Absorption Line Profiles for Differentially Rotating 2 $M_\odot$ Stellar Models

HAO

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+ Roberto Casini and Andy Skumanich!

Objective

- Stellar rotation has been an important subject in astrophysics for over 400 years. Galileo was the first to discover differential rotation.
- Our project strives to diagnose differential rotation in distant stars by analyzing line profiles.
- A greater understanding of differential rotation in other stars can unveil clues to stellar formation and evolution!

Self-Consistent Field Method (SCF)

- SCF is a method of treating rapid, differential rotation in stellar models.
- Though many others have used SCF for stellar modeling (ex. Ostriker et al. 1968), Jackson, MacGregor, and Skumanich (HAO) have had the most success.

SCF of Jackson, MacGregor, Skumanich

- Unlike their predecessors, Jackson, MacGregor, and Skumanich were able to obtain converged models for *all* main-sequence masses.
- These are the same models that I am using to study the absorption line morphology of 2 $M_{\odot}$ stars due to differential rotation.

So what do these SCF models give us?

- In the SCF models, angular momentum is a function of distance from the axis of rotation $\Sigma=\Sigma(r\sin 2)$. As such, centrifugal force is also a function of distance from the axis of rotation $M'_{cent}(\Sigma)=M(r\sin 2)$.
- The SCF models use an effective potential P, which is the gravitational and centrifugal potentials. $P = M_{grav} + M'_{cent}$ where M_{grav} is gravitational potential and M'_{cent} is centrifugal potential.
- $P = P(r)$, that is SCF assumes that the effective potential P, increases *monotonically* with *spherical radius*.

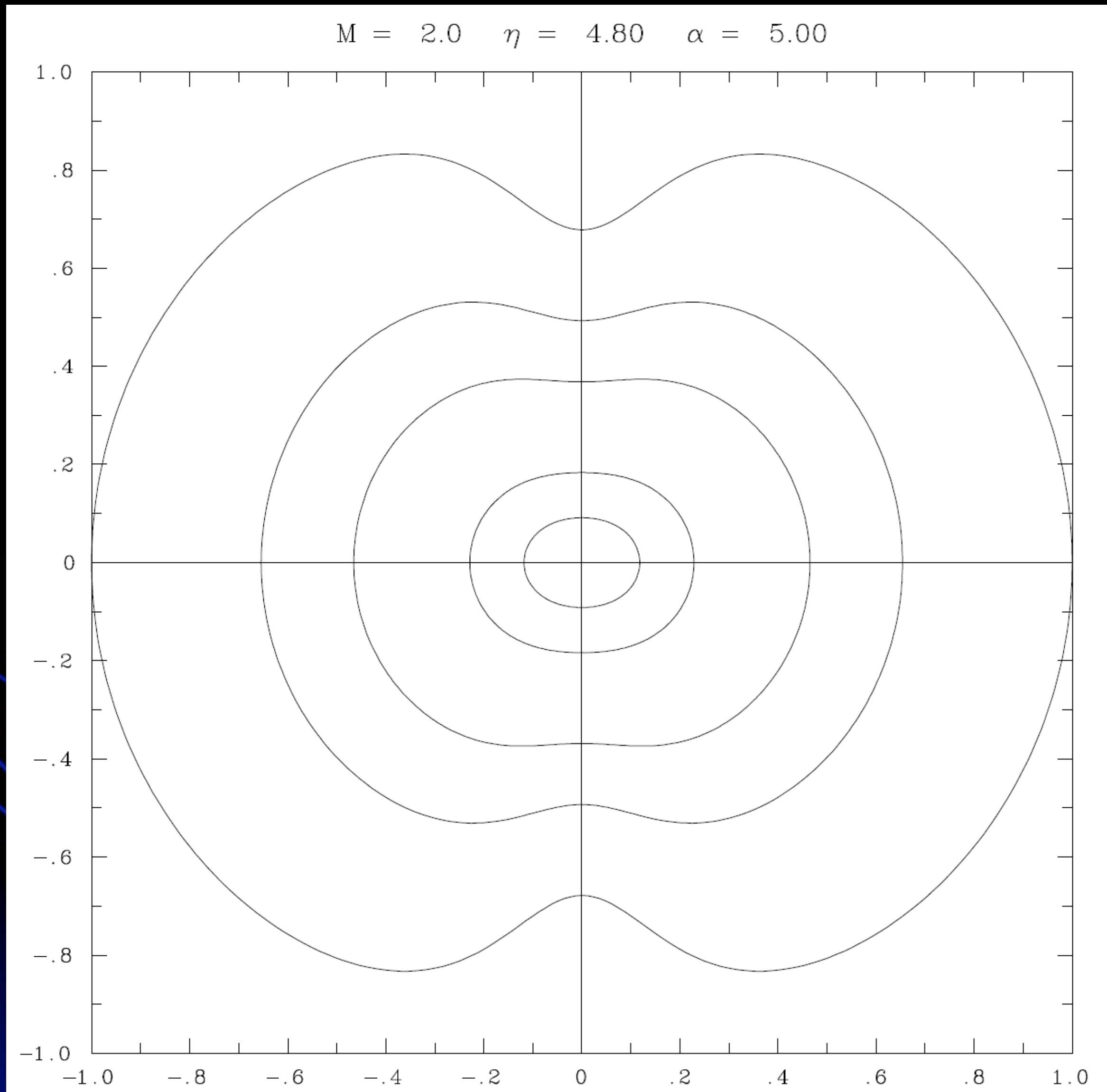
Furthermore…

So what do these SCF models give us? (continued)

- Pressure P, density Δ, and Temperature T are all functions of these *equal potential surfaces*
- P = P(P(r)) Δ = Δ(P(r)) T = T(P(r))

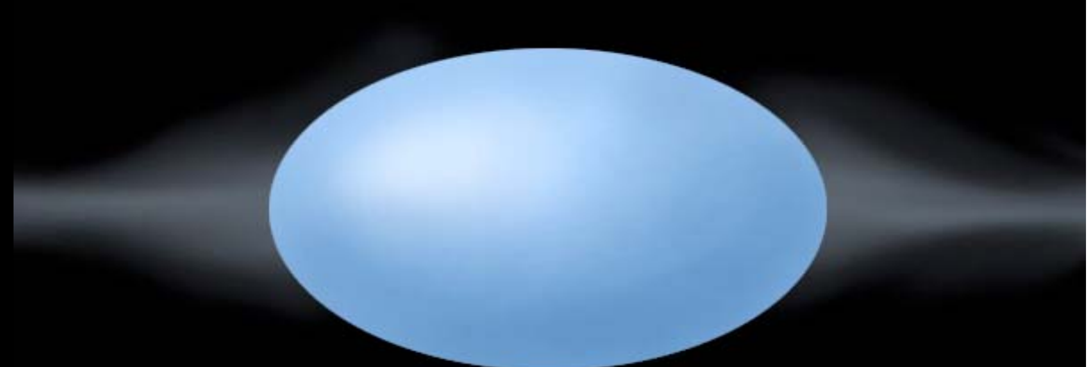
  so, P = P(r) Δ = Δ(r) T = T(r)
- The SCF models give us a stellar bodies with concentric *level surfaces* for effective potential, pressure, density, and temperature.

Concentric level surfaces…



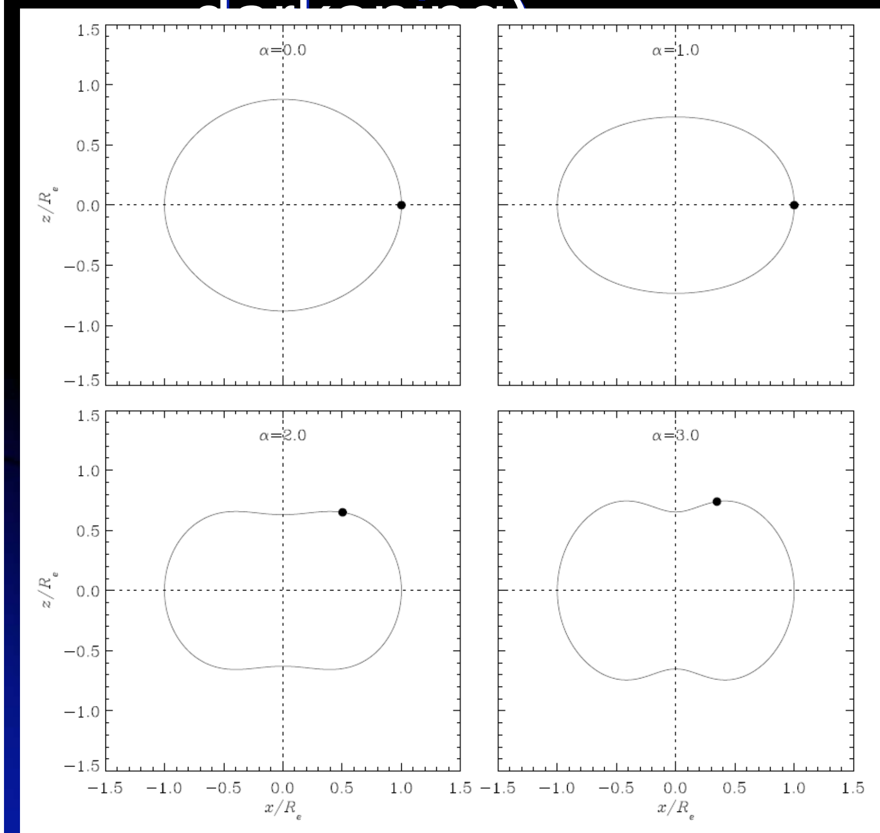
Important implication…

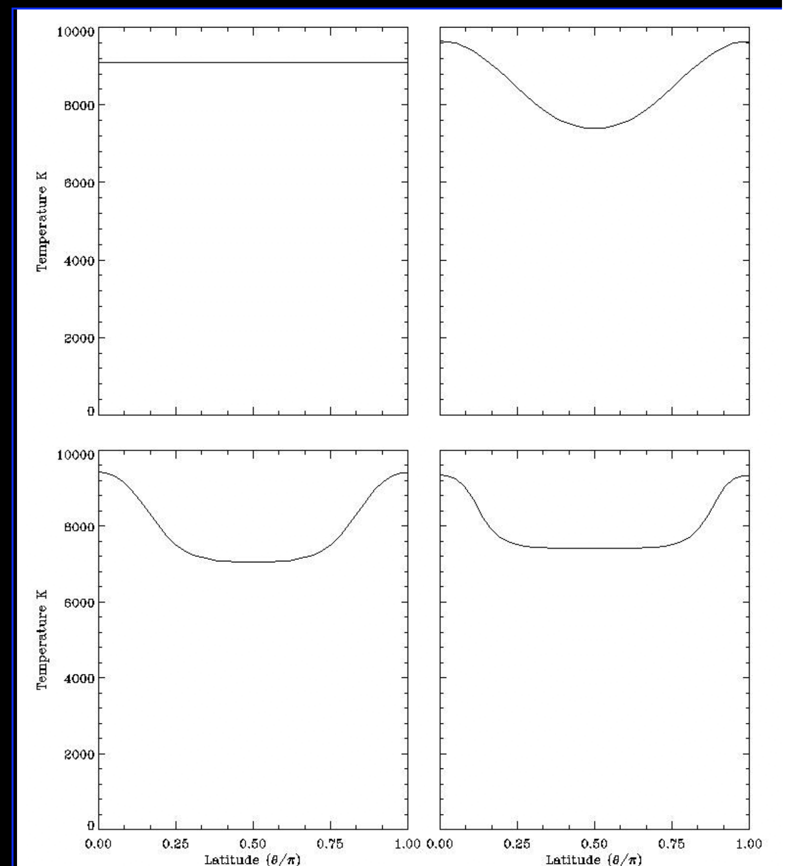
- The model photosphere is a level surface with constant P, P, Δ, and T.
- But we know that rapidly rotating stars have a large temperature gradient between the equators and the poles…



How do we impose a Temperature gradient?

- von Zeipel 1924, $T_{eff} \sim (M_{grav})^{1/4}$ (gravity darkening)



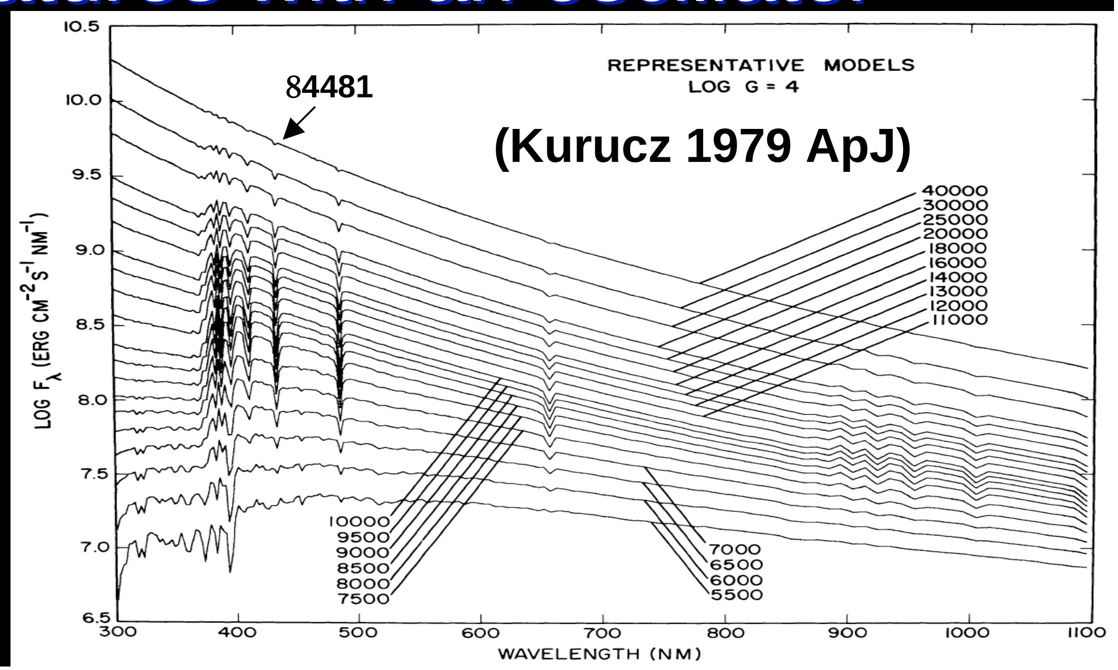


$\Sigma_o / \Sigma_e = 1 + \forall^2$

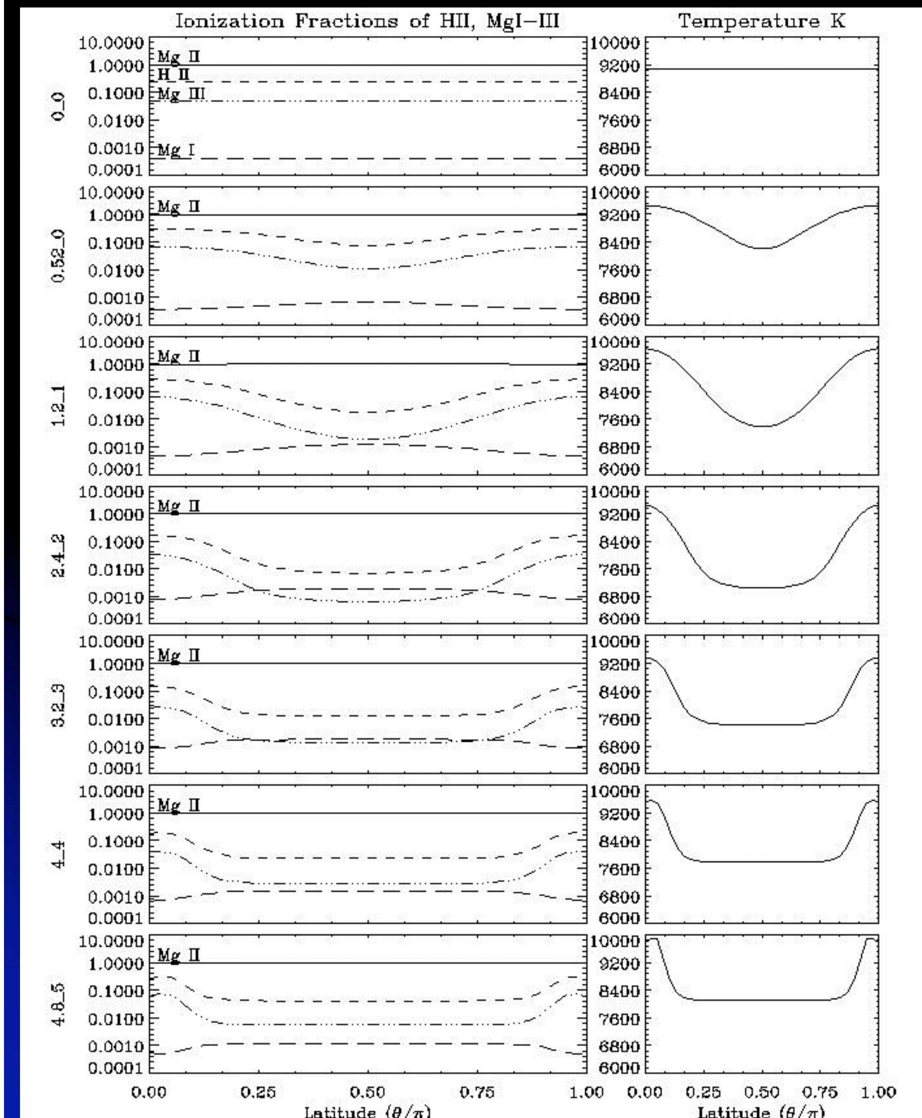
$0 = \Sigma_o / \Sigma_{cr}$

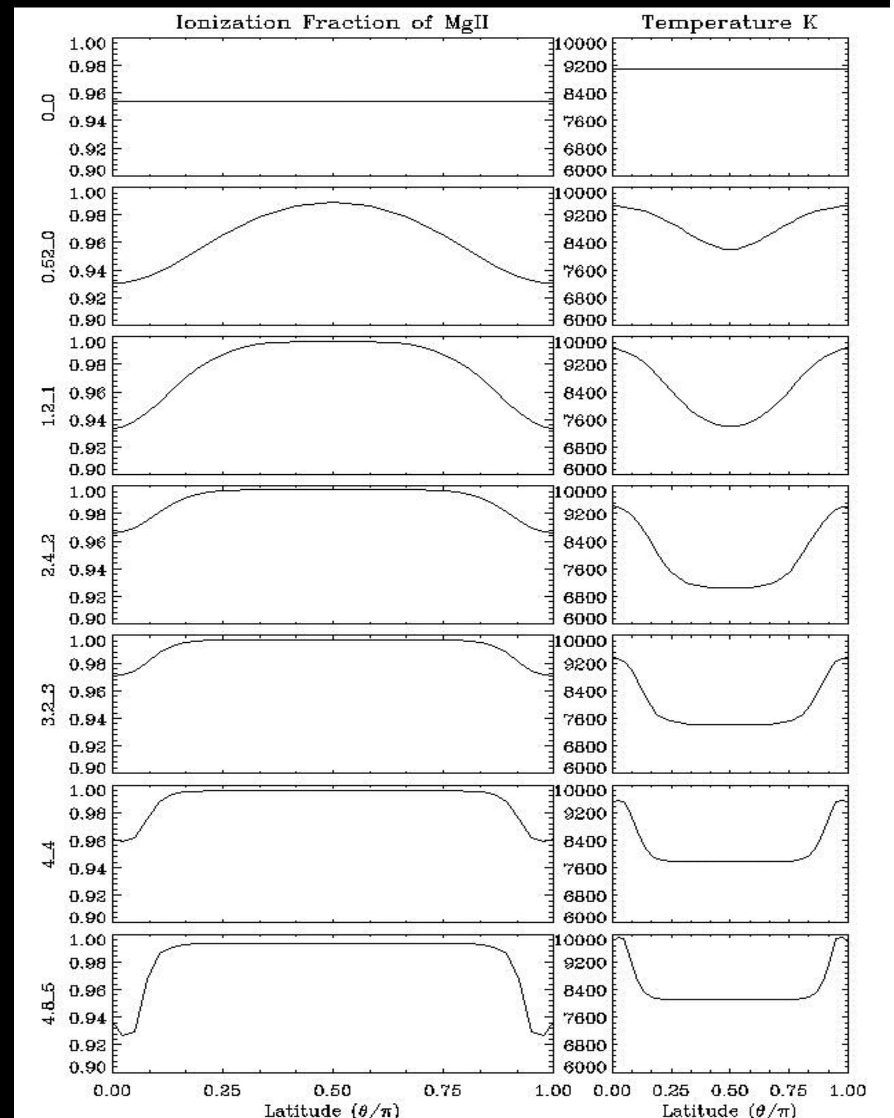
What line are we looking at and *why*?

- We are using the Mg II 84481 doublet.
- It exists as the $3d^2D \rightarrow 4f^2F$ transition.
- It is a prominent absorption line for a *vast* range of temperatures with an oscillator strength of 0.95.

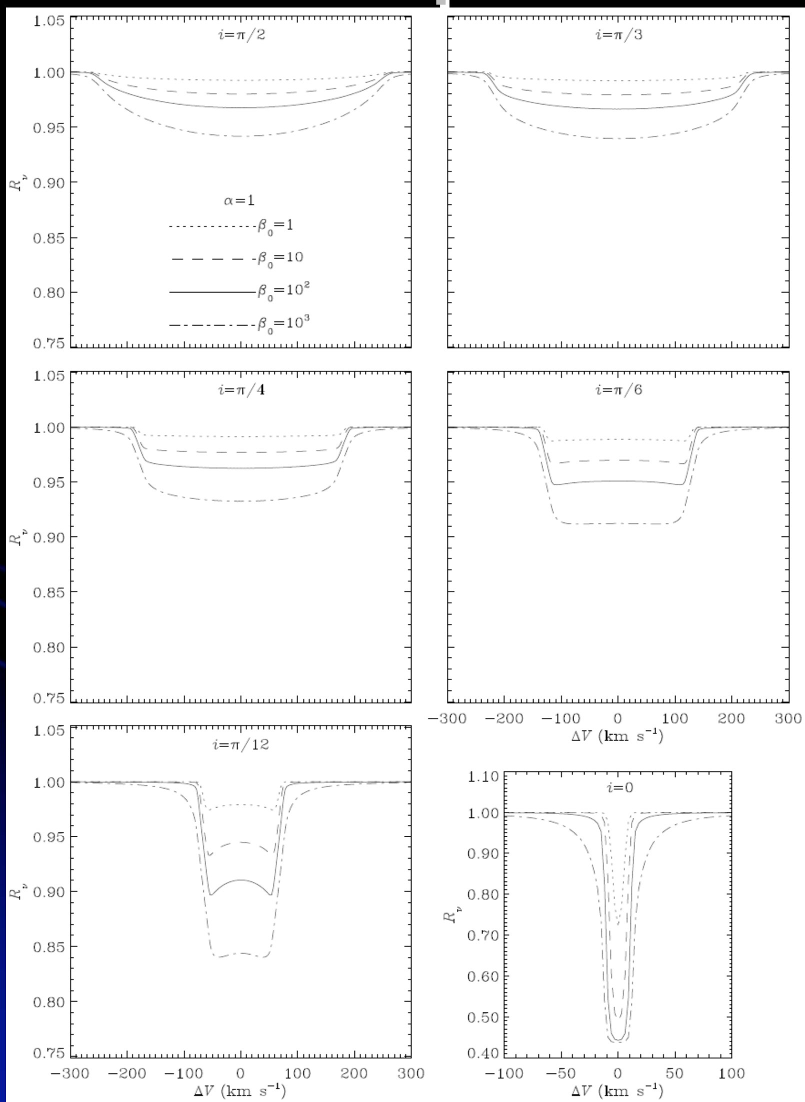


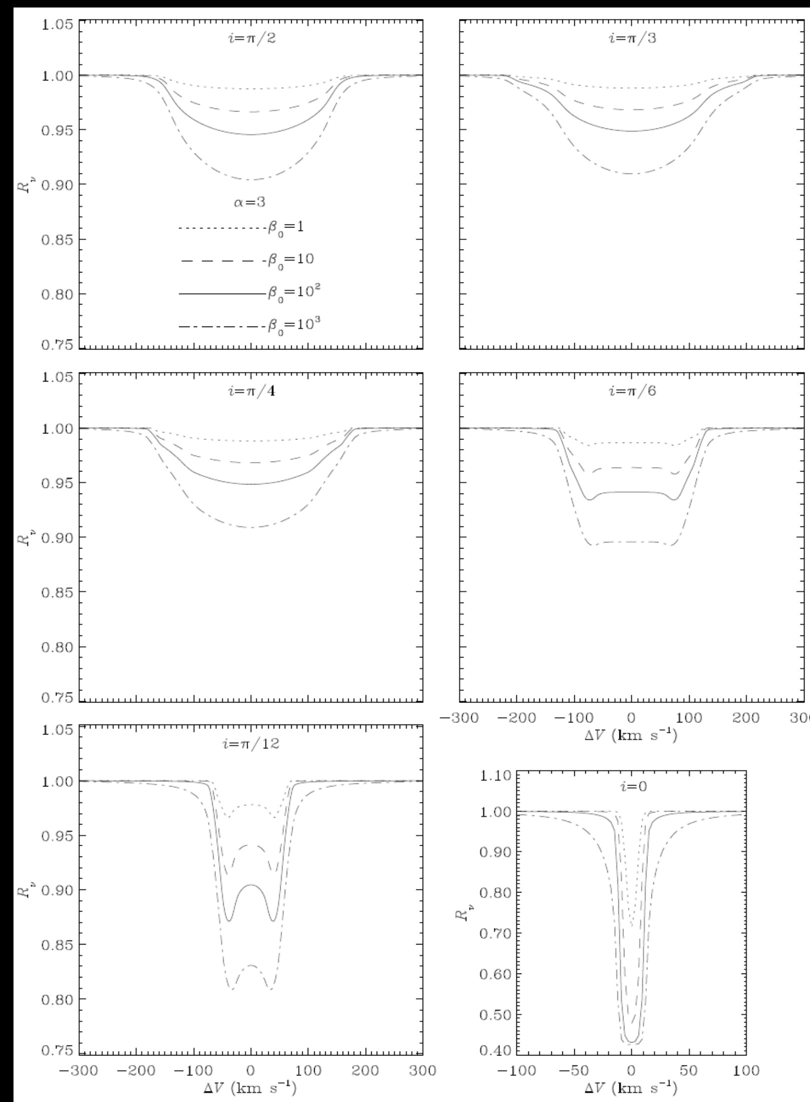
Abundances of HII and MgI-III in our models



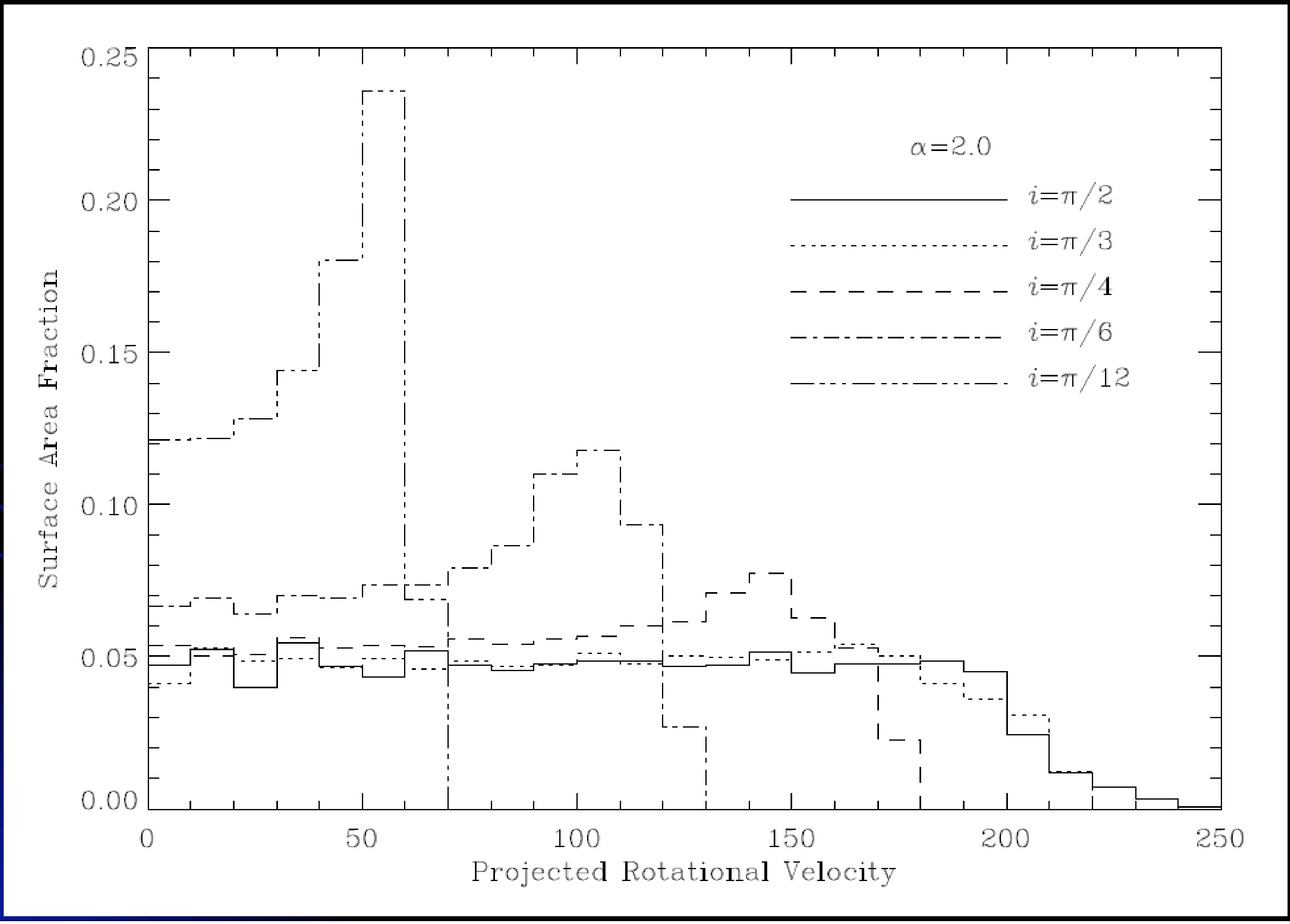


So what do these absorption profiles look like?





What accounts for the shape of these profiles…



Modifications to the profile code…

- I optimized the code for generating profiles by using a more sophisticated algorithm for calculating the Voigt function $H(a,\Lambda)$ (Humlíček 1981) and by exploiting inherent symmetries in the profile calculations.

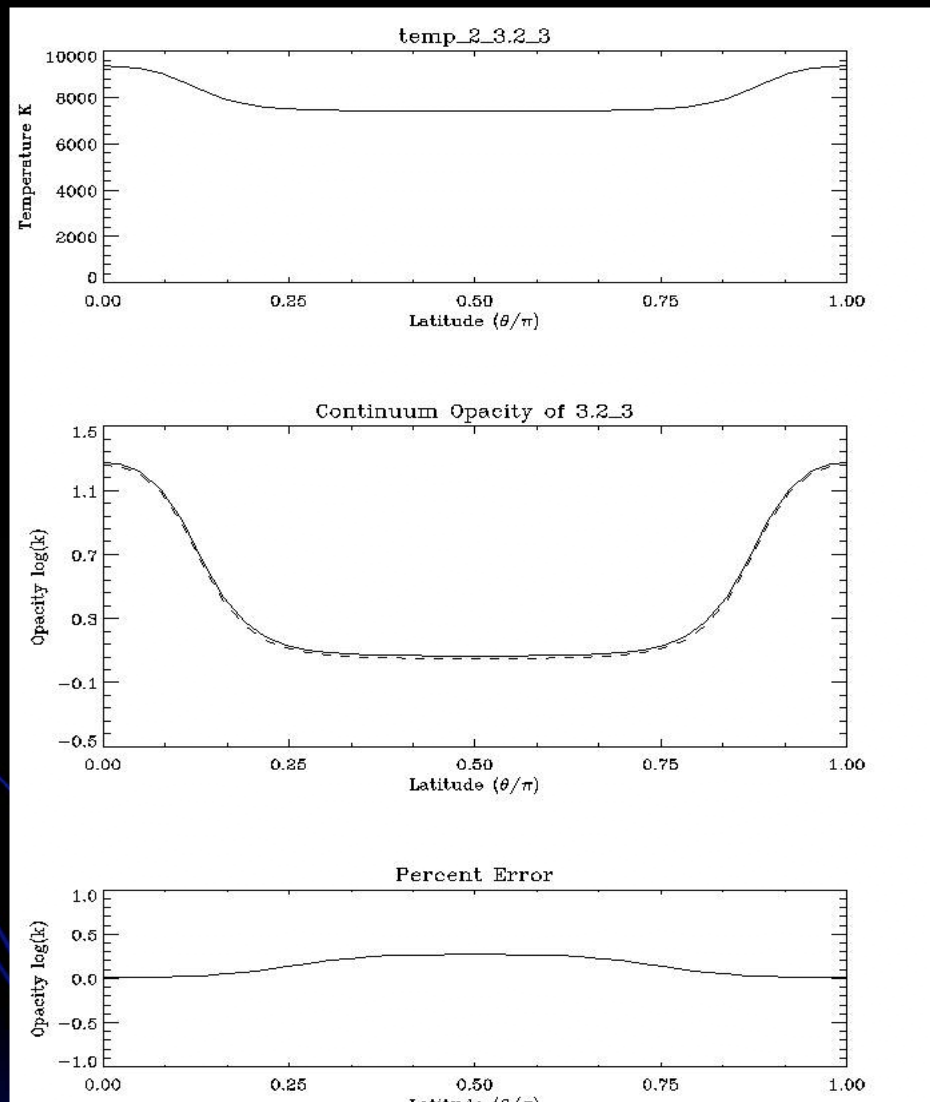
Profile code modifications (continued)

- I also calculated the line to continuum opacity ratio as a function of latitude. $\exists_o(2) = 6_\Lambda{}^l(2) / 6_\Lambda{}^c(2)$
- The continuum opacity was calculated using Rosseland mean opacities from both OPAL 1995 and Kurucz 1993

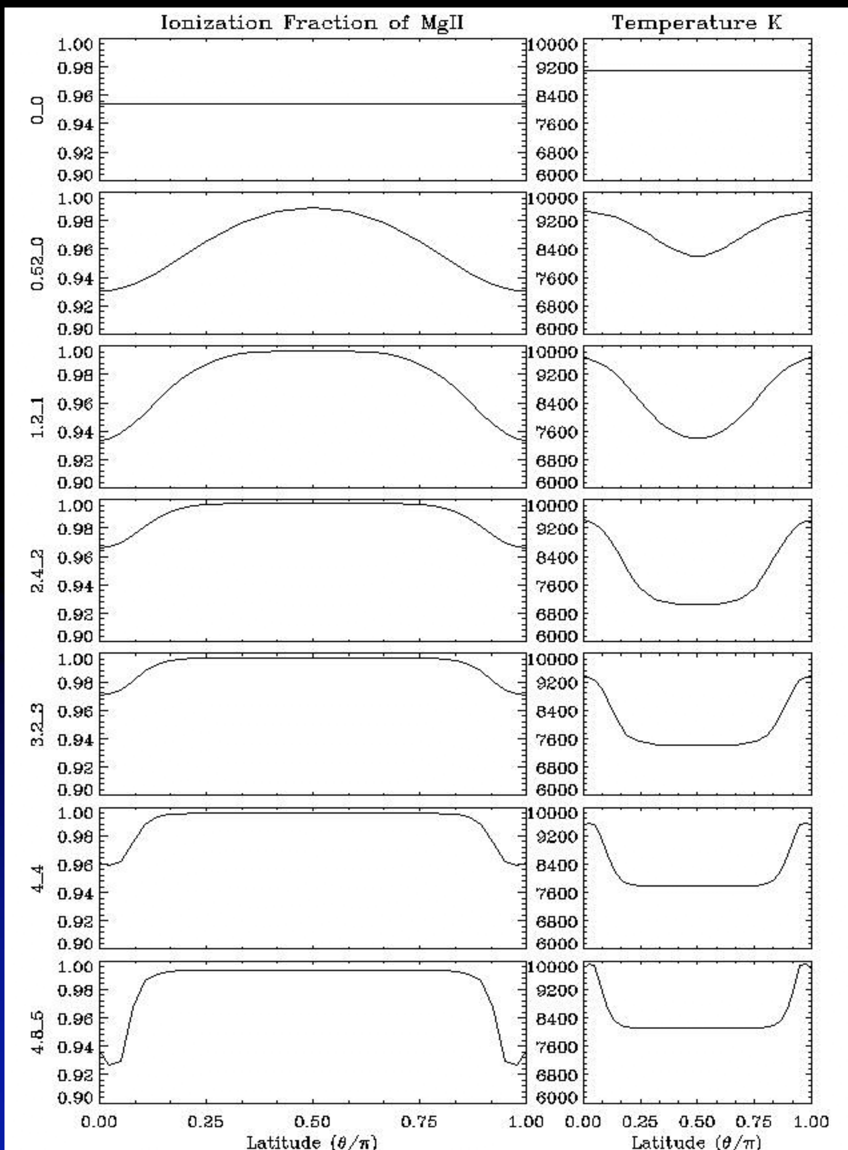
Continuum Opacity Interpolations

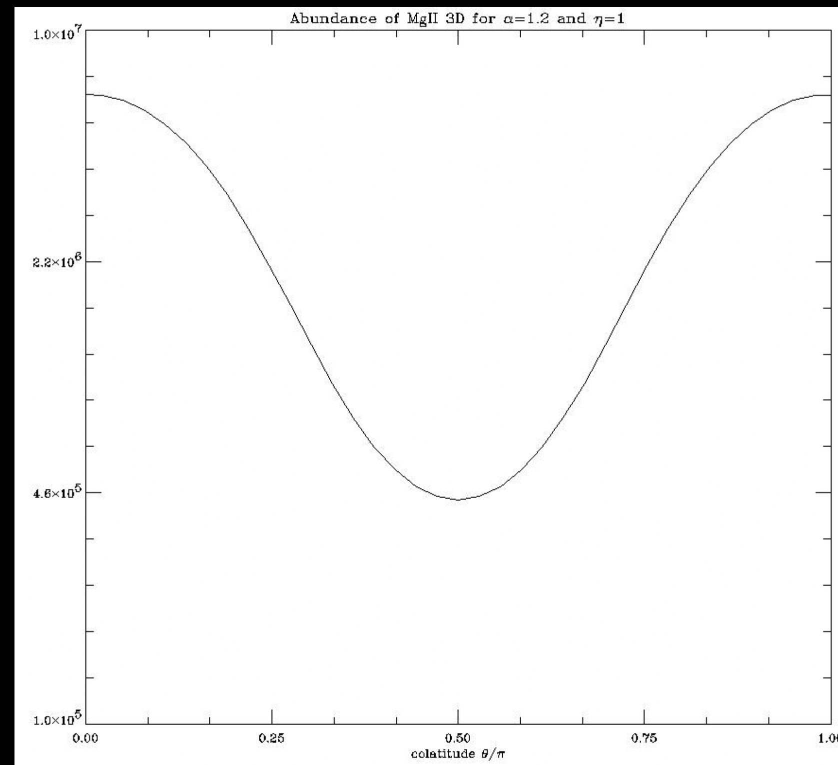
- The Kurucz mean opacities are interpolated from a table of values for temperature and pressure. $6_{Ross}(T,P)$
- The OPAL mean opacities are interpolated from a table of values for temperature and density. $6_{Ross}(T,\Delta)$
- For the OPAL table, we used a routine to convert Δ to P given H and He abundances as well as temperature.

Comparison of Continuum Opacity Models

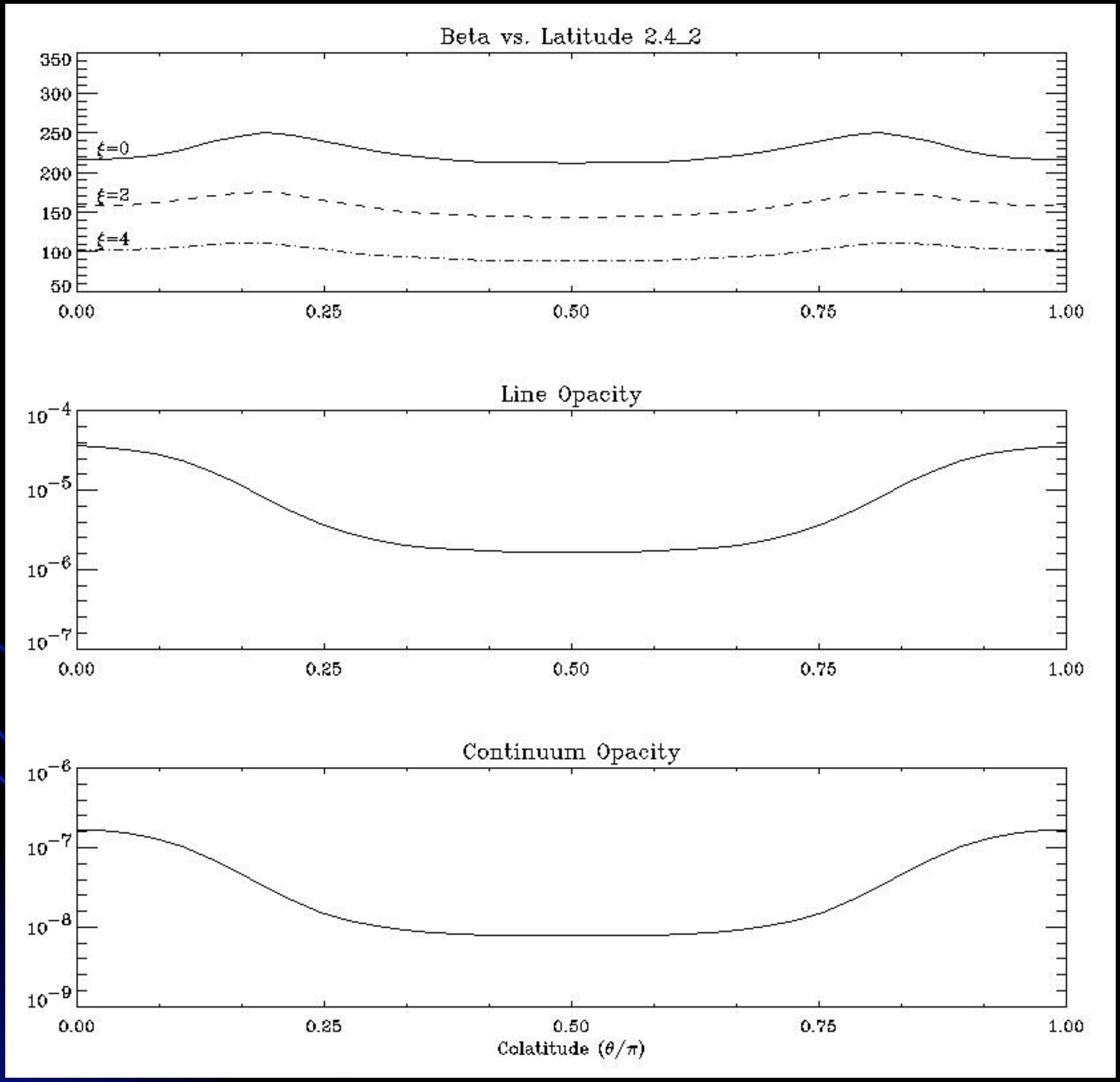


Line Opacity…





∃$_o$ is relatively constant!



Now to Principal Component Analysis…

Principal Component Analysis (PCA)

- We are currently using Principal Component Analysis (PCA) to analyze the morphology of a vast range of profiles with varying inclination angles i, degrees of absolute and differential rotation ∀ and 0, and varying degrees of microturbulence >.
- PCA is a pattern recognition technique whose eigenprofiles we are using to look at the "principal components" of our stellar spectra.

The Math Behind PCA

- PCA involves performing singular value decomposition on a covariance matrix C from N profiles M_n (observation matrix X).

$$X = M \begin{pmatrix} M_{11} & \cdots & M_{n1} & \cdots & M_{N1} \\ \vdots & & \vdots & & \vdots \\ M_{1n} & \cdots & M_{nn} & & \vdots \\ \vdots & & & \ddots & \\ M_{1N} & \cdots & \cdots & \cdots & M_{NN} \end{pmatrix} \quad (N \text{ columns})$$

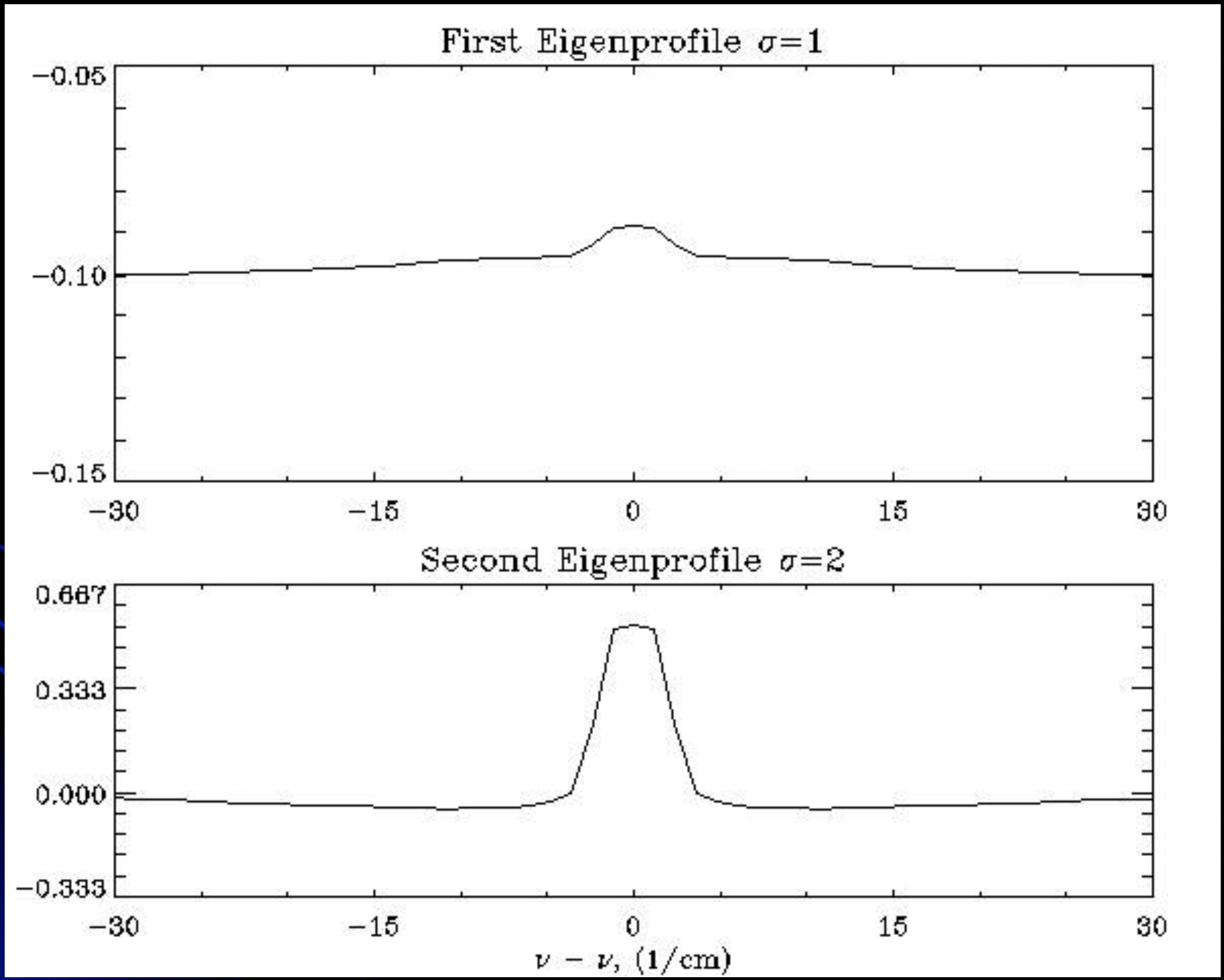
$$C = XX^T = UEV^T$$

- Where UEV^T is the singular value decomposition (SVD) of the covariance matrix C.

UEV^T…

- For the SVD of the covariance matrix $C = UEV^T$, E contains eigenvalues $\Phi_1 \ldots \Phi_n$, with corresponding eigenprofiles in U, for which C can be *reconstructed*.
- PCA attempts to use only the largest eigenvalues and their corresponding eigenprofiles to reconstruct observational profiles.
- These largest components are the "Principal Components."

An Example of the first two principle components while varying only inclination i, differential rotation ∀, and microturbulence >



Future Plans (Project)

- To be a viable resource, the PCA models need to be expanded to include other masses, absorption lines, and von Zeipel coefficients.

$$T_{eff} \sim (\text{Mgrav})^{∃=?}$$

- “In order to improve our fits, we explored an extension to the von Zeipel model, allowing the gravity darkening parameter ∃ to be a free parameter. We found that ∃=0.190 model significantly improved the goodness-of-fit”
  – (Monnier et al., 2007, on Altair)

“Significantly, a necessary aspect of this modeling is a determination of the gravity-darkening coefficient, which at a value of ∃=0.084 is consistent with a convective photosphere…”

-(van Belle et al., 2006, on Alderamin)

Future Plans (Me)

- I will continue working on this project for two more weeks thanks to funding from Emily and Keith.
- I am also organizing a research project at the Sommers-Bausch Observatory in which I will observe signs of gravitational darkening and rapid, differential rotation.
- I have enjoyed this exposure to stellar astrophysics and spectral modeling and hope to do further research in the field.

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