Examining the Impact of Prandtl Number and Heat Transport Models on Convective Amplitudes

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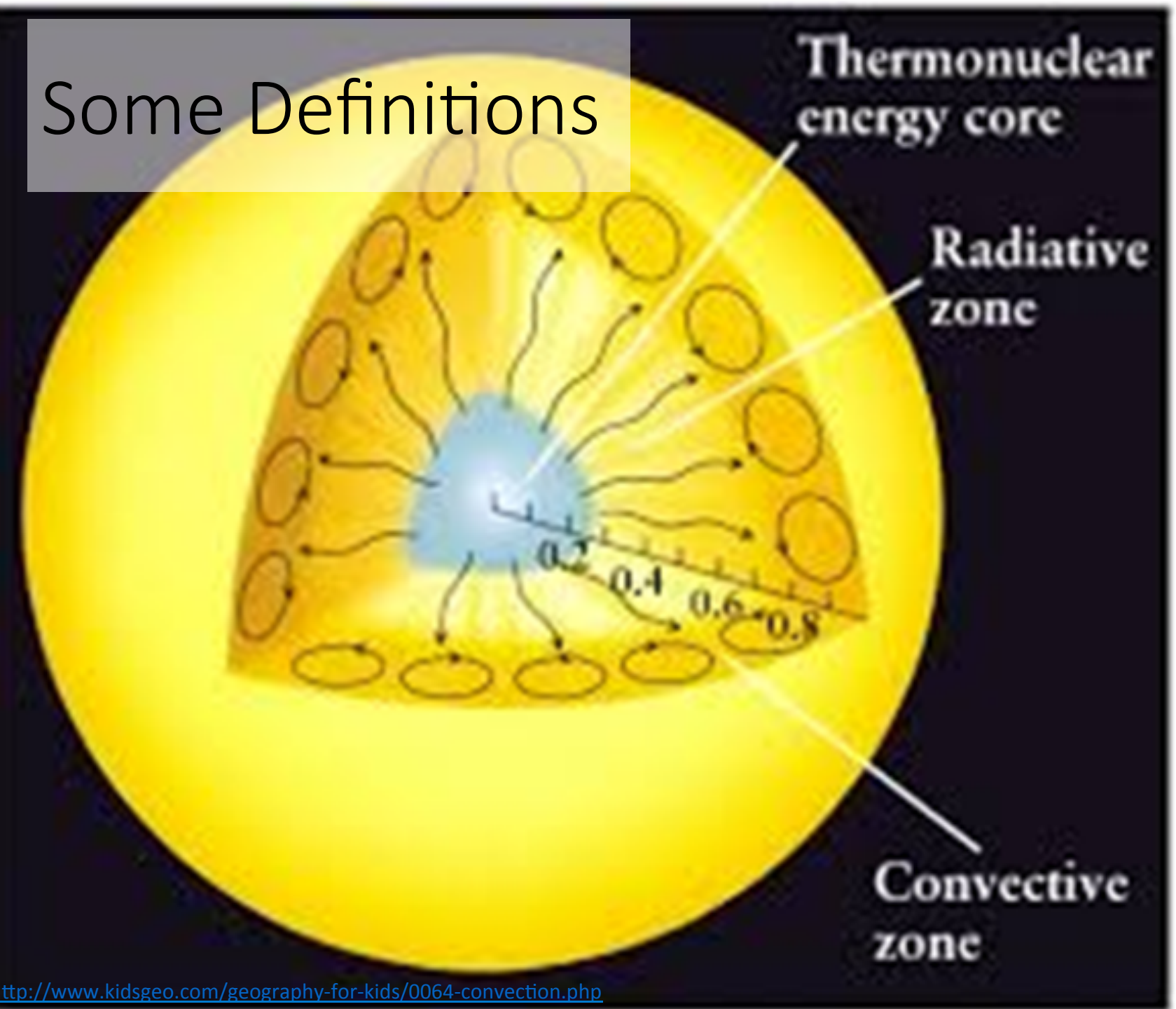
Some Definitions



ttp://www.kidsgeo.com/geography-for-kids/0064-convection.php

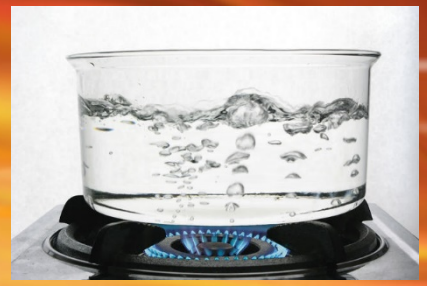
- Convection describes the transport of heat by the motion of a fluid.

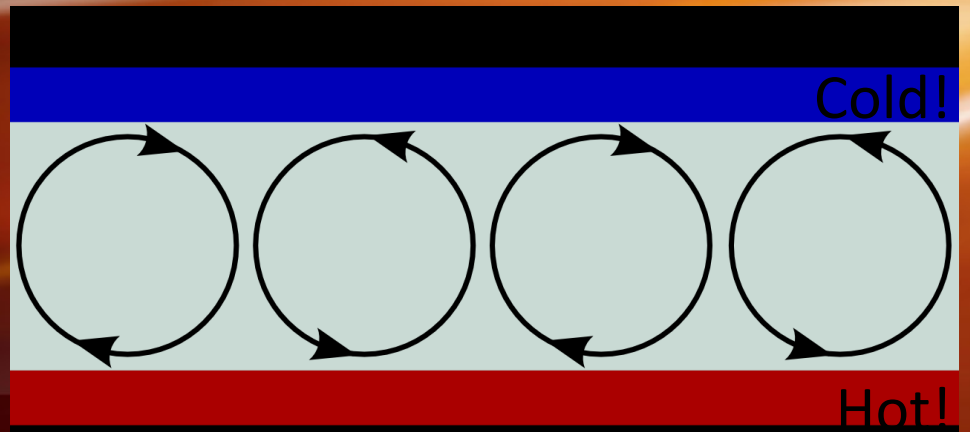
- Convective Amplitude refers to the root mean squared velocity (v_{rms}) of convection

- Recall:
  Kinetic Energy (KE) $\propto v_{rms}^2$

Why Study it?

- Convection: IT'S EVERYWHERE!!!





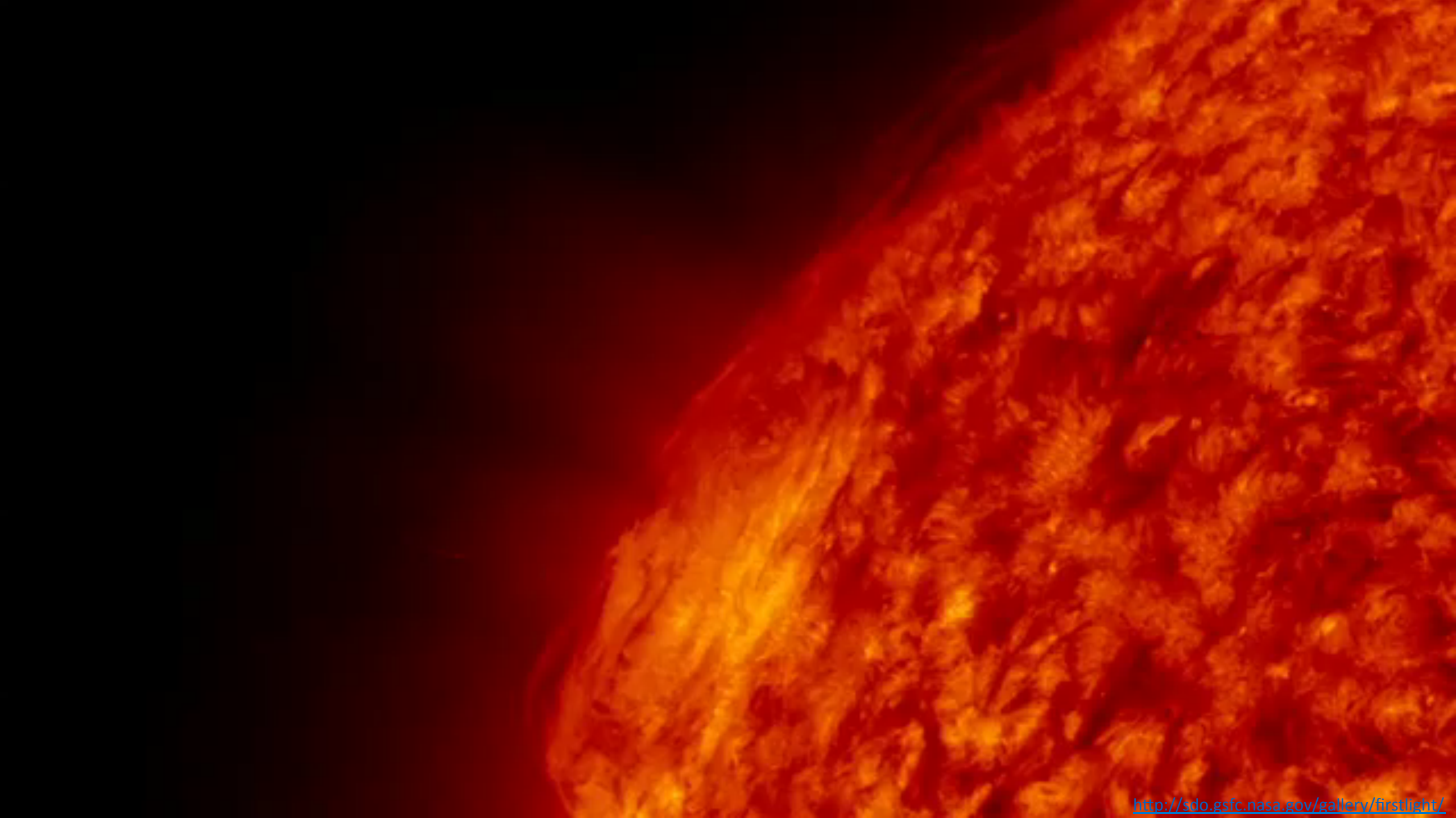
http://www.mis.mpg.de/applan/research/rayleigh.html

- The Rayleigh number describes heat transfer between a hot and cold plate

$$R_a = \alpha \Delta T g D^3 / P_r \kappa^2$$

Other reasons

- Magnetic Field Generation
- Coronal Mass Ejections
- Solar Flares



http://sdo.gsfc.nasa.gov/gallery/firstlight/

What is the Prandtl Number?

- Ratio of the viscous diffusivity to the thermal diffusivity, where:

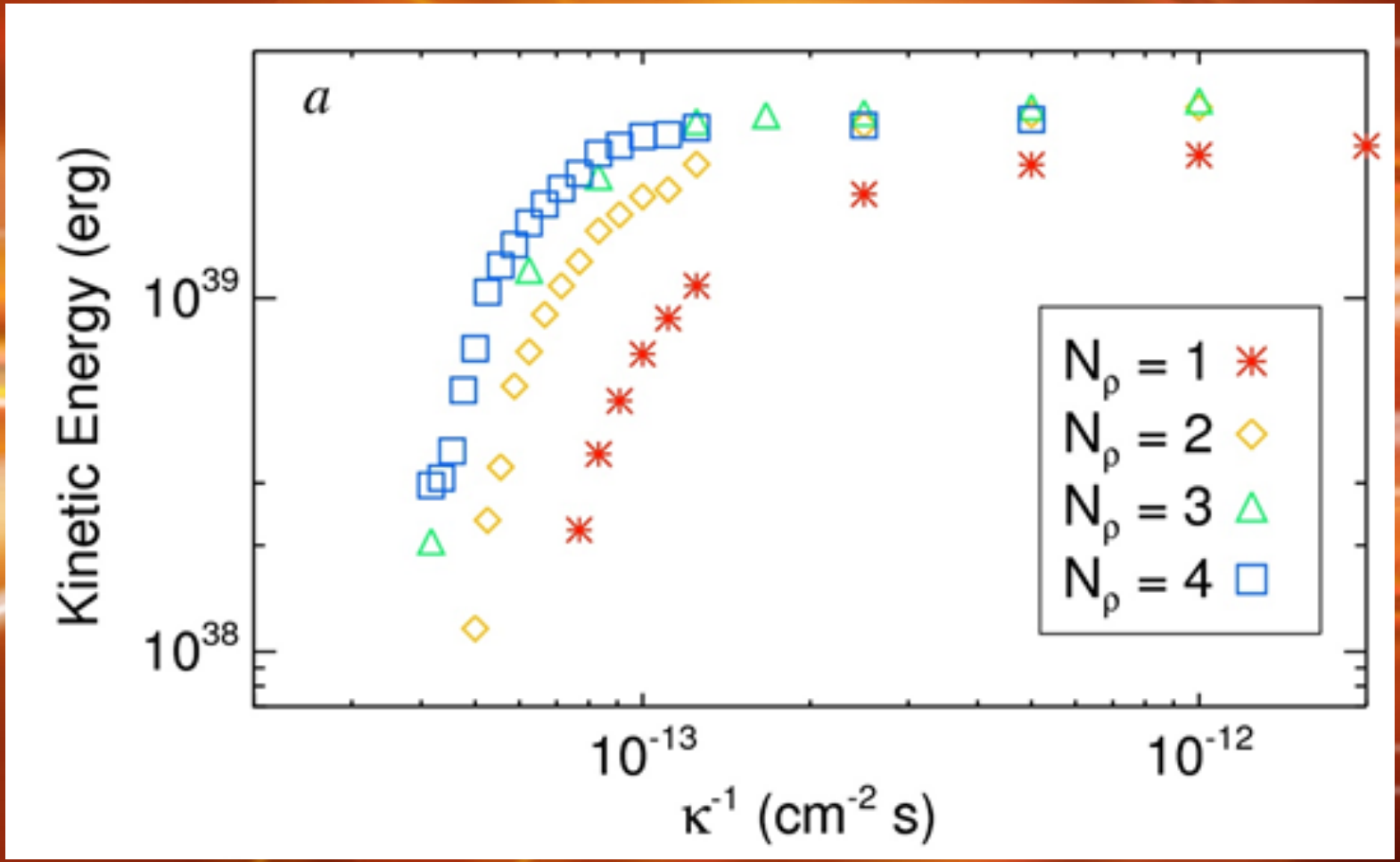
- $\nu = viscous\ diffusivity$
- $k = thermal\ diffusivity$

So (mathematically) $P_r = \nu / \kappa$

$R_a = \alpha \Delta T g D^3 / P_r \kappa^2$

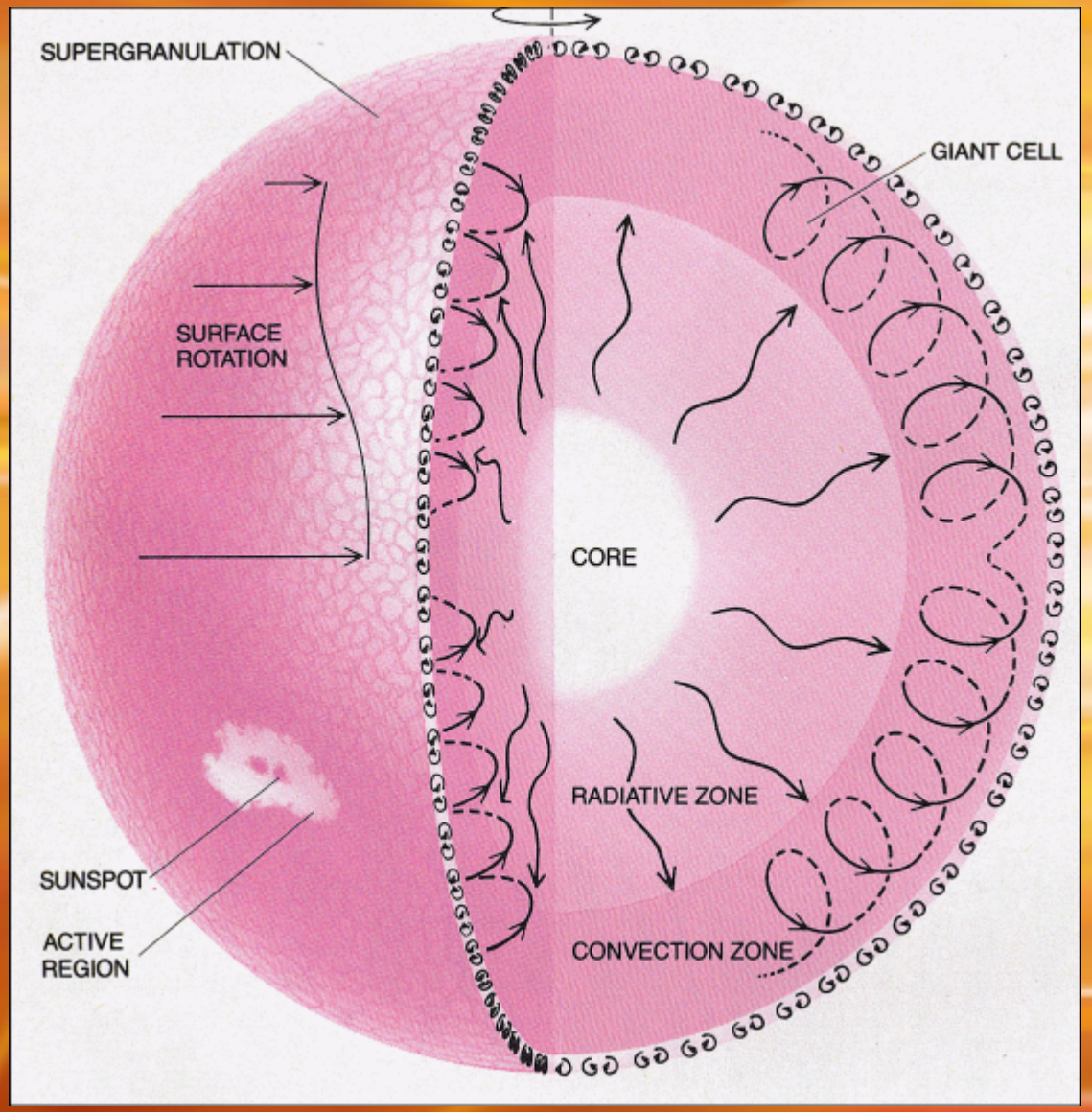
Which is inversely proportional to the Rayleigh number

- Typically, we expect to see an increasing KE with an increasing Rayleigh number and increasing $1/\kappa$



2 parts to Convection:

- Small scale surface convection
- Large scale deep convection
- Scales too far apart for any computer to replicate



Pleiades

NASA's fastest supercomputer

http://kepler.nasa.gov/news/nasakeplernews/index.cfm?FuseAction=ShowNews&NewsID=138

The Navier Stokes Equations under an anelastic constraint

- The Navier-Stokes equations describe the relationship among velocity, pressure, temperature, and density within a moving fluid (i.e, convection)

Based on the conservation equations:

- Conservation of Mass
- Conservation of Momentum
- Conservation of Energy

- Anelastic constraint:
  $\nabla \cdot (\rho v) = 0$

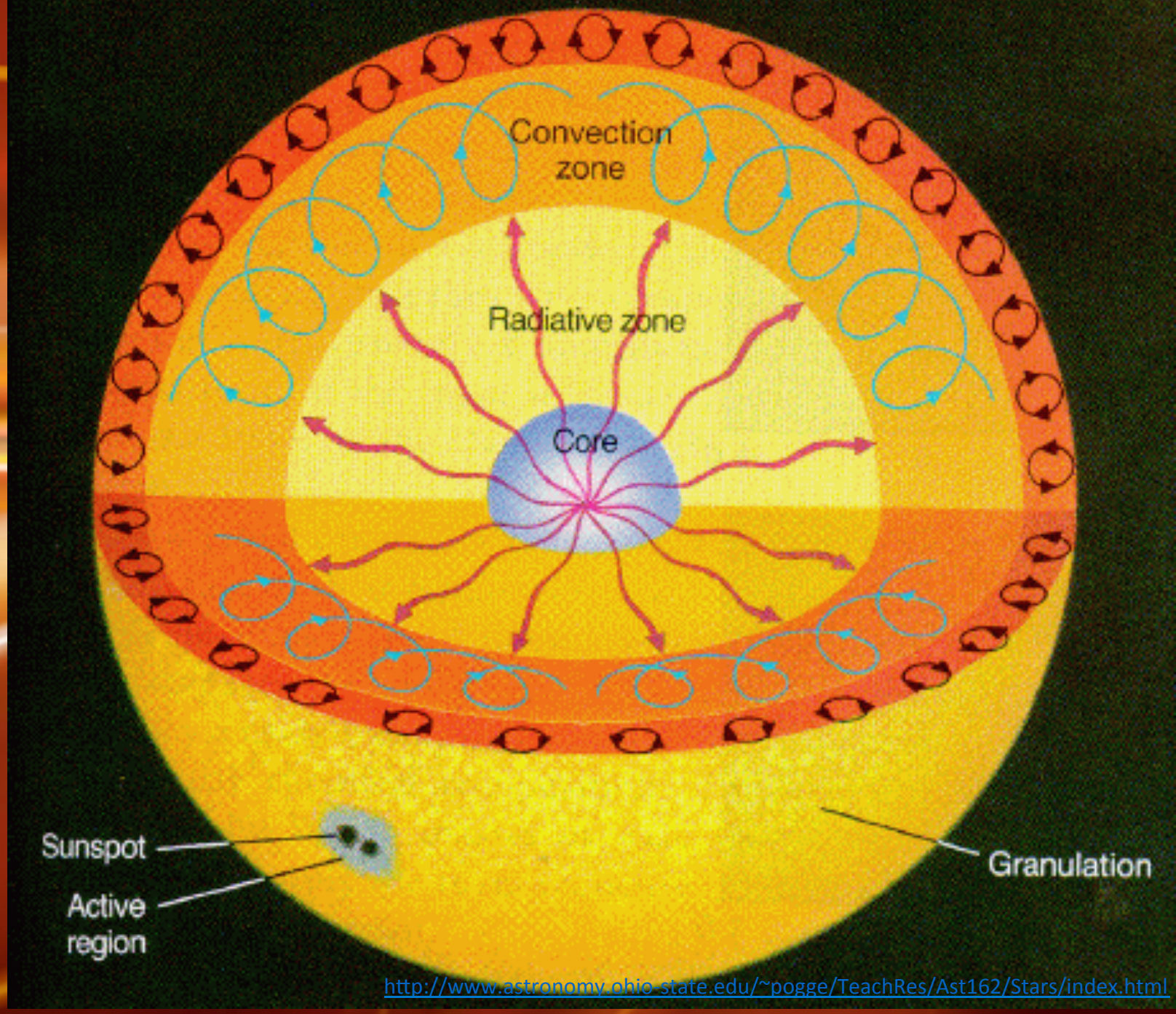
Fluxes in the Sun

- Heat transport can be described with the fluxes:
  - Radiative
  - Enthalpy & Kinetic Energy
  - Conductive

The sum of these fluxes adds to constant:

$L_\odot / 4\pi r^2$ (or) $F_\odot$

$L_\odot$ = Luminosity of the Sun



http://www.astronomy.ohio-state.edu/~pogge/TeachRes/Ast162/Stars/index.html

Fluxes in the Sun

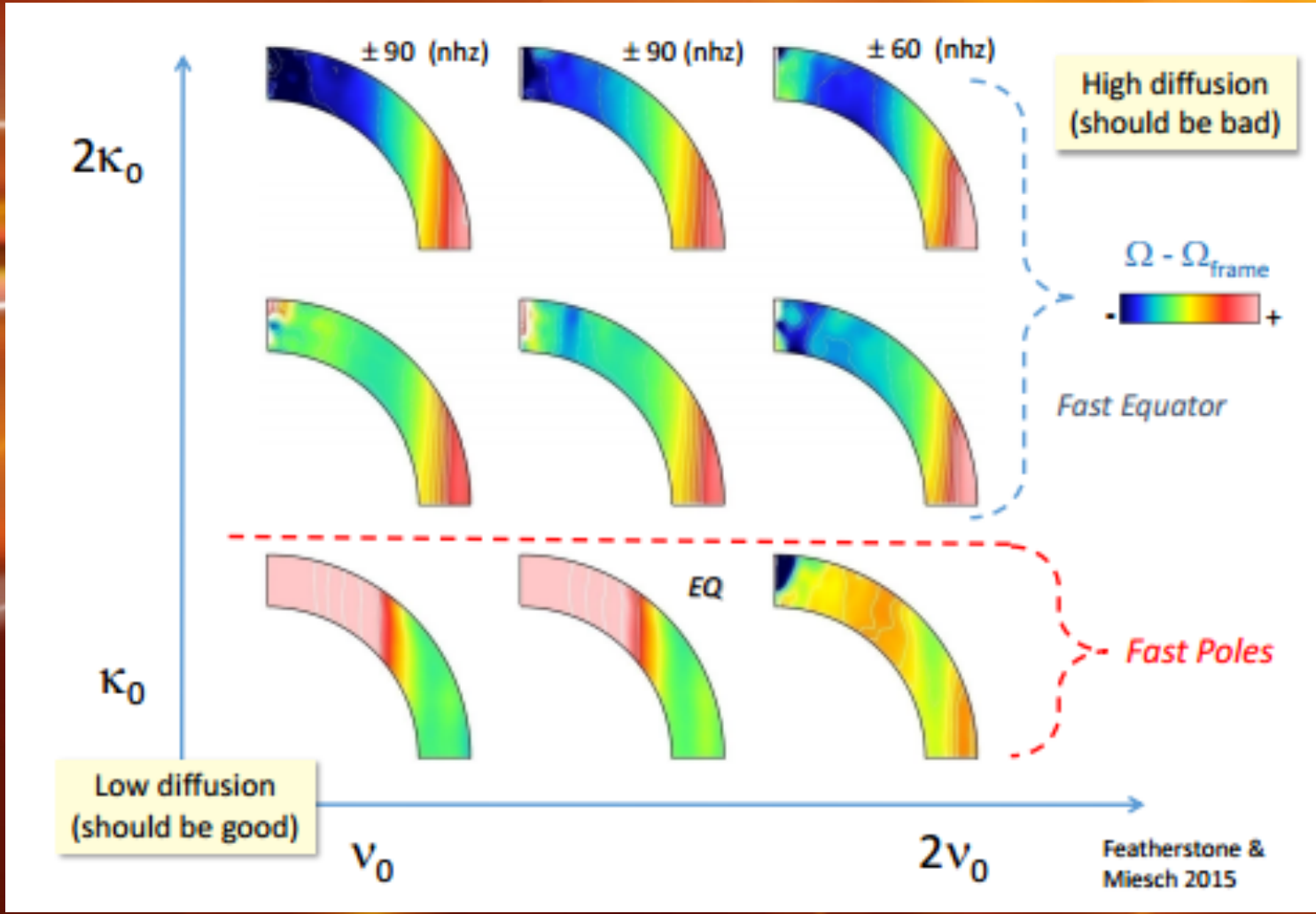
Conservation of Energy:

- $\rho T \partial S/\partial t = -\nabla\cdot(F_{rad} + F_e + F_k + F_{cond})$

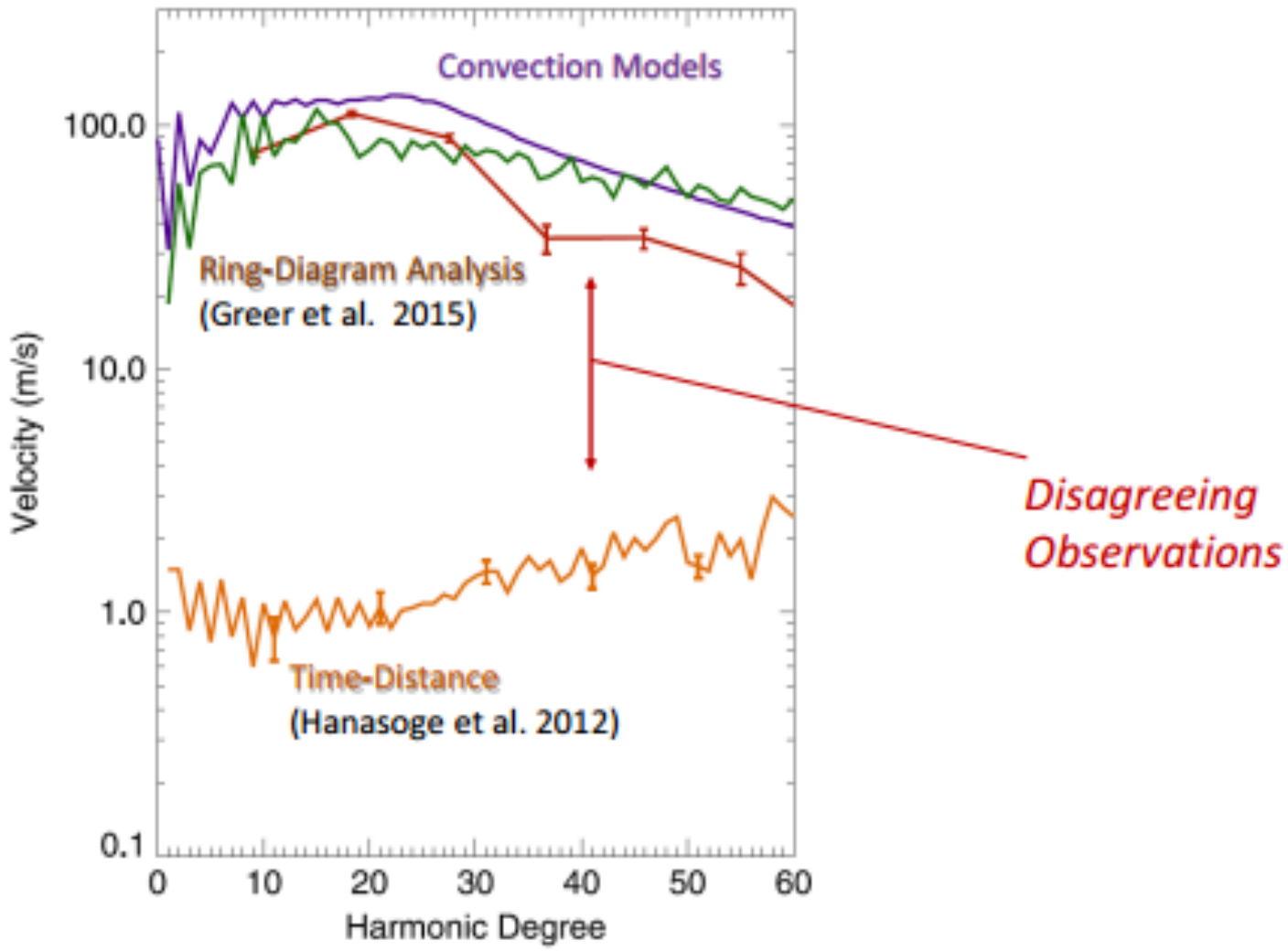
Recall: F_{cond} represents small scale convection

Problem: $F_{cond} \propto -\kappa dS/dr$ → As κ decreases, v_{rms} increases, and no longer matches observations of the sun ($\kappa_{sun} \sim 10^7$, $\kappa_{model} \sim 10^{12}$)

For Example…


±90 (nhz)
±90 (nhz)
±60 (nhz)
High diffusion
(should be bad)
$2\kappa_0$
$\Omega - \Omega_{frame}$
-
+
Fast Equator
EQ
Fast Poles
κ_0
Low diffusion
(should be good)
ν_0
$2\nu_0$
Featherstone &
Miesch 2015

Also…

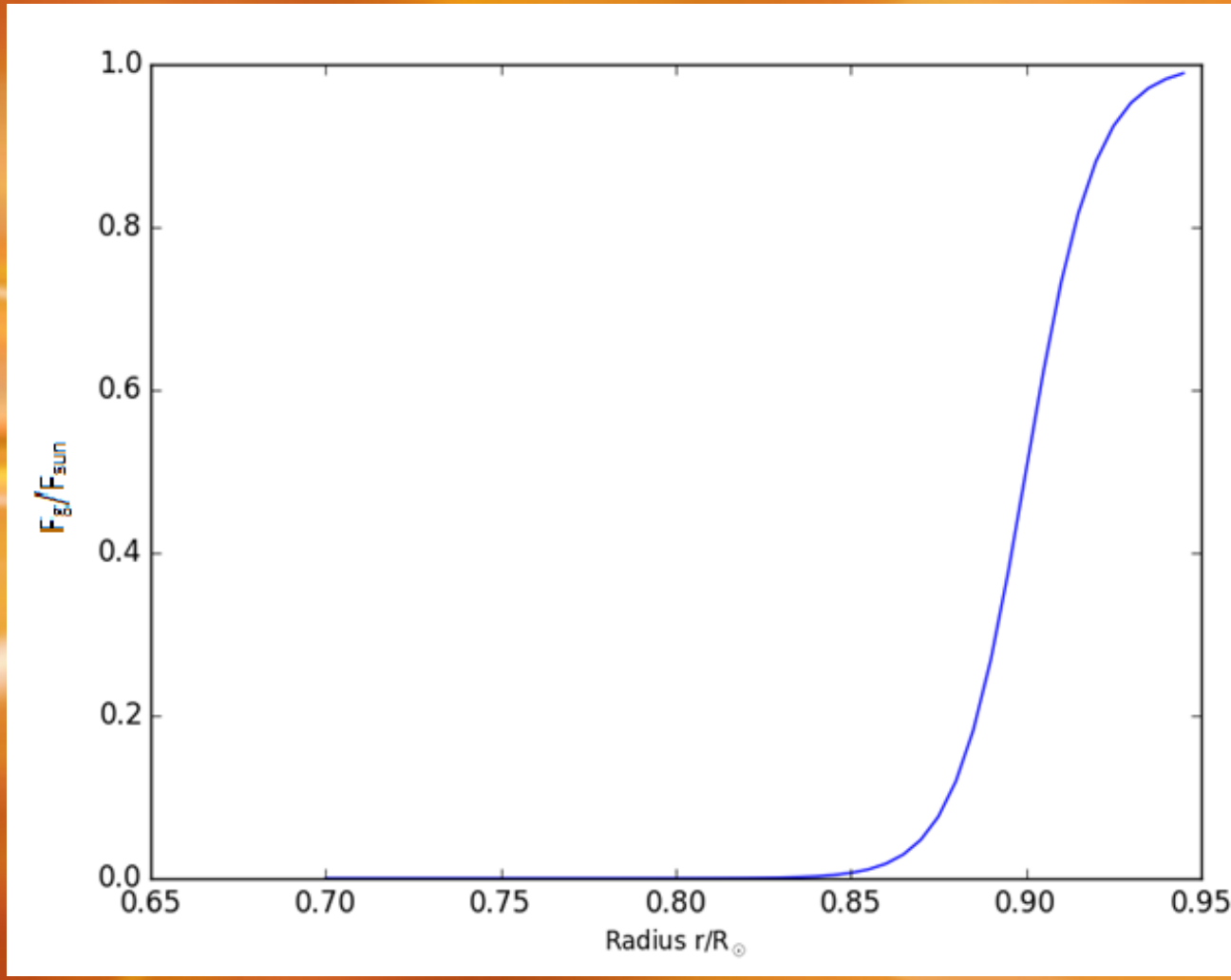


Conclusion: We might need a new model to replace F_{cond}

New model

- We add a new flux, the granulation flux, to F_{cond}
  $F_g = F_\odot\,(0.5+0.5\tanh x)$
- Where $x = r - r_g / d_g$
  $(r_g \text{ and } d_g \text{ are constants})$
- $0.5+0.5\tanh x$ satisfies the boundary conditions:

  $\lim_{x\to 0} F_g = 0$ and $\lim_{x\to\infty} F_g = F_\odot$
- Advantage: F_g *is not proportional to* $-\kappa dS/dr$
  so we can lower κ without raising the velocity

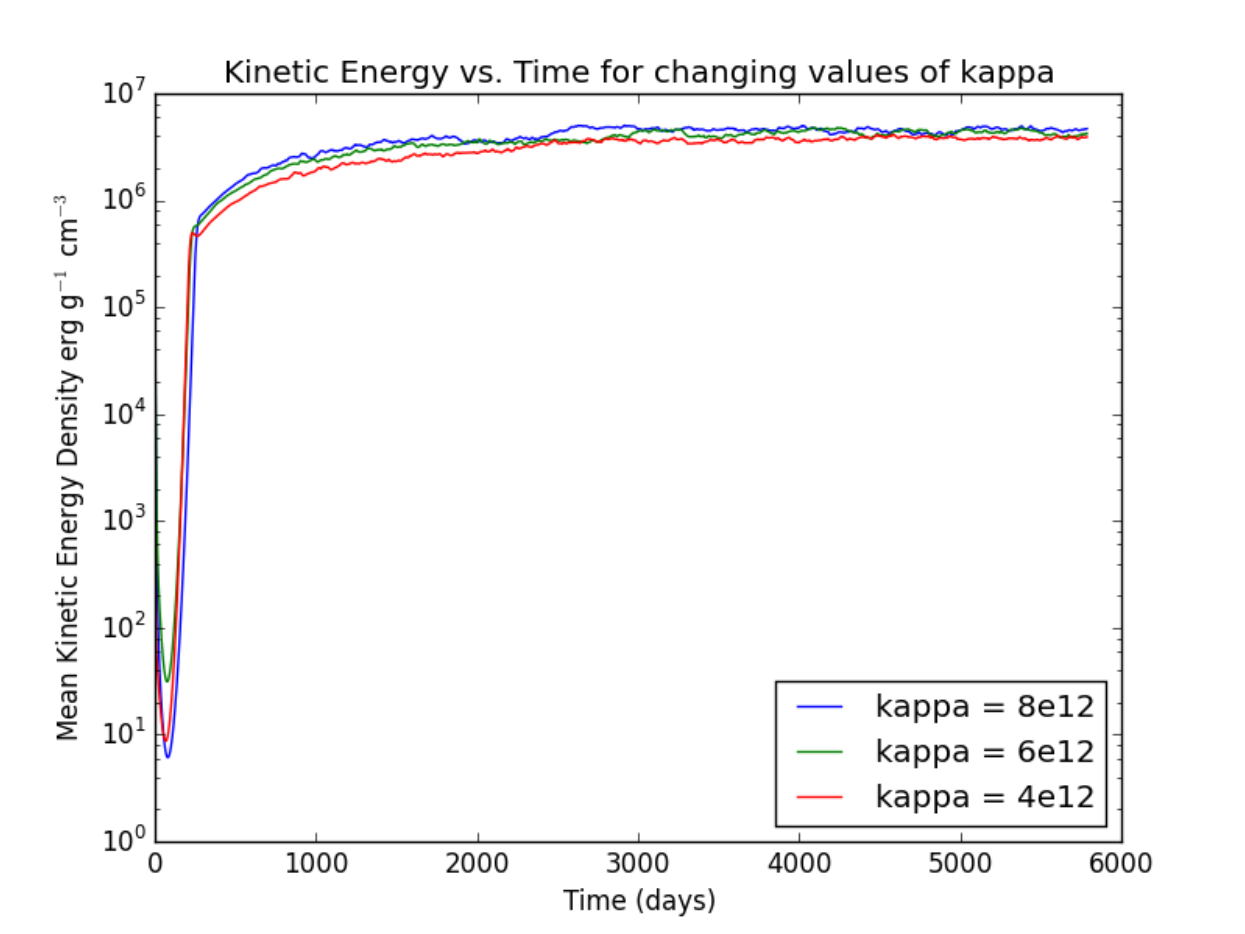


Comparing the new vs. the old

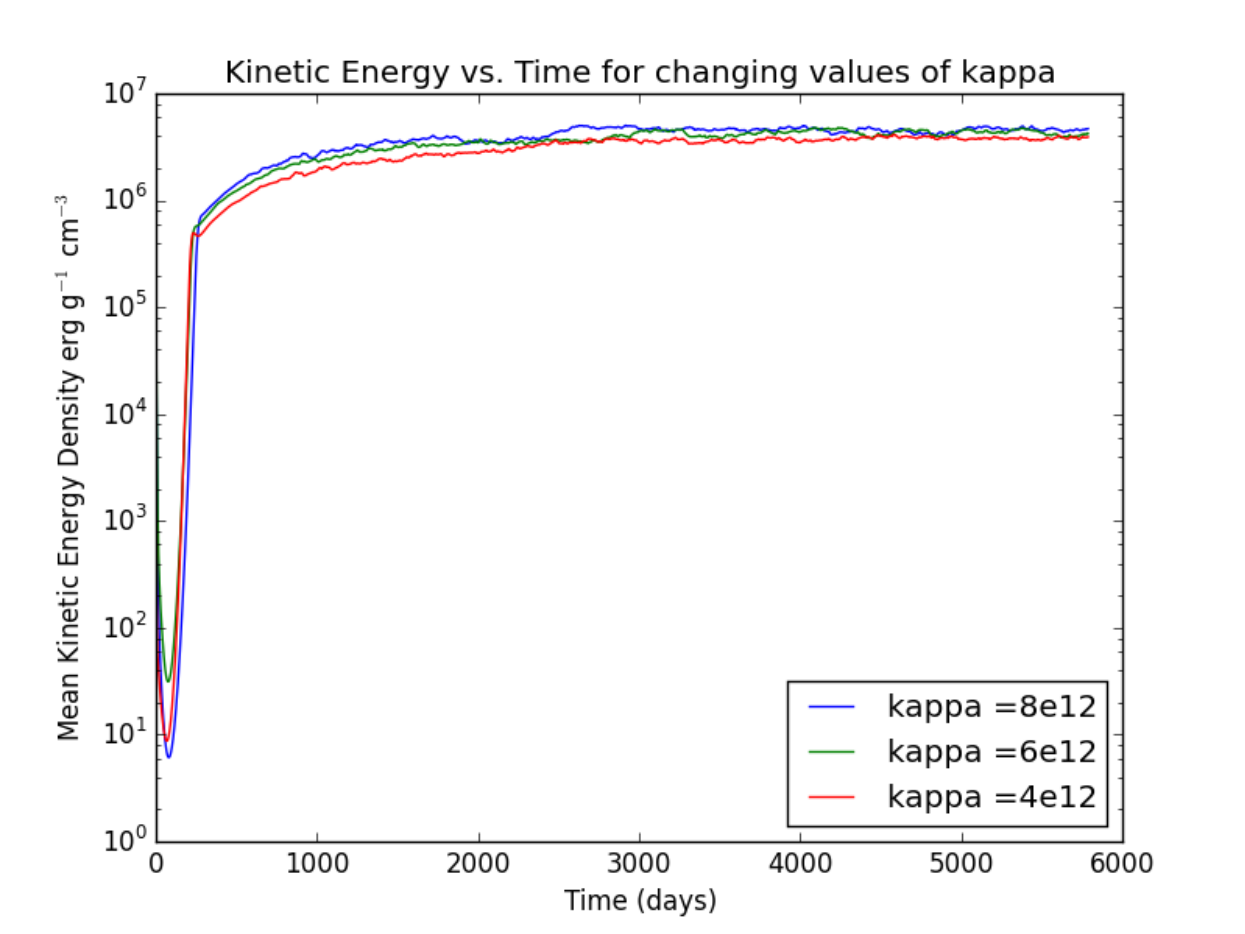
- KE vs time
- Flux balance
- Power spectra
- Convection structure
- $v\downarrow rms$ vs. $1/\kappa$

KE vs. Time

- Without $F\downarrow g$

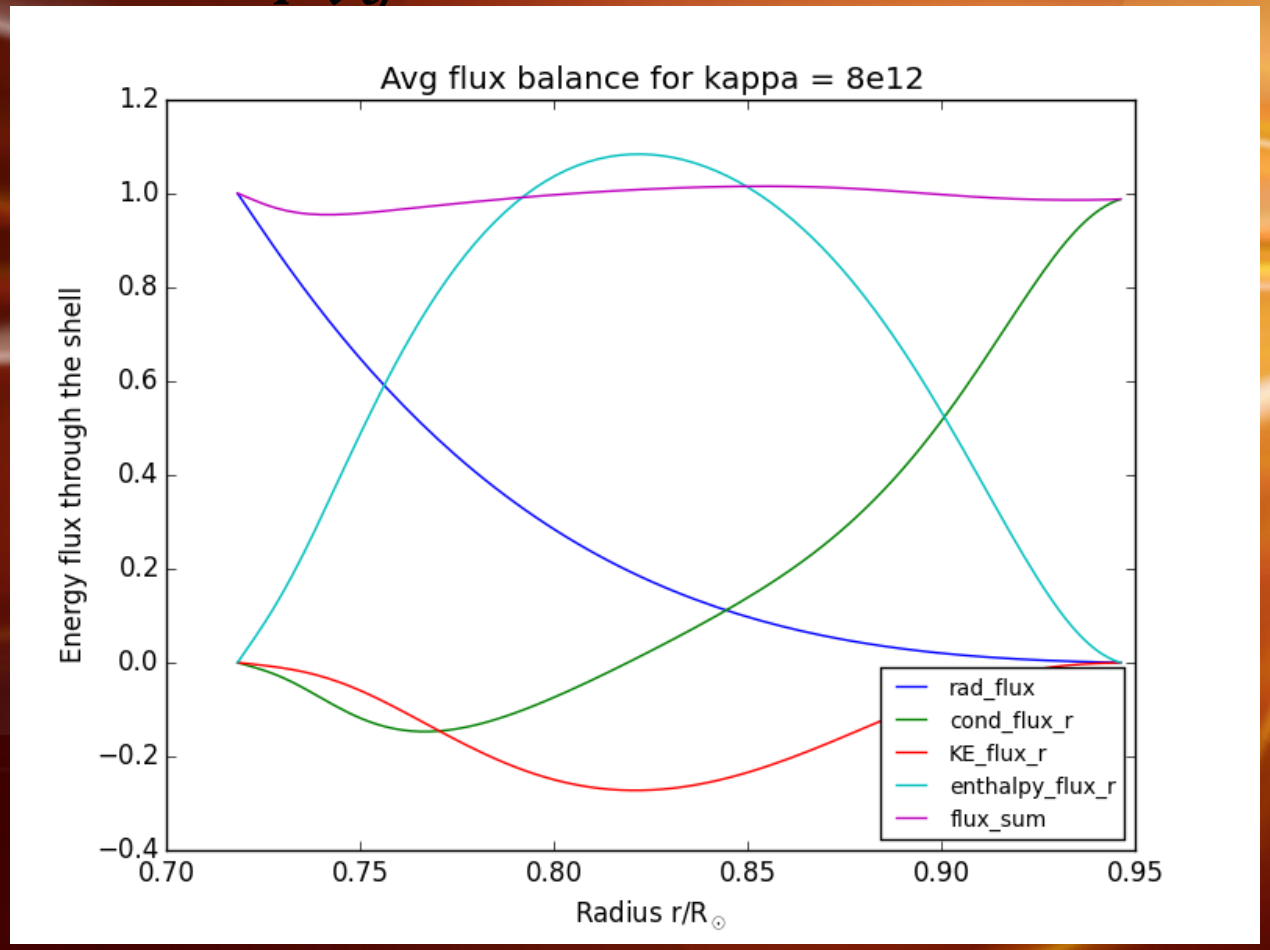


- With $F\downarrow g$

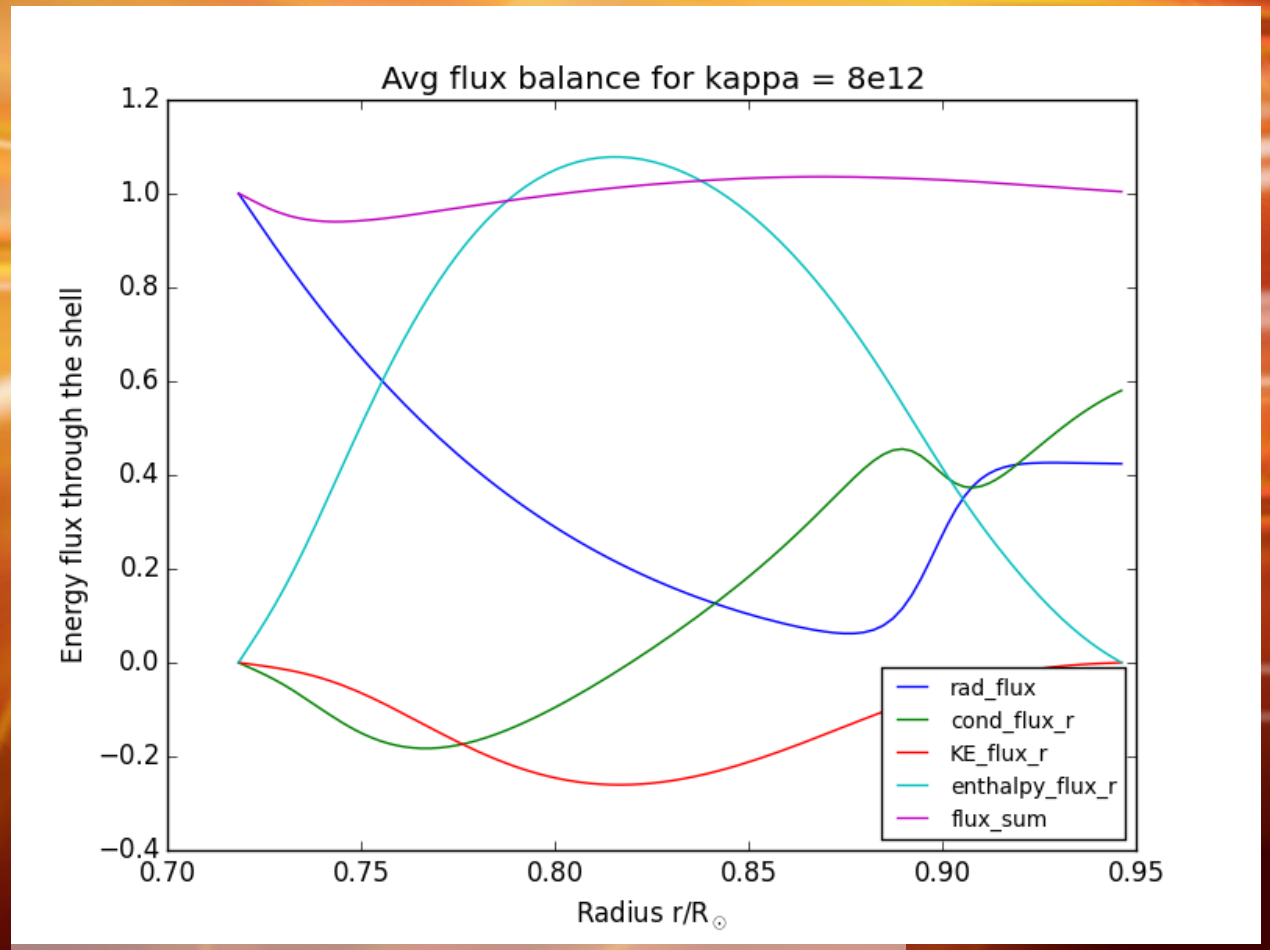


Flux Balances

- Without $F\downarrow g$



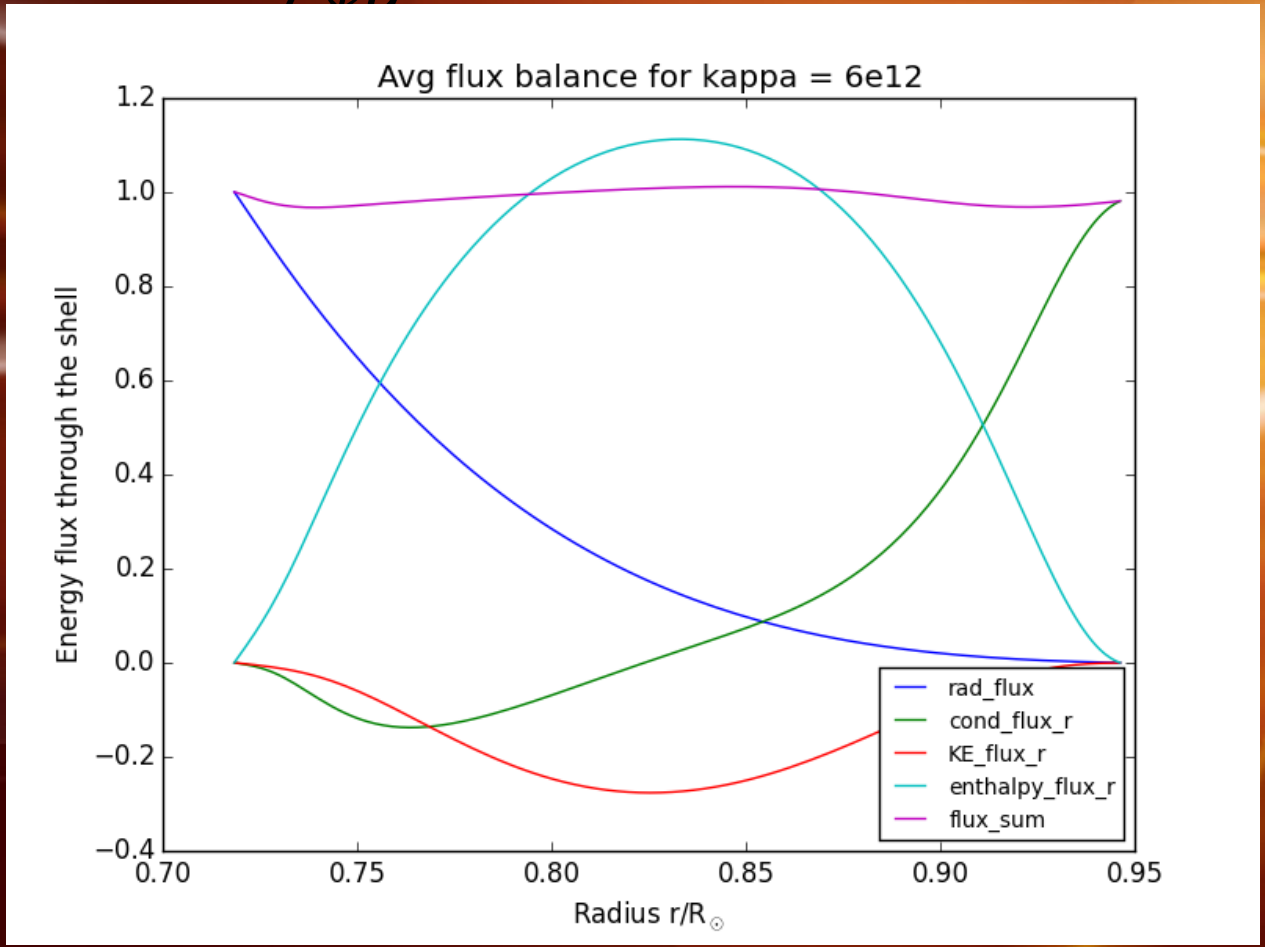
- With $F\downarrow g$



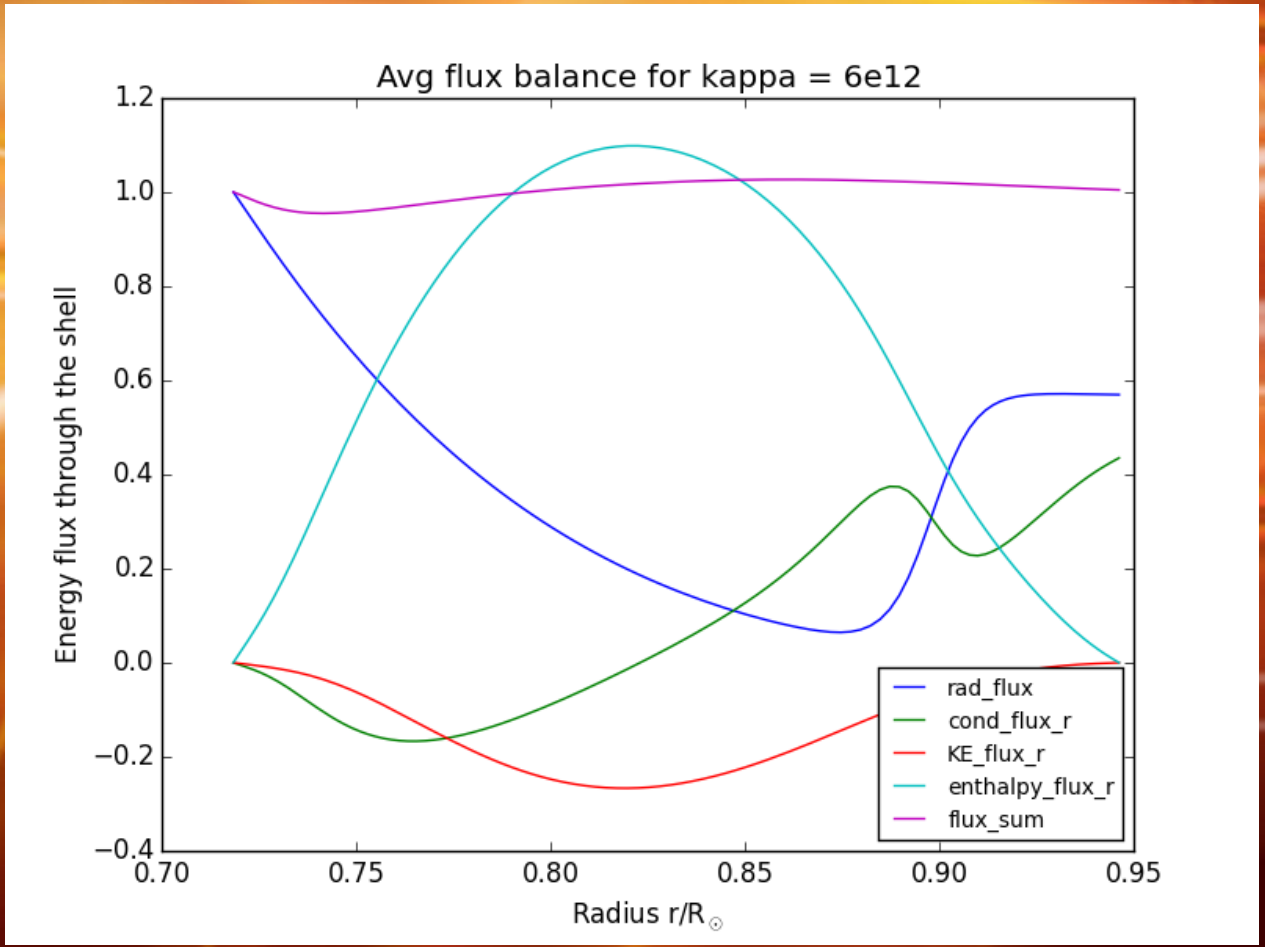
Note: $F\downarrow g$ has been added to $F\downarrow rad$

Flux Balances

- Without $F_{\downarrow g}$

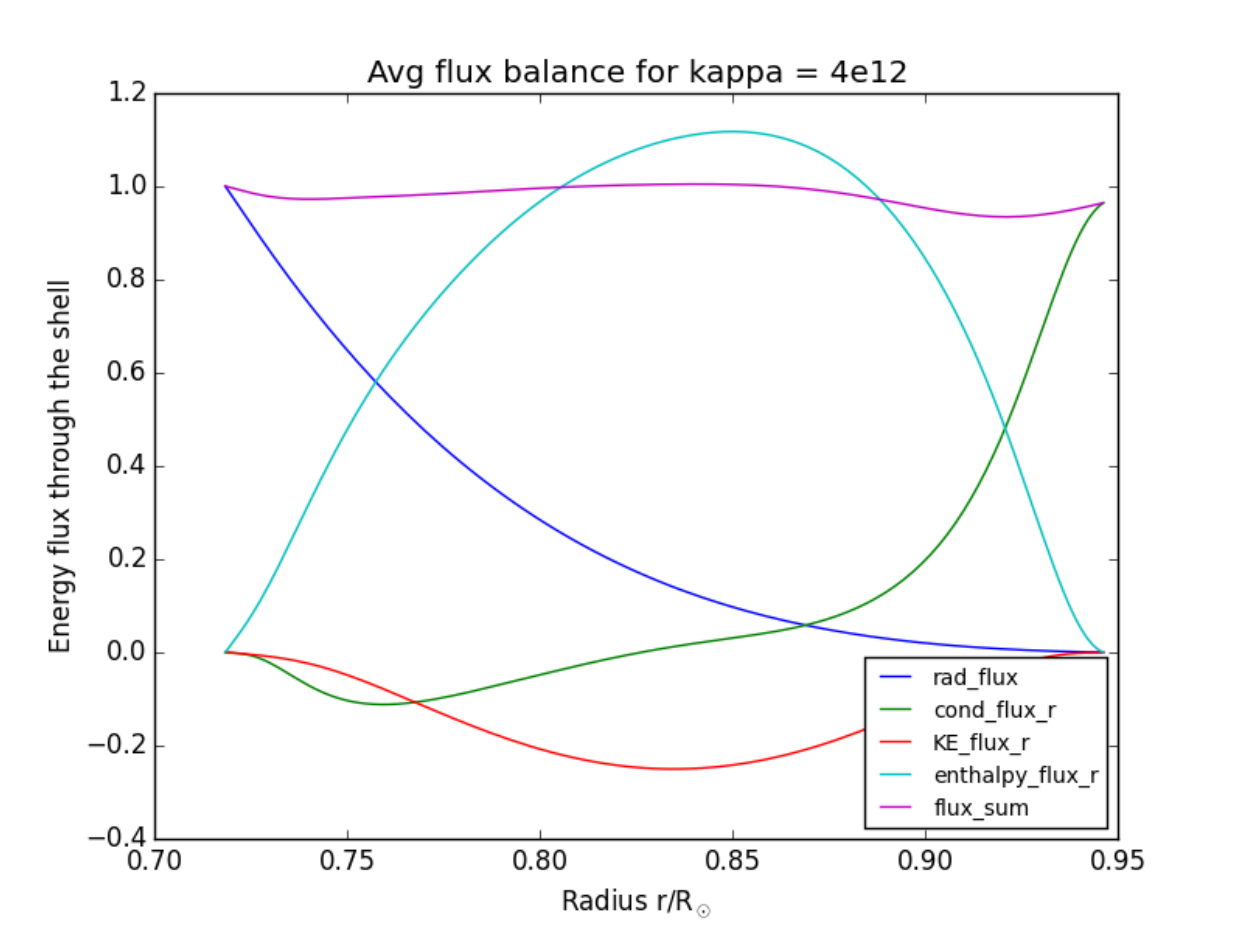


- With $F_{\downarrow g}$

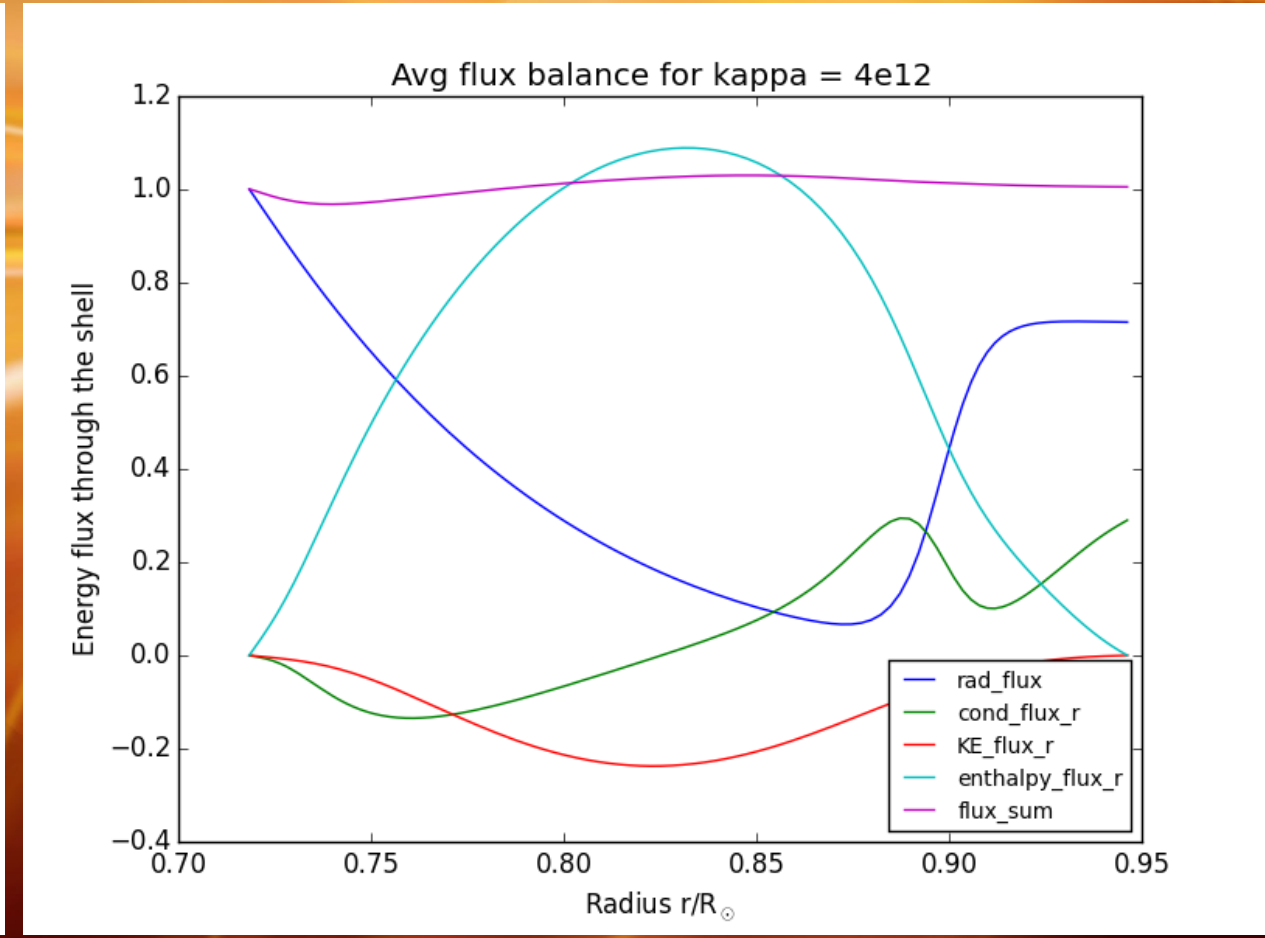


Flux Balances

- Without $F_{\downarrow g}$

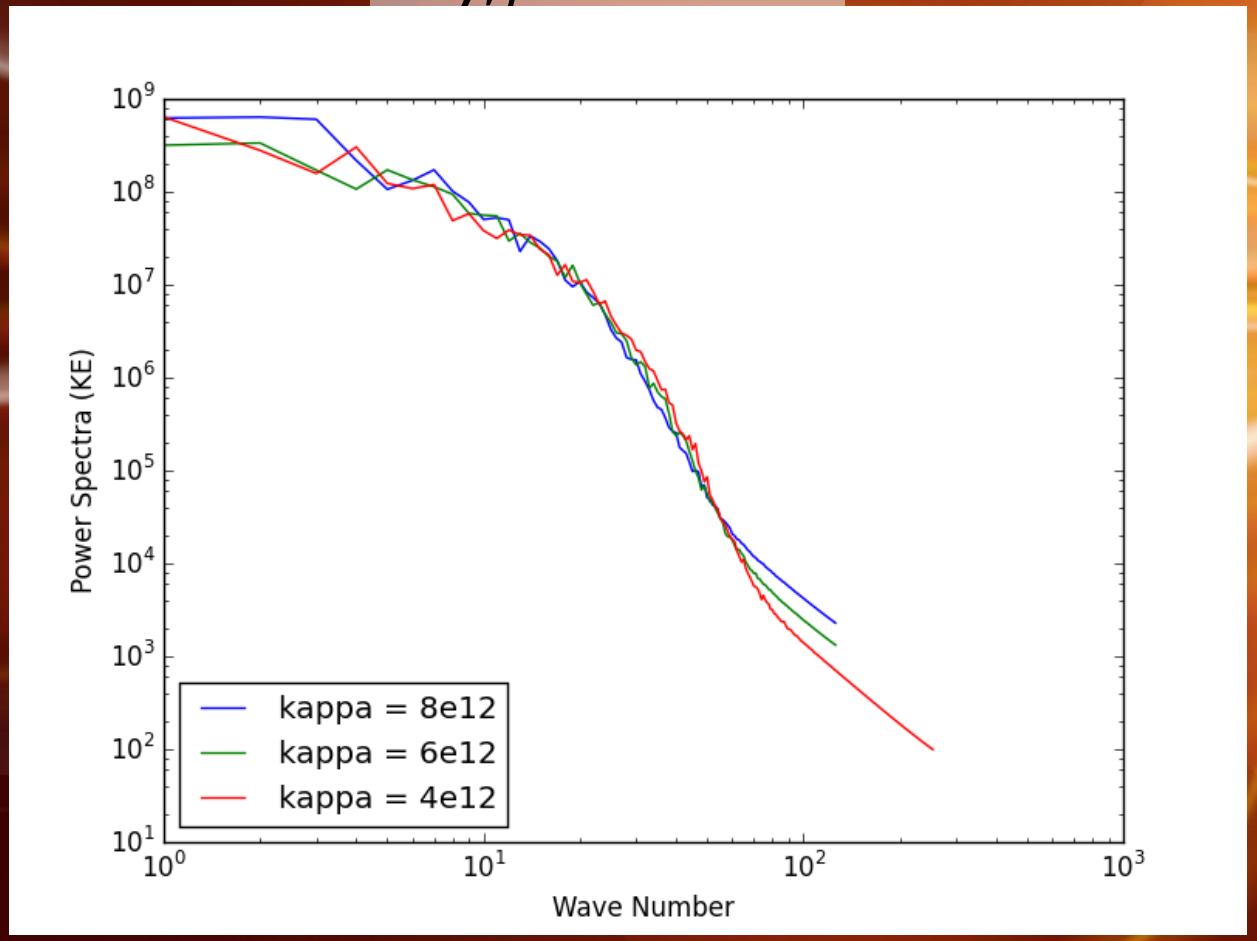


- With $F_{\downarrow g}$

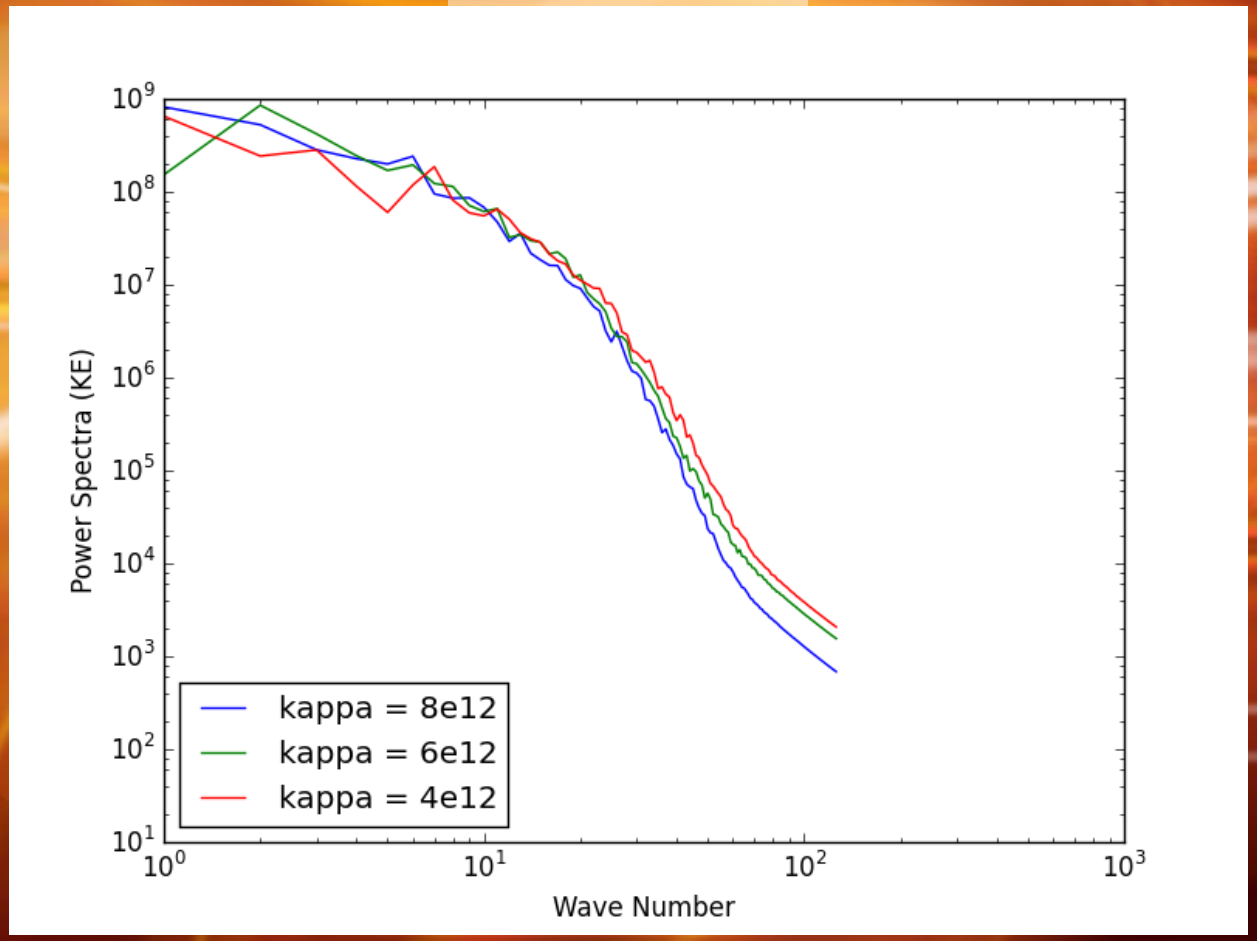


Power Spectra

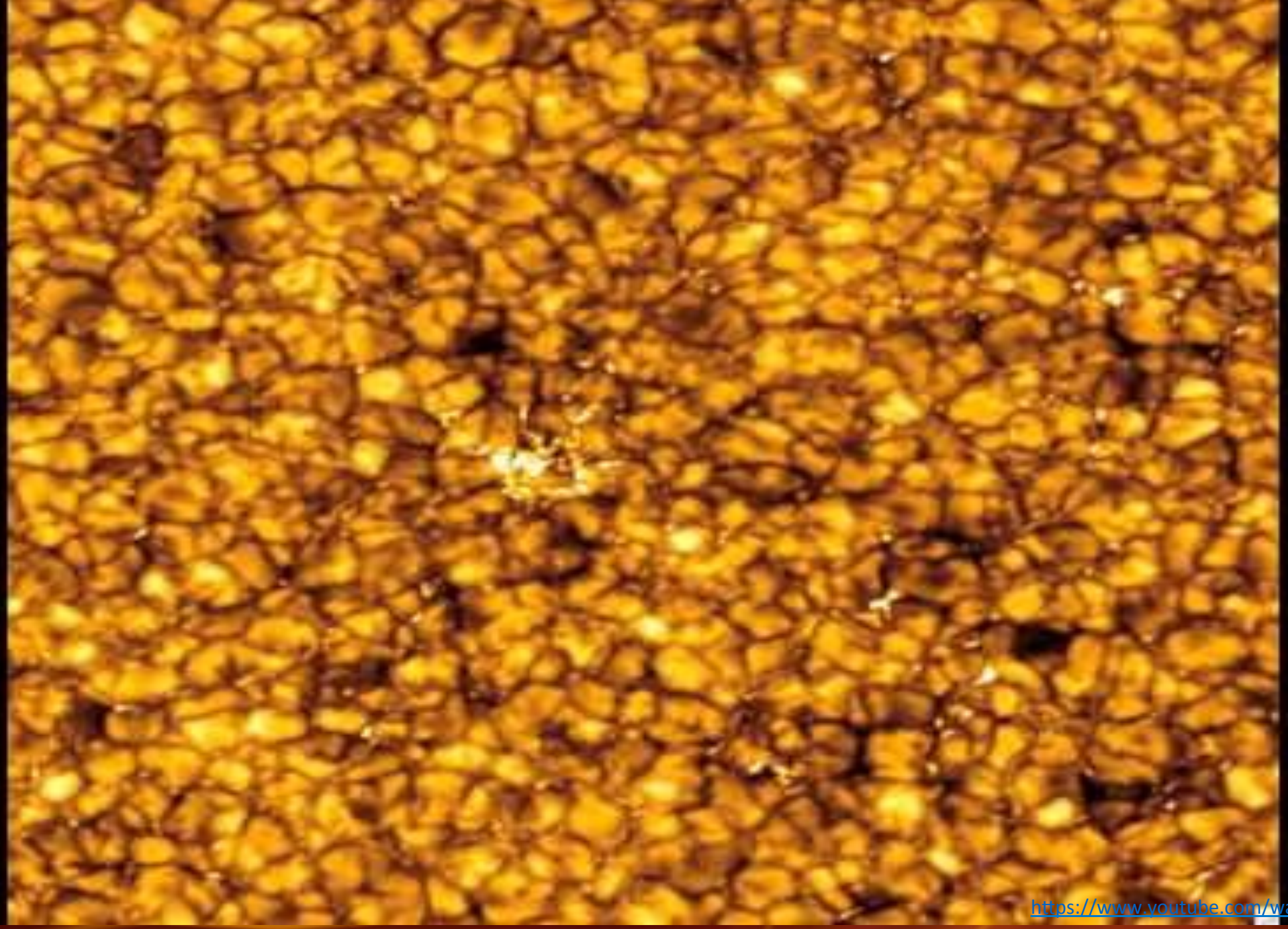
- Without $F\downarrow g$



- With $F\downarrow g$



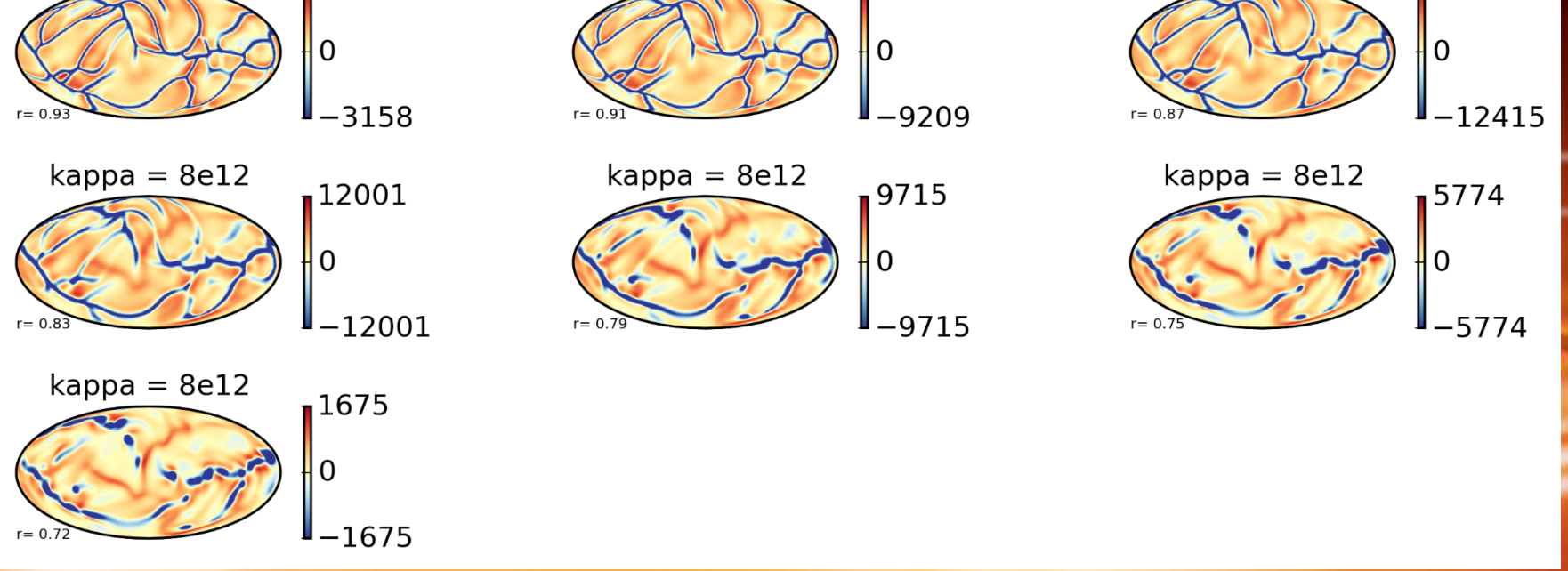
$Wave\ number = 2\pi/L$, Where L is the length of an eddy



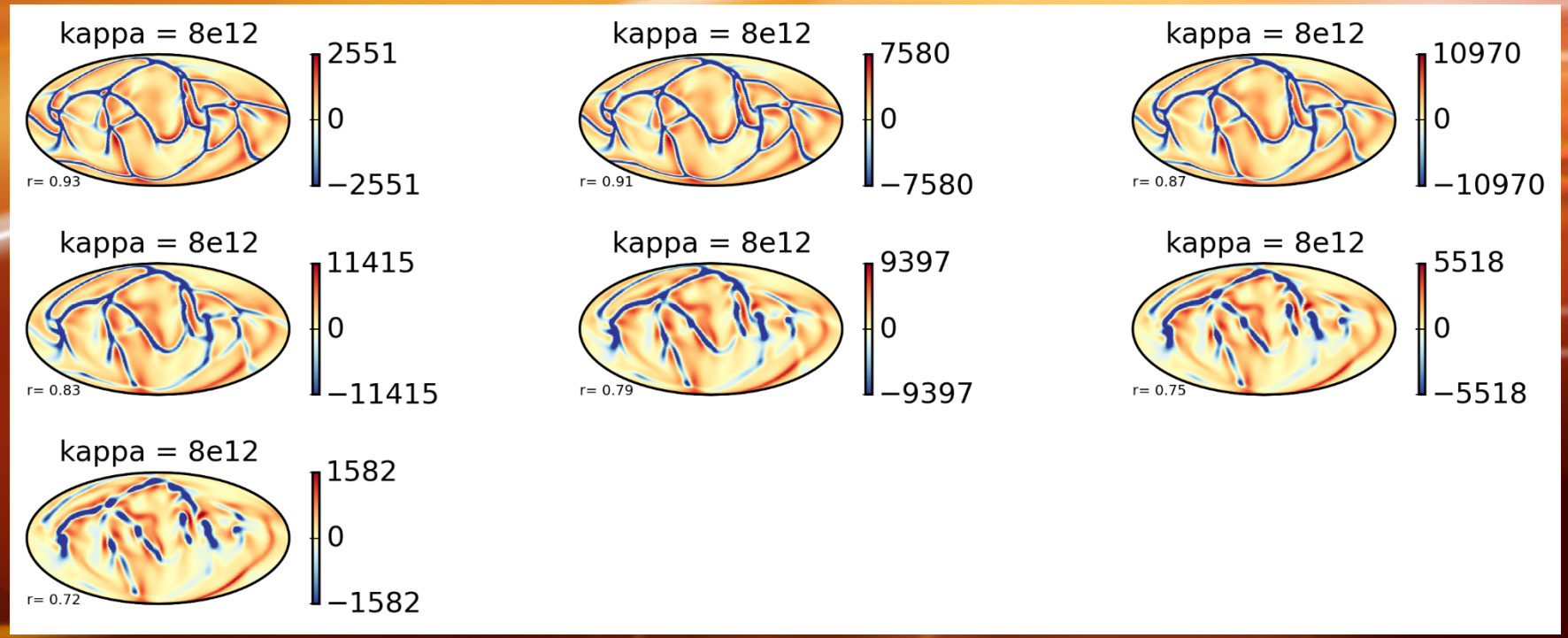
https://www.youtube.com/watch?v=W_Scoj4HqCQ

Convection Structure

- Without F_g



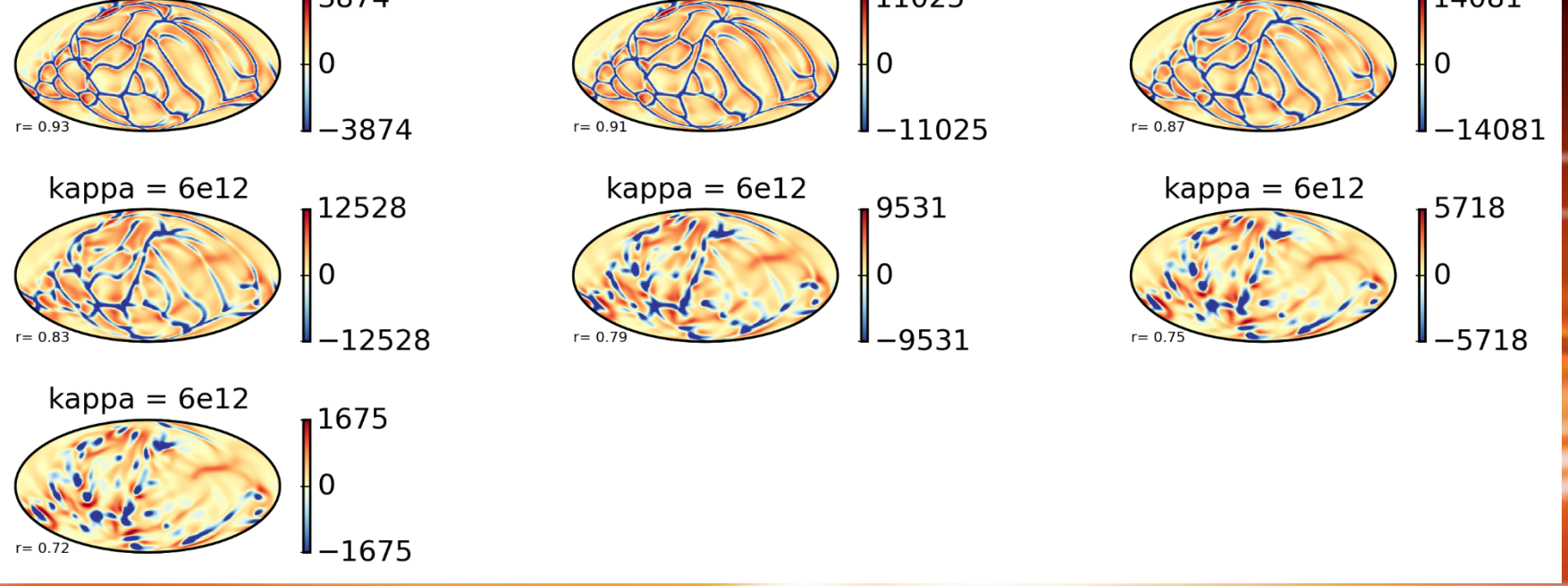
- With F_g



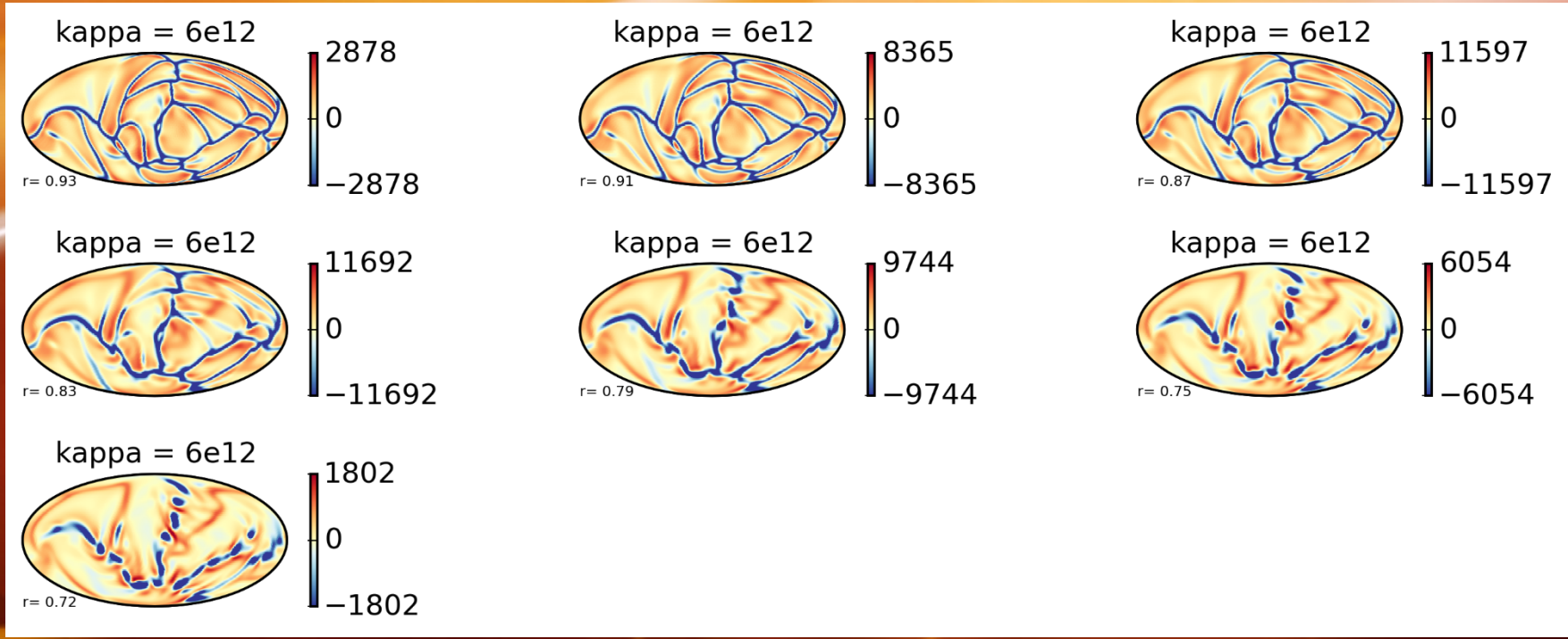
- Colored bars show v_{rms} value (cm/s)

Convection Structure

- Without $F_{\downarrow g}$

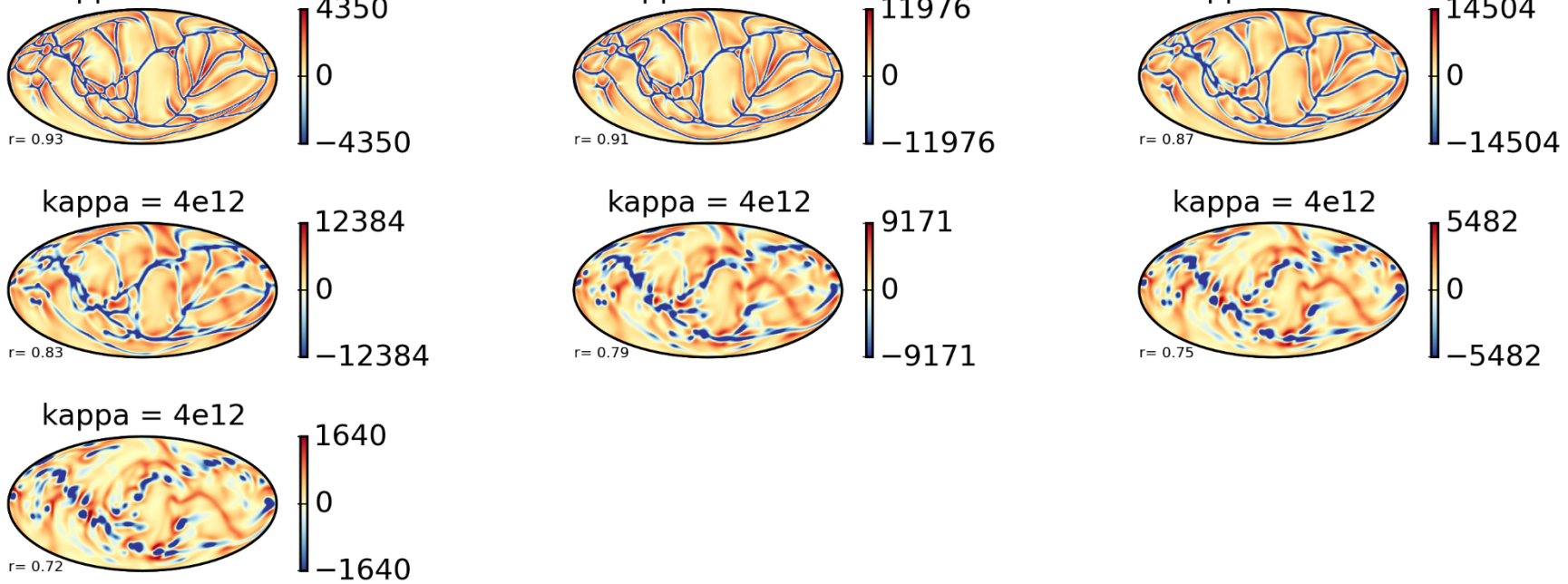


- With $F_{\downarrow g}$

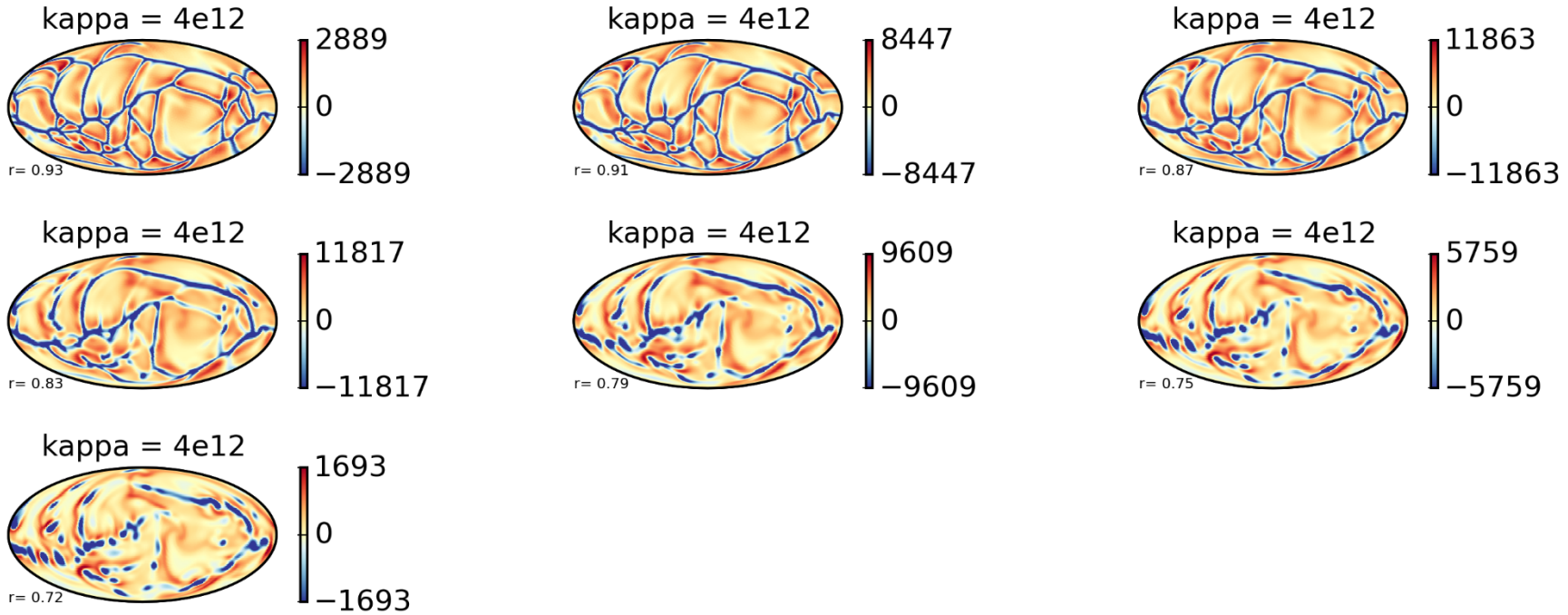


Convection Structure

- Without $F\downarrow g$

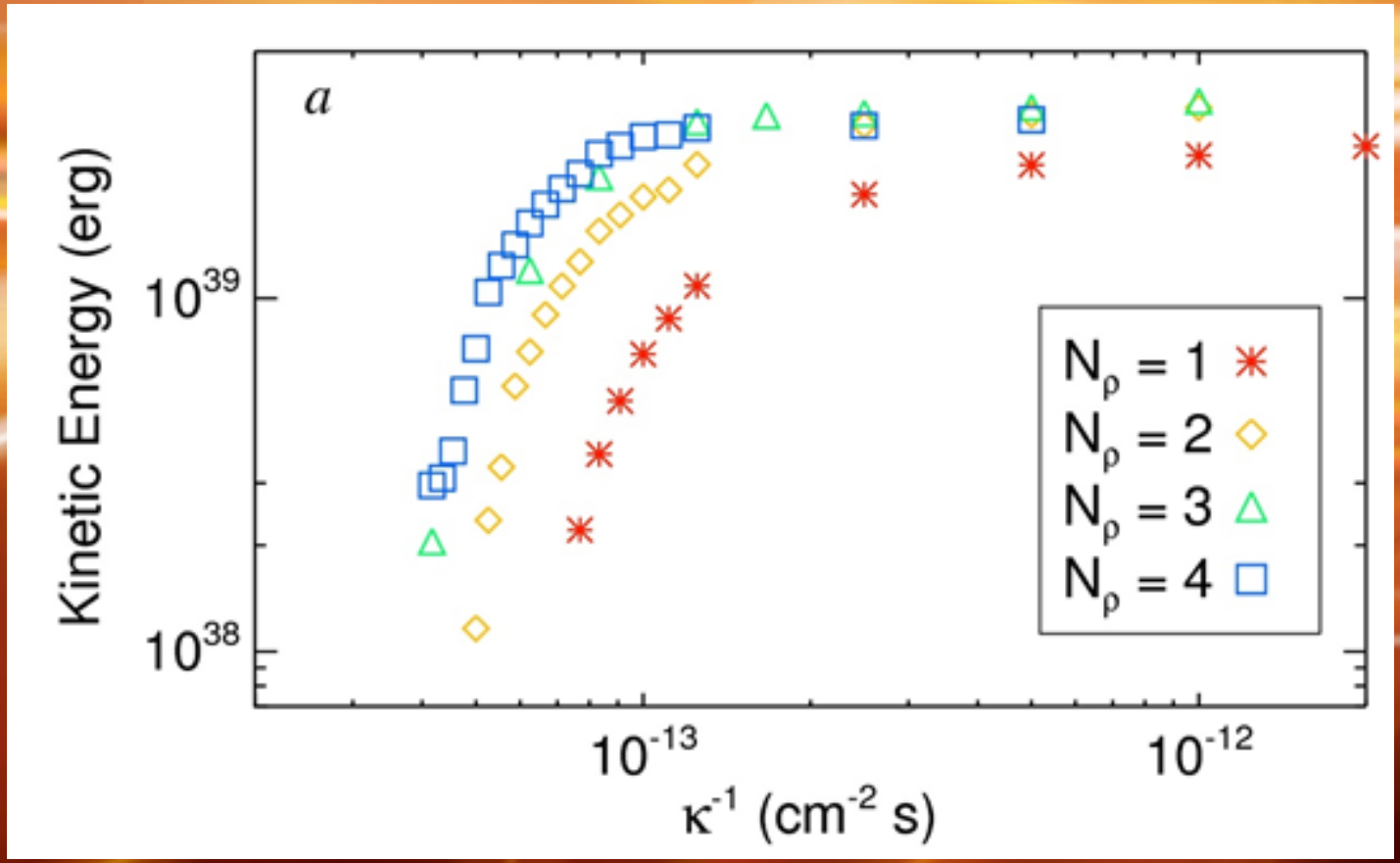


- With $F\downarrow g$



Kinetic Energy vs. 1/kappa

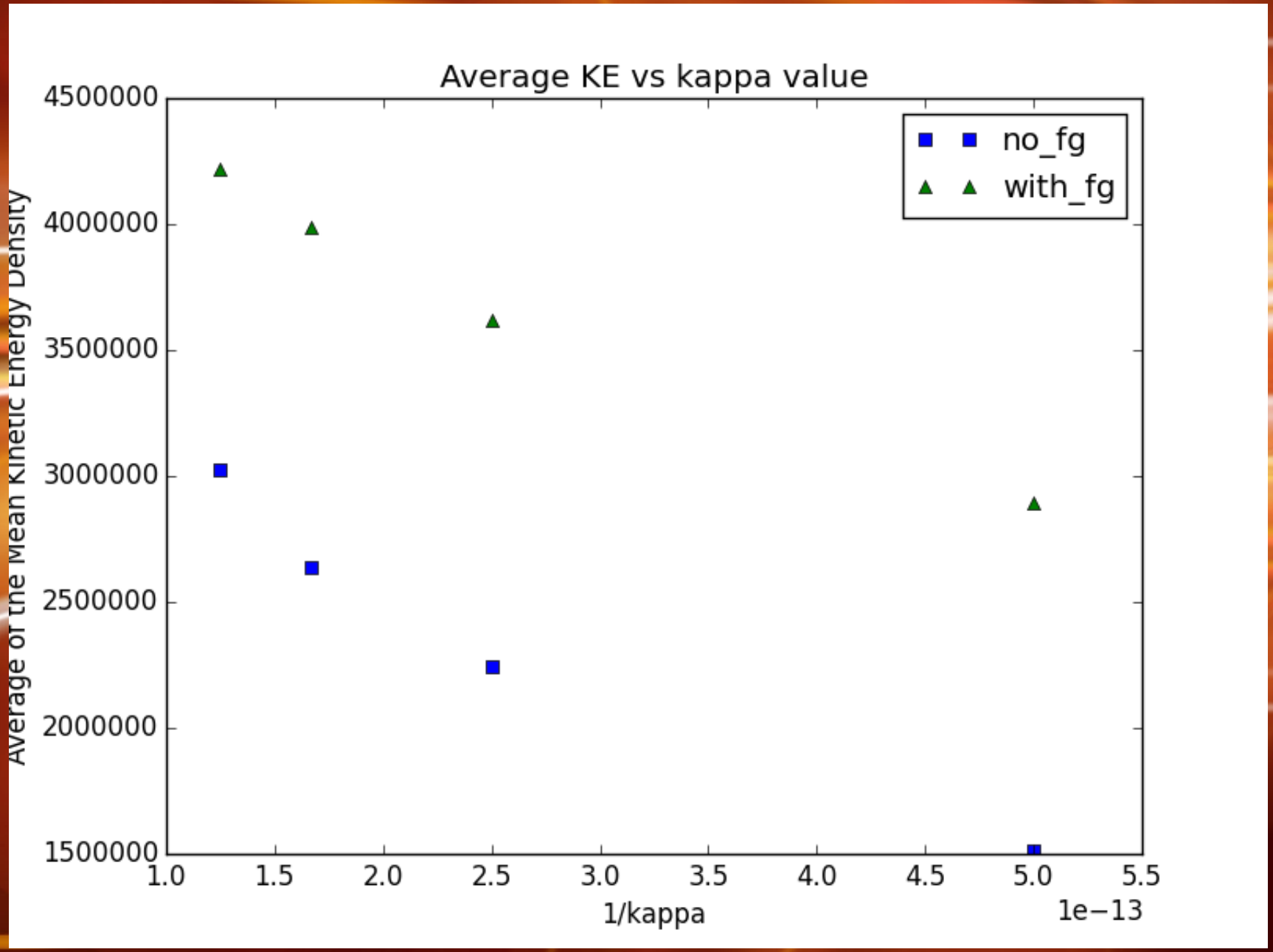
- Recall:
  $R\downarrow a \propto KE \propto 1/\kappa$



Results:

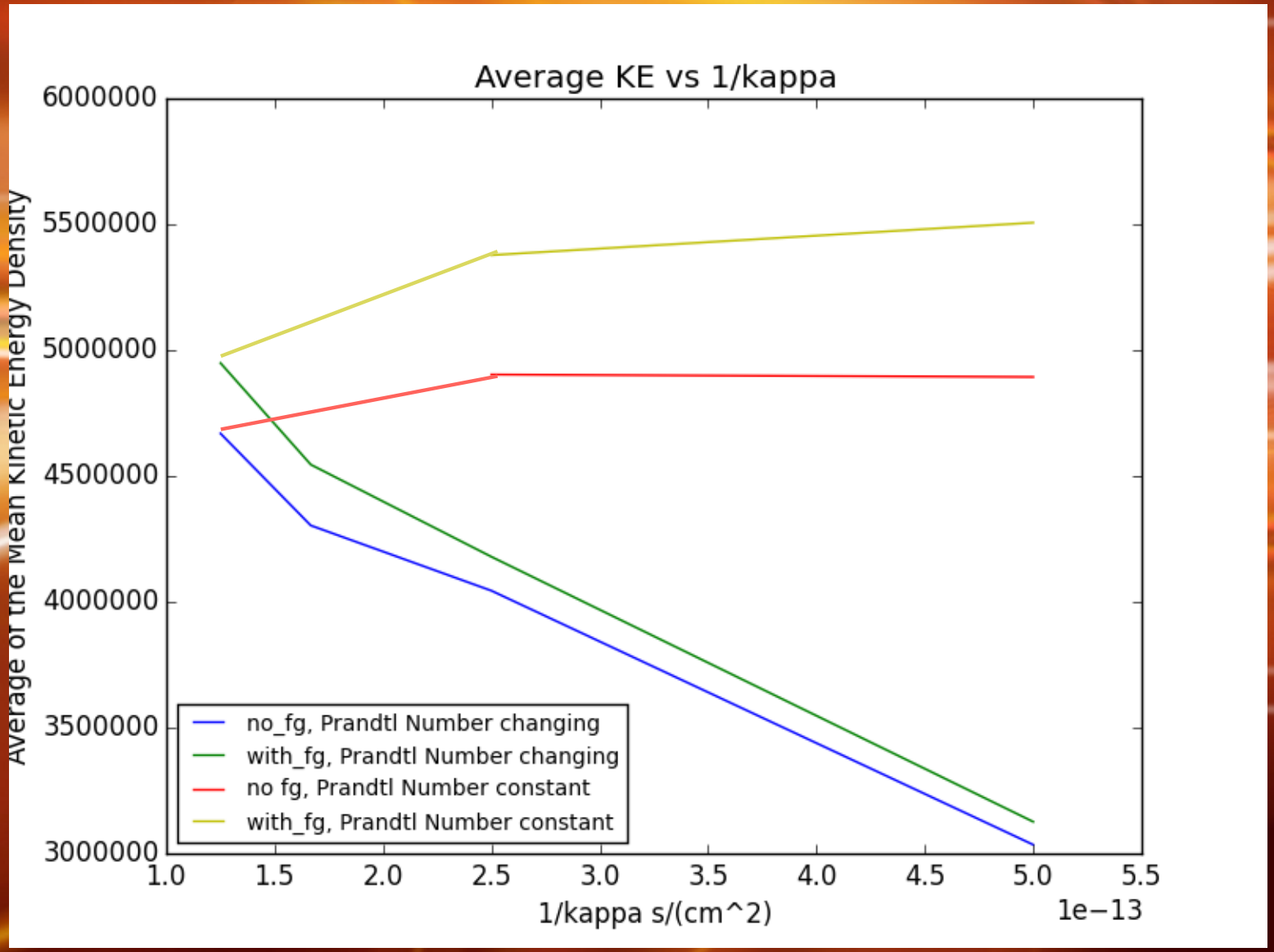
- Opposite from expected

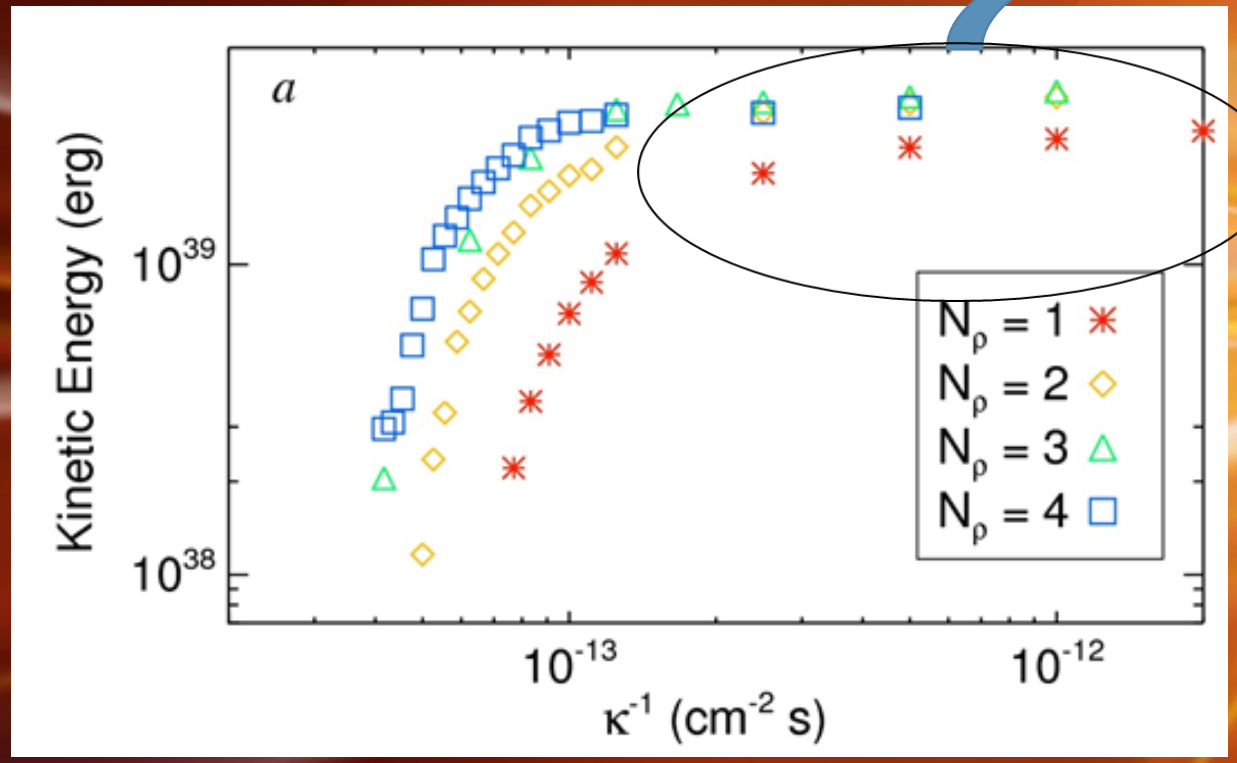
Why?

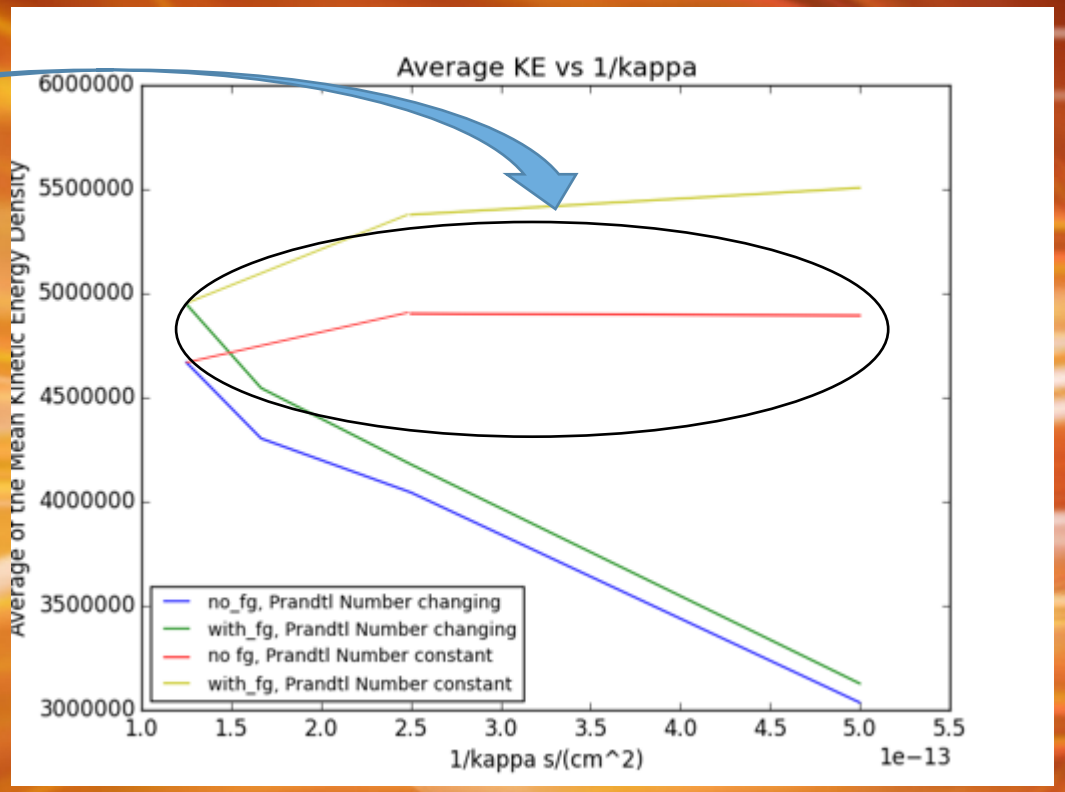


Prandtl Number Effect Tests

- Recall $P\downarrow r = \nu/\kappa$
- Ran tests varying ν and κ to observe the Prandtl number effect on convective amplitude

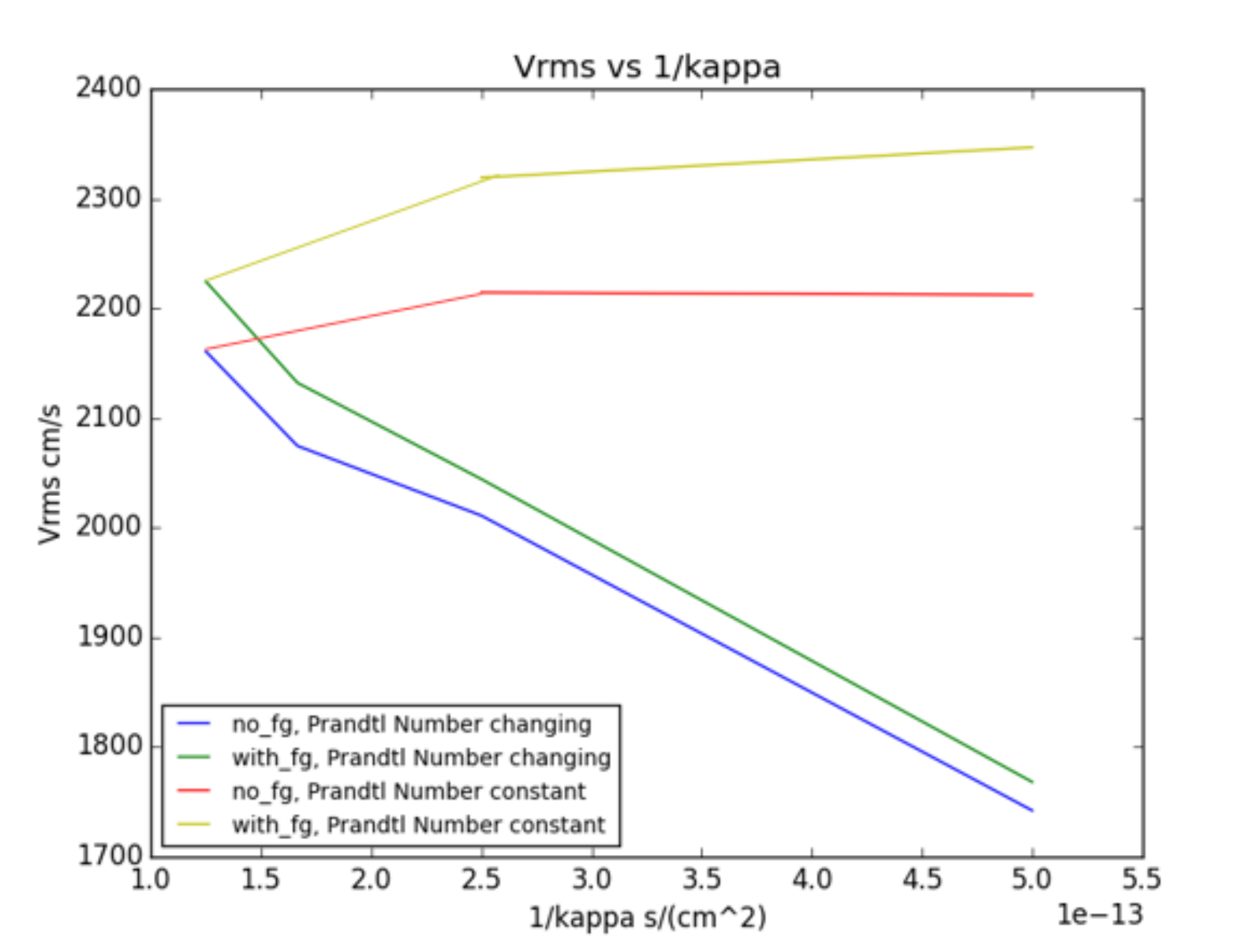




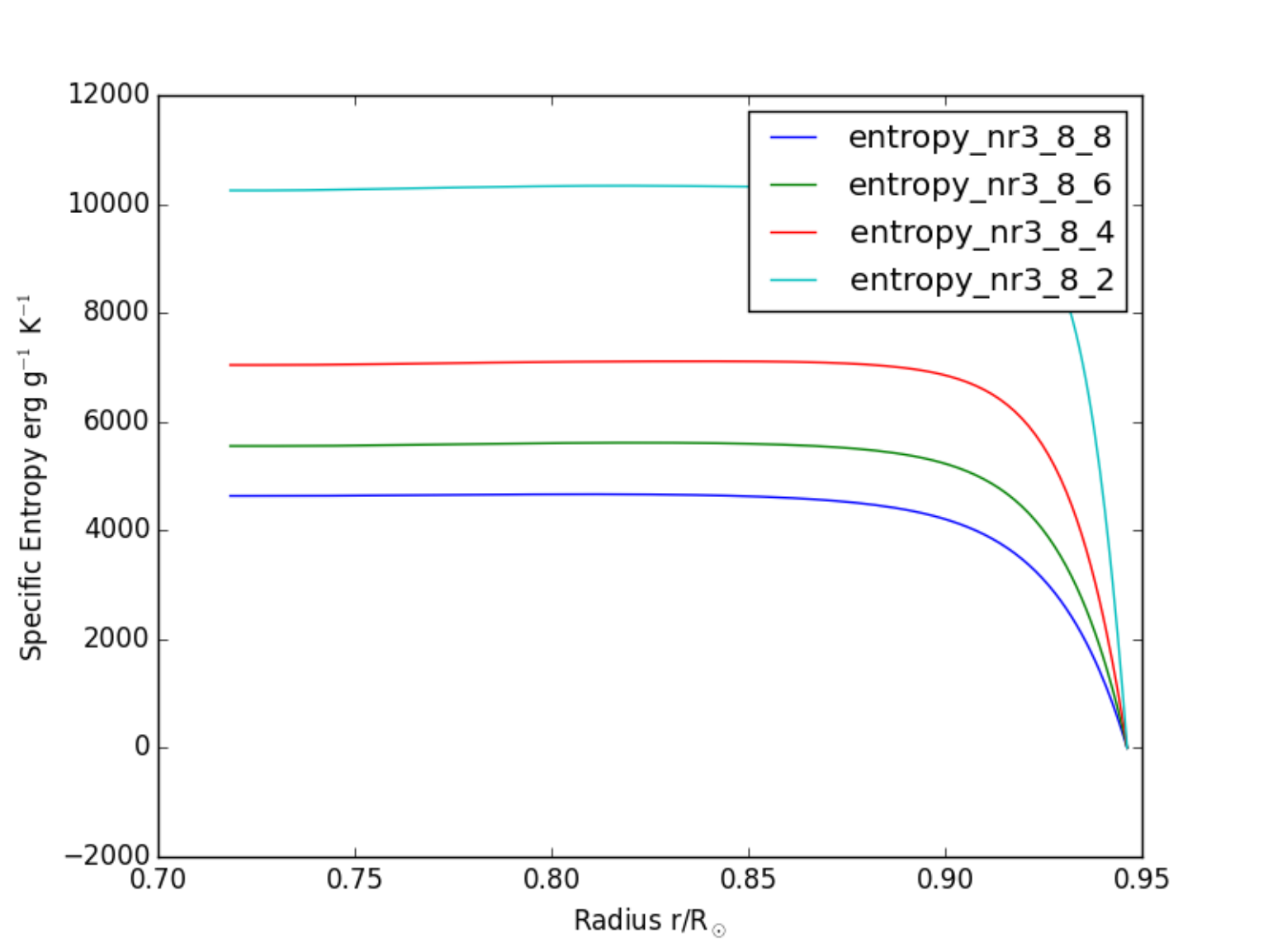


- Models correspond when Prandtl number is constant

- We can plot against the v_{rms} (convective amplitude) instead of KE to get a better look at the Prandtl Number Effect
- The convective amplitude (i.e, v_{rms}) decreases with increasing Prandtl number



Entropies


entropy_nr3_8_8
entropy_nr3_8_6
entropy_nr3_8_4
entropy_nr3_8_2
Specific Entropy erg g^{-1} K^{-1}
Radius r/R$_\odot$
12000
10000
8000
6000
4000
2000
0
−2000
0.70
0.75
0.80
0.85
0.90
0.95

Summary

- The structure and amplitude of deep convection is similar for both models of surface convection considered (conduction and fixed-flux)
  - Suggests that deep convection is not very sensitive to the details of the surface convection
- The Prandtl number makes a big difference on the convective amplitude!
  - As kappa is decreased, holding Pr constant, vrms increases
  - As kappa is decreased, holding nu constant (increasing Pr), vrms decreases! (surprise!)
  - The second situation (increasing Pr) is very promising because vrms in current convection simulations might be too big and the real kappa of the Sun is very small

What's next?

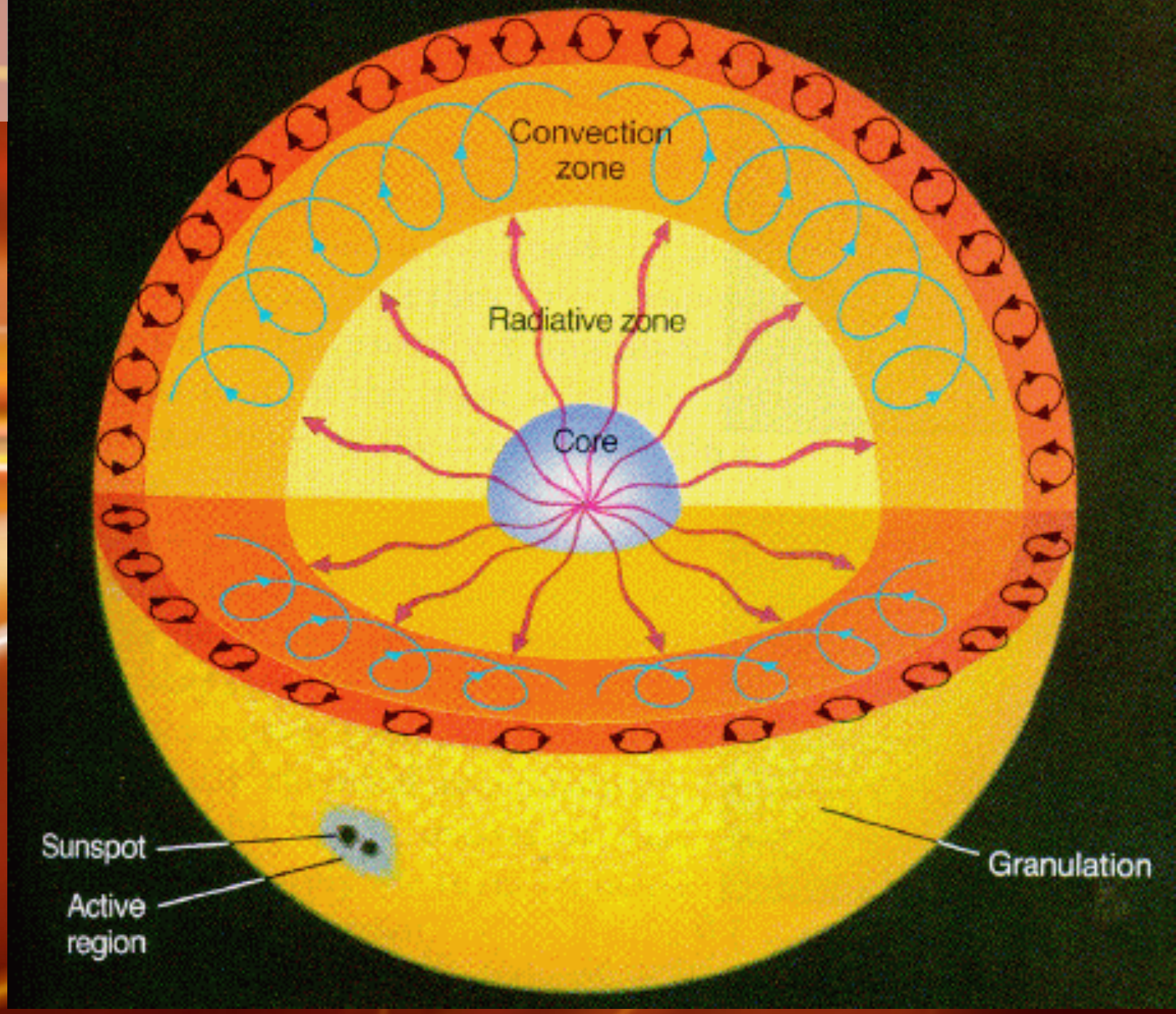
- Testing the following Hypothesis:
  - $T\uparrow$, *in the middle of the convection zone, increases as you decrease* κ.
  - Where $F\downarrow e \propto v\downarrow rms\, T\uparrow \approx F\downarrow\odot$
  - If true, explains decreasing $v\downarrow rms$ with increasing Prandtl number

Questions?

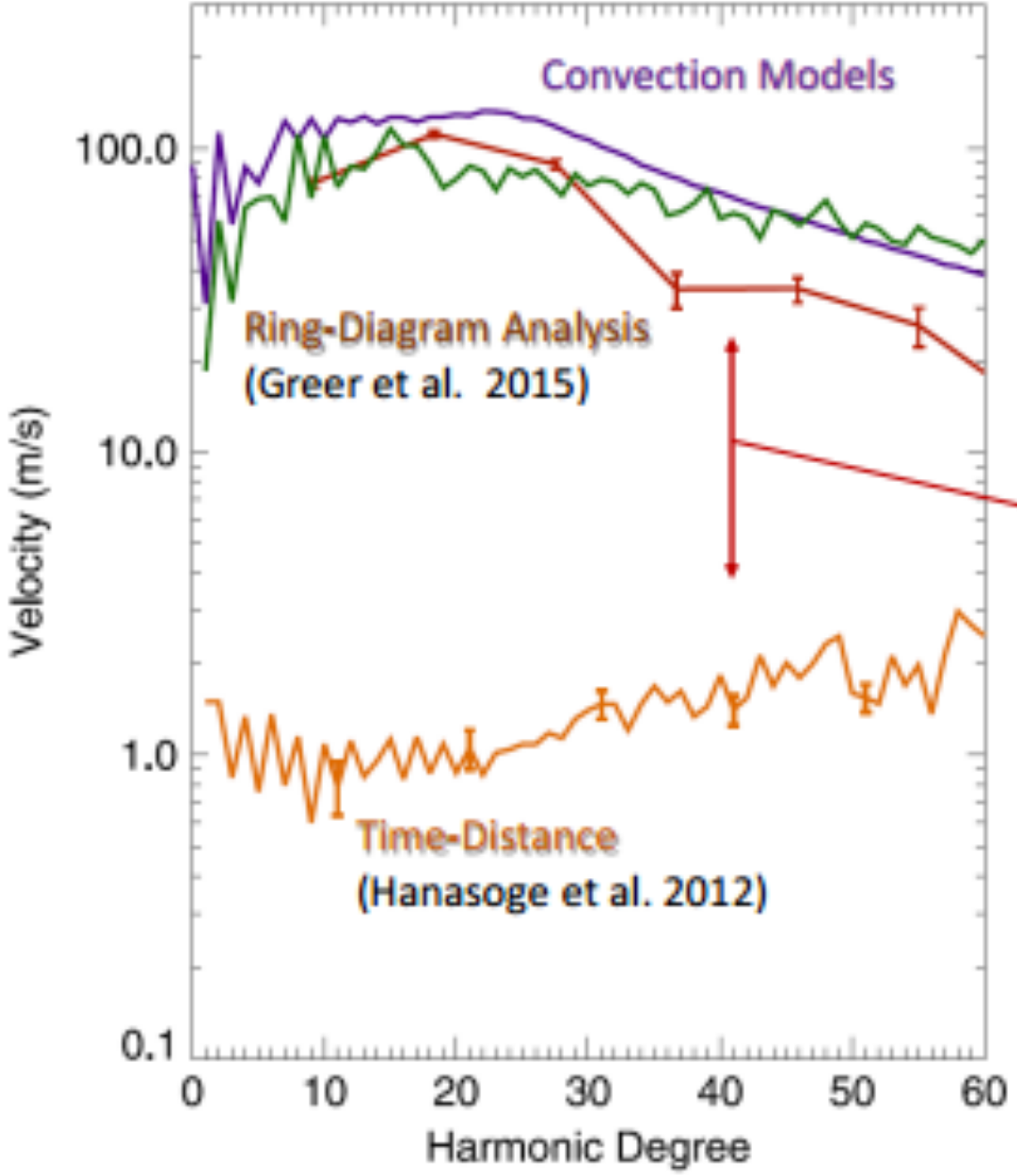
Flows in the Sun

- The Solar Dynamo generates magnetic fields from the flows:
  - Convection
  - Differential rotation
  - Meridional Circulation

Fusion
mass → radiation & thermal energy
Convection
thermal Energy → kinetic energy
Dynamo
kinetic energy → magnetic energy



- Tips: highlight important words in different colors



The whole story?

Maybe not…

This is exciting!

Disagreeing Observations

Something new under the Sun?