Small Particle Population in Saturn's Rings from Self-Gravity Wake Observations

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Introduction

$$n(a) = n_0 (a / a_0)^{-q}, \;\; a_{\min} \le a \le a_{\max}$$

- Parameters can be determined using optical depth measurements at different wavelengths. [Marouf et al. 1983, Zebker et al. 1985]
- The presence of self-gravity wakes introduces viewing geometry dependence.
- 173 stellar occultations: 57 (VIMS), 116 (UVIS)

VIMS, UVIS FOV/Pixel Size

- VIMS pixel:
  0.25mrad x 0.50mrad
  (Nicholson, Hedman 2010)
  λ_{VIMS} = 2.92μm

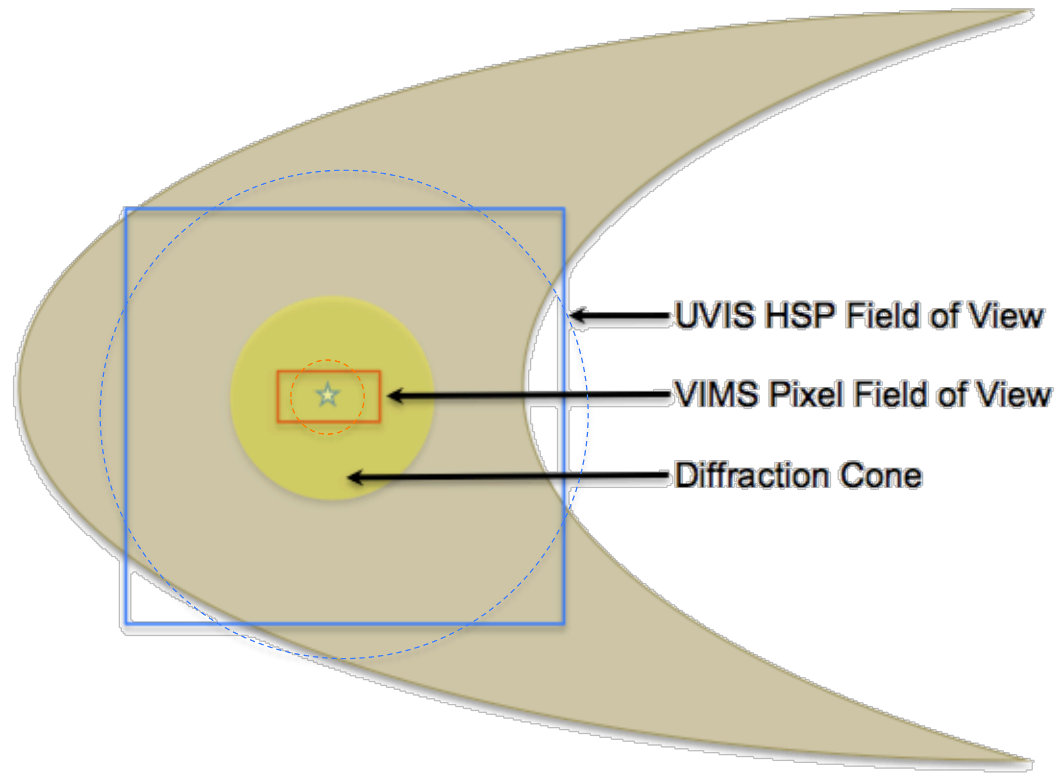
- UVIS FOV:
  6.4mrad x 6.0mrad
  λ_{UVIS} = 0.15μm

- Effective angular radius of circular field of view:

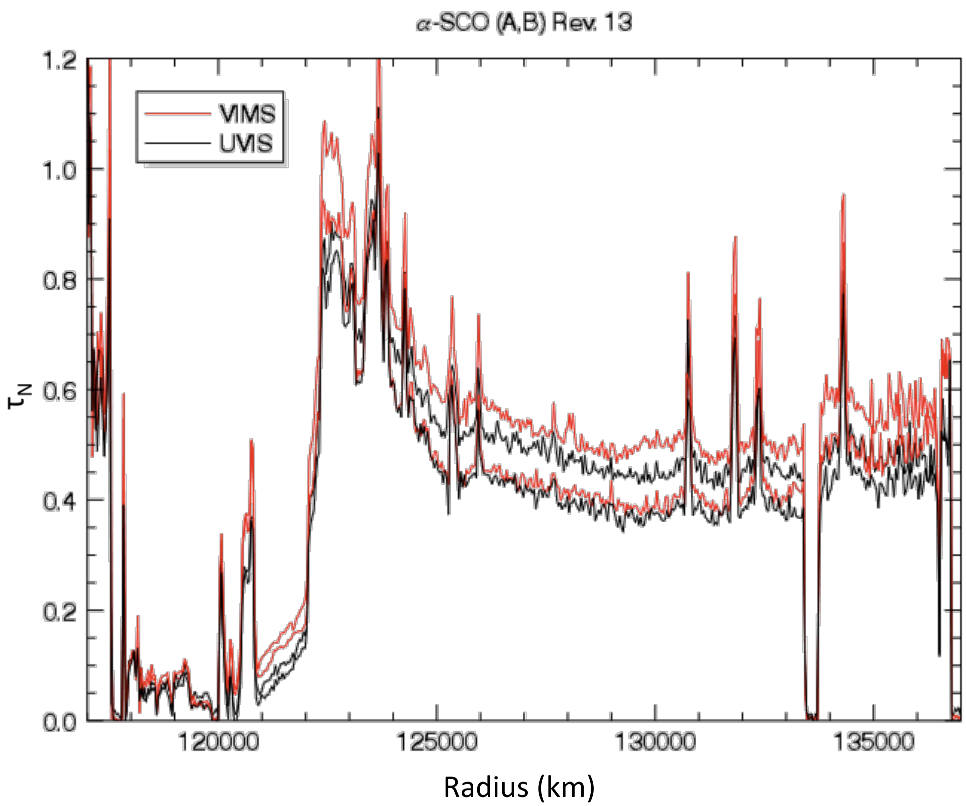
$$\theta_{eff} \equiv \frac{L+W}{\sqrt{4\pi}}$$

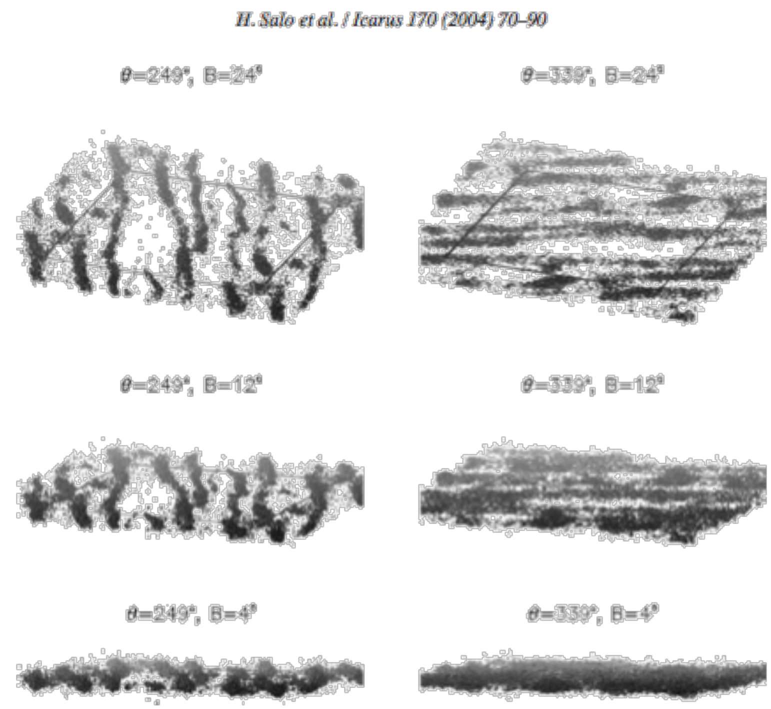
- Critical particle radius at which diffraction effects become apparent:

$$a_{crit} = 1.22\frac{\lambda_{VIMS}}{2\theta_{eff}} \approx 8.86mm$$



Self-Gravity Wakes and Differential Optical Depths

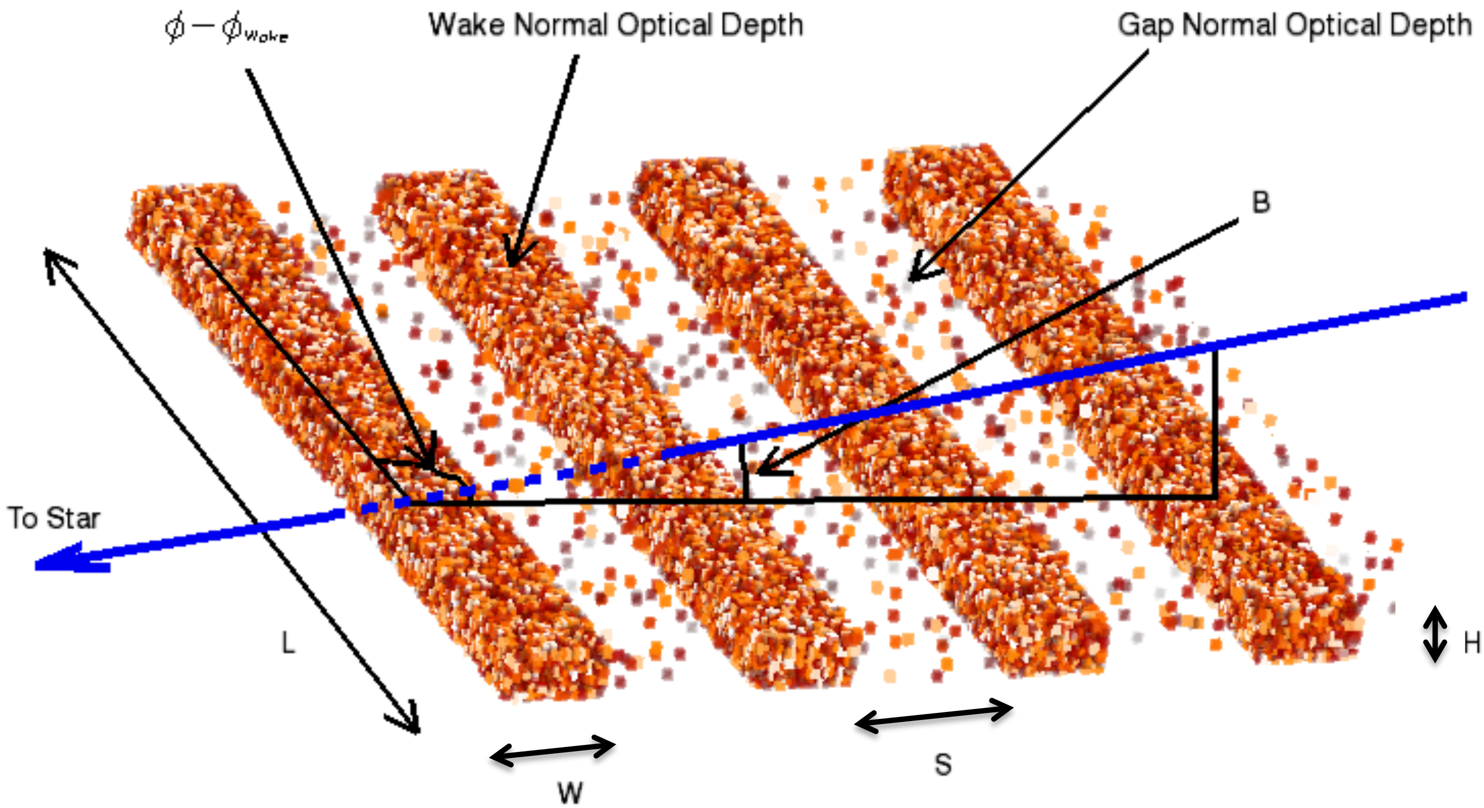




*Numerical Simulations from Salo et al. 2004

- Viewing geometry dependence
- Wavelength dependence

Rectangular Cross-Section Wake Model



- Self-Gravity wakes are essentially opaque when compared with the gaps between them.
- We introduce a new free parameter, τ_{small}, to the "Granola Bar" model of Colwell et al. 2006, 2007
- τ_{small} represents the additional optical depth in the wake gaps seen by VIMS occultations.

Determination of a_{min} from free parameter, τ_{small}

$$\tau = \int_0^{a_{\max}} \pi a^2 Q_e(a,\lambda) n(a)\, da$$

$$Q_{\mathrm{UVIS}} \approx 1: \qquad \tau_{UVIS} = 1 \cdot \frac{\pi n_0}{a_0^{-q}} \int_{a_{\min}}^{a_{\max}} a^{2-q}\, da = \pi n_0 a_0^q \left[\frac{\left(a_{\max}^{3-q} - a_{\min}^{3-q}\right)}{3-q} \right]$$

$$a < a_{crit}: \ Q_{\mathrm{VIMS}} \approx 2: \ \tau_{VIMS} = \frac{\pi n_0}{a_0^{-q}} \left[2 \cdot \int_{a_{\min}}^{a_{crit}} a^{2-q}\, da + 1 \cdot \int_{a_c}^{a_{\max}} a^{2-q}\, da \right] = \pi n_0 a_0^q \left[\frac{a_{\max}^{3-q} + a_{crit}^{3-q} - 2a_{\min}^{3-q}}{3-q} \right]$$

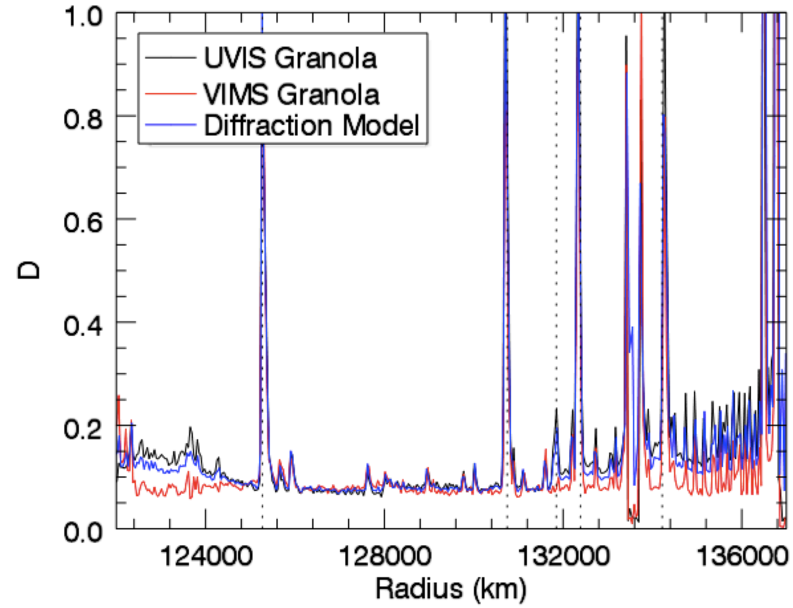
$$\tau_{small} = \tau_{VIMS} - \tau_{UVIS}$$ (Difference in gap optical depths)

$$\Rightarrow a_{\min} = \left(a_{crit}^{3-q} - \frac{(3-q)a_0^{-q}}{\pi n_0} \tau_{small} \right)^{\frac{1}{3-q}}$$

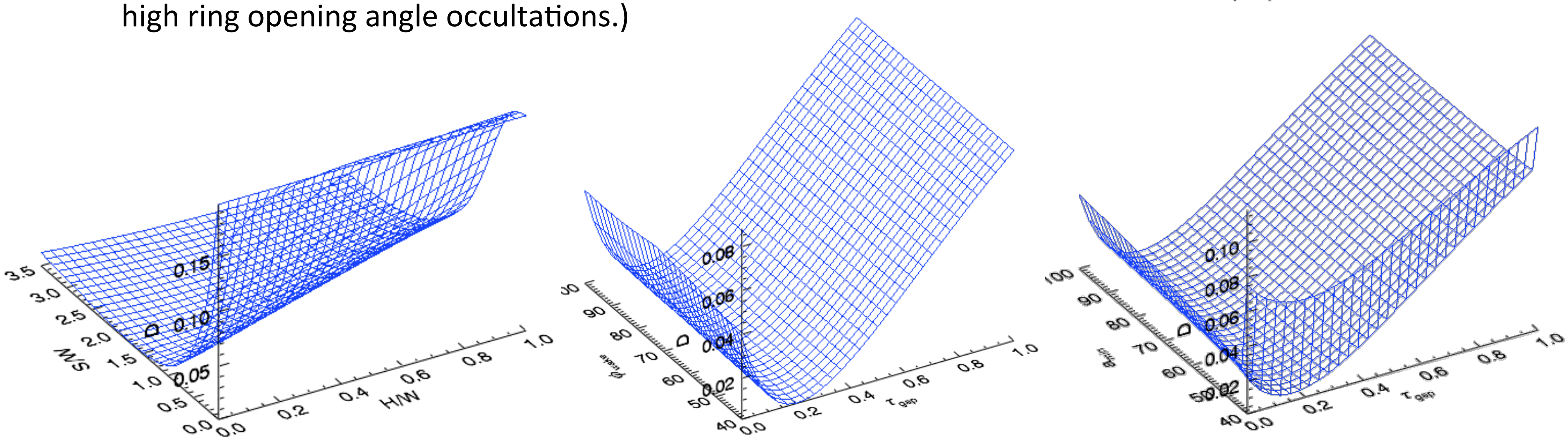
Determination of Best-fit Model Parameters

$$D = \frac{1}{\nu}\sum_i \left(\tau_{data} - \tau_{comp}\right)^2$$

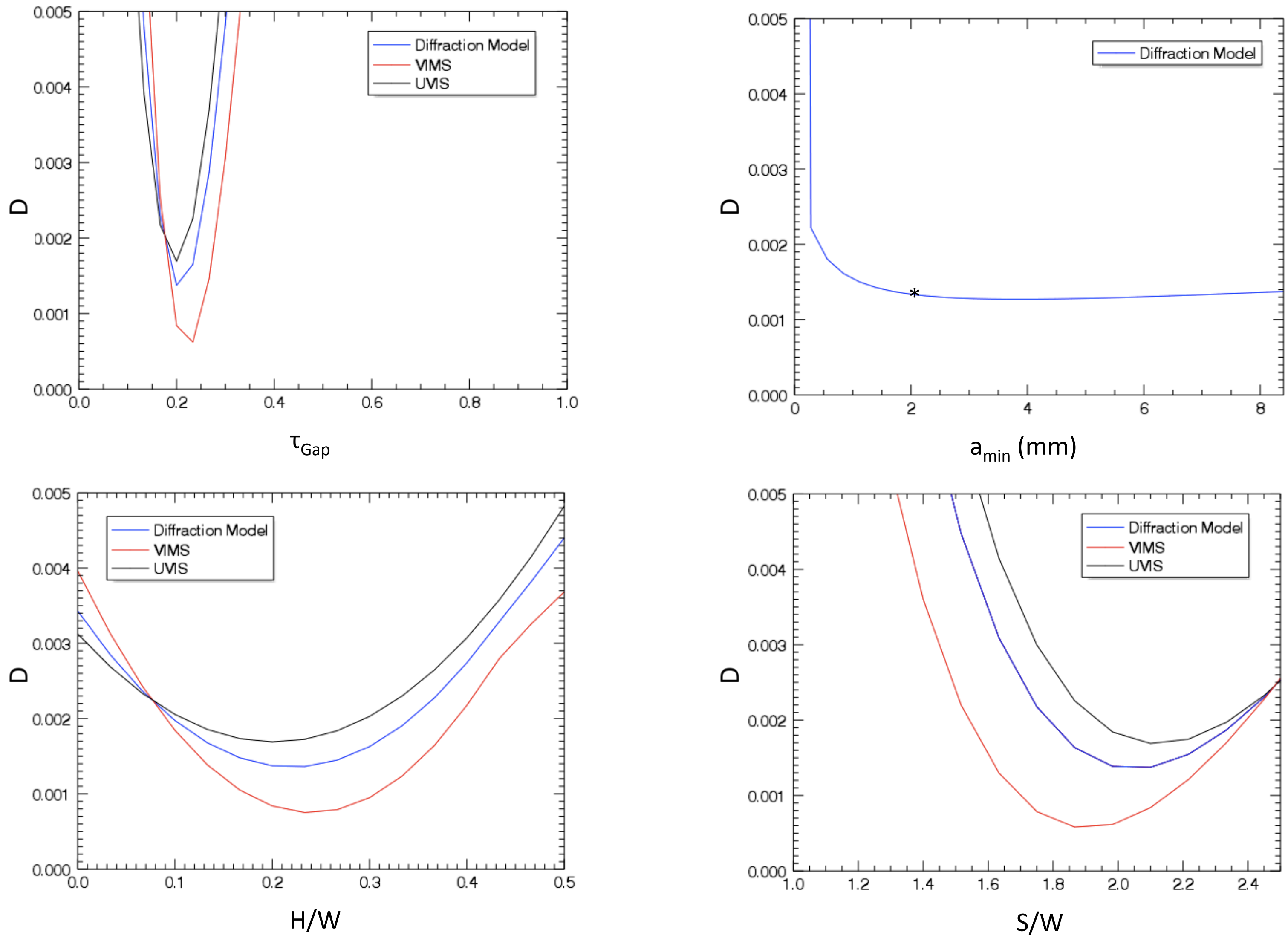
ν = # Deg. of Freedom



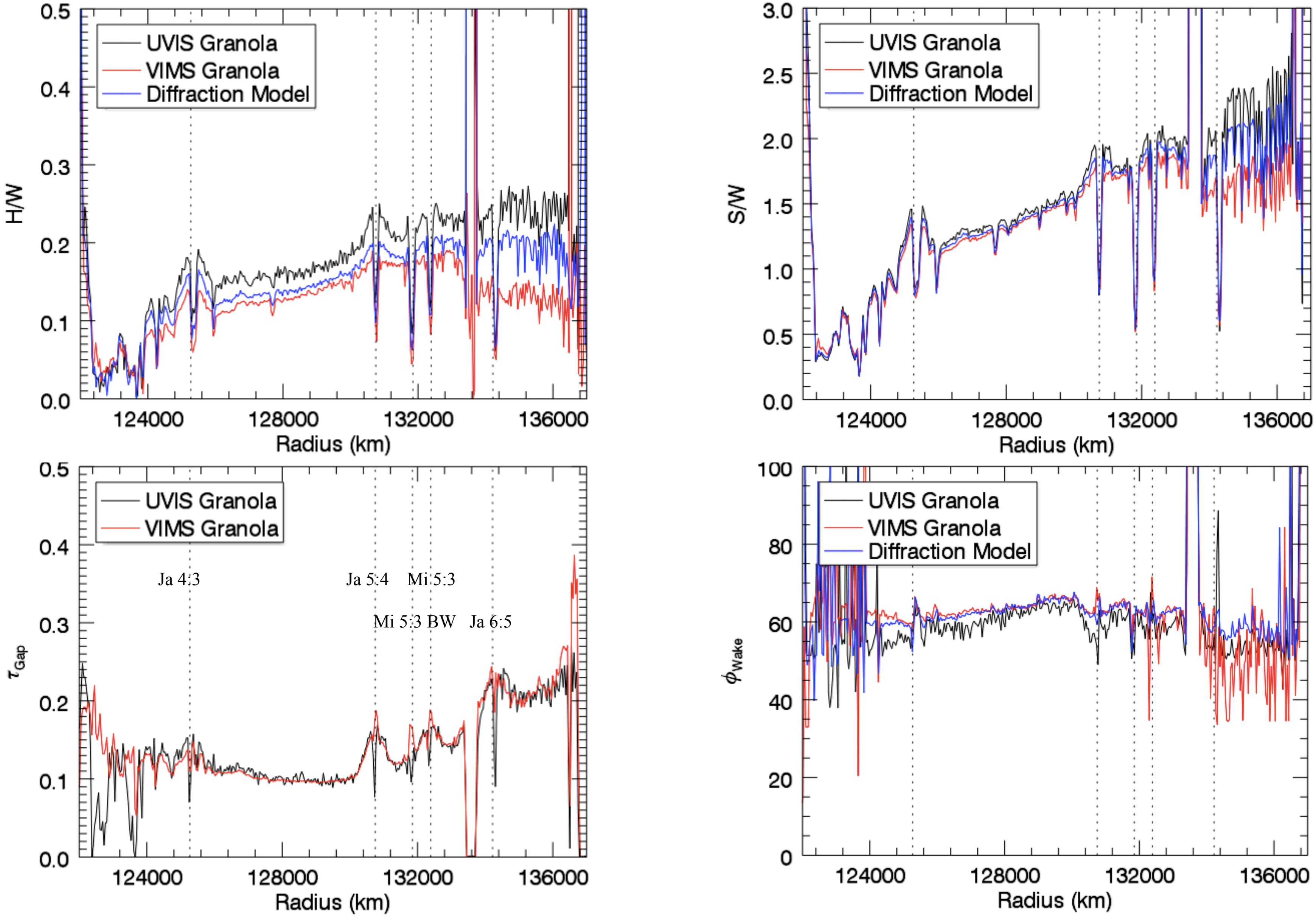
- Model is more sensitive to some parameters than others.
- Less sensitive to φ_{wake} (Due to large number of high ring opening angle occultations.)



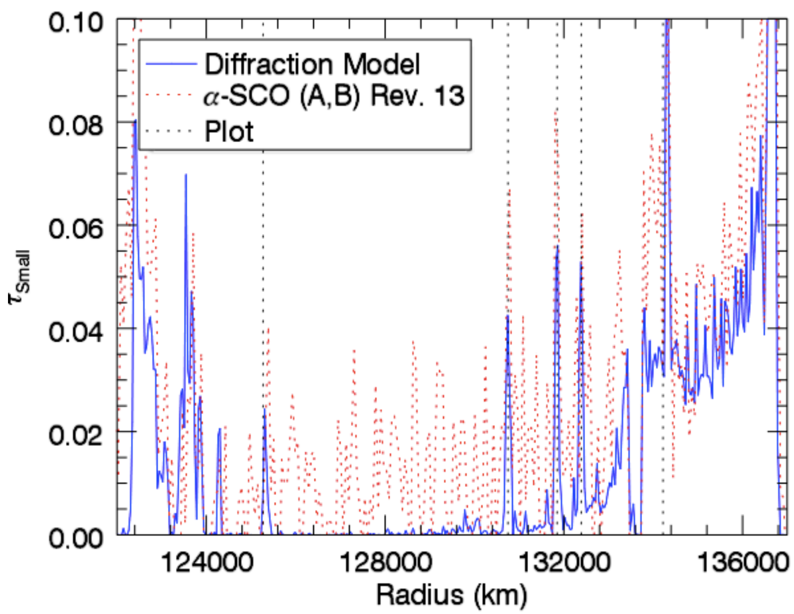
Cross-Sections of 'D' at R = 135,000 km

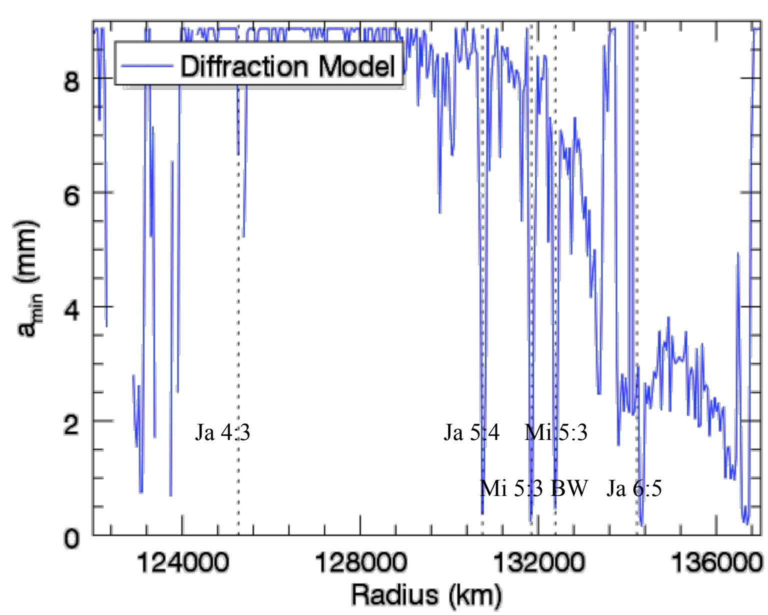


Model Results (A Ring)



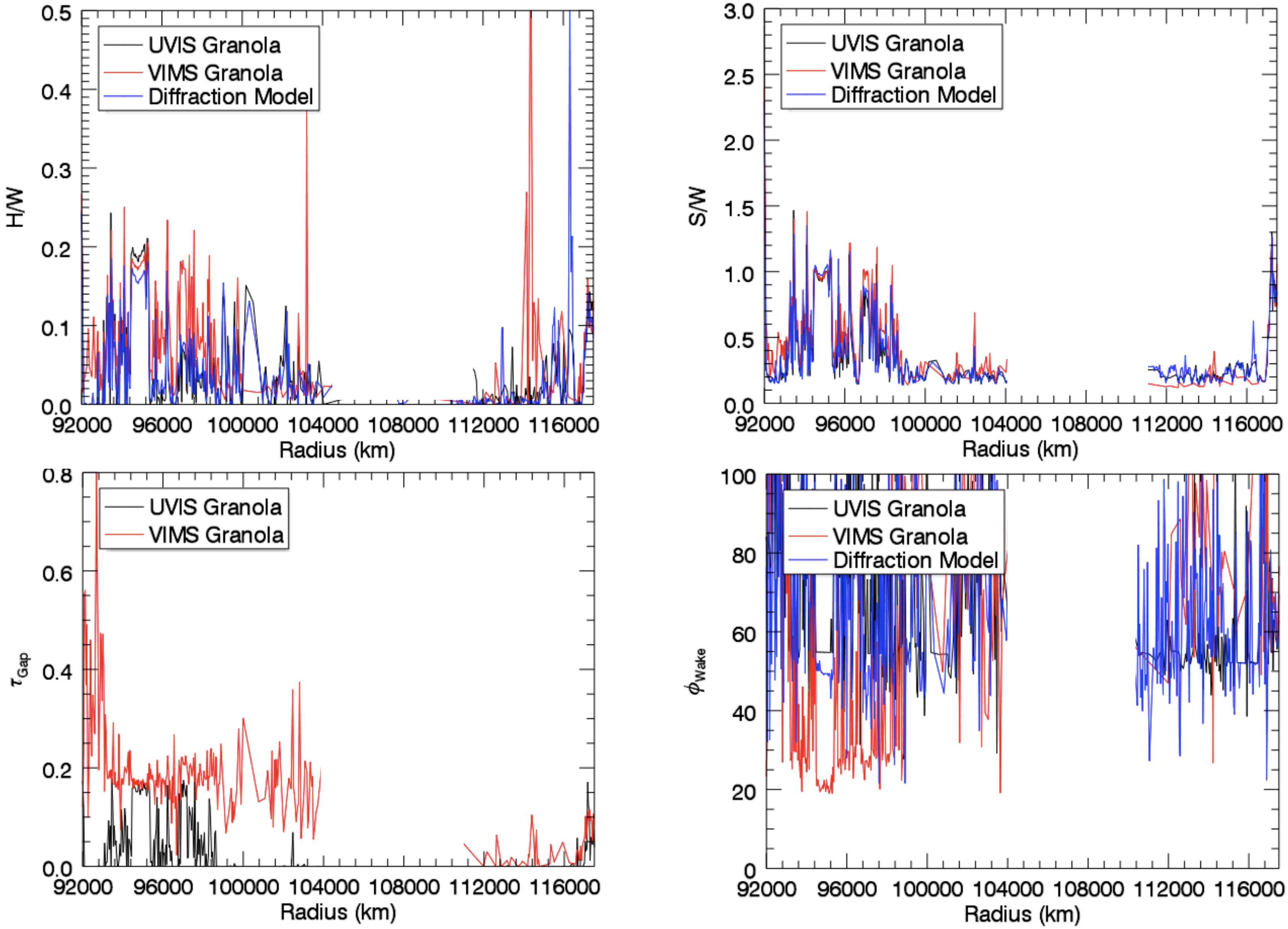
Model Results (A Ring)



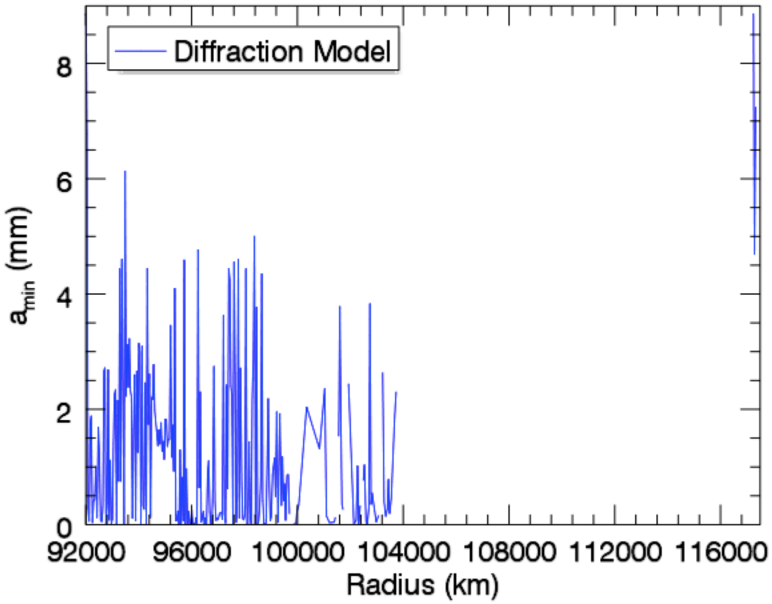


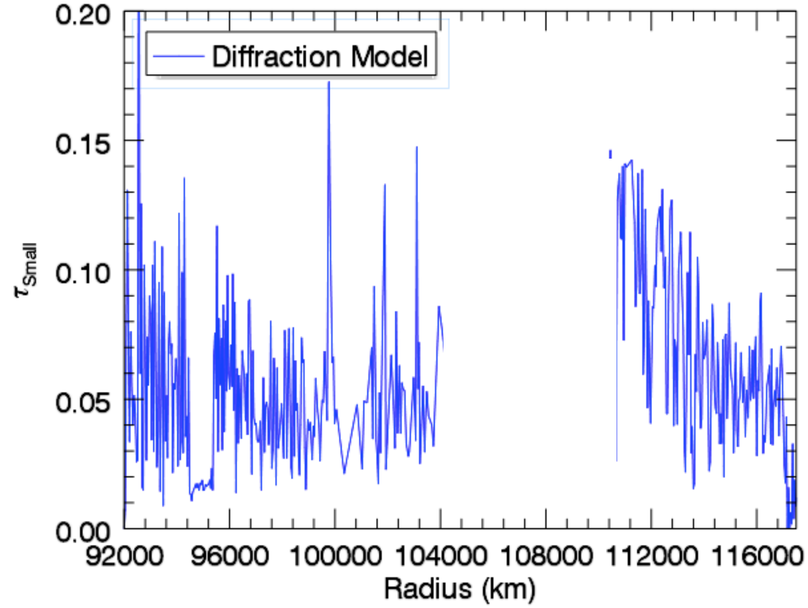
- τ_{small} similar to $\Delta\tau$ for α-SCO (13) occultations.
- Increasing number of sub-cm particles outward through A Ring.
- Sub-mm particles in outer A Ring.

Model Results (B Ring)

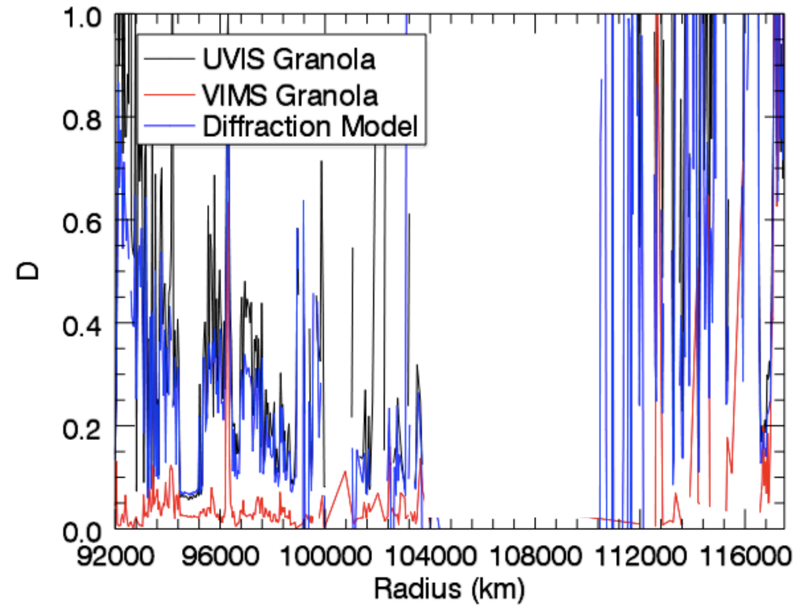


Model Results (B Ring)



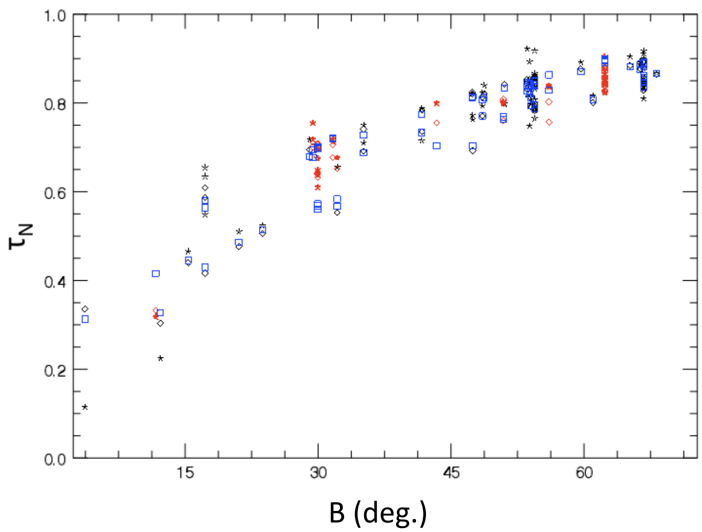


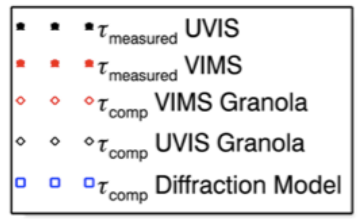
- Sub-cm particles in B1.
- 2mm particles in lower optical depth region at 95,000km.



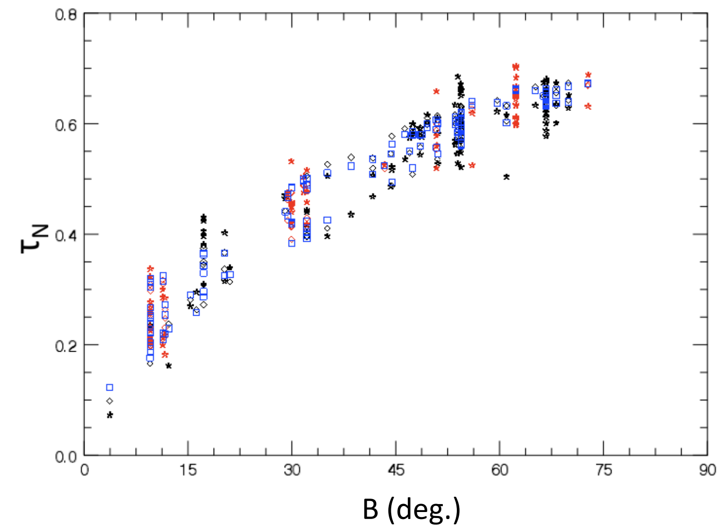
Computed Optical Depths

R = 95,000 km

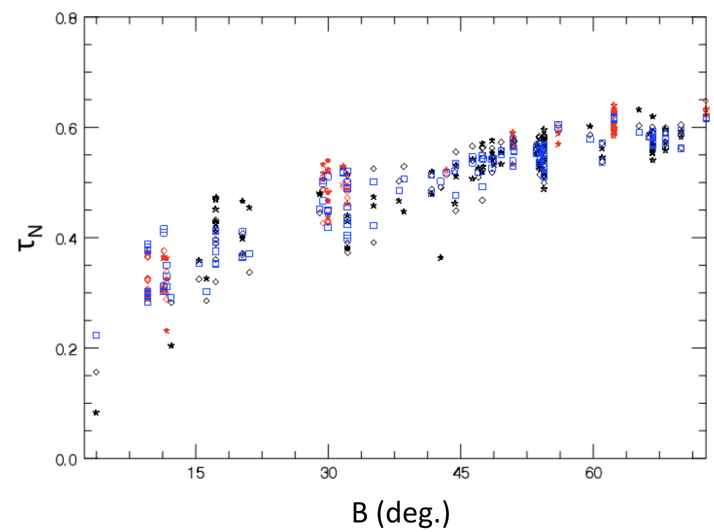




R = 128,000 km



R = 135,000 km



- Not matching at B angles below ~3°.

Conclusions

- Trend of increasing number of sub-cm particles outward throughout A Ring.
- Sub-cm particles in B1, Innermost and outermost portions of A Ring.
- Particles ranging from 2mm down to <0.5mm in Trans-Encke region.
- A ring wake parameters consistent with previous studies: Colwell et al. 2006, 2007, Hedman et al. 2007, Nicholson, Hedman 2009.

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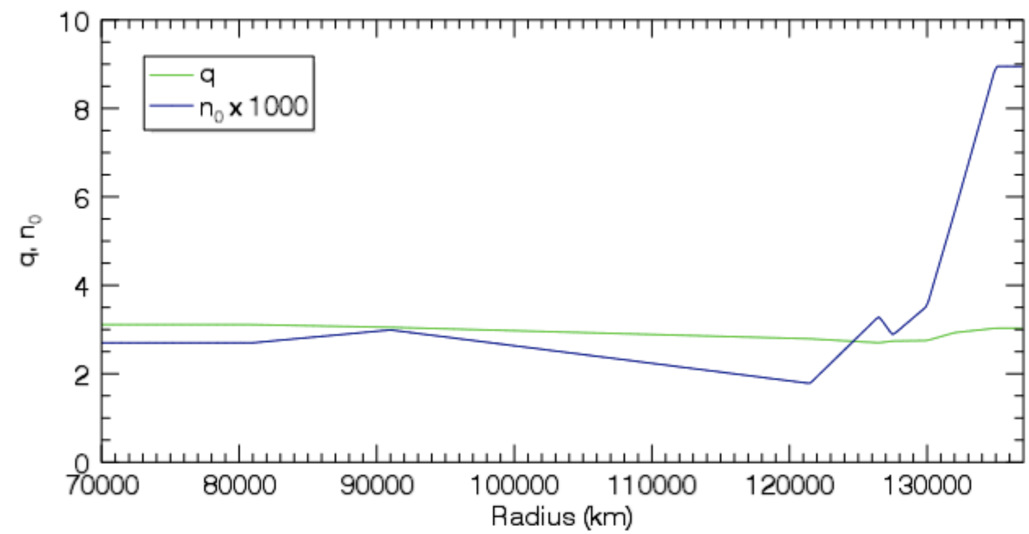
Other Fixed Parameters

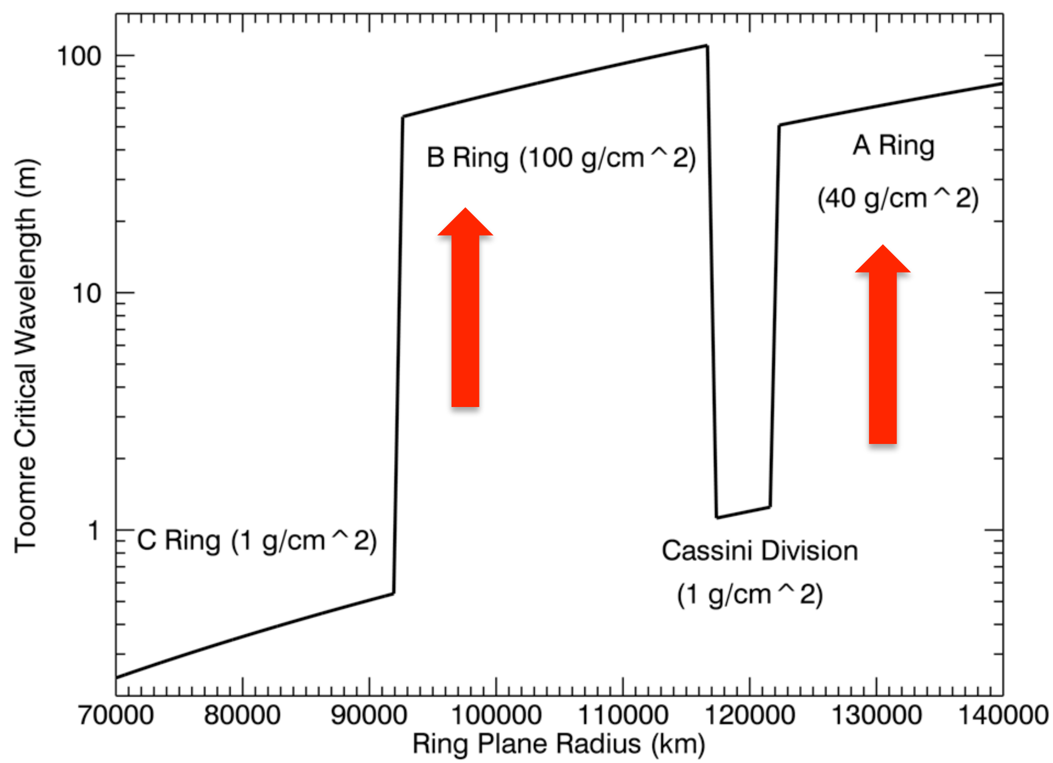
- q and n_0 in differential power law size distribution estimated from Voyager RSS data (Cuzzi et al. 2009):



- Analytic Rectangular Cross-section Wake Analytic Model:

$$T = e^{-\frac{\tau_N}{\mu}} = \frac{S/W - H/W\left|\sin(\phi - \phi_{wake})\right|\cot B}{S/W + 1} e^{-\frac{1}{\mu}(\tau_{Gap} + \tau_{small})}$$

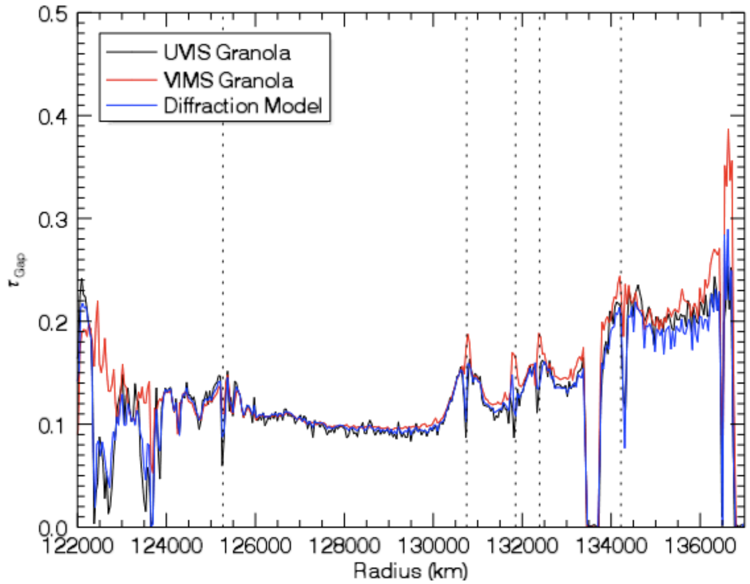
Self-Gravity Wakes

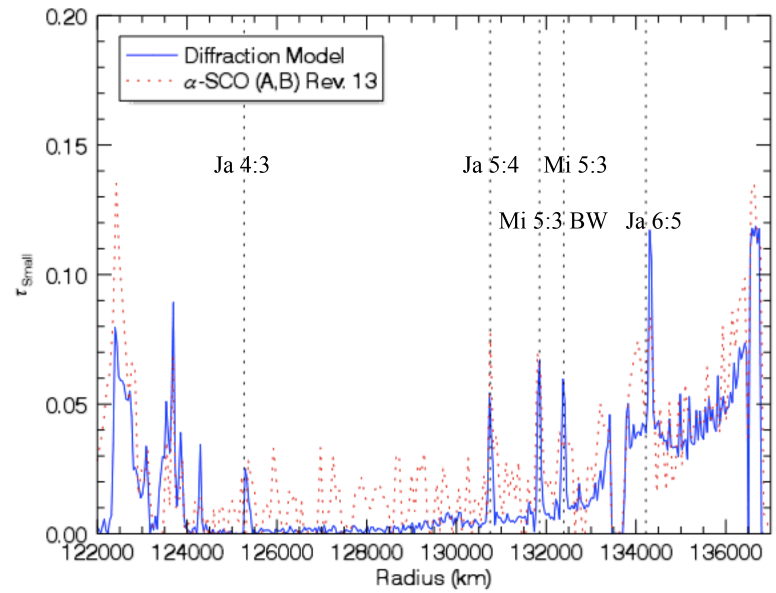


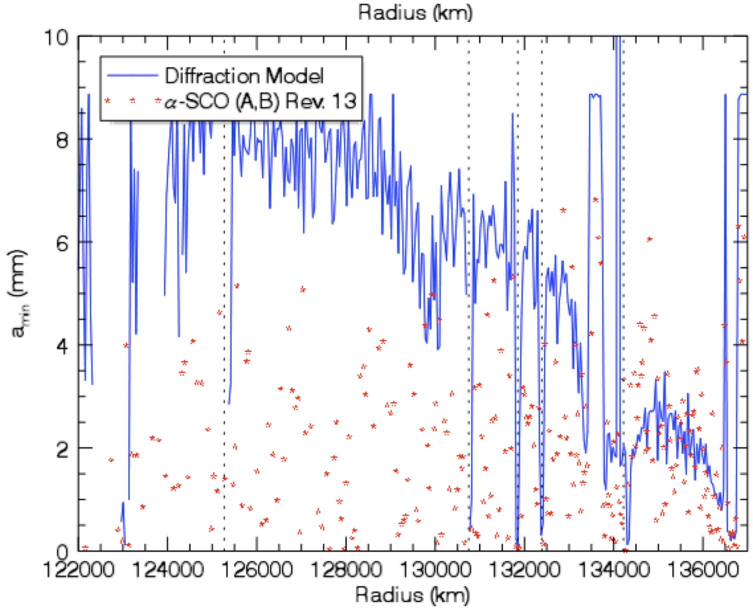
The Toomre Critical Wavelength (most unstable wavelength for gravitational collapse) in Saturn's rings. Only in the A and B rings is λ_{crit} significantly larger than individual ring particles.

$$\lambda_{crit} = 4\pi^2 G\sigma / \kappa^2 \approx 1-100 \text{ m}$$

Model Results (A Ring)



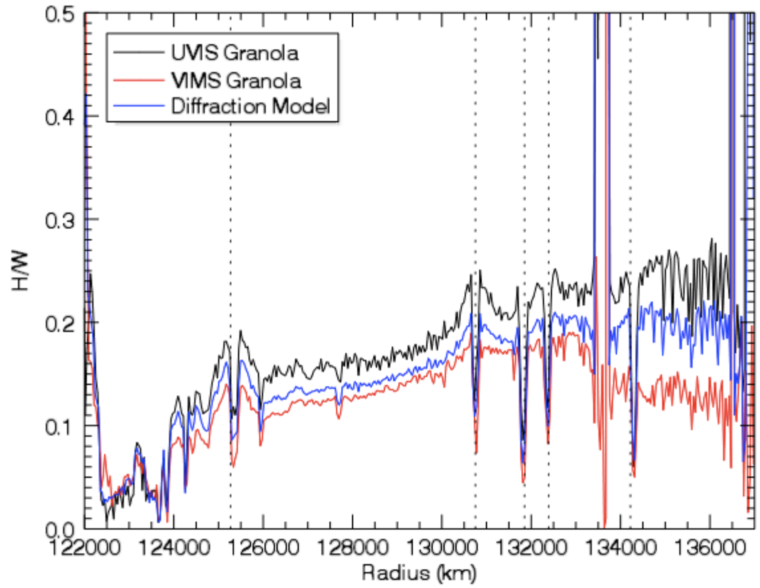


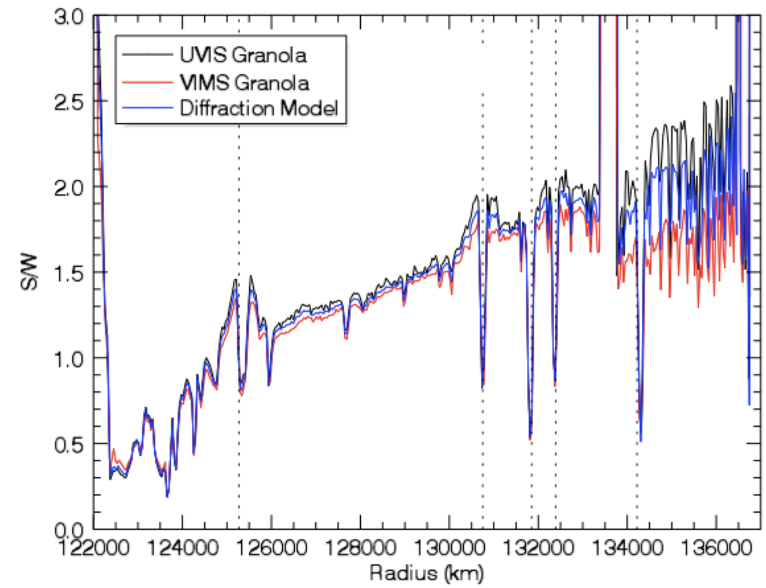


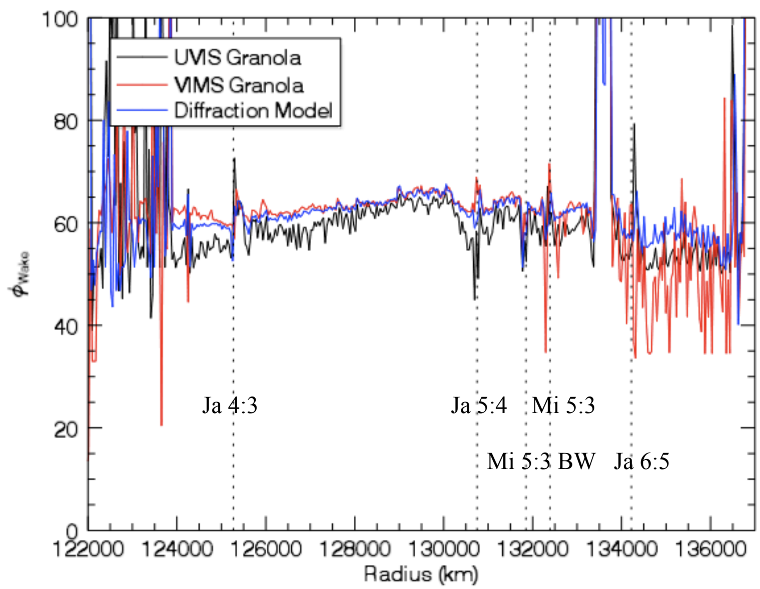
- τ_{Gap} consistent between VIMS/ UVIS in central A ring but diverges in Trans-Encke region and at prominent resonances.
- τ_{small} similar to $\Delta\tau$ for α-SCO (13) occultations.

- Evidence of sub-mm particles in outer A Ring.

Model Results (A Ring)

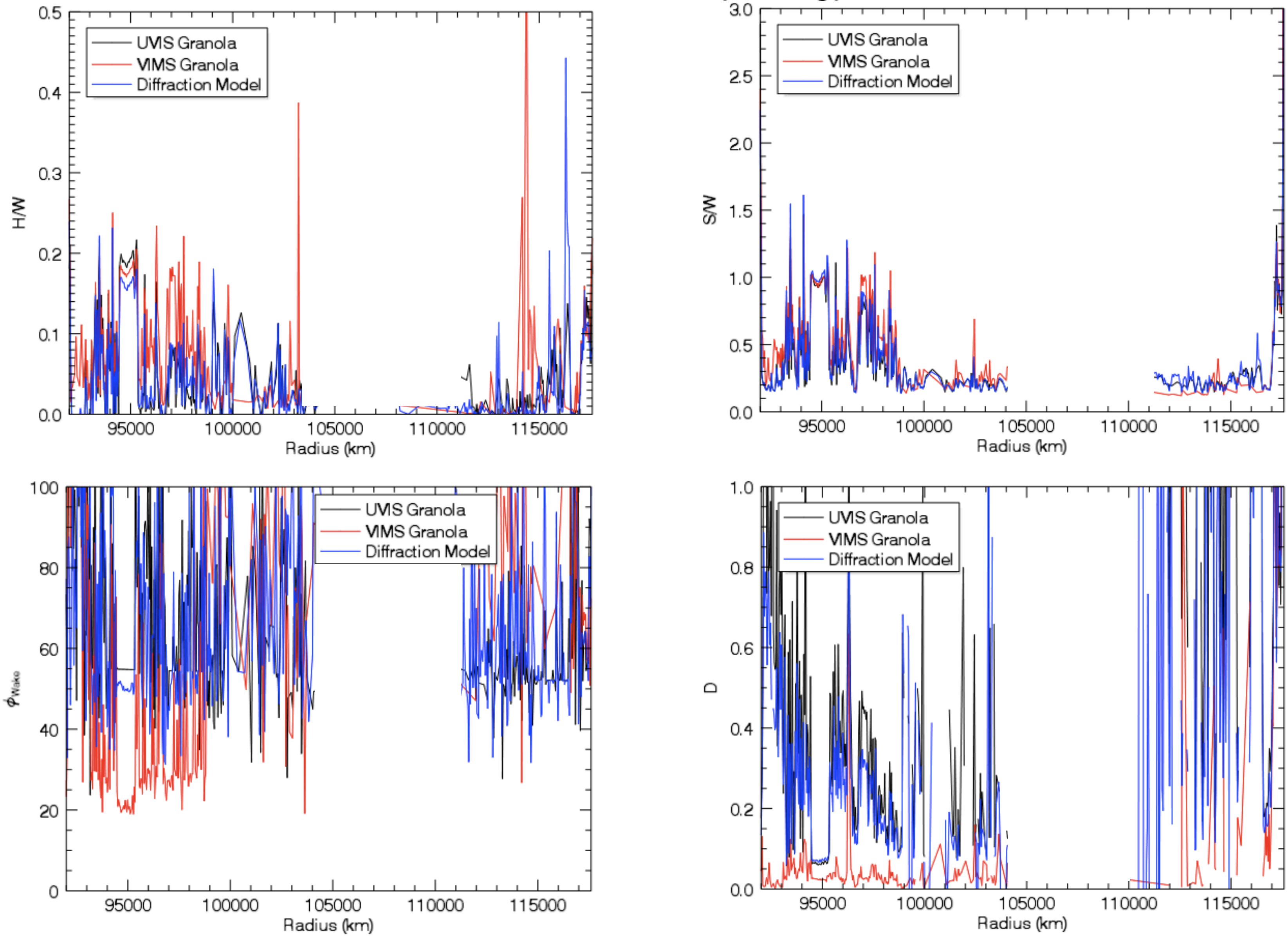






- Self Gravity Wake parameters generally inline with previously published results: Colwell et al. 2006, Hedman et al. 2007, Nicholson, Hedman 2009.
- H/W significantly different between VIMS and UVIS, Diffraction Model splits the difference.

Model Results (B Ring)



Model Results (B Ring)