Comment on “Parallel electric fields in the upward current region of the aurora: Numerical solutions” [Phys. Plasmas 9, 3695 (2002)]

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In a recent paper Ergun *et al.* [Phys. Plasmas **9**, 3695 (2002)] endeavor to obtain realistic 1-D numerical solutions of the Vlasov-Poisson equations for an oblique strong double layer in the space environment, given a set of data based constraints. Here it is argued that their numerical methods are overly simplified and that a fully dynamic, multi-dimensional analysis is needed even for the lowest-order solution.

The objective of Ergun *et al*.[1] is to derive self-consistent quasi-neutral particle distributions in an oblique strong double layer while incorporating particle and electric field data obtained with the Fast Auroral SnapshoT (FAST) satellite, specifically those illustrated in Figure 7 of a companion paper[2]. As a first step,[1] a smooth function $d\Phi/dz$ is fitted to the measured magnetic field-aligned electric field along the satellite path, where the path is parallel to the horizontal x-axis and perpendicular to the magnetic field-aligned z-axis. Assuming that the double layer is planar with its electric field vector $(E_x,0,E_z)$ at 63° angle with respect to the upward directed z-axis (while neglecting the measured E_x), this yields a potential $\Phi(z)$ that is monotone, thus greatly simplifying the problem at hand. Given this $\Phi(z)$, the authors proceed to find a suitable combination of ion and electron velocity distributions that allows quasi-neutrality throughout the z-range of the oblique electric field structure, including approximated measured distributions.

The principal free parameter is the velocity distribution of the low-energy electrons, which enter from below and become reflected within the upward electric field. Ambient electron measurements are only available for energies above those of satellite photo electrons (for energies above 100 eV). Resulting particle distributions versus z with different assumptions about the up-flowing ion mass composition are

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shown in Figures 6, 7 and 8 of Ref. 1. The underlying numerical approximations lend themselves to the following three points of commentary.

1. While the quoted particle-in-cell numerical simulations [3] have yielded scale sizes in Debye lengths that are virtually independent of both double layer angle and ion gyro radii, other results, mainly analytical, suggest that oblique scale sizes are controlled by ion gyro radii.[4,5] In any case, the simulated oblique structures have been found to substantially increase the magnetic moment of ions entering from the high potential side when these ions are treated as test particles.[6]

2. The assumed infinitely magnetized up-flowing protons in Figure 6 of Ref. 1, having started out at $\Phi = 0$ as a drifting Maxwellian with $v_d \approx 50$ km s^{-1}, $v_d/v_{th} = 2$ and a density of 4.0 cm^{-3}, are reduced in density by half at $\Phi \approx -39$ V due to acceleration. The matching reduction in "cold" ionospheric electron density (hotter particles contribute ~ -0.6 e cm^{-3}) is a factor of $1.4/3.4 \approx 0.4$. Since the density of a reflected near-Maxwellian electron distribution in a static negative Φ falls of as

$$n(\Phi) \sim n_0\, exp(e\Phi/kT) \qquad (1)$$

this factor 0.4 requires that $kT \sim 43$ eV. This is unrealistic and disagrees with the temperature of ~3 eV listed in the figure's table. However, with only 3 eV temperature these electrons would have lost half their density at $\Phi \approx -2$ V, where the up-flowing protons would still have a density of about 3.7 cm^{-3}, leaving a net positive charge ≈ 1.4 e cm^{-3}. Where the protons have been reduced to half their initial density, at $\Phi \approx -39$ V, the "cold" electron density is negligible, and the net positive charge is again ≈ 1.4 e cm^{-3}, having been larger in between, up to a maximum of 2.4 e cm^{-3}. Using the planar approximation

$$\Delta\Phi \approx -q(\Delta s)^2/(2\, \varepsilon_0) \qquad (2)$$

where $\Delta\Phi$ is the potential difference of -37 V, q is an average charge density of 1.9 e cm^{-3}, Δs is the distance into the charge layer, parallel to the electric field vector, and ε_0 is the vacuum dielectric constant, the distance Δs comes to about 46 m. The corresponding increase in absolute electric field strength is ΔE ~1.6 V m^{-1}, and the increase in perpendicular ($\cdot sin(63°)$) electric field is $\Delta E_x \sim 1.4$ V m^{-1}. A proton traveling upward at the initial 50 km s^{-1} drift speed over a distance of 46 m /cos(63°) will thus experience an increase in ambient perpendicular electric force of $e\Delta E_x \sim 1.4$ eV m^{-1} in a mere 2 ms. This increase is almost four times larger than the magnetic force $ev_{\perp}B \approx 0.36$ eV m^{-1} on a proton with typical 3 eV gyro energy ($v_{\perp} = v_{th}$ =50/2 km s^{-1} assumed) in the local $B \approx 14{,}340$ nT, and it occurs over less than half a gyro period ($\tau_g \approx 4.6$ ms). Since the protons are in fact being accelerated upward and tend to deflect in the direction of the electric field, the actual rate of increase in the ambient perpendicular electric force is even greater than that. This all conspires to violate the protons' first invariant.

3. The adjustment for polarization and **E×B** drifts of the up-flowing ions in Figures 7 and 8 of Ref. 1 is based on the 2-D geometric outline in Figure A1 of same paper and assumes that the drifts preserve the first invariant. The inferred lowest order adjustment includes an initially faster reduction in ion density with decreasing Φ near the ionospheric side. This would seem to abate the problem raised in point 2 above, but it is misleading. The lowest-order effect of polarization drift in the assumed static field configuration is to divert upward ion flow along the z-axis, a flow associated with decreasing ion density, into horizontal **E×B** drift along the y-axis (third dimension), where density is preserved ($\partial n/\partial y = 0$). The net effect is a small increase in positive charge at fixed Φ, not a reduction. This can be illustrated in kinetic terms in the manner of the attached Figure 1, which assumes time-invariant fields (and uniform magnetic field).

With regard to panel b of this figure, the following should be noted. While the preservation of magnetic moment does not preclude some spatial variation of the electric field over the course of a single gyration, the **E×B** drift of each ion at a given point is defined by the local electric field alone. Adding a common (lowest-order) polarization drift velocity in the x-direction leaves the inequality Y′ > Y intact. To make the ion density fall off more rapidly with decreasing potential Φ, initially, and better match the electron density in Eq. (1) above, it is necessary to reduce the local ion velocity volume by removing ions with certain pitch and gyro-phase angles from potential Φ (*cf.* Figure 1 in Ref. 5). This, however, still means that the ion magnetic moments are in fact altered, because it requires that ions starting from zero potential at different gyro phases experience substantially different electric forces on the way.

ACKNOWLEDGMENT

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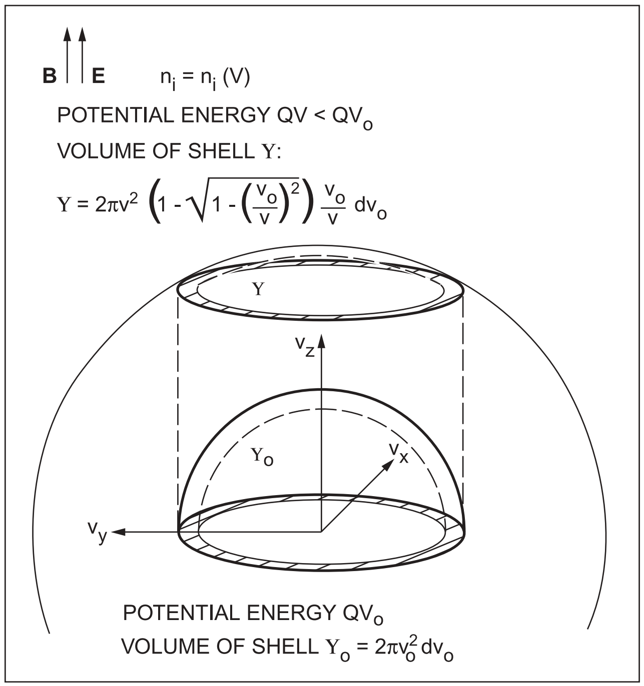
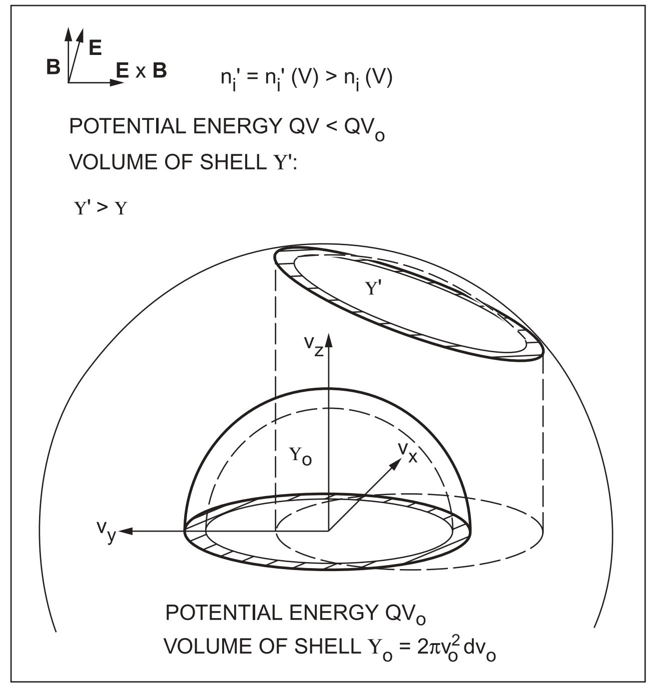
FIG. 1. (a) Transformation of ion velocities in a static parallel electric field, given a narrow range of (arbitrary) initial energy. (b) Same for oblique electric field, assuming the magnetic moment is preserved (adapted from Ref. 5).

(a)

(b)



B
E
$n_i = n_i (V)$
POTENTIAL ENERGY $QV < QV_o$
VOLUME OF SHELL Y:
$Y = 2\pi v^2 \left(1 - \sqrt{1 - \left(\frac{v_o}{v}\right)^2}\right) \frac{v_o}{v}\, dv_o$
Y
v_z
Y_o
v_x
v_y
POTENTIAL ENERGY QV_o
VOLUME OF SHELL $Y_o = 2\pi v_o^2\, dv_o$



B
E
E x B
$n_i' = n_i' (V) > n_i (V)$
POTENTIAL ENERGY $QV < QV_o$
VOLUME OF SHELL Y':
$Y' > Y$
Y'
v_z
Y_o
v_x
v_y
POTENTIAL ENERGY QV_o
VOLUME OF SHELL $Y_o = 2\pi v_o^2\, dv_o$