Collisions between Gravitational Aggregates in the Tidal Field

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Hyodo & Ohtsuki 2014, ApJ

Tidal Field

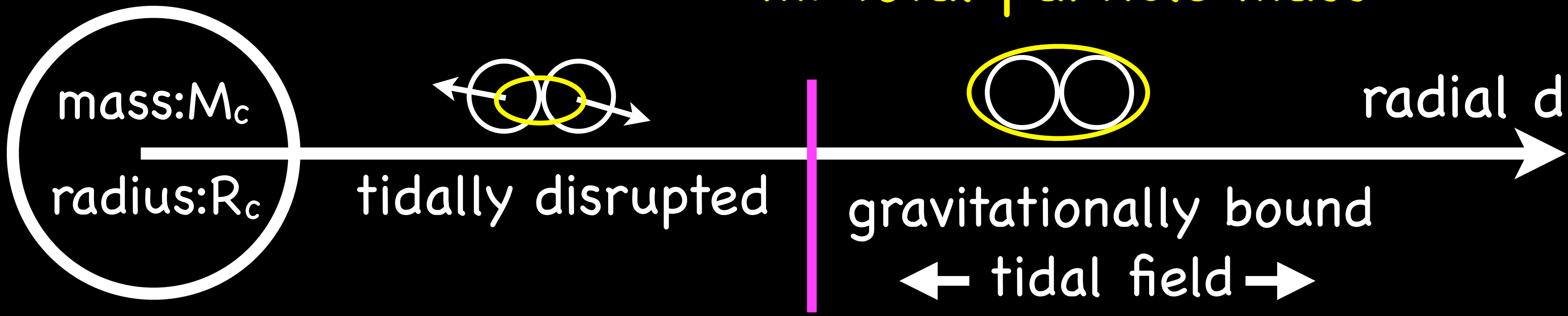
central planet

density: ρ_c

sphere of influence

Hill radius: $R_H = (m/3M_c)^{1/3} a$

m: total particle mass



Roche limit (for particles radially aligned)

$$a_R = 2.29(\rho_c / \rho_p)^{1/3} R_c$$

particle density: ρ_p

Universal Law

(Stewart & Leinhardt 2009)

$$m_{lr}/m_{tot}=-0.5(Q_R/Q_{RD}^{*}-1)+0.5$$

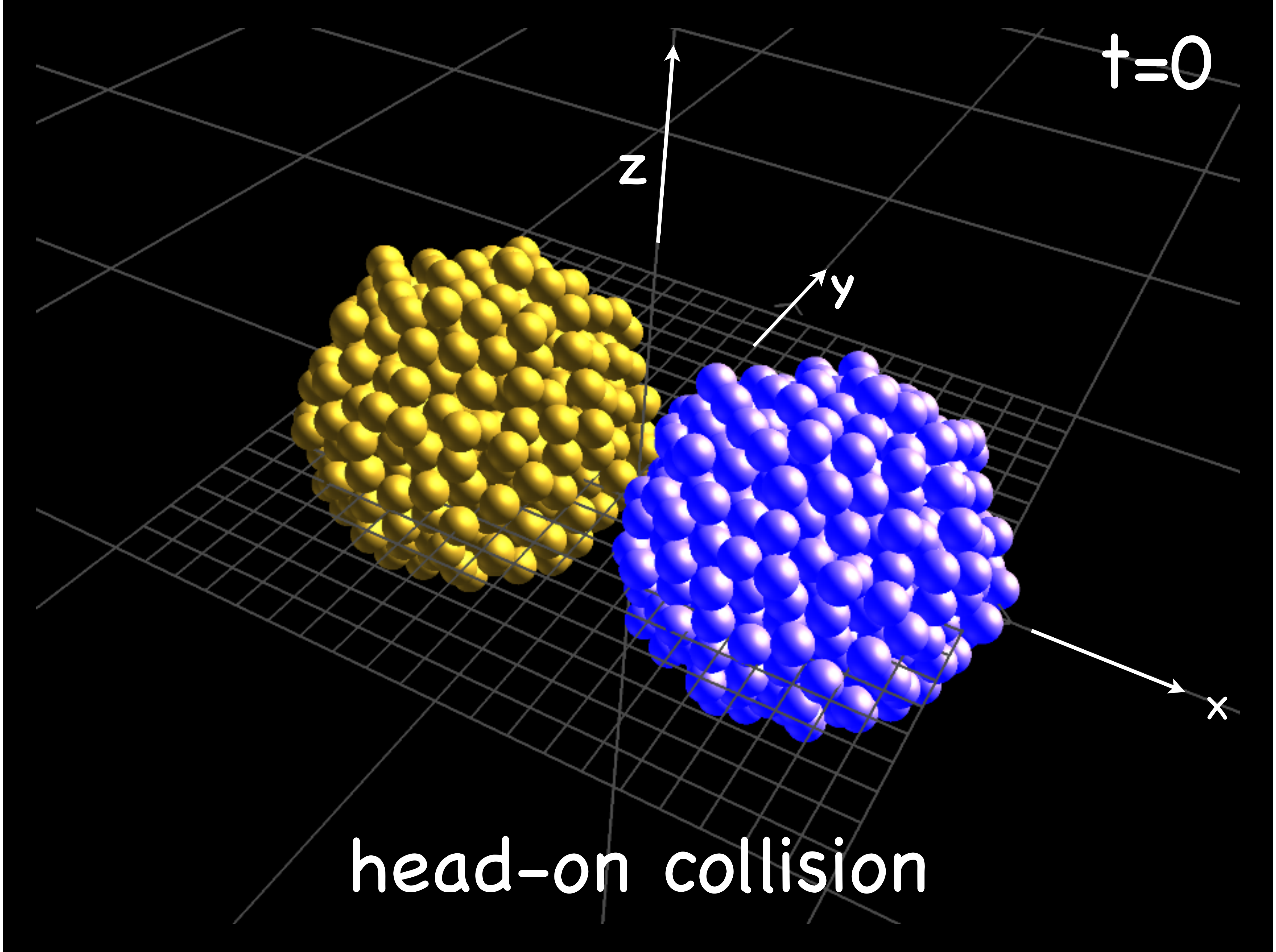
$Q_R=0.5\mu(v_{impact})^2/m_{tot}$

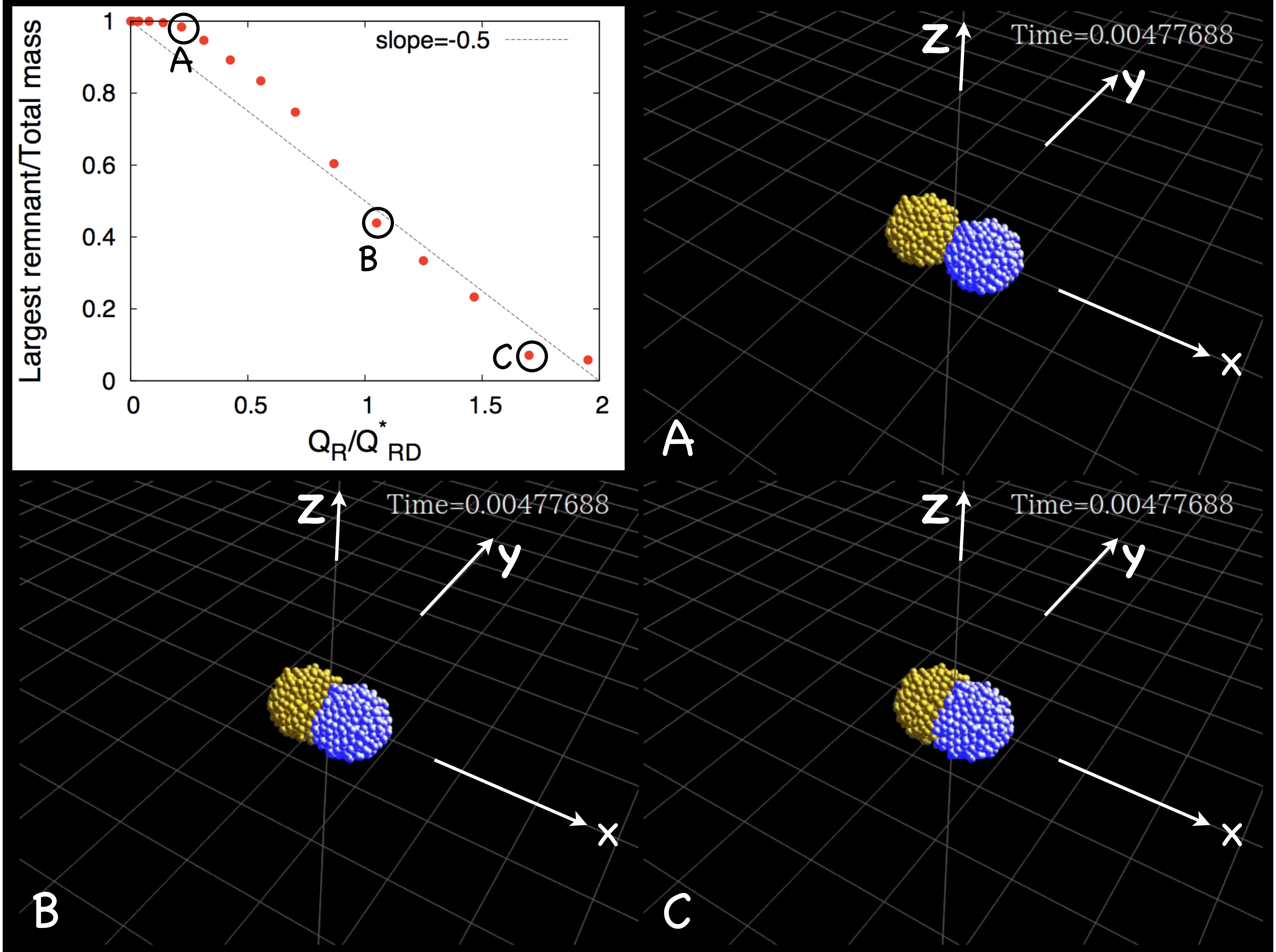
$\mu=m_{impactor}m_{target}/m_{tot}$

Q_{RD}^{*}: Q_R required to disperse half the total mass, m_{tot}

m_{lr}/m_{tot}: monotonically decreasing function of Q_R

Collisions in Free Space


t=0
z
y
x
head-on collision

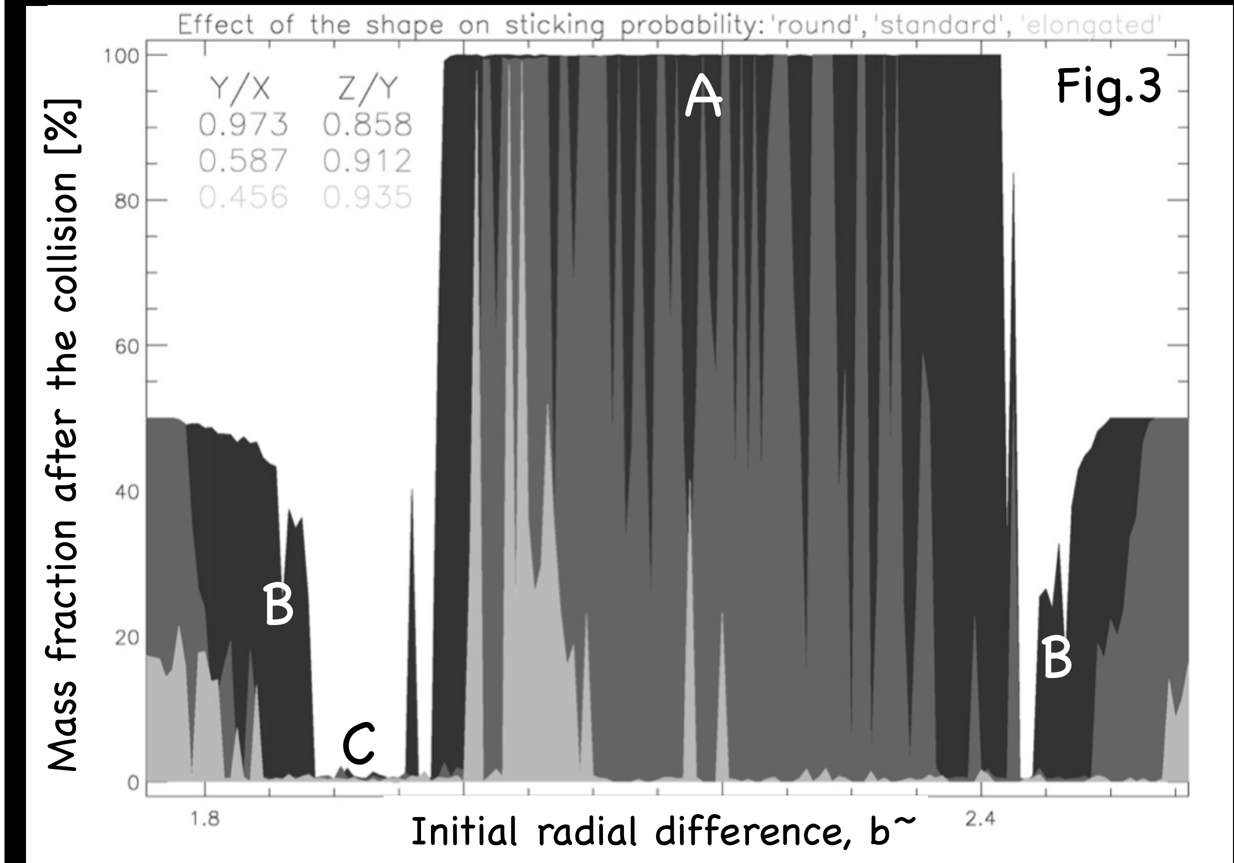


Largest remnant/Total mass
slope=-0.5
A
B
C
Q_R/Q^*_{RD}
0
0.2
0.4
0.6
0.8
1
0.5
1.5
2
Time=0.00477688
z
y
x
A
B
C

Collisions in the Tidal Field

Previous Work

- Karjalainen (2007)
  - Aggregate collision in the tidal field
  - Aggregates are initially circular orbits
  - Examine accretion efficiency
    -> more elongated is more prone to be destroyed



Saturnian System:
a=140,000km (F ring)

Outcomes varies
- A: total accretion
- B: partial disruption
- C: total disruption

This Work:

Investigating dependences of collisional disruption on various

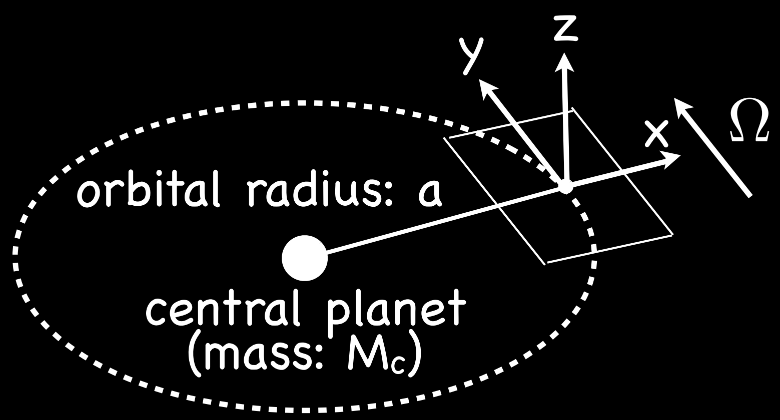
- Impact velocities
- Impact directions
- Radial distances from the central planet

Method: Local N-body simulation

Numerical Method

- Local N-body simulation (Hill coordinate)
  - Equations of motion

$$m_i\left(\ddot{x}_i - 2\Omega\dot{y}_i - 3\Omega^2 x_i\right) = F_x$$
$$m_i\left(\ddot{y}_i + 2\Omega\dot{x}_i\right) = F_y$$
$$m_i\left(\ddot{z}_i + \Omega^2 z_i\right) = F_z$$



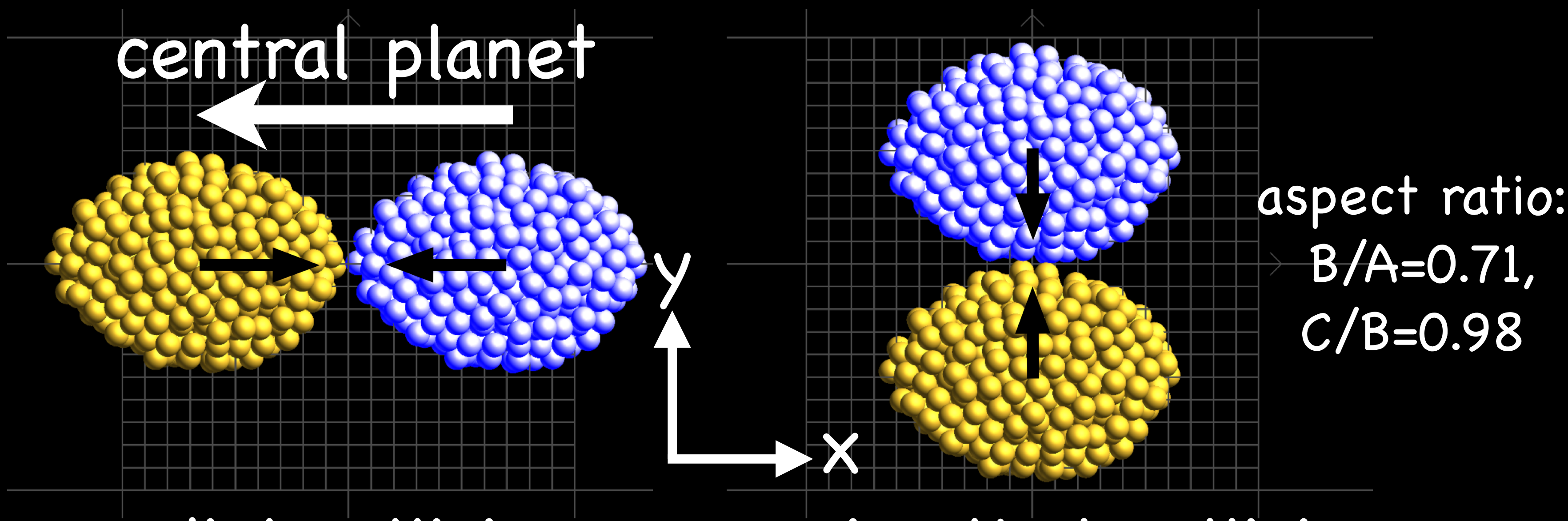
$$\boldsymbol{F}_i = -\sum_{j\neq i}^{N} m_i m_j \frac{\boldsymbol{r}_{ij}}{|\boldsymbol{r}_{ij}|^3}, \qquad \boldsymbol{r}_i = (x_i, y_i, z_i), \qquad \Omega = \sqrt{\frac{GM_c}{a^3}}$$

  - 2nd-order symplectic leapfrog (Quinn et al. 2010)
  - hard-sphere model (smooth particles with normal coefficient of restitution ϵ_n=0.25)

- We assume Saturnian system

Collision Model

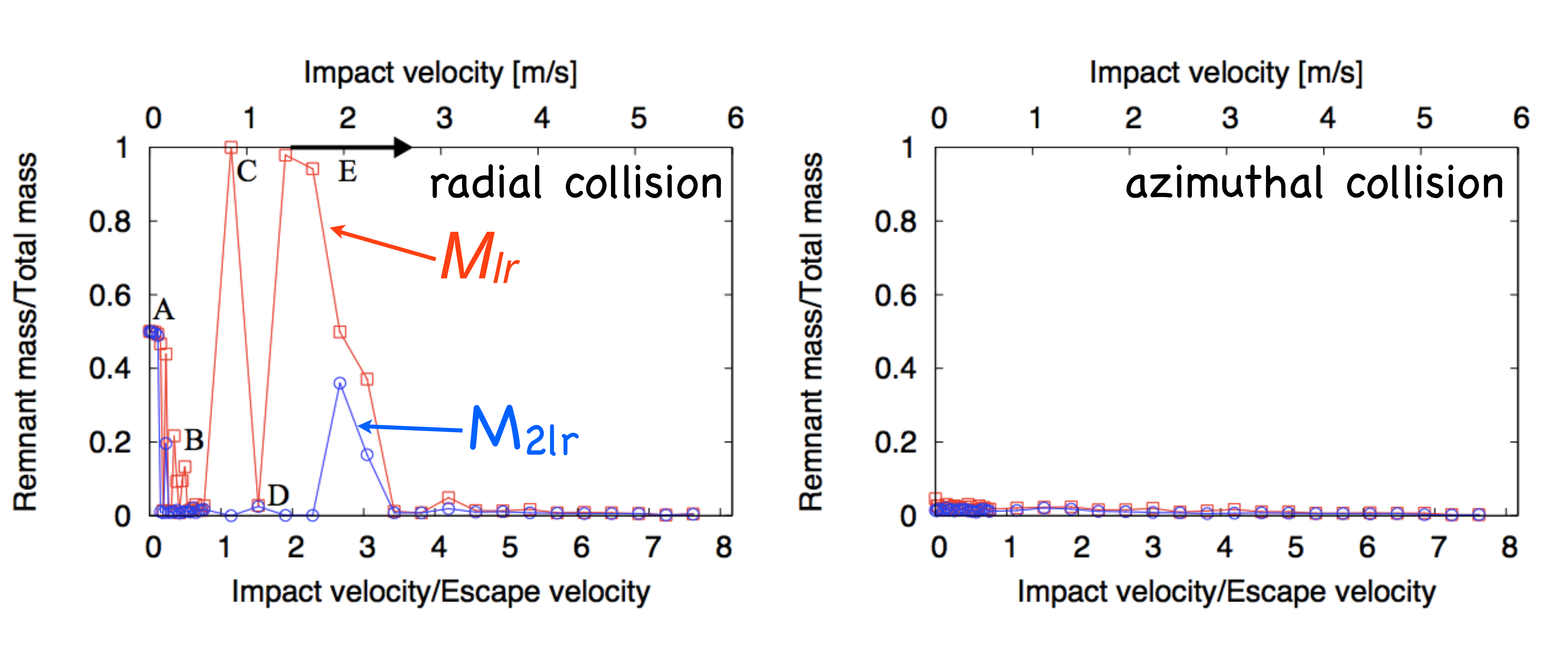
- Using equal sized tidally elongated aggregates
  - -originally 1km-radius spherical aggregate
  - -particle density is 0.9 g/cm^3 (icy particle)
- Investigate the dependence on collision directions
  - -with various radial distances and impact velocities



radial collision azimuthal collision

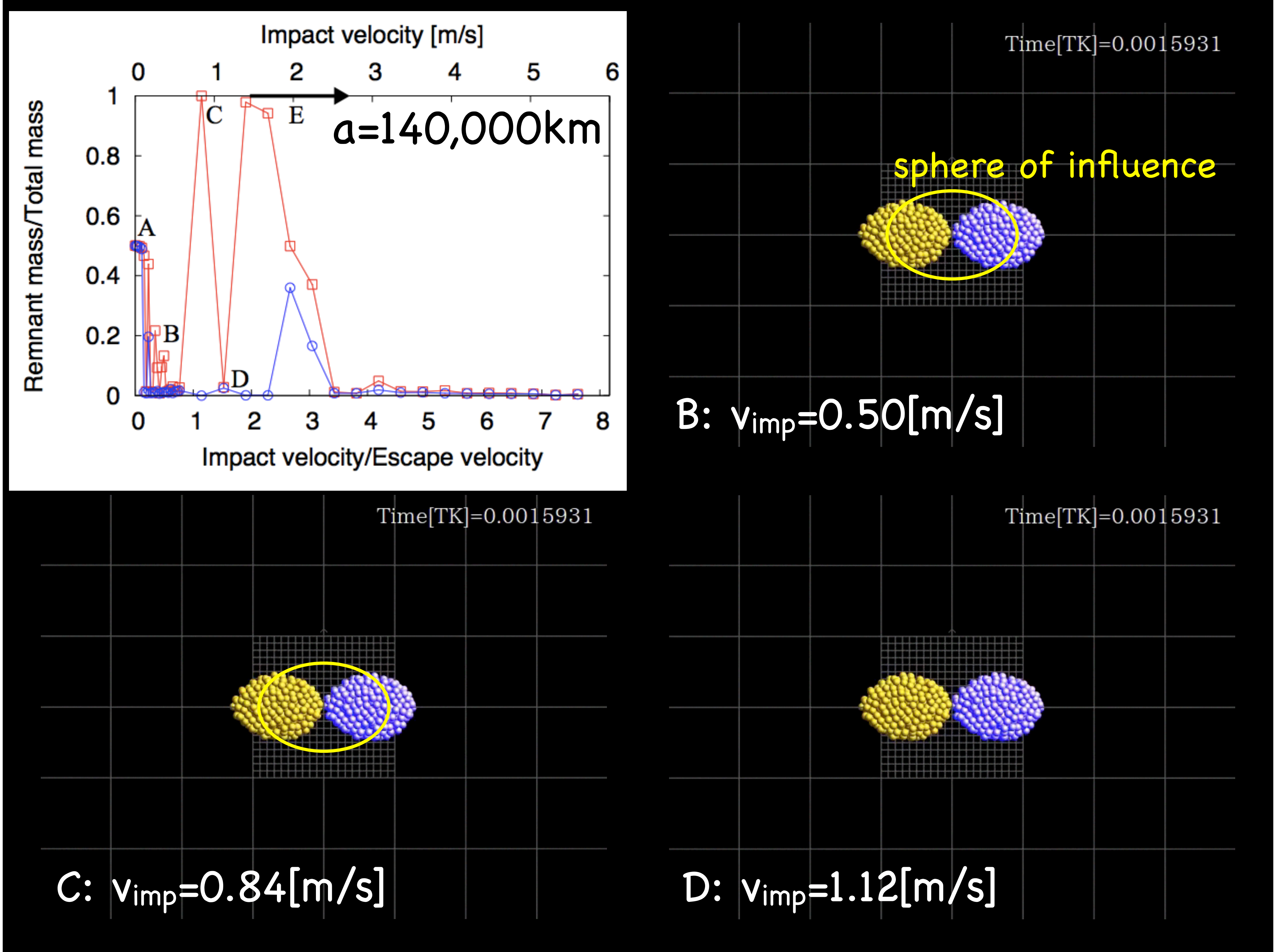
Strong Tidal Field:
Applications to F ring (a=140,000km)

Upper horizontal axis: Case of 1km-radius sphere with density 0.9 g cm^{-3}

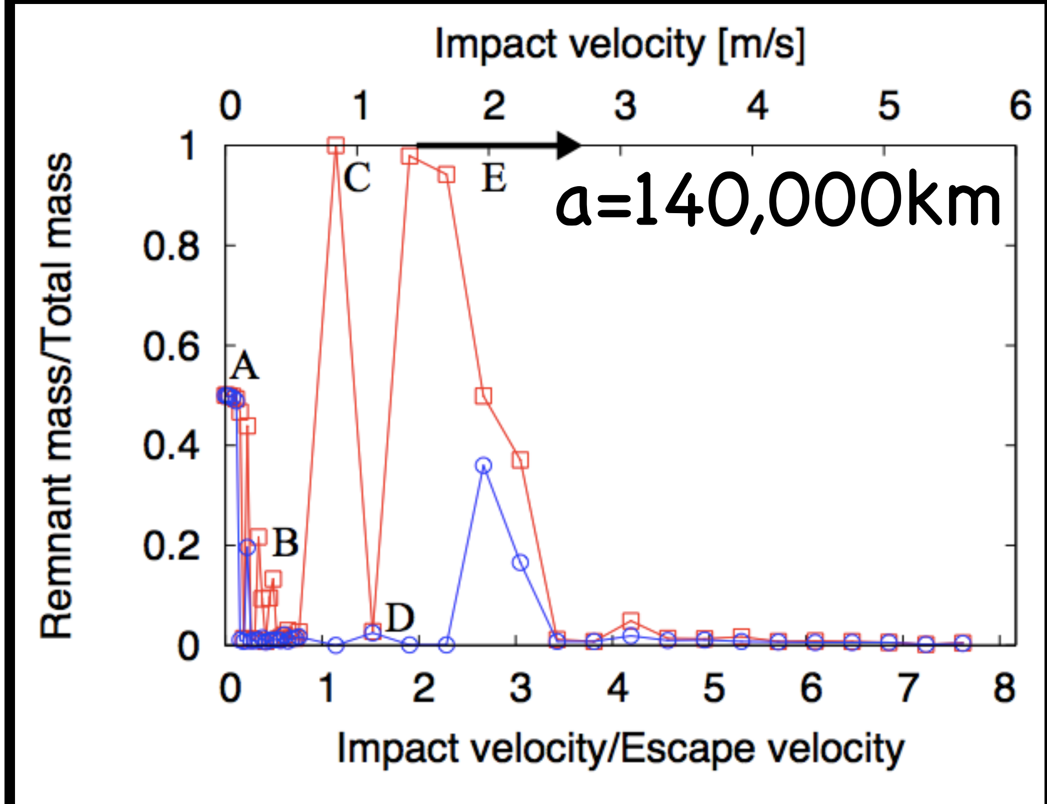


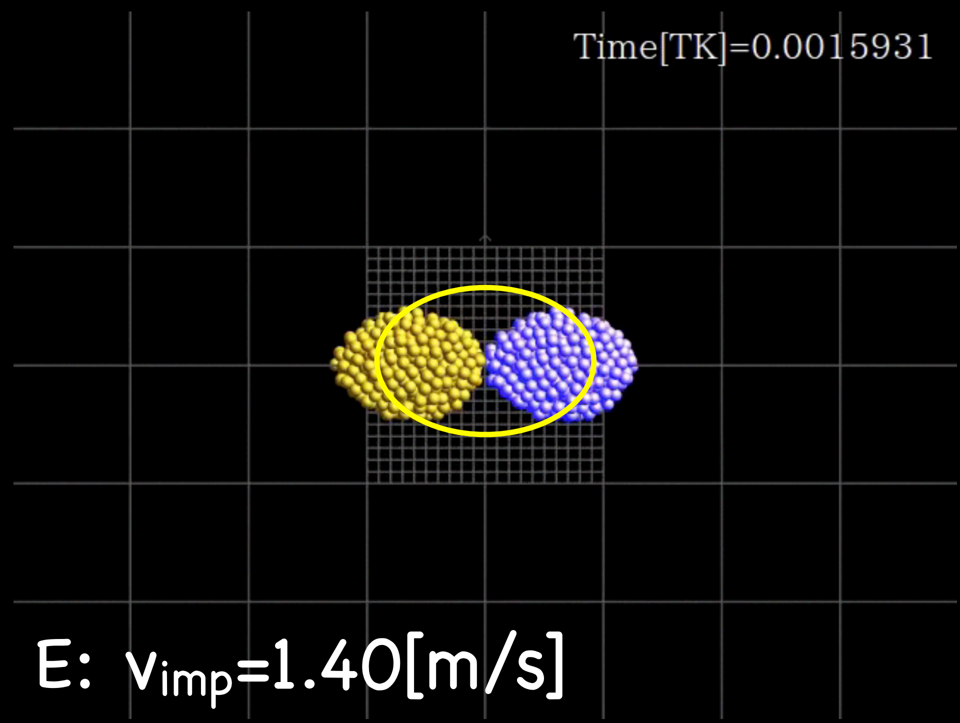
Escape velocity : that of a sphere which has aggregate mass

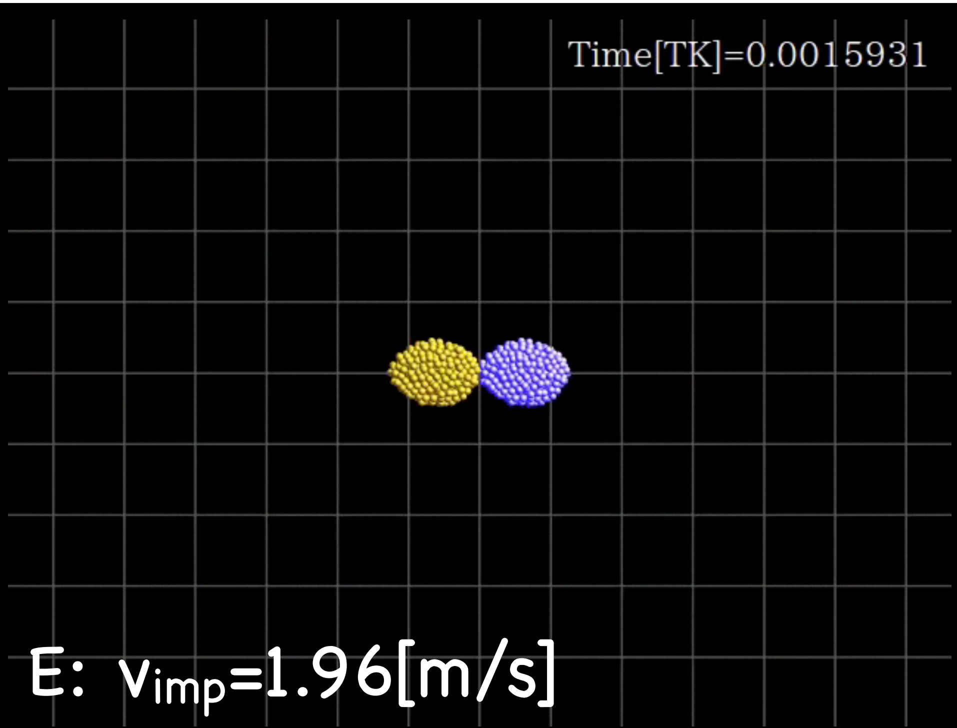
M_{lr}: largest remnant, M_{2lr}: 2nd largest remnant

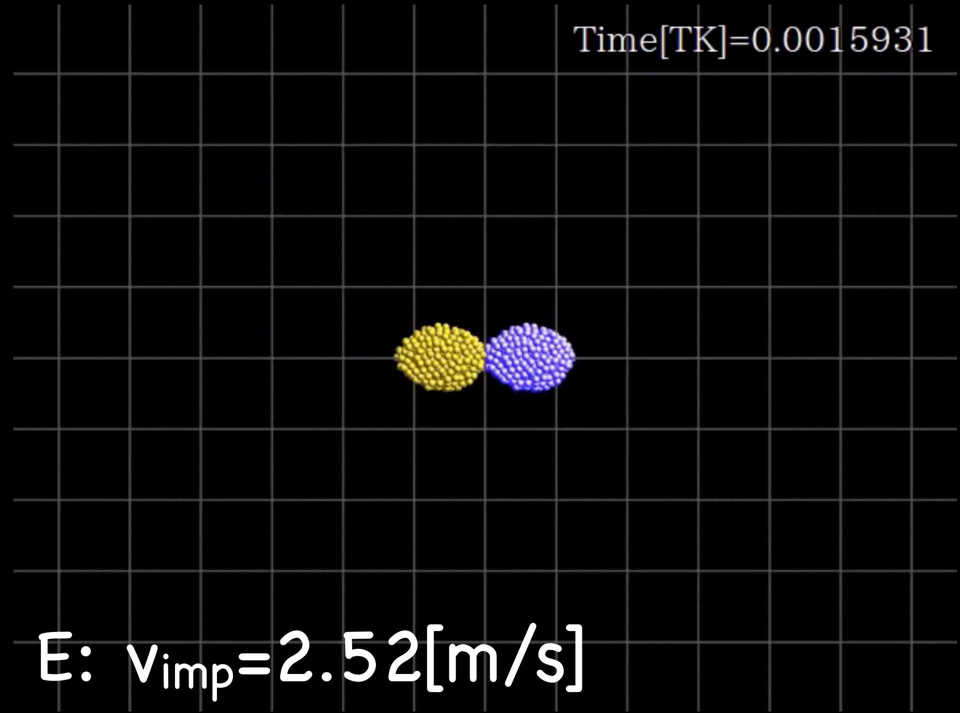


Impact velocity [m/s]
Remnant mass/Total mass
Impact velocity/Escape velocity
a=140,000km
A
B
C
D
E
Time[TK]=0.0015931
sphere of influence
B: vimp=0.50[m/s]
Time[TK]=0.0015931
C: vimp=0.84[m/s]
Time[TK]=0.0015931
D: vimp=1.12[m/s]

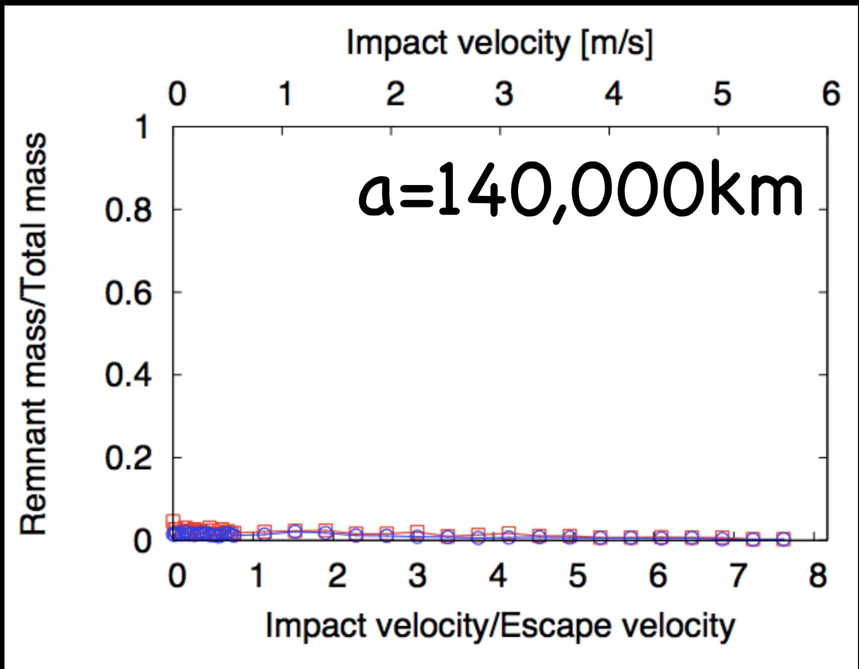

Impact velocity [m/s]
0
1
2
3
4
5
6
Remnant mass/Total mass
1
0.8
0.6
0.4
0.2
0
A
B
C
D
E
a=140,000km
0
1
2
3
4
5
6
7
8
Impact velocity/Escape velocity


Time[TK]=0.0015931
E: v_{imp}=1.40[m/s]

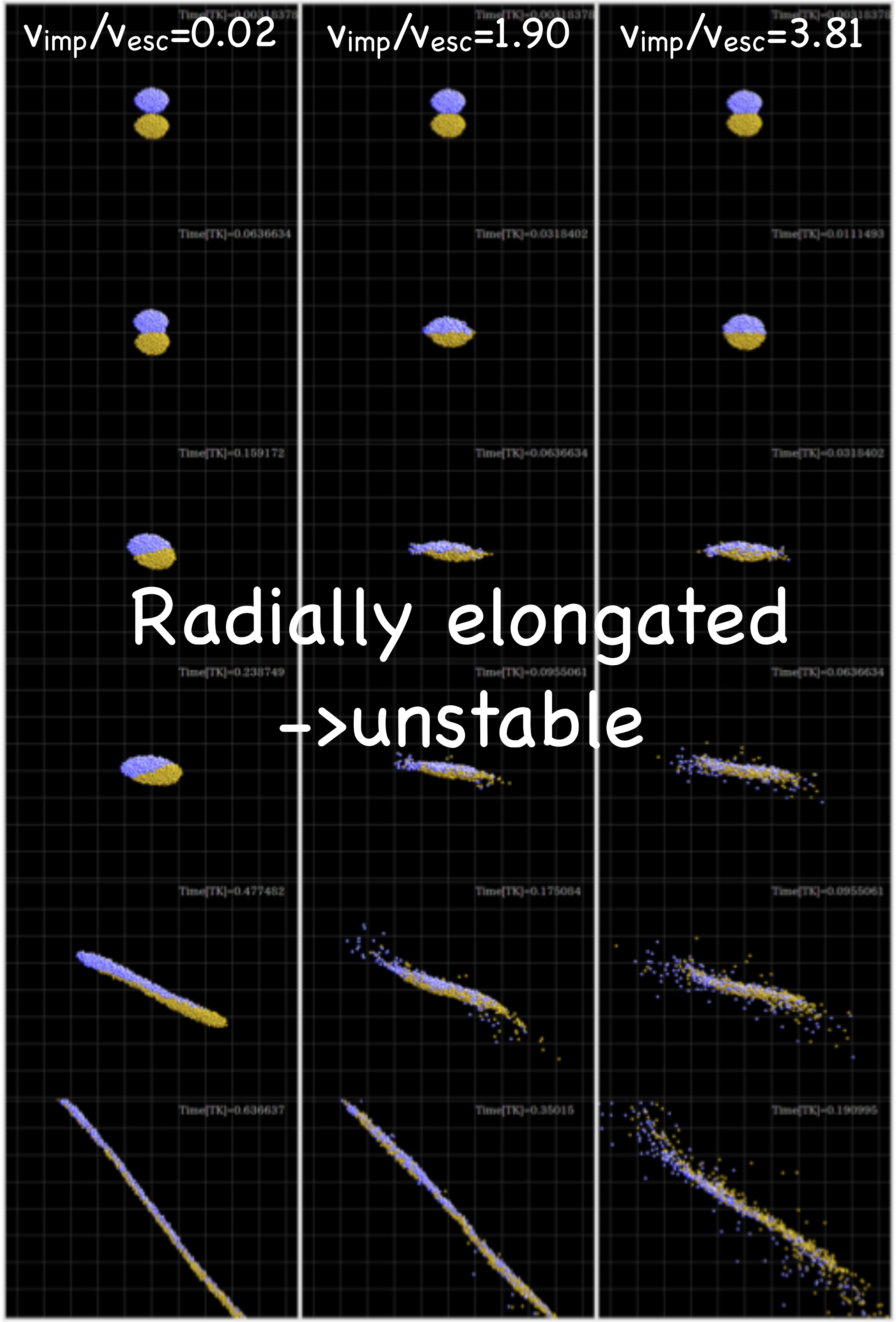

Time[TK]=0.0015931
E: v_{imp}=1.96[m/s]


Time[TK]=0.0015931
E: v_{imp}=2.52[m/s]

Case of azimuthal collisions

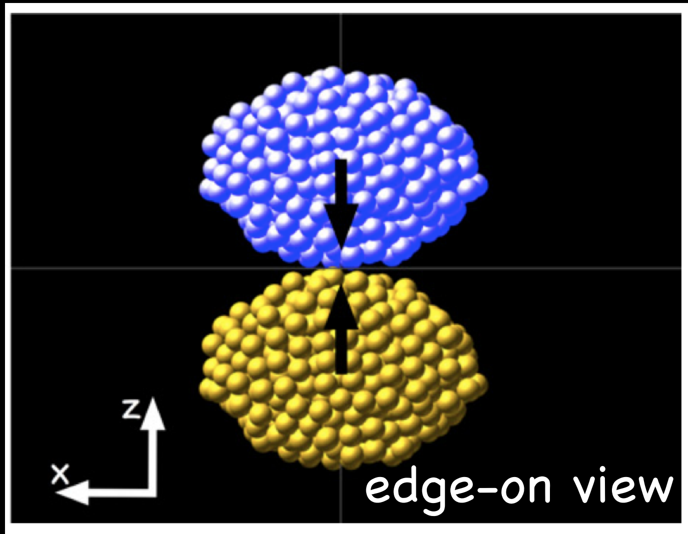


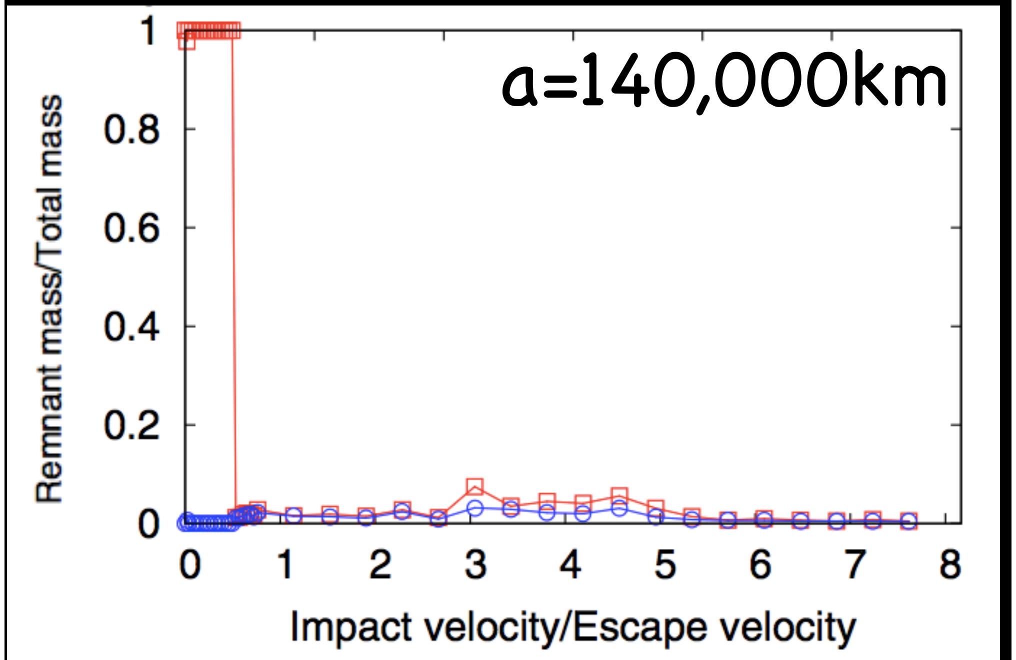
Much more destructive than radial collision

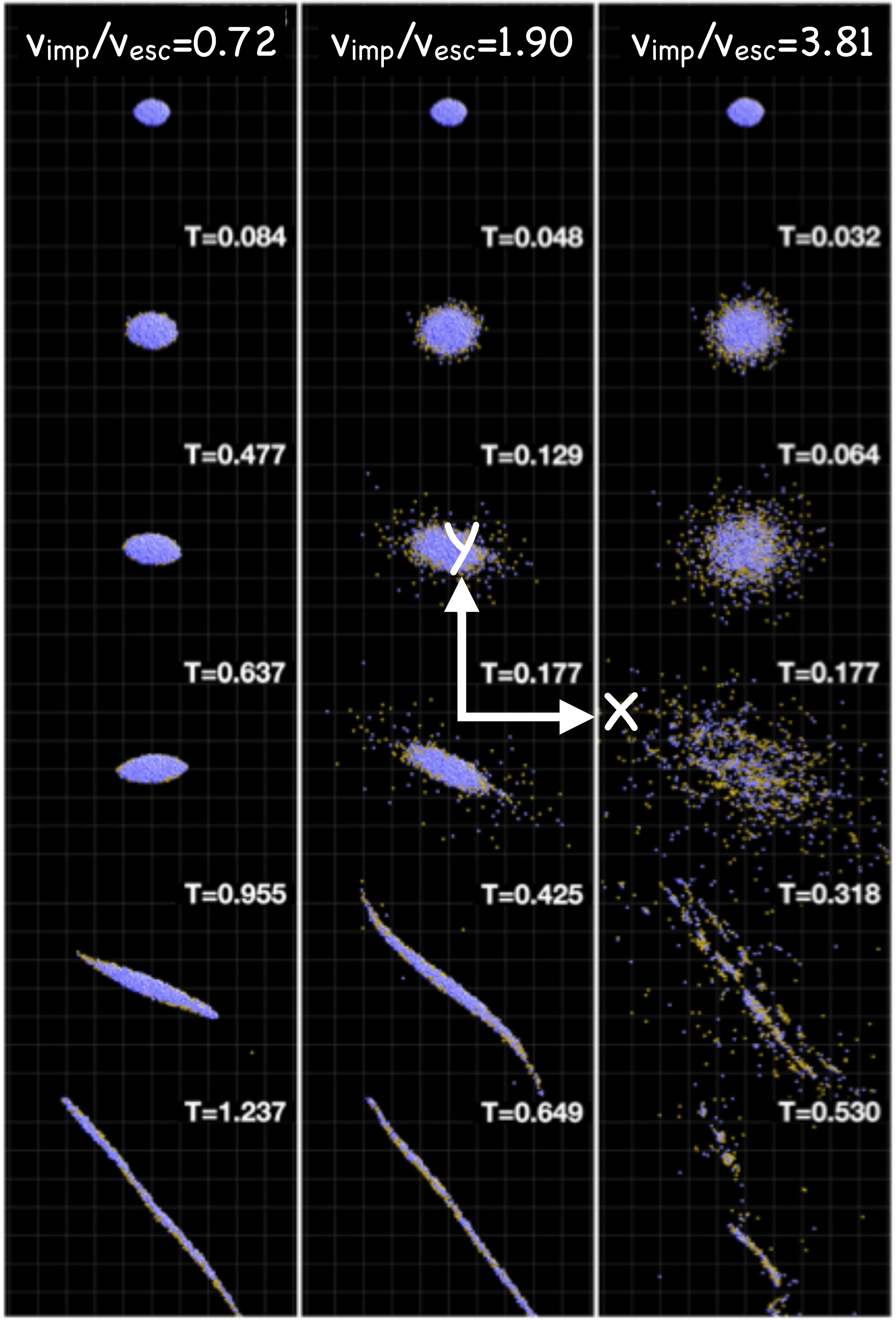


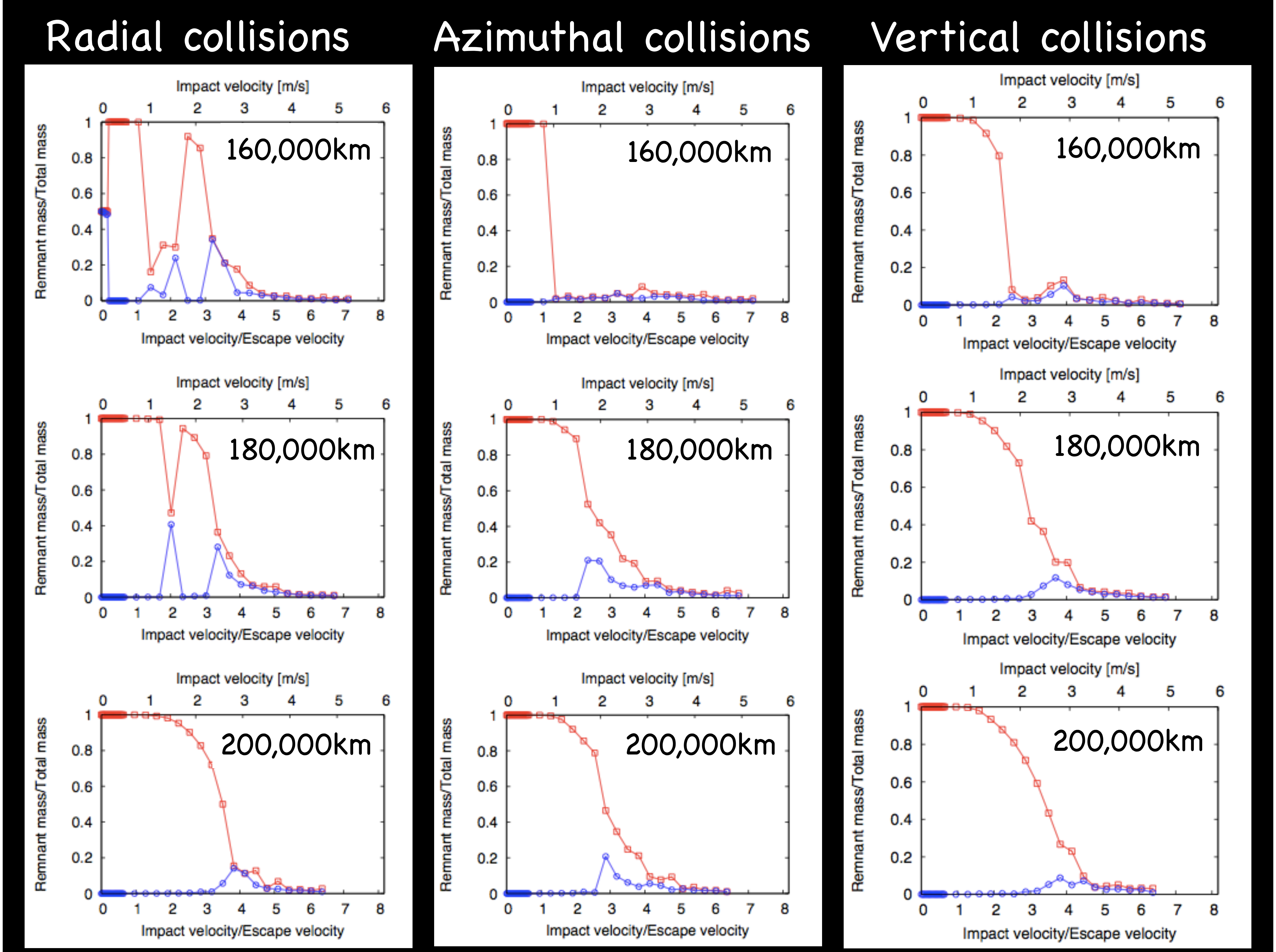
Radially elongated
->unstable

Case of vertical collisions

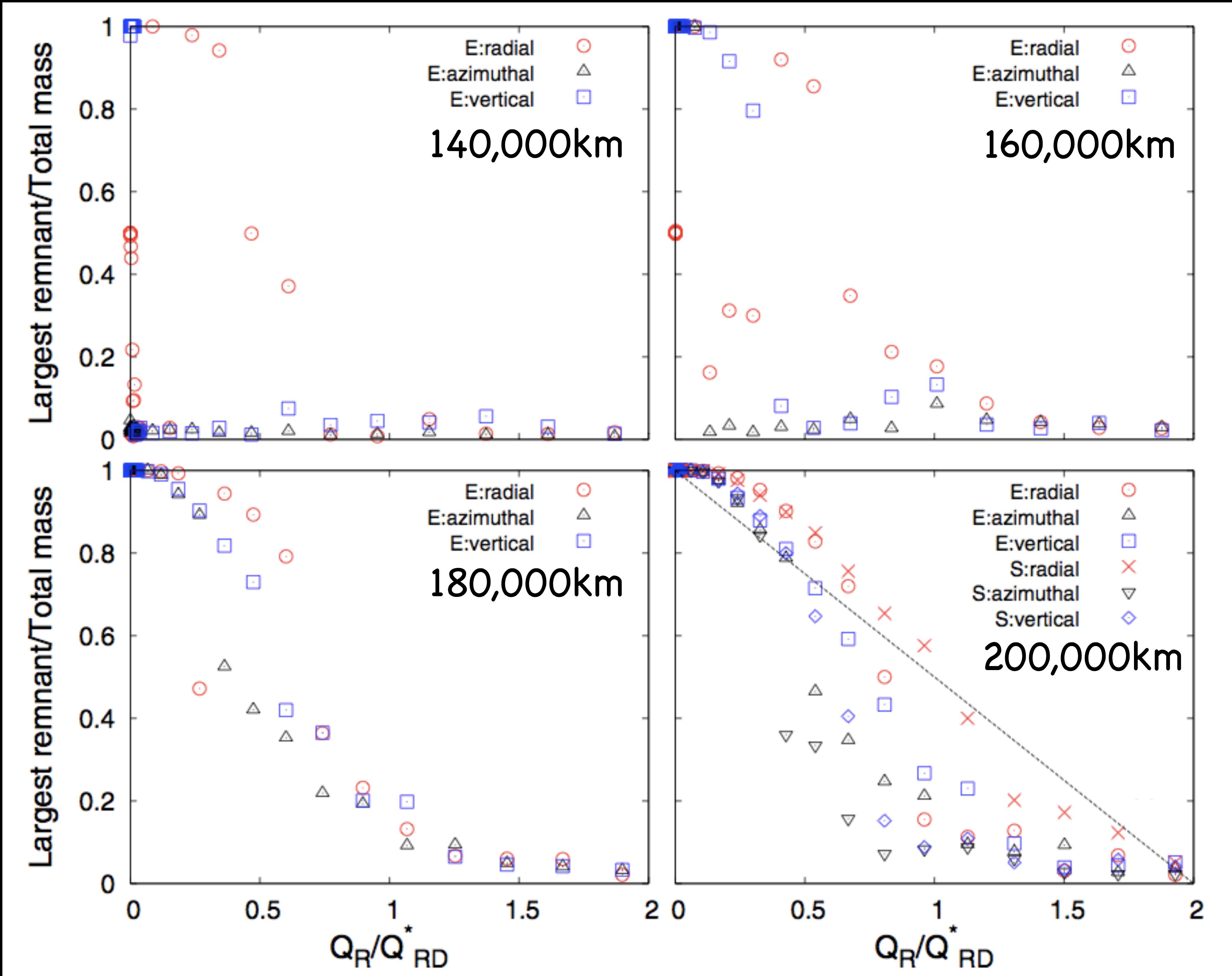







Radial collisions
Azimuthal collisions
Vertical collisions
160,000km
180,000km
200,000km
Impact velocity [m/s]
Remnant mass/Total mass
Impact velocity/Escape velocity

Validity of Universal Law



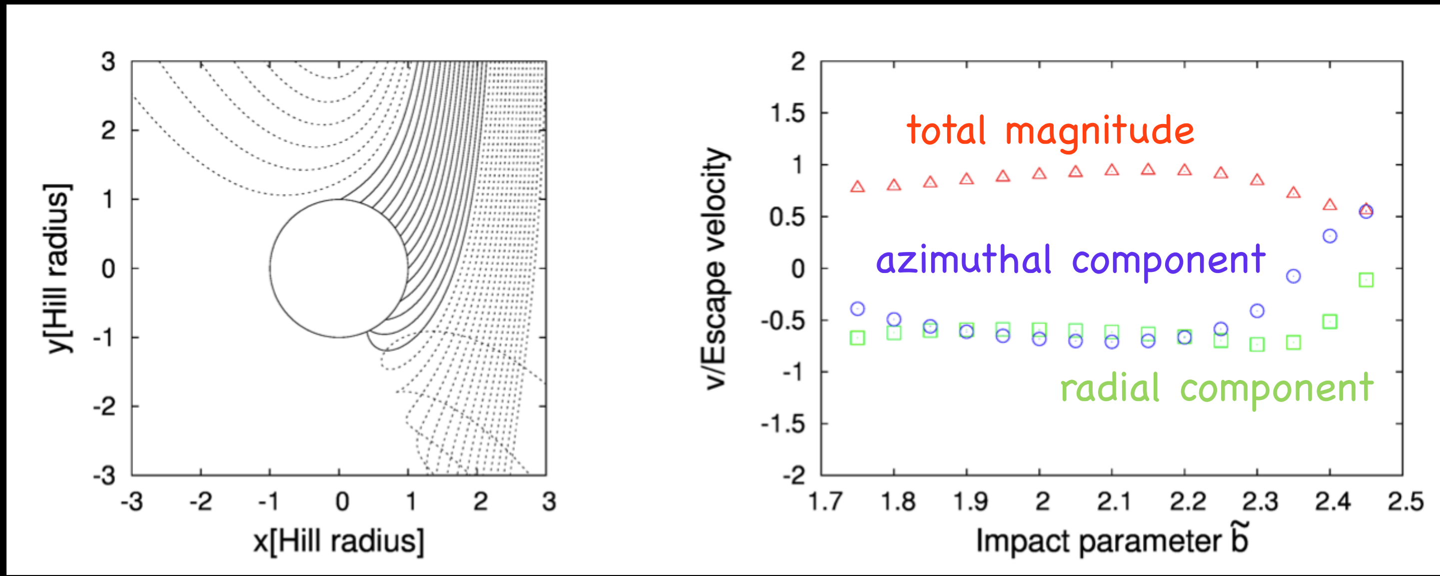
Conclusions

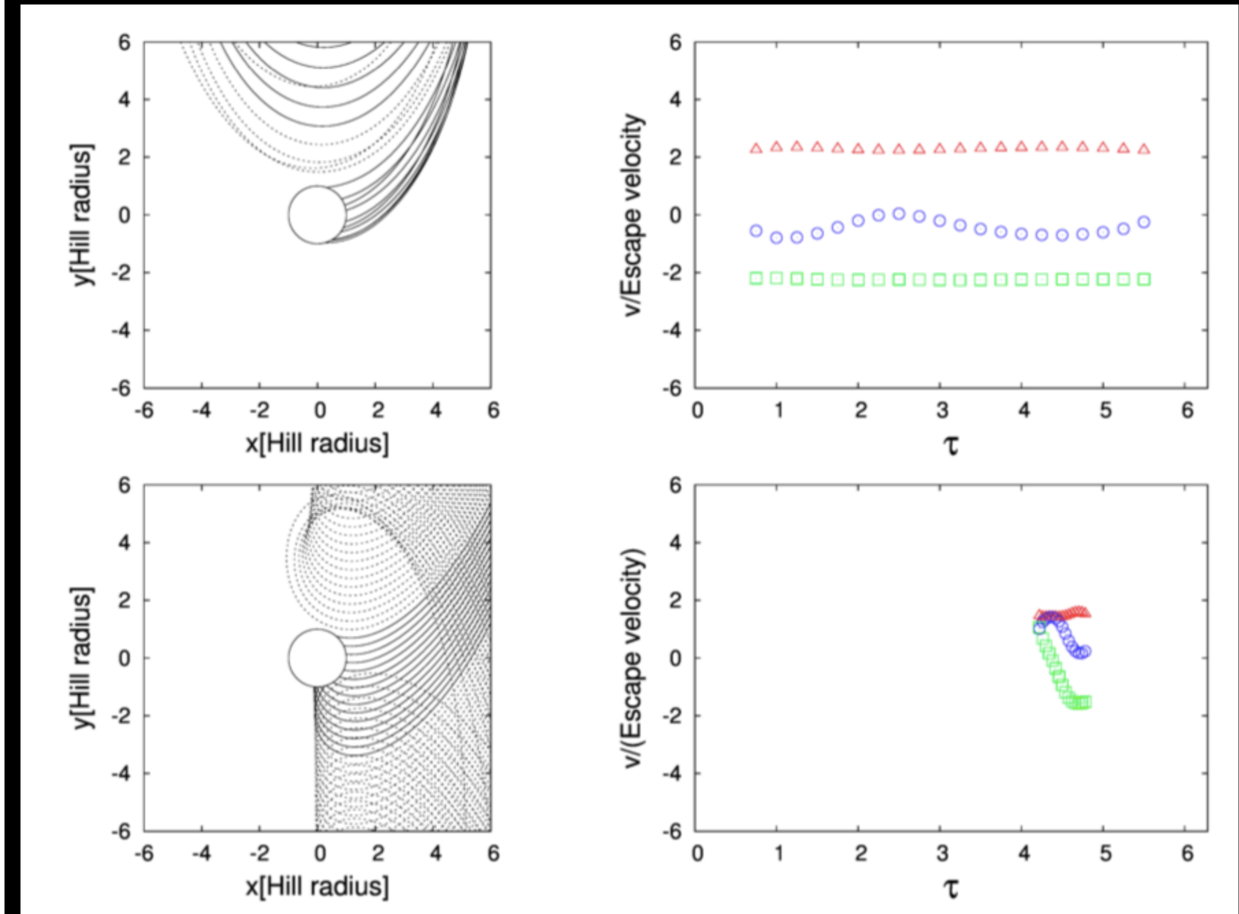
In the (strong) tidal field :
corresponding to Saturn's F ring

- Outcomes strongly depend on collision directions
- Even much smaller impact velocities than escape velocity result in total disruption
- Universal law is NOT applicable

Appendix

Three-body Orbital Calculations





circular orbits ($\tilde{e}=0$, $\tilde{r}_p=1$)

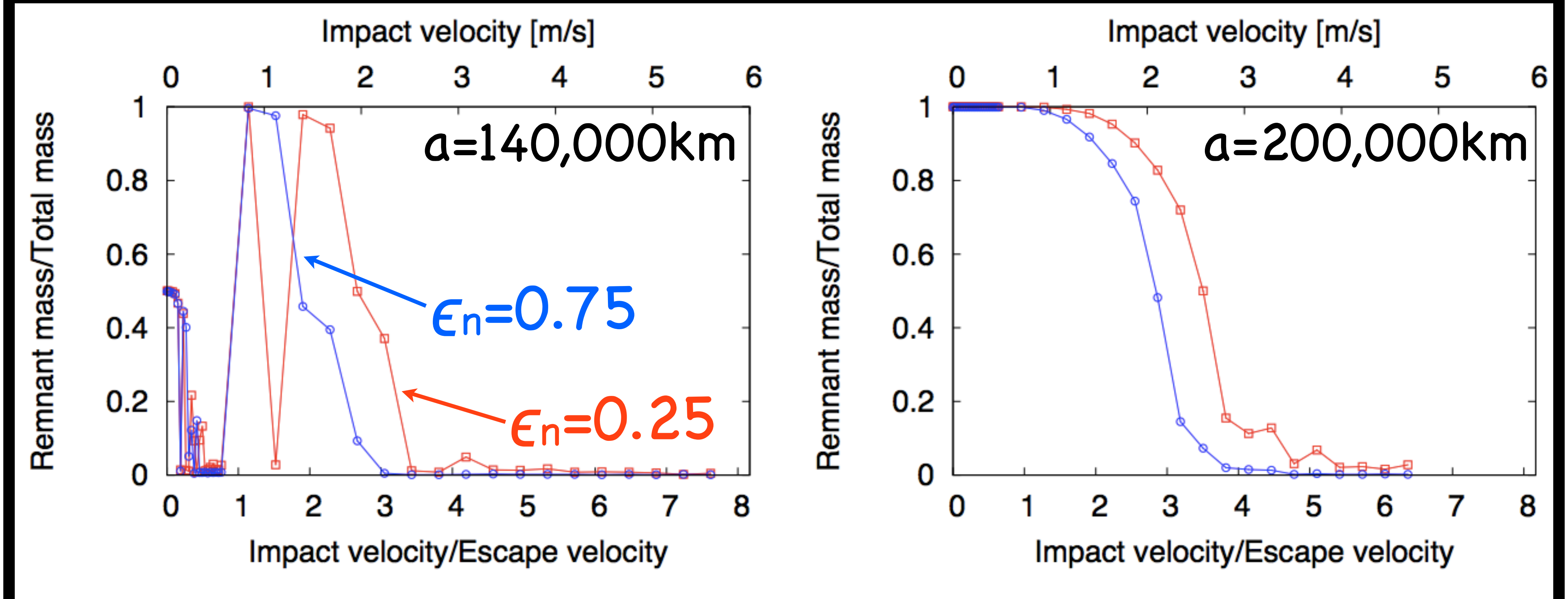
$\tilde{b}=1.15$

elliptical orbits ($\tilde{e}=5$, $\tilde{r}_p=1$)

$\tilde{b}=5.0$

τ : horizontal phase angle

Dependence on Coefficient of Restitution



*results of radial collisions