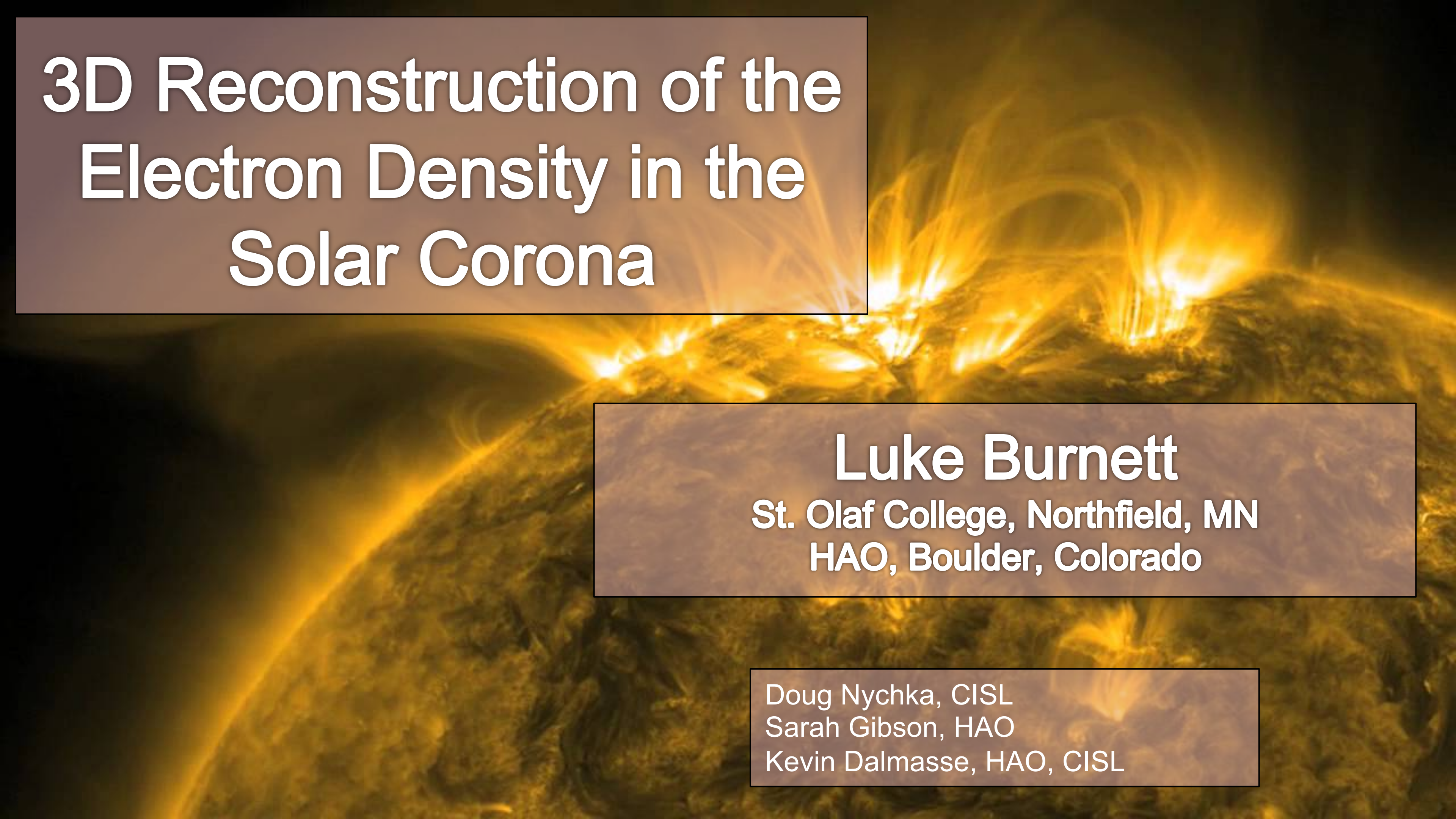

3D Reconstruction of the Electron Density in the Solar Corona

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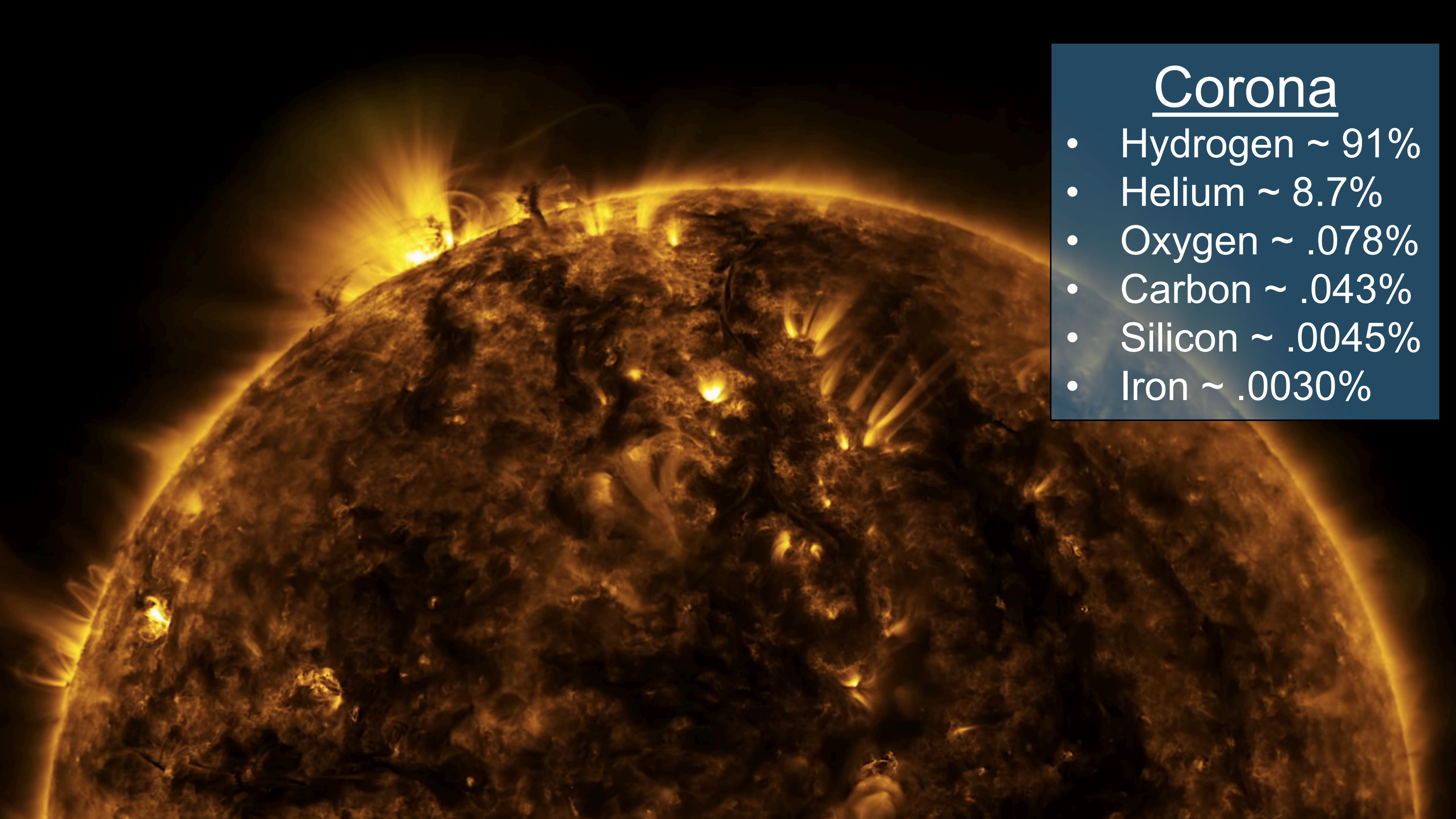
Kevin Dalmasse, HAO, CISL

Variations over time

- Flares
- CMEs
- Small-scale variation



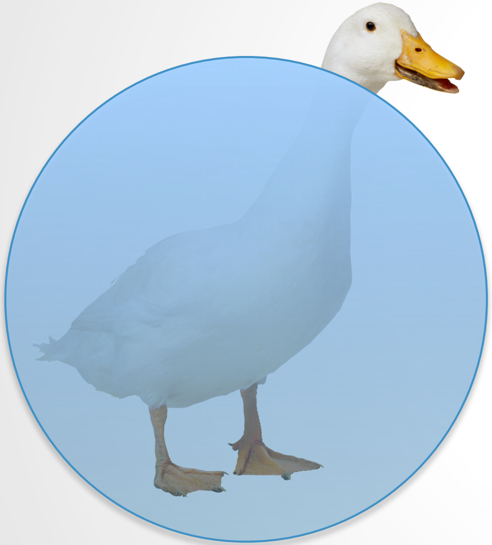
Corona

- Hydrogen ~ 91%
- Helium ~ 8.7%
- Oxygen ~ .078%
- Carbon ~ .043%
- Silicon ~ .0045%
- Iron ~ .0030%

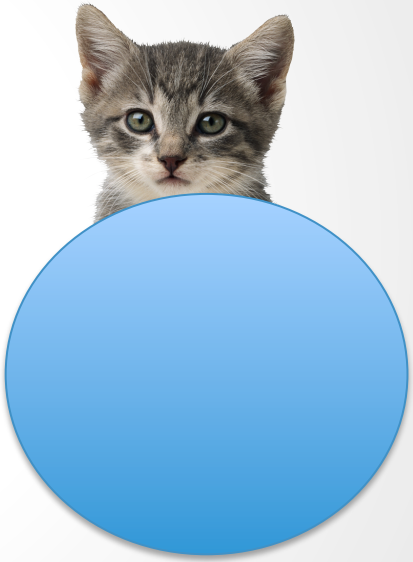
What we have to work with:

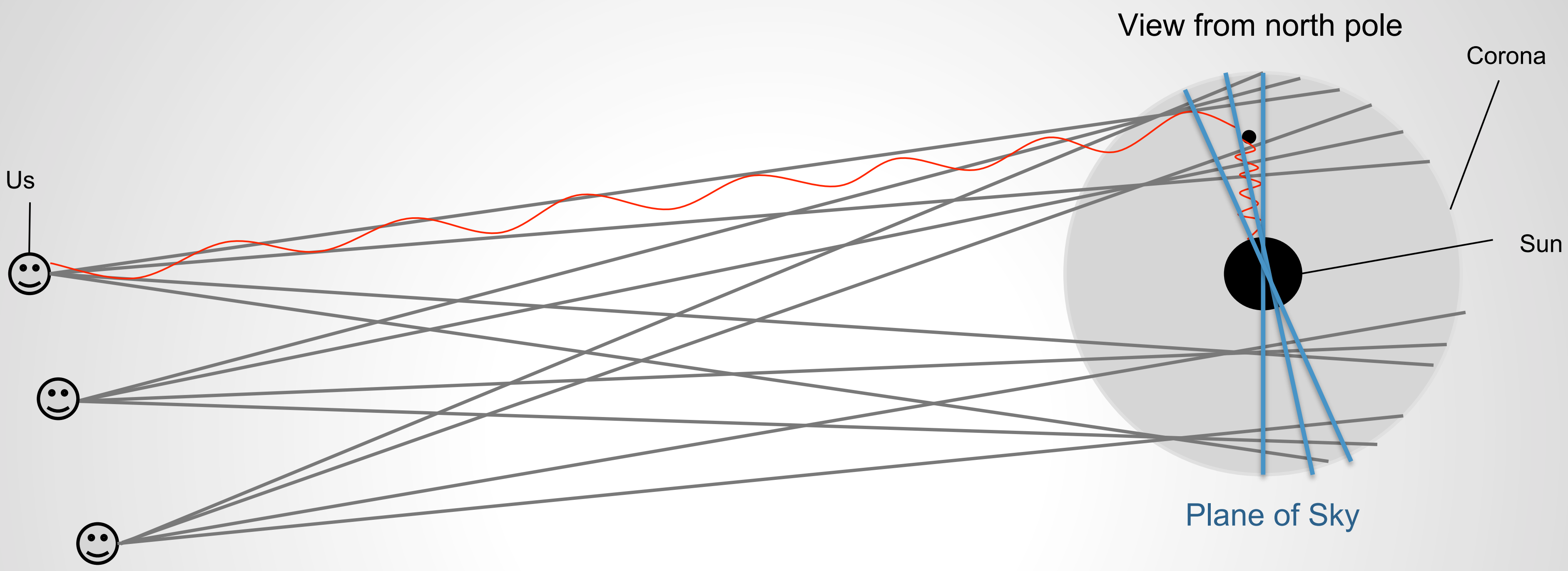
- White-light images
- 2-D images at different angles
- Projection on the "plane of sky" (POS defined by observer's location)

Optically Thin



Optically Thick

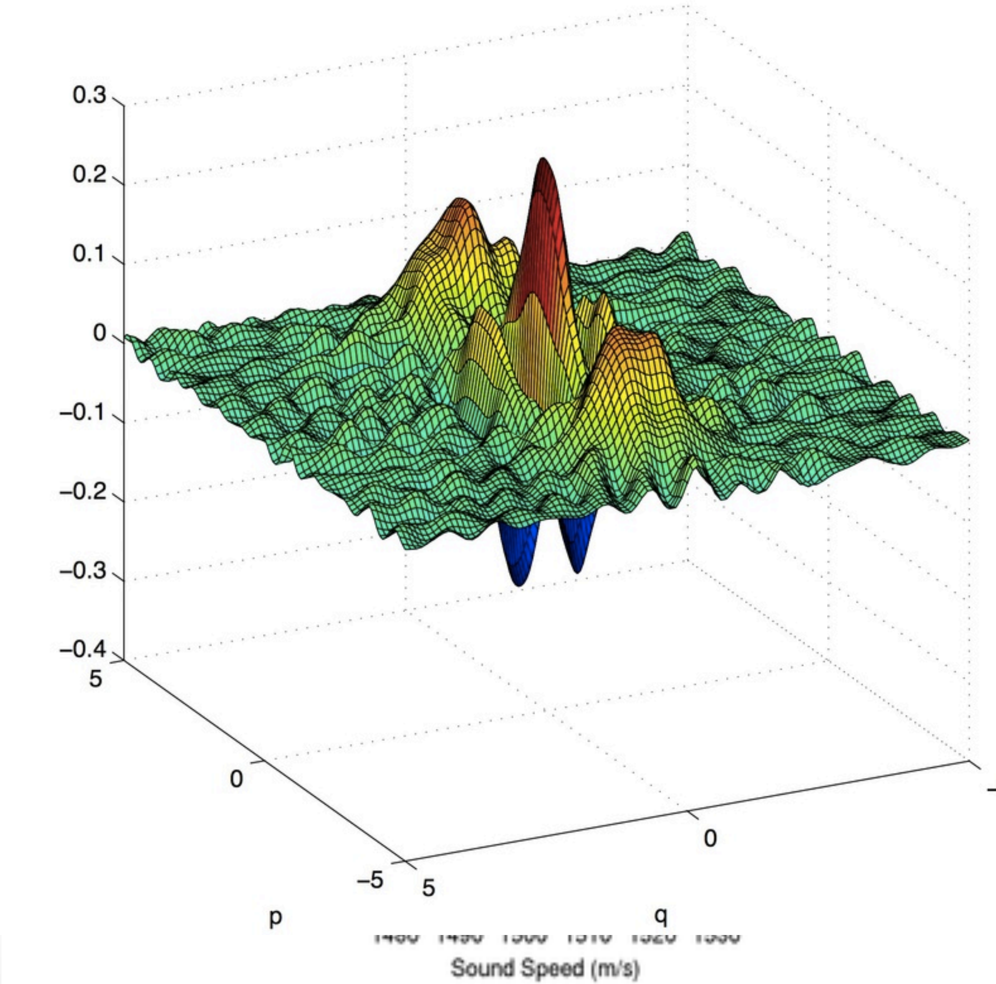




$$pB_{\mathrm{LOS}} = \int_{\mathrm{LOS}} N_e(x) \cdot S(x)\,dl$$

Tomography

The reconstruction of an object of N dimensions through a series of M-dimensional slices or observations where M < N.



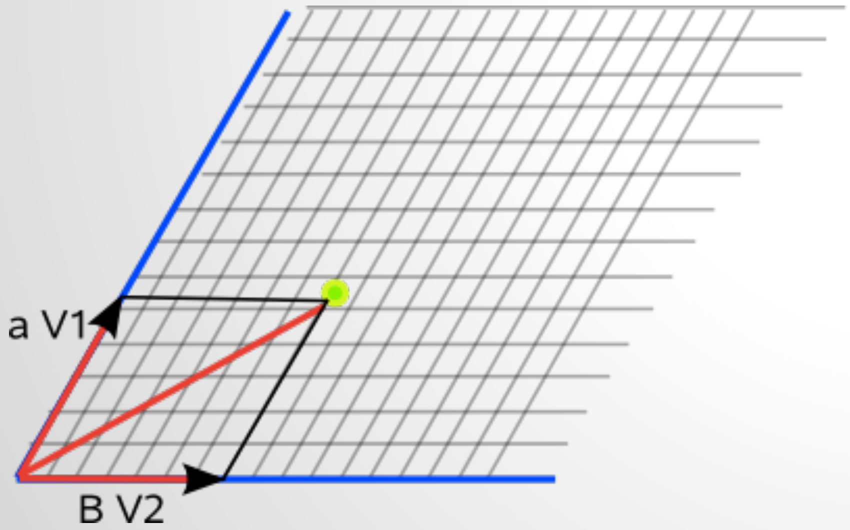
Examples:

- MRI
- Ocean Acoustic Tomography
- Quantum State Tomography

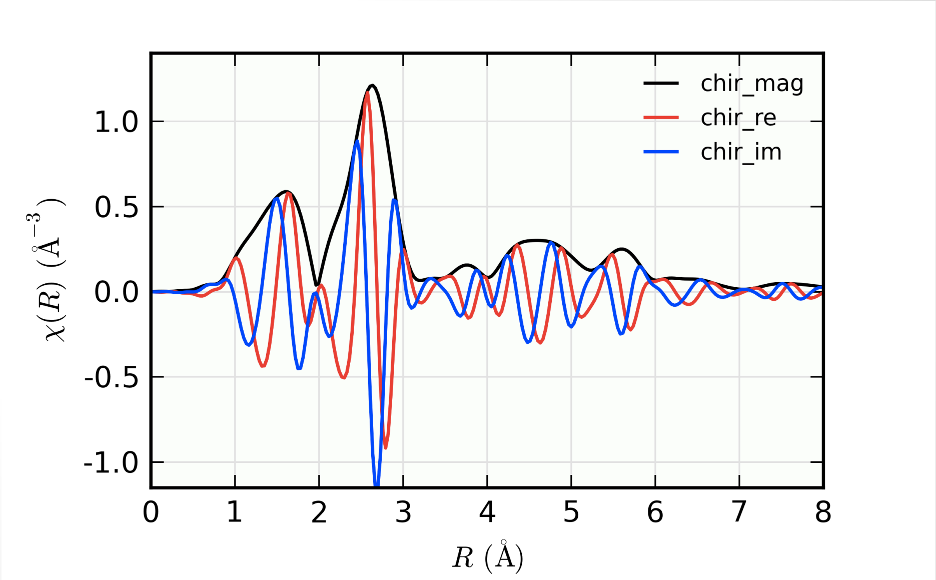
Radial Basis Functions

Combination of functions X constants

Basis vectors spanning a vector space

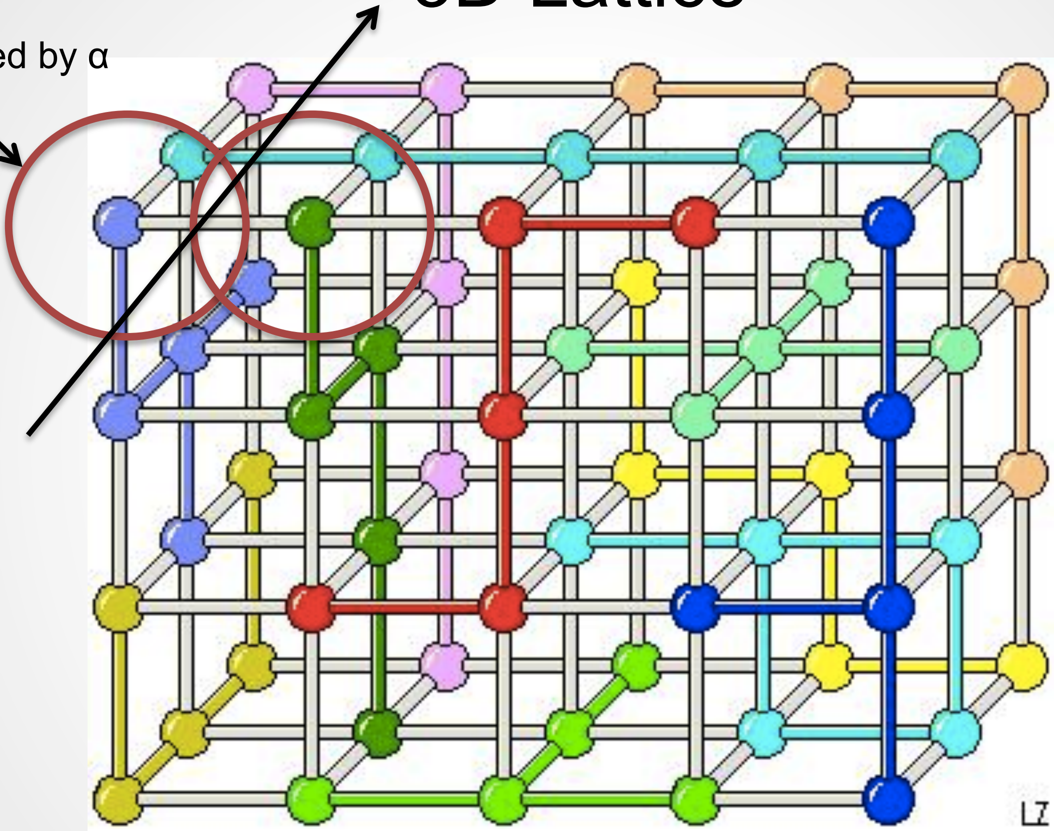


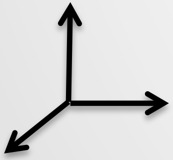
Sines and cosines describing a function



3D Lattice

“radius” defined by α





Spatial Dimensions

Basis Functions

Point in space

Basis function node location

$$x = (r_1, \theta_1, \phi_1) \qquad b = (r_2, \theta_2, \phi_2)$$

$$d = \left(\frac{r_1 - r_2}{\alpha_r}\right)^2 + \left(\frac{\theta_1 - \theta_2}{\alpha_\theta}\right)^2 + \left(\frac{\phi_1 - \phi_2}{\alpha_\phi}\right)^2$$

Distance in normalized space

α = “radius” of influence

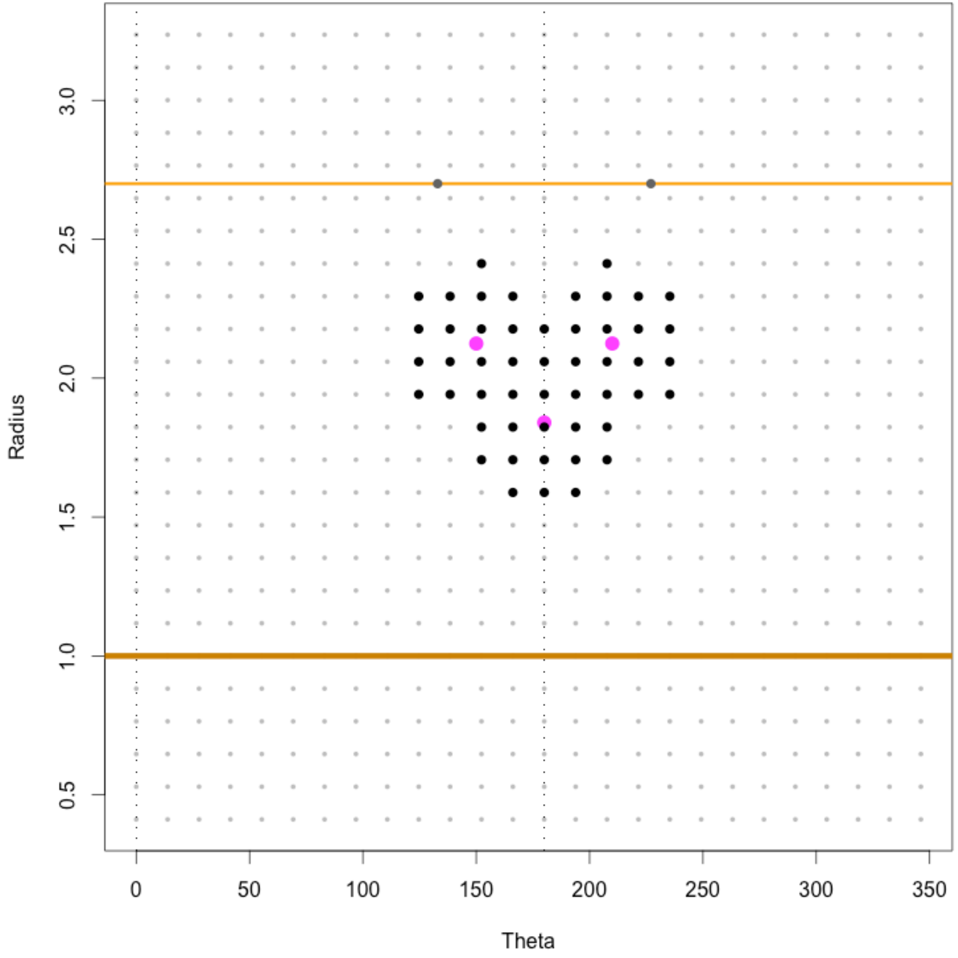
$$\psi_i(x) = \begin{cases} 0 & d \geq 1 \\ (1 - d^2)^3 & d < 1 \end{cases}$$

Value of basis function

Looking down from north pole (polar plot)

x (points in space)

b (node locations)



Integral and Sum Form

$$N_e(x) \approx N_e^*(x) = \sum_{i=1}^{n} \psi_i(x) \cdot c_i$$

Basis Functions

$$pB_k = \int_k N_e(x) \cdot S(x) dl$$

K[th] line of sight

$$pB_k = \sum_{i=0}^{n} \left[\int_k \psi_i(x) \cdot S(x) dl \right] \cdot c_i$$

Matrix Form

$$pB_k = \sum_{i=1}^{n} c_i \int_k \psi_i(x) \cdot S(x)dl$$

$$pB_k = c_1 \int_k \psi_1(x) \cdot S(x)dl + c_2 \int_k \psi_2 \cdot S(x)dl + \cdots + c_n \int_k \psi_n \cdot S(x)dl$$

$$\begin{bmatrix} pB_1 \\ pB_2 \\ \vdots \\ pB_m \end{bmatrix} = \begin{bmatrix} \int_1 \psi_1 \cdot S(x)dl & \int_1 \psi_2 \cdot S(x)dl & \dots & \int_1 \psi_n \cdot S(x)dl \\ \int_2 \psi_1 \cdot S(x)dl & \int_2 \psi_2 \cdot S(x)dl & \dots & \int_2 \psi_n \cdot S(x)dl \\ \vdots & \vdots & \ddots & \vdots \\ \int_m \psi_1 \cdot S(x)dl & \int_m \psi_2 \cdot S(x)dl & \dots & \int_m \psi_n \cdot S(x)dl \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

$$\boxed{pB = A\vec{c}}$$

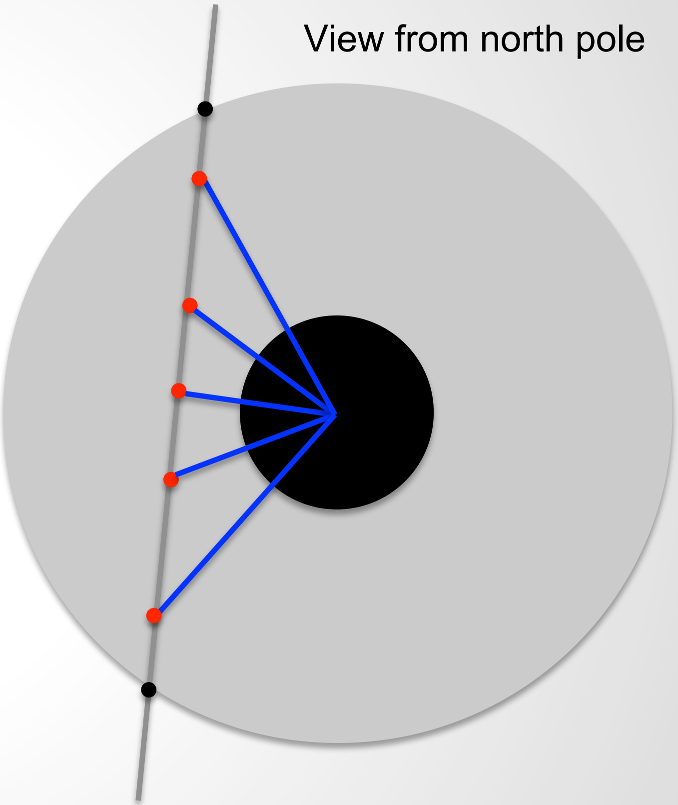
Computing A

$$A = \begin{bmatrix} \int_1 \psi_1 \cdot S(x) dl & \int_1 \psi_2 \cdot S(x) dl & \dots & \int_1 \psi_n \cdot S(x) dl \\ \int_2 \psi_1 \cdot S(x) dl & \int_2 \psi_2 \cdot S(x) dl & \dots & \int_2 \psi_n \cdot S(x) dl \\ \vdots & \vdots & \ddots & \vdots \\ \int_m \psi_1 \cdot S(x) dl & \int_m \psi_2 \cdot S(x) dl & \dots & \int_m \psi_n \cdot S(x) dl \end{bmatrix}$$

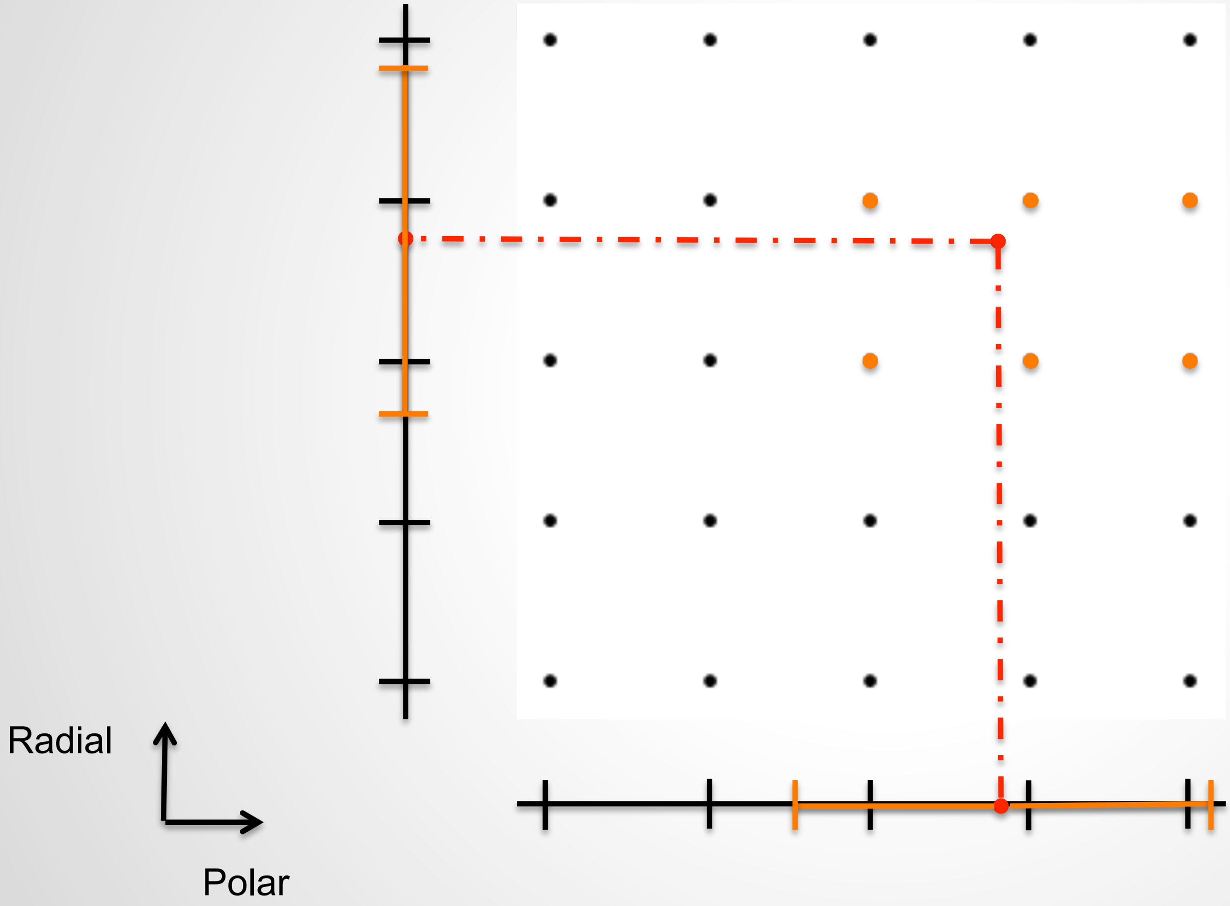
Major algorithms in program
• Determine LOS sample points • Find basis functions in range • Compute integral

Determining LOS sample points

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} t$$



Find Basis Functions in Range

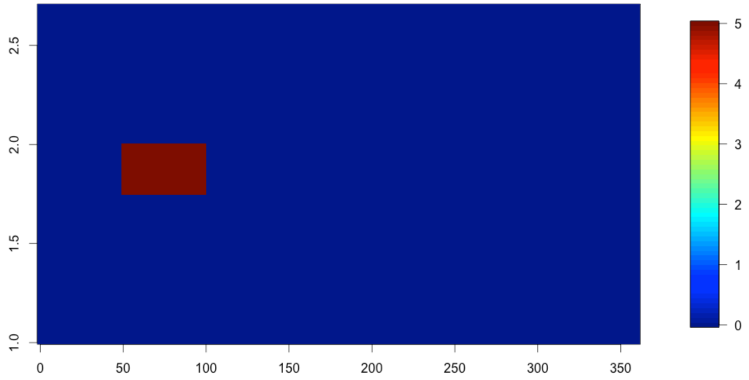


$$p\vec{B} = A\vec{c} \qquad N_e^*(x) = \sum_{i=0}^{n} \psi_i(x) \cdot c_i$$

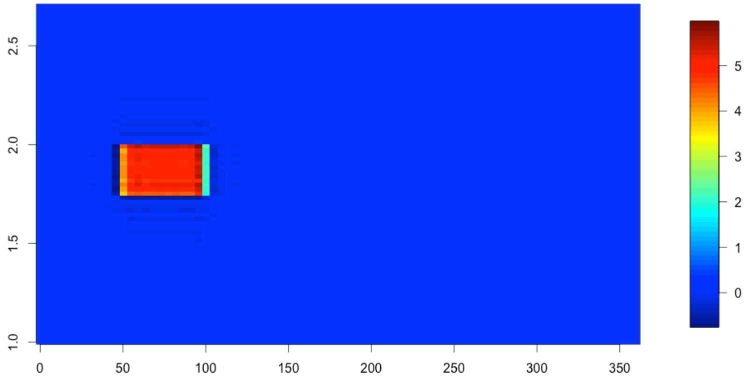
1. Can invert pB to get c, and thus N_e
   - Minimize: $(pB - Ac^*)^T(pB - Ac^*) + \lambda(c^*)^T Q c^*$
2. Allows us to check accuracy of the modeling method
   - Model with known density
   - Get c coefficients $(N_e - \Psi c)^T(N_e - \Psi c) + \lambda c^T Q c$
   - Use c coefficients to get pB
   - Do inversion with pB to get new set of c coefficients (c*)
   - Compare c to c*

Step 1: Model with known density
Step 2: Obtain c coefficients

Ground truth

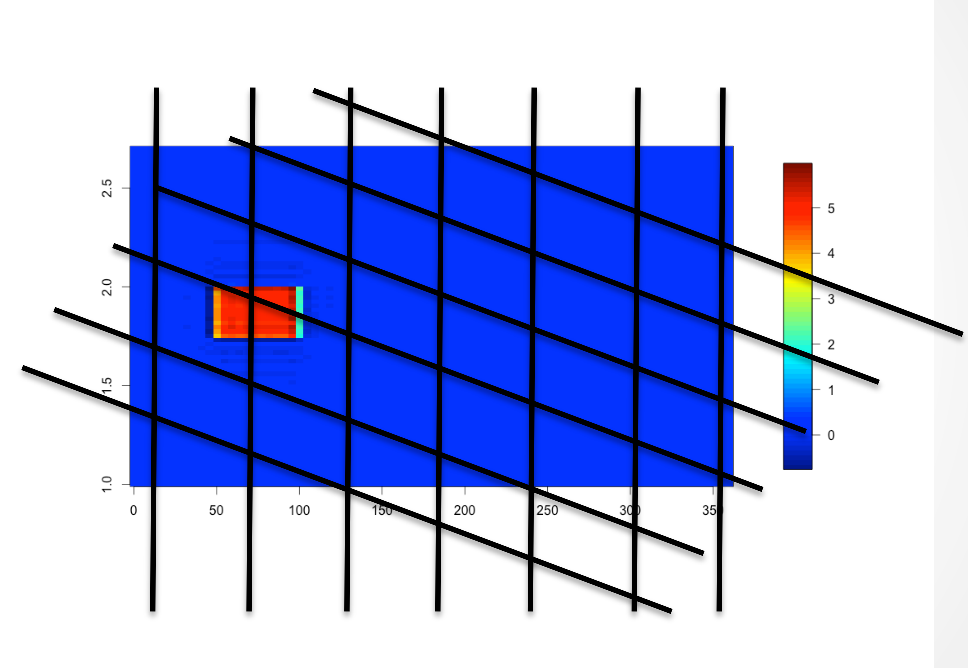


Reconstructed density with 24522 basis functions $N_e(c)$



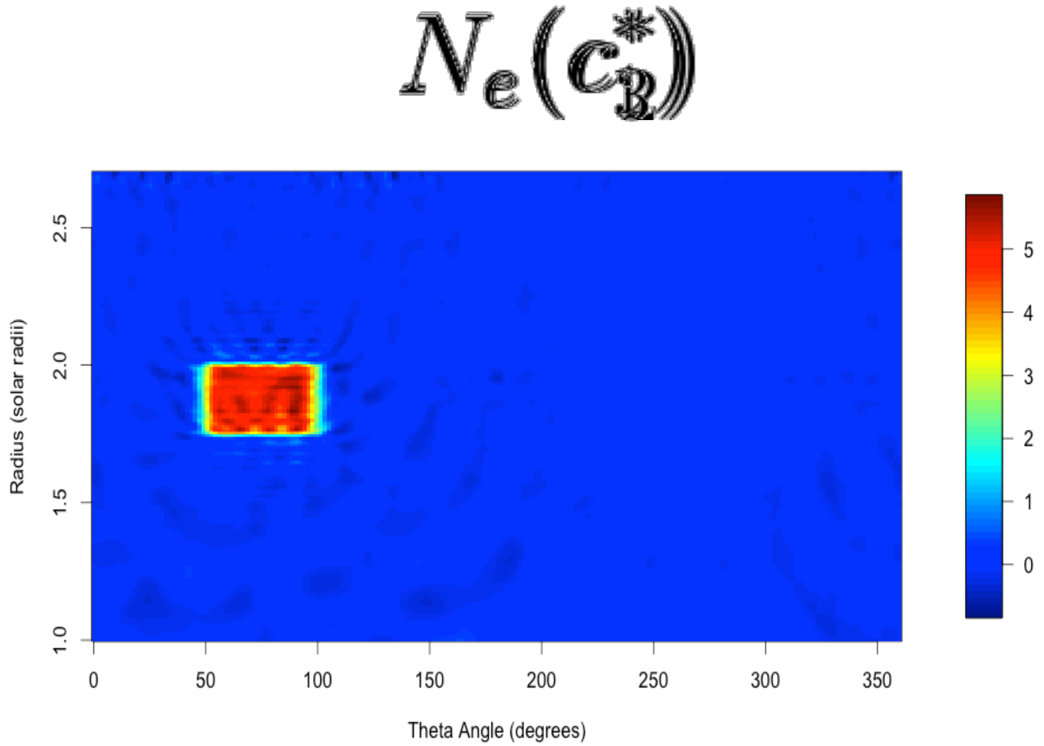
$$N_e(x) \approx \sum_i \psi_i(x) \cdot c_i$$

Step 3: Use c coefficients to get pB



$$A\vec{c} = p\vec{B}$$

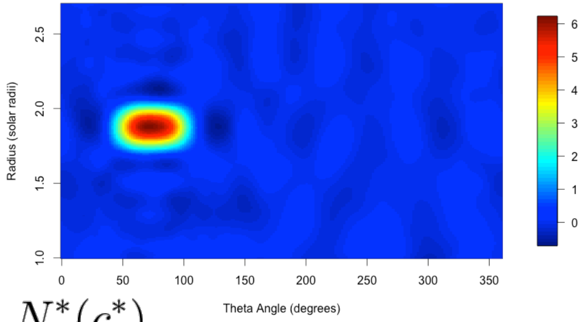
Step 4: Do inversion with pB to get new set of c coefficients (c*)



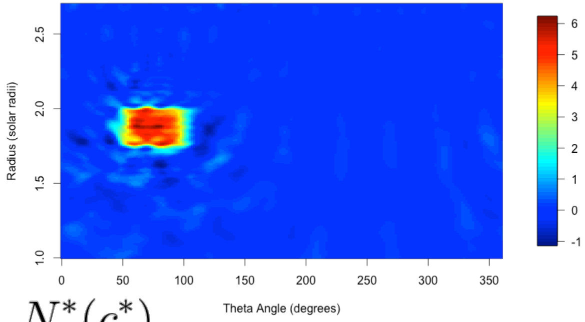
14672 Basis Functions

18 Viewing angles

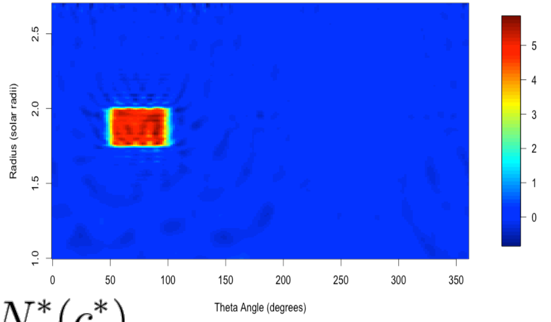
360 LOS per angle



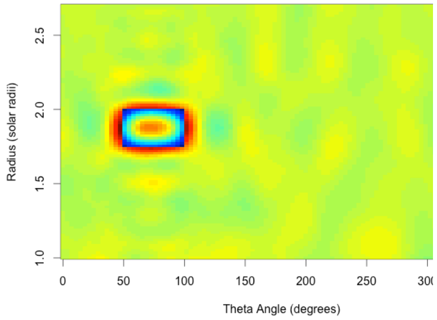
$N_e^*(c_1^*)$



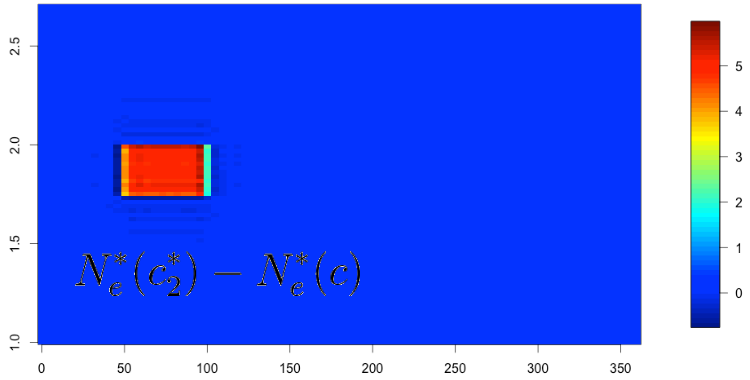
$N_e^*(c_2^*)$



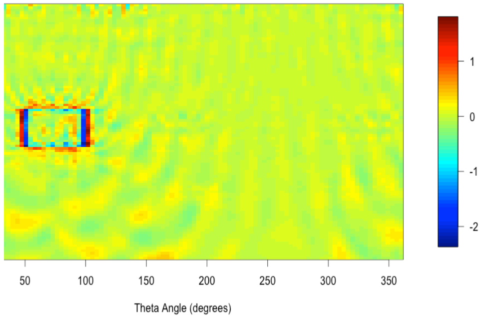
$N_e^*(c_3^*)$



$N_e^*(c_1^*) - N_e^*(c)$



$N_e^*(c_2^*) - N_e^*(c)$



$N_e^*(c_3^*) - N_e^*(c)$

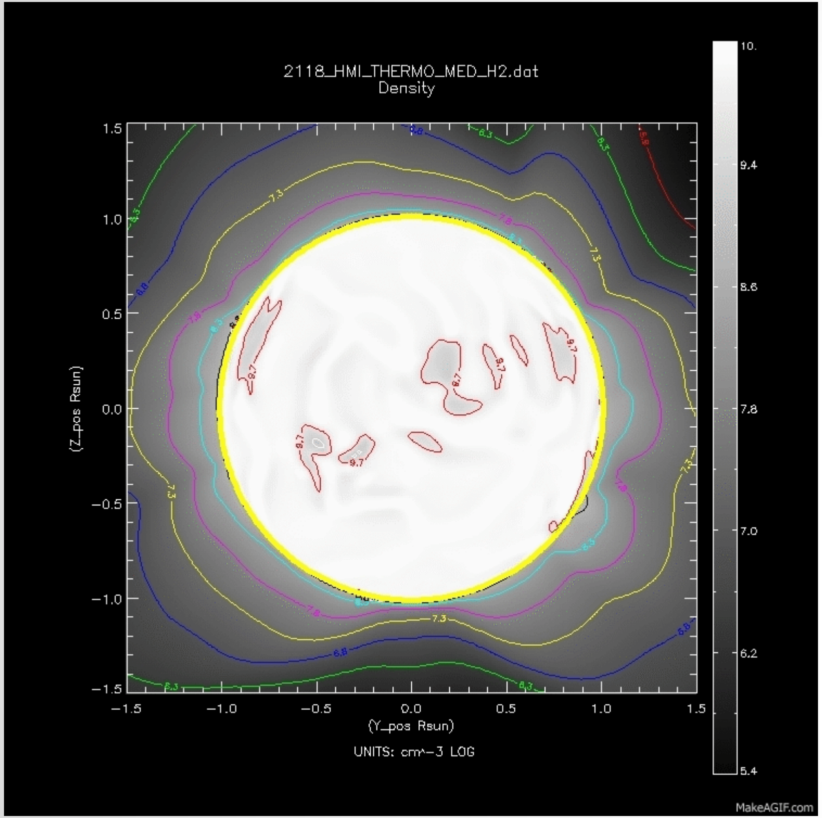
Mean square error

Average error: -.2

$N_e^*(c)$

ean square error: .012

verage error: -.00014

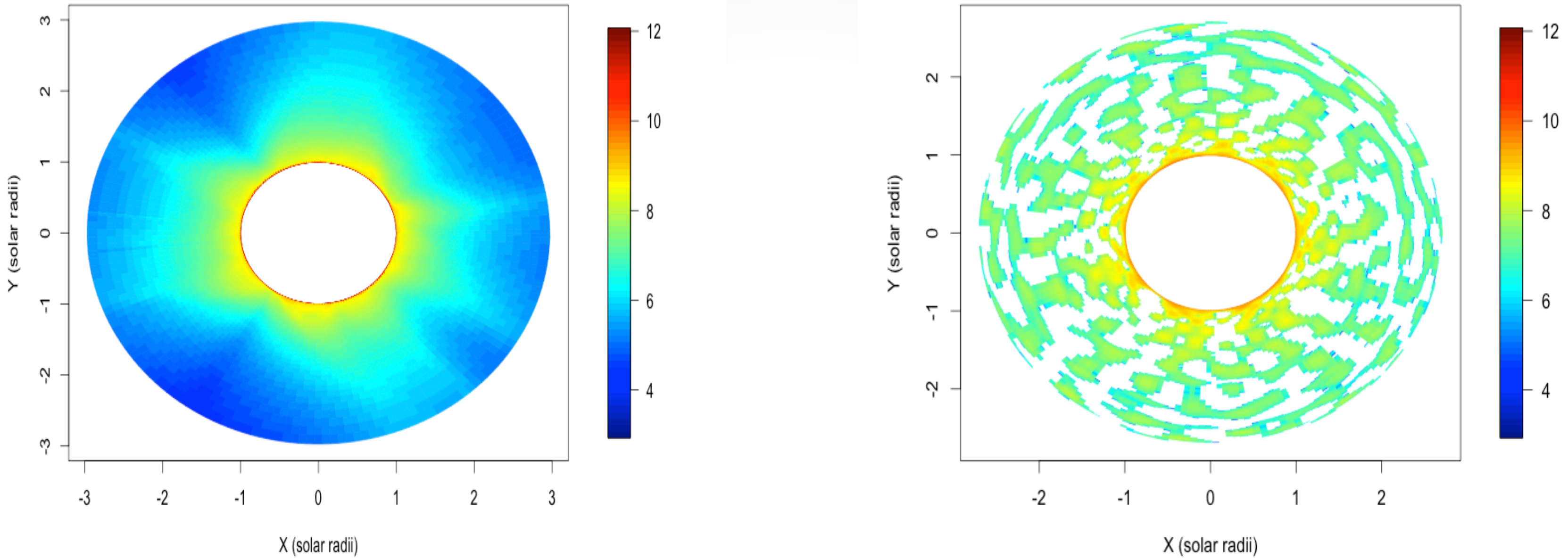


View from Earth

Predictive Science Model

- Boundary-driven model
- Solve through MHD equations
- Provides density at all points
- Datacube available for each solar rotation (chosen to match data)

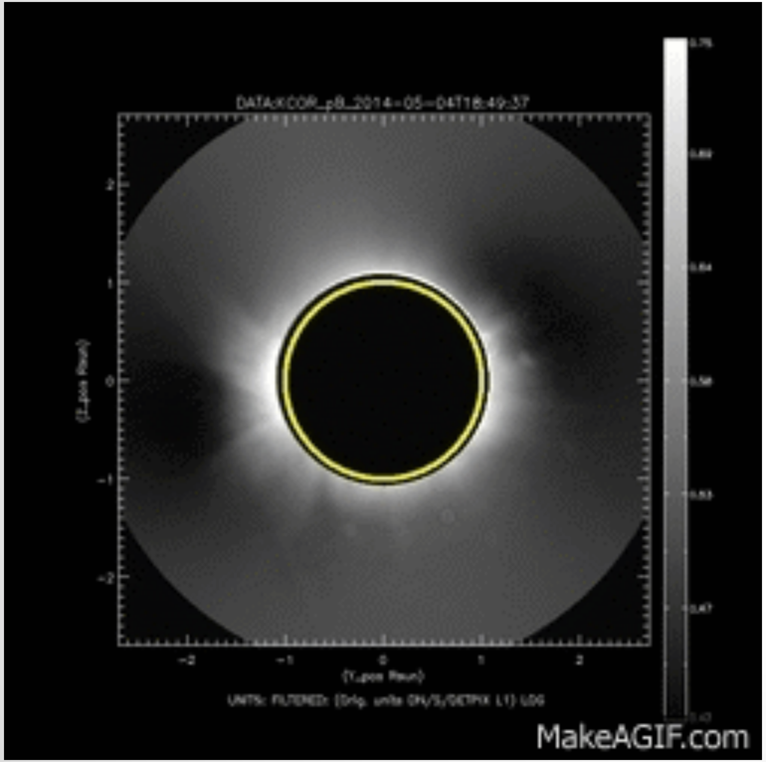
View from north pole



Mean square error: 1.01 e8
Average error: -2.6 e6

Final Milestone

Applying our method to actual data



View from Earth (pB)

Completed

- Some assessment of accuracy
- Data Collection
- Program to build A-matrix

Future Work

- Finish 2D testing
- Extend testing to three dimensions
- Finish R-C interface
- Perform method with real data
- Consider further improvements (?)