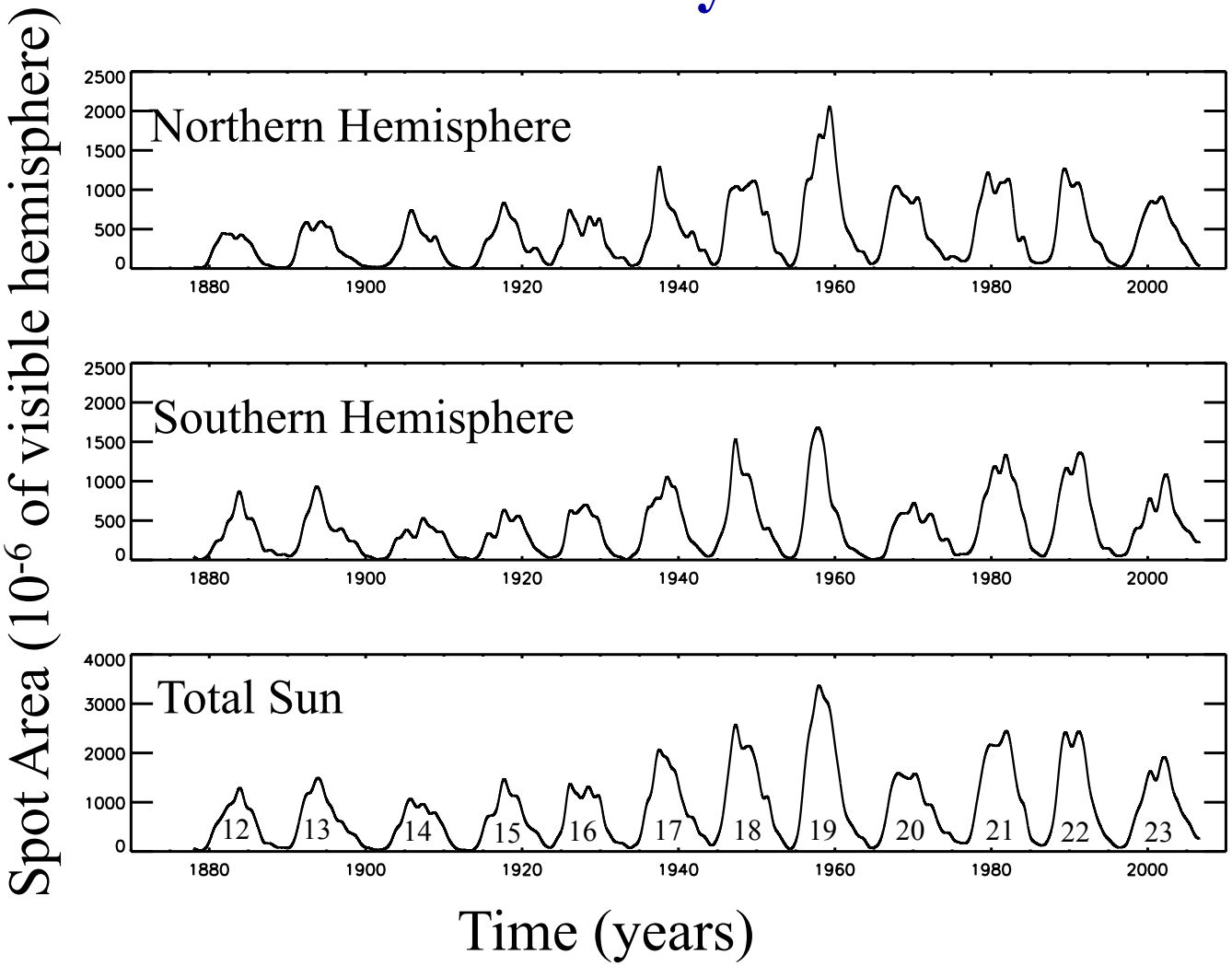
Solar dynamo

• Sunspots are thought to form from strong toroidal flux tubes which rose to the surface due to their magnetic buoyancy

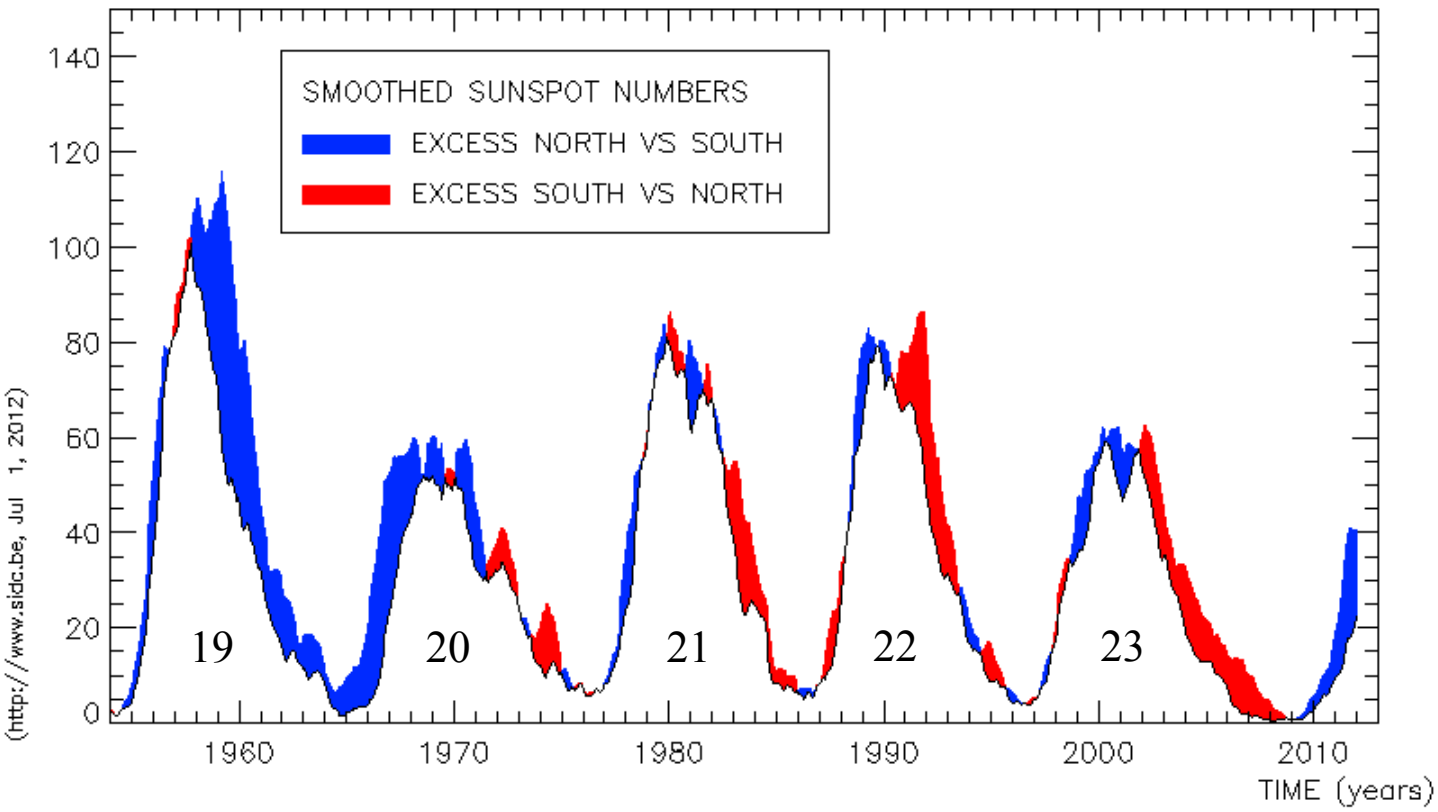
• Equatorward migration of sunspot-belt is explained by an equatorward propagation of the subsurface toroidal fields by the dynamo wave and equatorward return flow of meridional circulation



Spot-area cycle in North & South hemisphere and total cycle

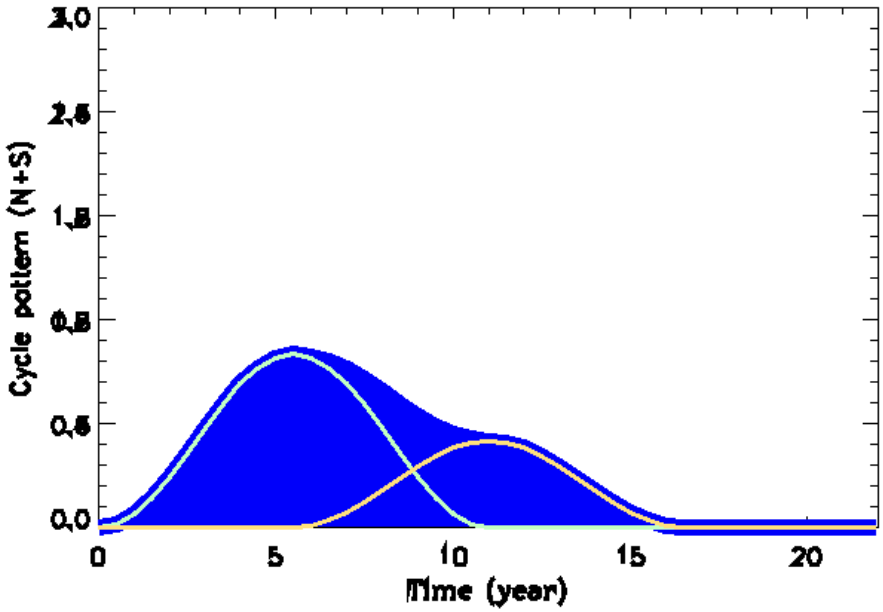


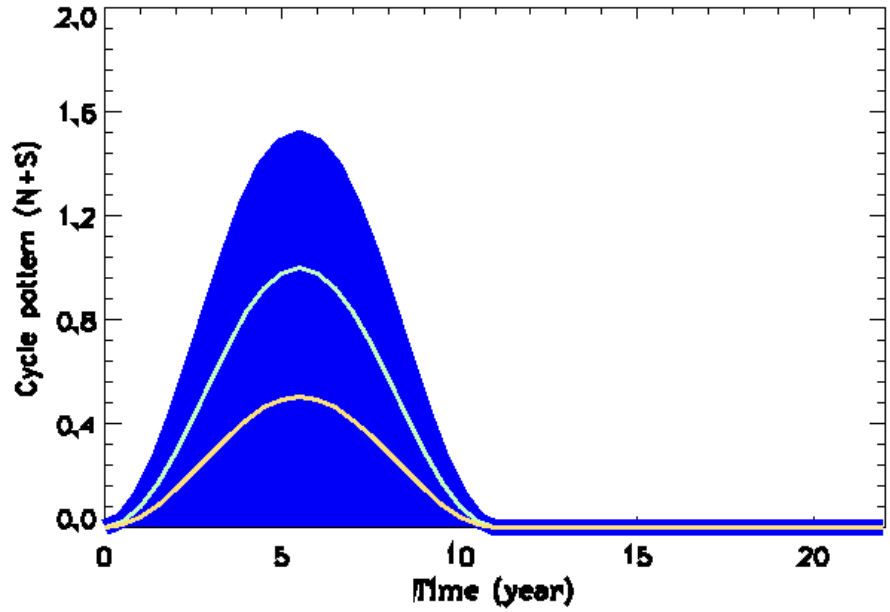
Excess North versus South



- **Can we characterize communication between the North and South?**
- **Do the North and South cycles have any systematic "beating" patterns ?**
- **If yes, can we predict the "beating" patterns of the ascending cycle 24 from the long-term patterns in the past?**

Total amplitude heavily depends on the phase and amplitude of North cycle and South cycle





Knowledge of "beating" patterns would help understand the cross-talk between the two hemispheres

Hemispheric cross-talk, like solar cycle amplitude and shape, plays important role in influencing certain aspects of space weather, such as geoeffectiveness,

Approach: Simple Fourier Analysis

Prescribe the cycles in the form:

$$Cy_n(t) = \frac{a_{n0}}{2} + \Sigma_{k=1}^{l} a_{nk} \left(\sin^2(k\,\omega_n t + \delta_{nk})\right),$$

$$Cy_s(t) = \frac{a_{s0}}{2} + \Sigma_{k=1}^{l} a_{sk} \left(\sin^2(k\,\omega_s t + \delta_{sk})\right).$$

Cast them in Fourier modes:

$$\mathcal{F}(t) = \frac{a_o}{2} + \sum_{j=1}^{N} \left[a_j \cos\left(\frac{j\pi t}{\tau}\right) + b_j \sin\left(\frac{j\pi t}{\tau}\right)\right]$$

$$\mathcal{F}(t) = \sum_{j=0}^{N} \left[a_j \cos\left(\frac{j\pi t}{\tau}\right) + b_j \sin\left(\frac{j\pi t}{\tau}\right)\right]$$

Orthogonality relations:

$$a_j = \frac{1}{\tau}\int_0^{\tau} dt\mathcal{F}(t) \cos\left(\frac{j\pi t}{\tau}\right)$$

$$b_j = \frac{1}{\tau}\int_0^{\tau} dt\mathcal{F}(t) \sin\left(\frac{j\pi t}{\tau}\right)$$

If we express a_j and b_j as:

$$a_j = A_j \cos\delta_j$$

$$b_j = -A_j \sin\delta_j$$

Amplitude and phase come out to be

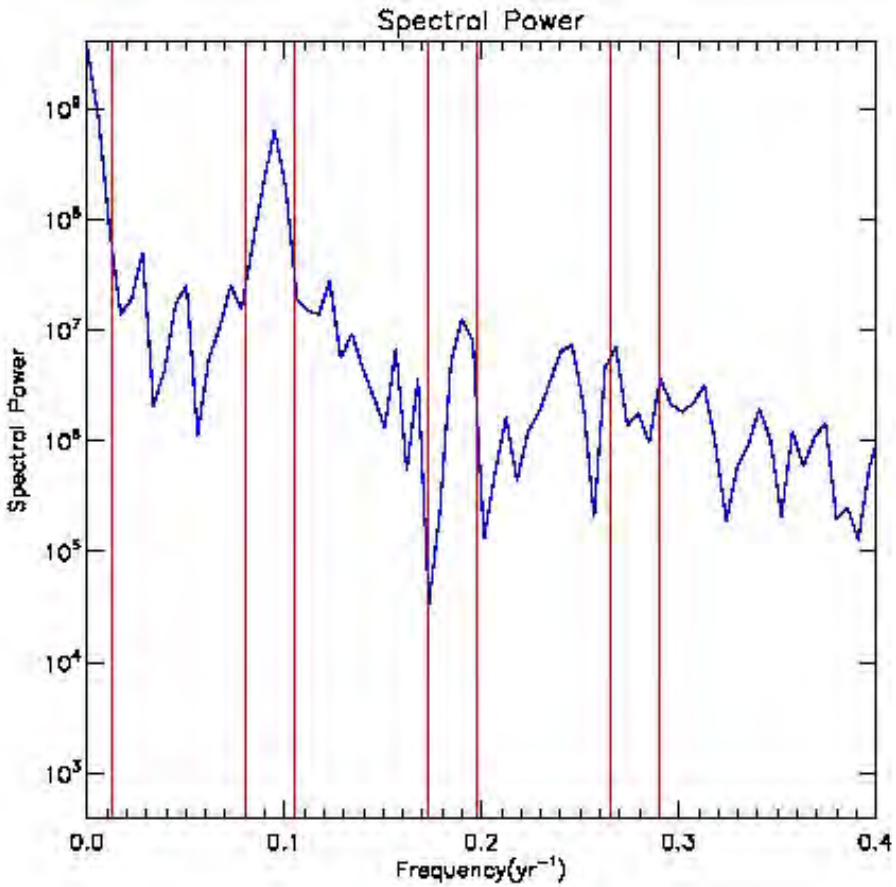
$$A_j = \sqrt{a_j^2 + b_j^2}$$

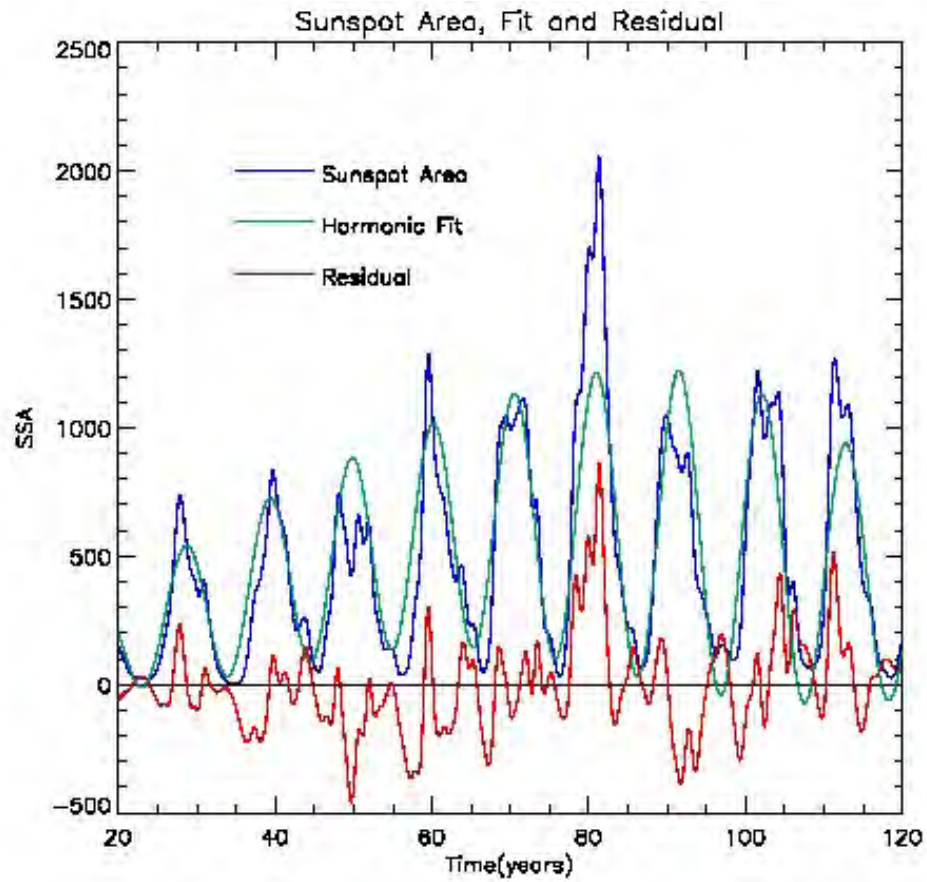
$$\delta_j = \tan^{-1}\left(\frac{-b_j}{a_j}\right).$$

Final form

$$\mathcal{F}(t) = \sum_{j=0}^{N} A_j \cos\left(\frac{j\pi t}{\tau} + \delta_j\right)$$

A look at the whole time series

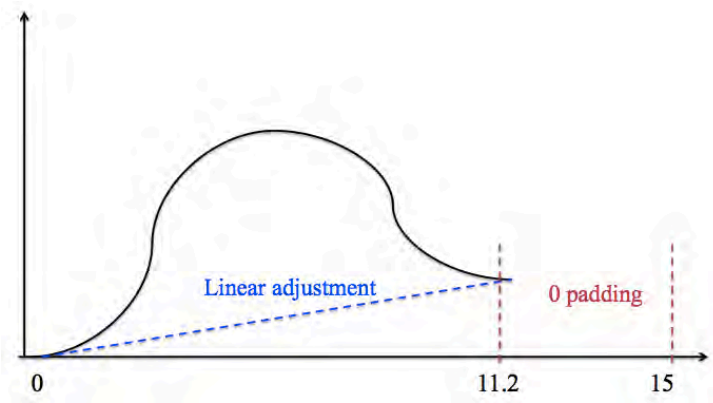




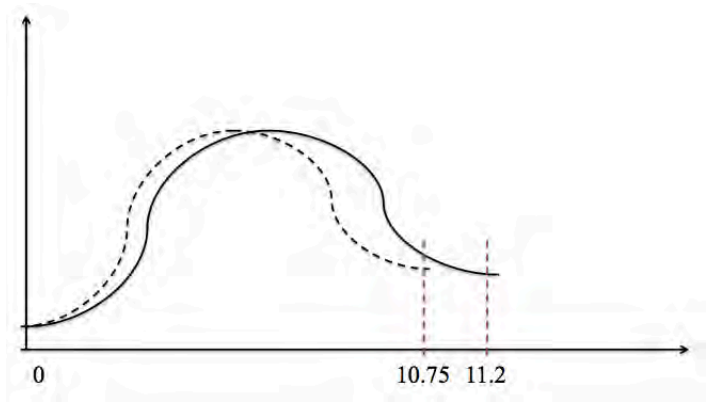
- Most of the power is at the very low frequencies or around 11 years
- There is no apparent pattern in the residual modes

Two Main Approaches to Individual Cycles

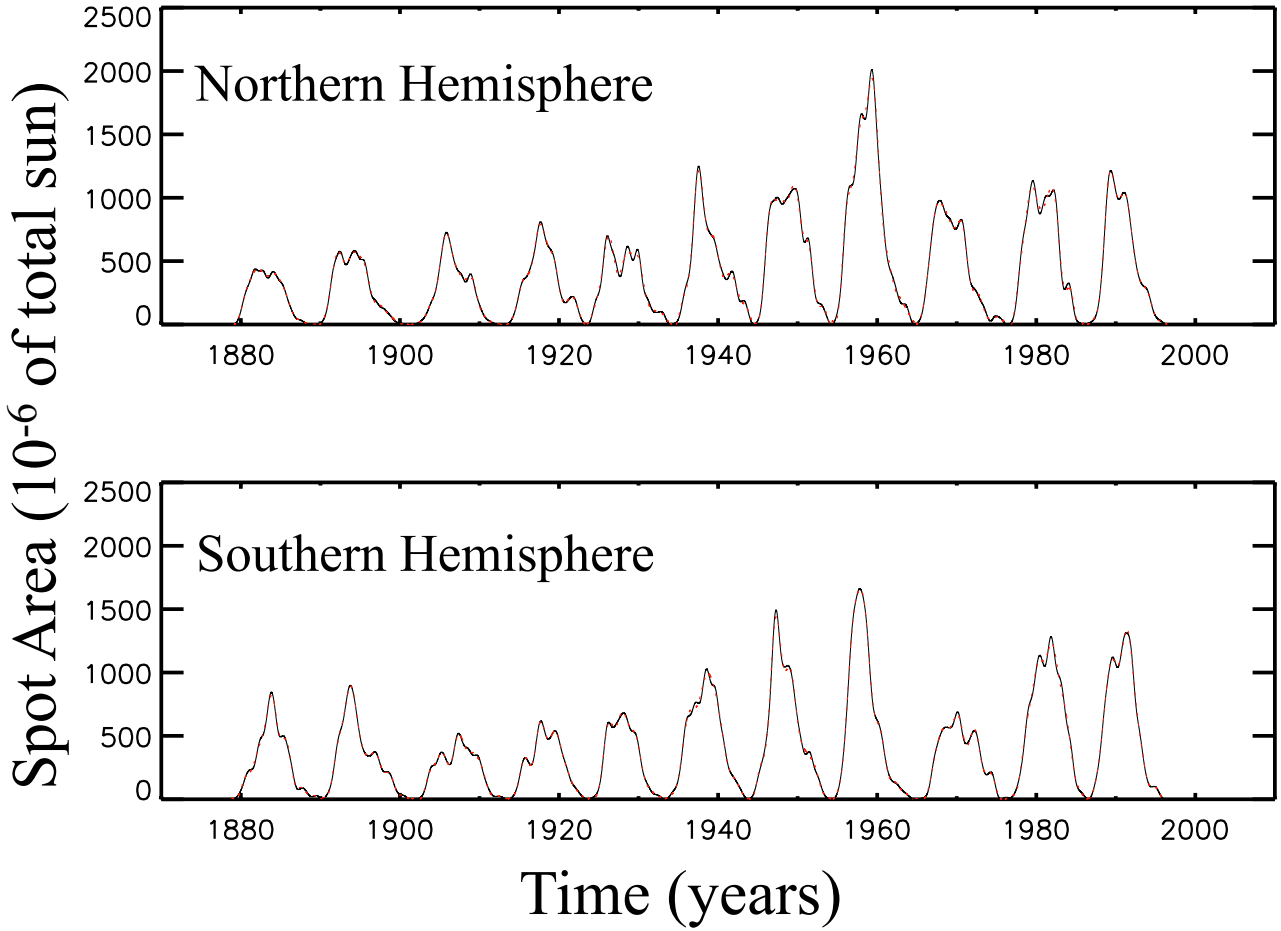
- 'Expansion':
  - Adding 0s till you reach fundamental
  - Needs a fundamental initially bigger than any individual cycle (15 years used)
  - Every cycle has the same number of 'time' points



- 'Stretching/Compression':
  - Changing the size of the interval between time points to give make the difference between beginning time and end time the same
  - Fundamental is the average fundamental of each cycle (10.75 years use here)
  - Every cycle has a different number of 'time' points

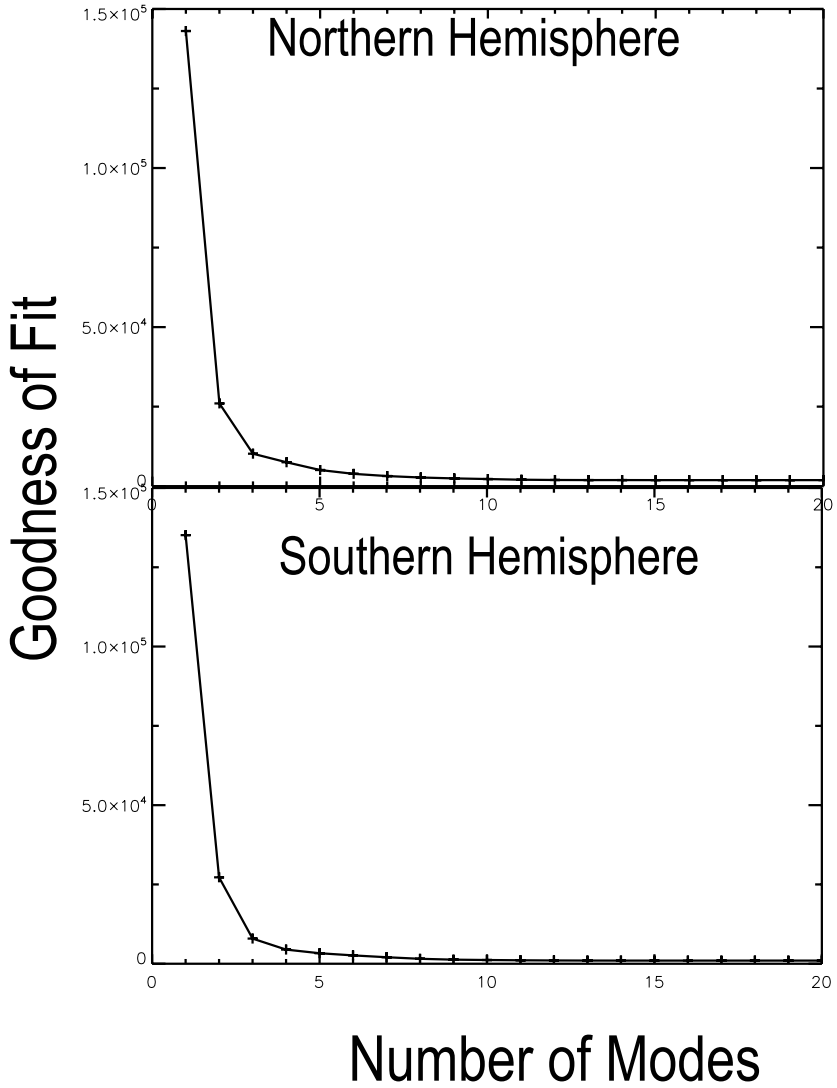


Reconstructed cycles



- **10 modes (including 0th) used**
- **Fitting individual cycles rather than all together**
- **This is obtained using expansion method**
- **Mode 0 is mean**
- **Mode 1 (fundamental) is 15 years**
- **Mode 2 is 7.5 years**
- **Mode 3 is 5 years**
- **Etc.**

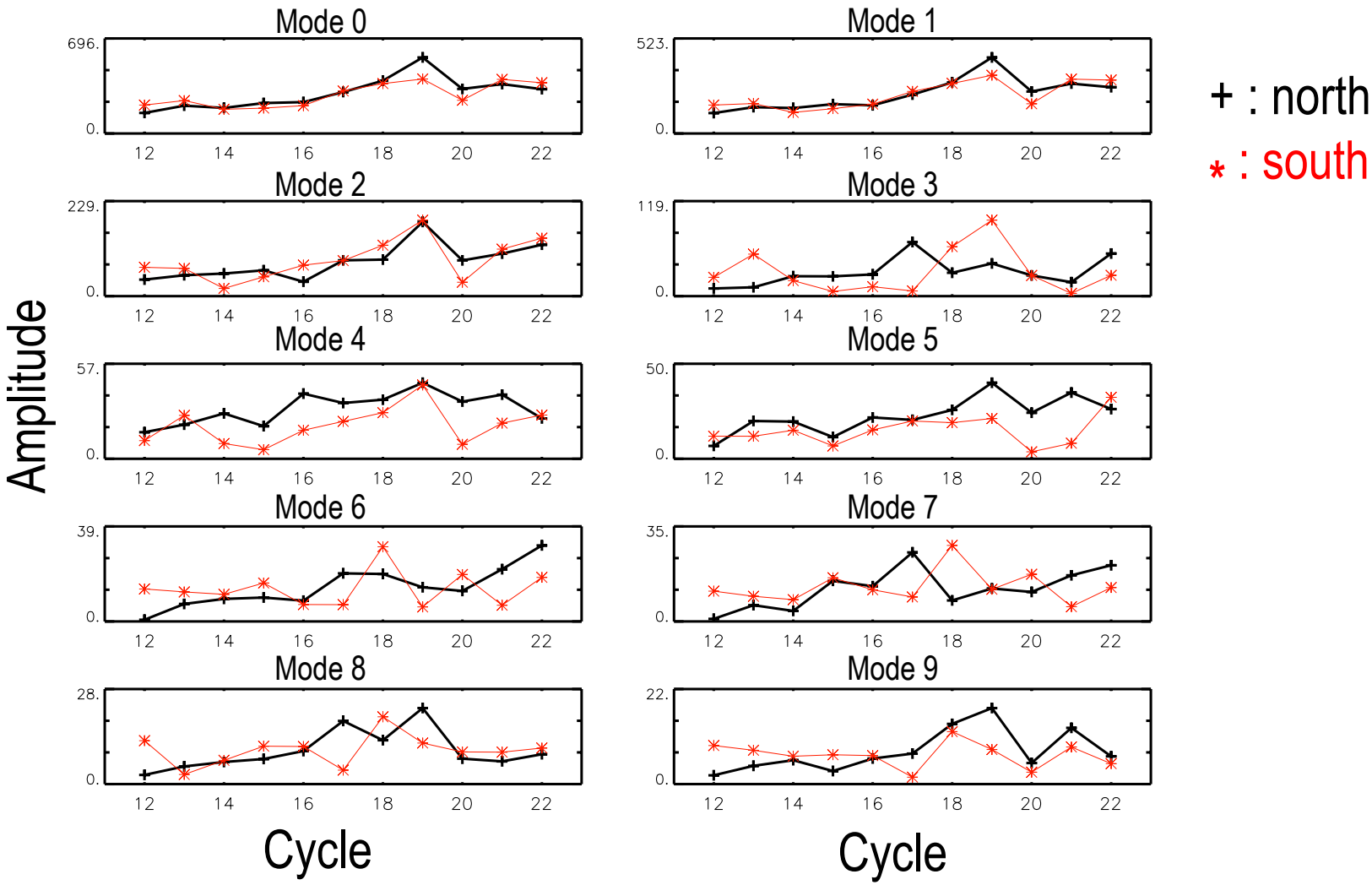
X^2 ('Goodness of Fit Estimation for North and South')



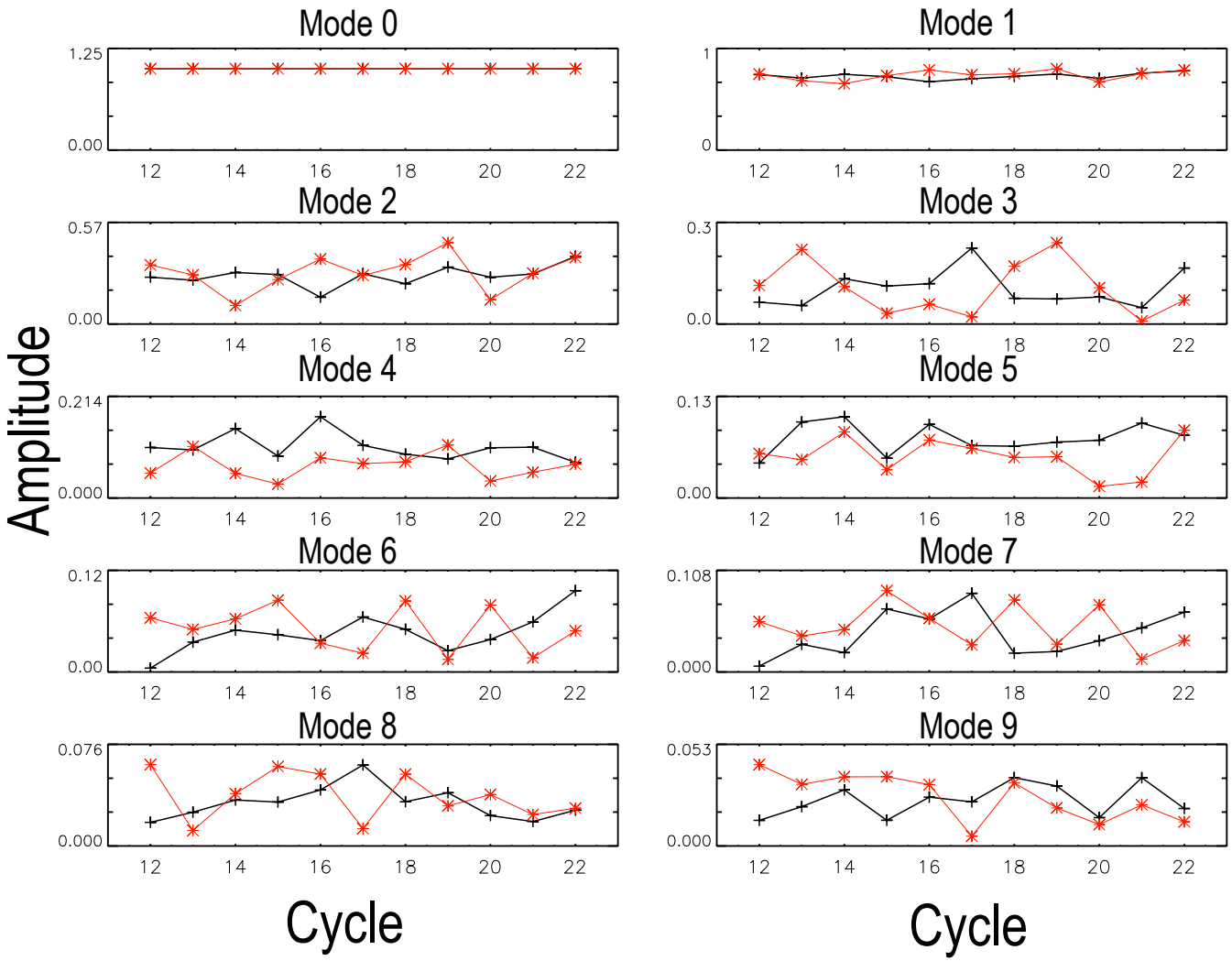
$$X^2 = \frac{1}{n}\sum_{i=1}^{n}(f_i - g_i)^2$$

Where f is the original function and g is the function reconstructed by modes

Amplitude pattern in North vs South



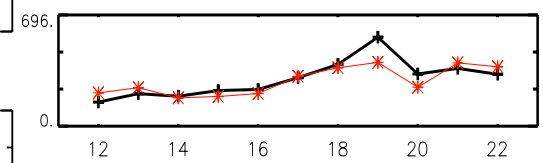
Amplitude, Accounting for Fundamental Amplitude



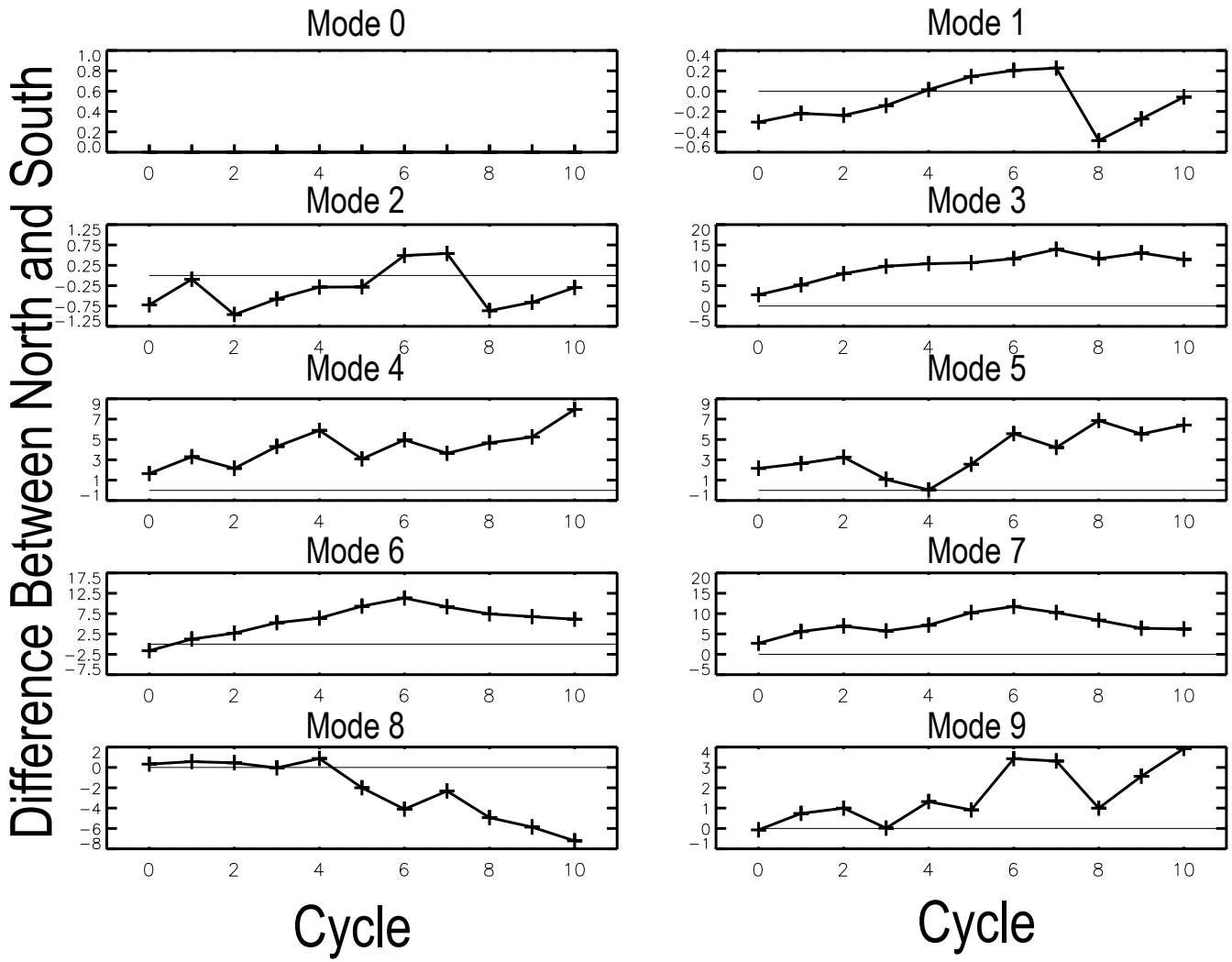
+ : north

* : south

Original Amplitude, For Comparison



Phase pattern in N & S



Phase difference:

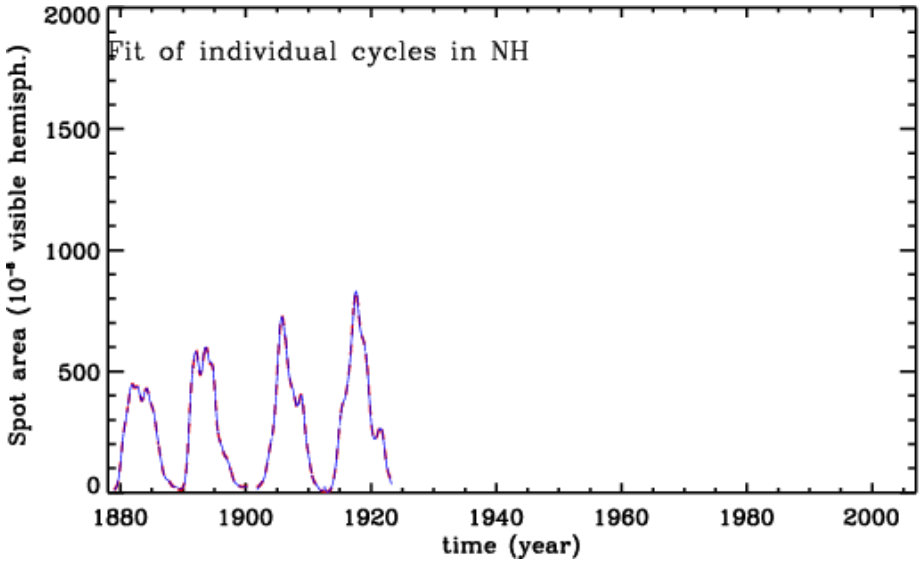
$$\delta_n - \delta_s$$

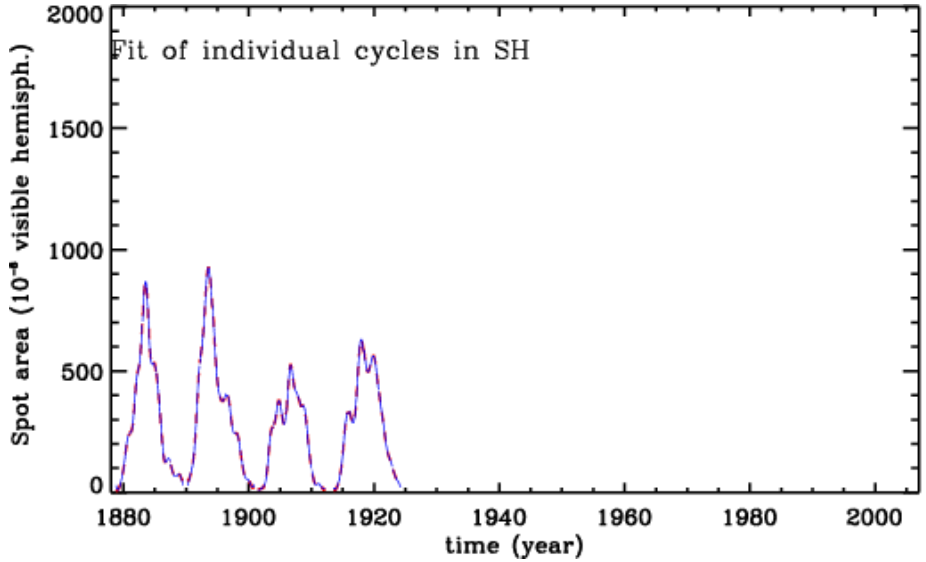
- Positive indicates that south is leading north
- Negative means that north is leading south

Results

- Individual Cycles can be well-represented by as few as 5 modes.
- The relative strength of higher harmonics shows no discernible correlation with cycle strength
- The 0th and first harmonic are well correlated between hemispheres
- The first and second harmonics reveal a lead or lag between the N and S phases which switch back and forth within a few cycles

Reconstructed Cycles





Summary

- Individual Cycles can be well-represented by as few as 5 modes.
- The relative strength of higher harmonics shows no discernible correlation with cycle strength
- The 0th and first harmonic are well correlated between hemispheres
- The first and second harmonics reveal a lead or lag between the N and S phases which switch back and forth within a few cycles

- Future goal: We have analyzed the expansion method which preserves timing, and now we would like to finish analysis of stretched method that preserves the minima level

References:

- Bracewell, R.N. (2000), *The Fourier Transform and its Applications* (3rd ed.), Boston: McGraw-Hill

- Dikpati, M, Gilman, P.A., Toma, G, Ghosh, S.S., P.A.: 2007, *ApJ*, **245**

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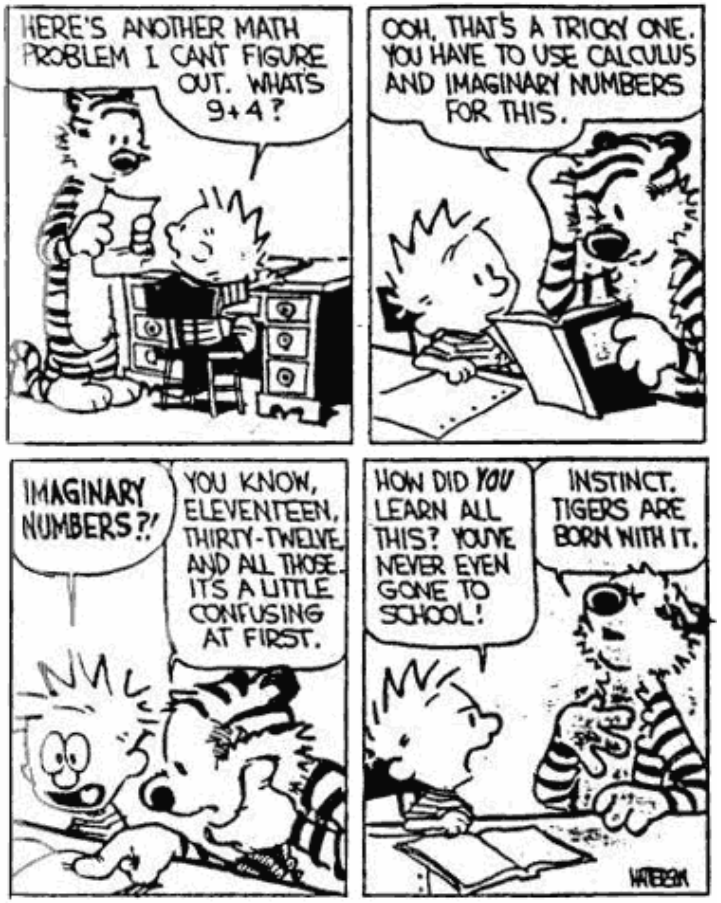
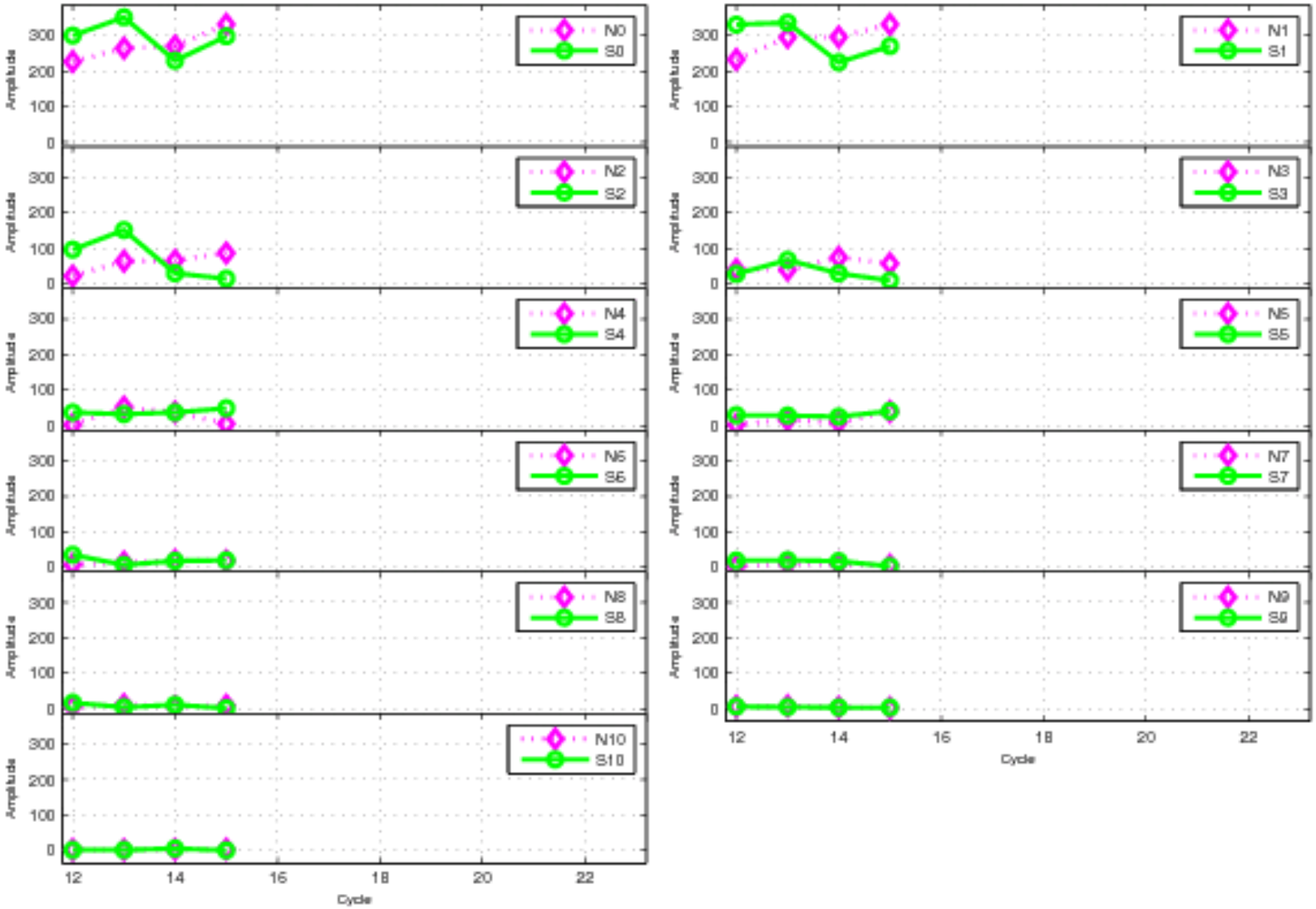
•Thanks to HAO for providing a wonderful research environment

•Thanks to Mausumi Dikpati, Nick Featherstone, and Charles Lindsey for showing me an incredible amount of patience and giving me an incredible amount of time this summer.

Questions?


HERE'S ANOTHER MATH PROBLEM I CAN'T FIGURE OUT. WHAT'S 9+4?
OOH, THAT'S A TRICKY ONE. YOU HAVE TO USE CALCULUS AND IMAGINARY NUMBERS FOR THIS.
IMAGINARY NUMBERS?!
YOU KNOW, ELEVENTEEN, THIRTY-TWELVE, AND ALL THOSE. IT'S A LITTLE CONFUSING AT FIRST.
HOW DID YOU LEARN ALL THIS? YOU'VE NEVER EVEN GONE TO SCHOOL!
INSTINCT. TIGERS ARE BORN WITH IT.
WATTERSON

Amplitude pattern



Phase pattern