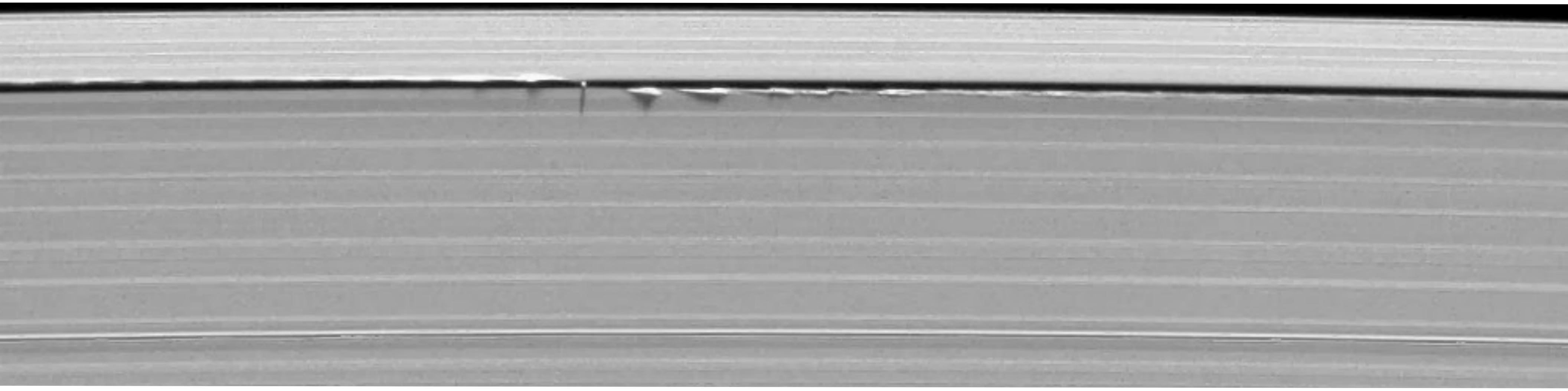

PARTICLE CLUSTERING IN PERIODICALLY FORCED PLANETARY RINGS

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Background and Goal

- Analytic models of rings predict a variety of structure, but the models are limited.
- *N*-body simulations have their own assumptions, but allow one to calculate many parameters and view structure that develops given those assumptions and parameters.
- Esposito *et al.* (2012) predicted with an analytic model that structure in the rings follows a predator-prey situation between mass aggregates and mean-square velocity.

~ We want to test that model with *N*-body simulations.

METHOD (IDEALIZED FORCING)

- REBOUND *N*-body code. (Rein & Liu, 2011)
- *Adjusted integrator to add forcing to a_x: $+= q \cdot \sin(\delta \cdot t) \cdot \sin(2\pi x/L_x)$
  - q sets the magnitude of the forcing
  - δ sets the forcing so it is not at resonance with the orbital period
  - second sin() sets forcing at 0 at cell boundaries to simplify ghost cells
- Calculate every 1/40th orbit: Viscosity, mean-squared velocity, mass aggregates.
- Esposito *et al.* (2012) observed predator-prey behavior after 4 forced orbits; we ran 6.

*Does **not** track azimuthal forcing. **Does** conserve angular momentum.

Parameter Space

- Location: Outer edge B ring, a_0 = 117.56 Mm
- $L_x = L_y = 10 \cdot \lambda_{crit}$ (cell sizes 340 – 1360 m)
- N = 37,000 – 1,169,000 (largest haven't finished yet; N = 493,00 have)
- **Four** simulations run for *every* parameter set: 192 simulations, ≈15,300 CPU hours (1.74 yrs). (Robbins *et al.* (2010) used 27,000 CPU hrs)

τ		ρ		q^*		δ
1.0		0.225		0		0.7
1.5	x	0.450	x	20x	x	1.3
2.0				50x		
				100x		

*Multiples of the RMS particle acceleration at steady-state (orbit 6.000). q = 0 is unforced.

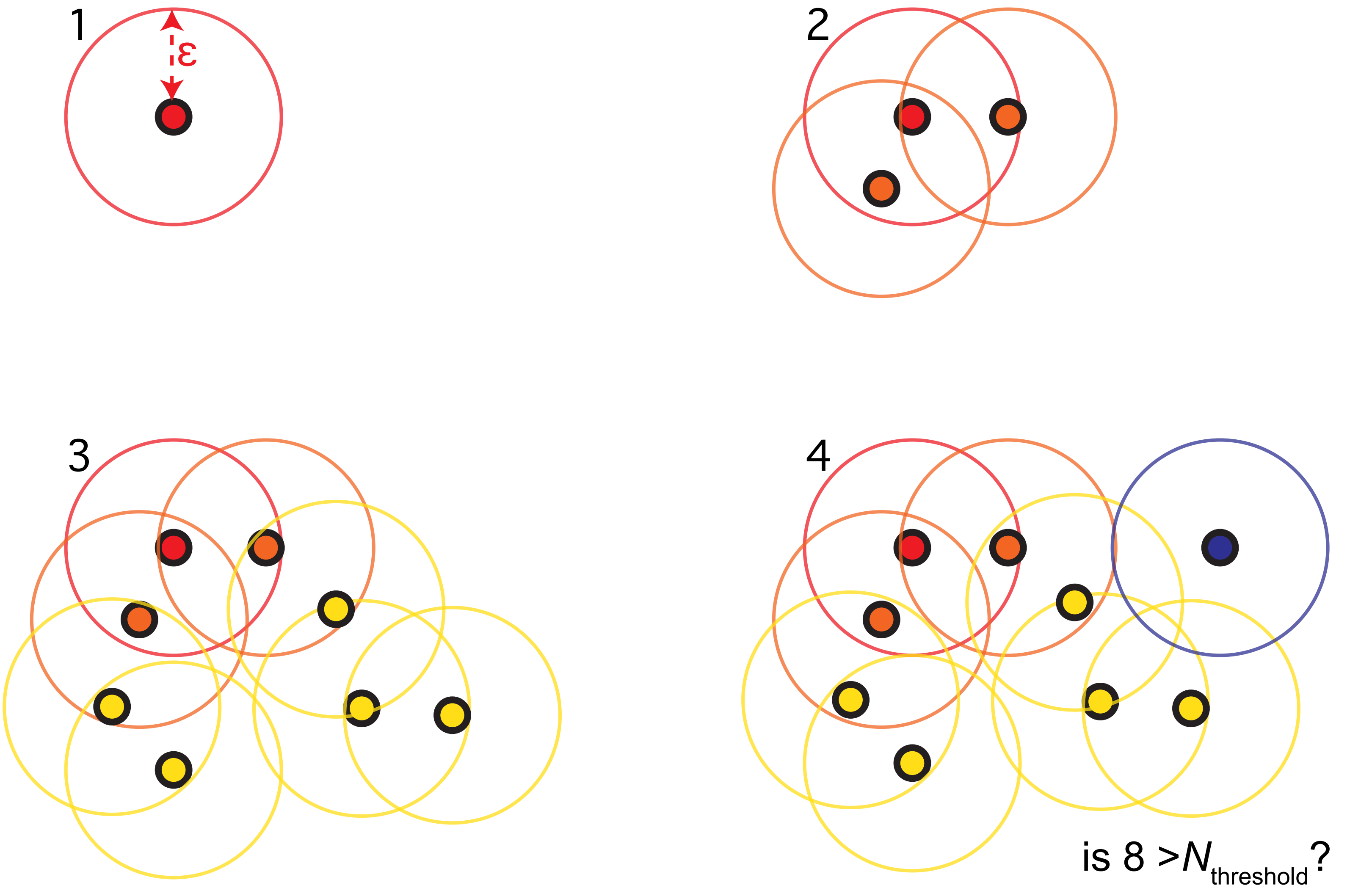
MASS AGGREGATES

- First variable is mean-square velocity.
- Second variable is mean aggregate mass (second moment of mass distribution):

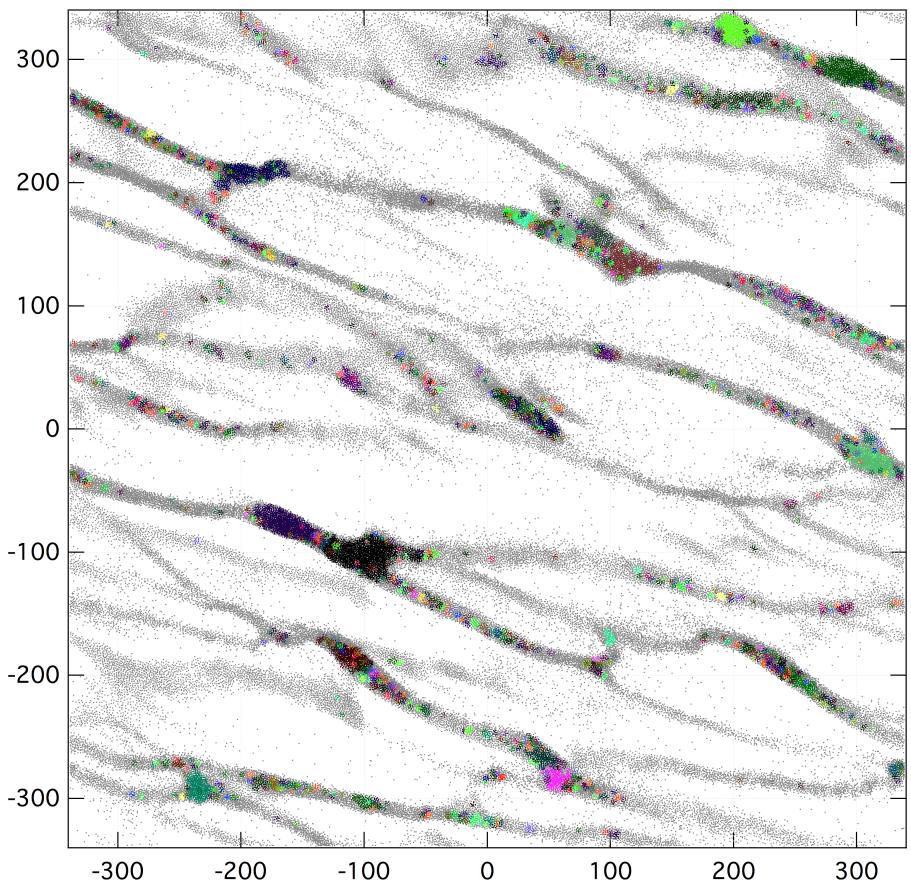
$$\mathcal{M} = \frac{\sum_{j}^{\text{number of clumps}} \left(\sum_{i}^{\text{particles in clump}} m_i \right)^2}{\sum_{j}^{\text{number of clumps}} \left(\sum_{i}^{\text{particles in clump}} m_i \right)}$$

- But: Need a method to identify clumps.
  - needs to be a quantitative method
  - needs to be a hard cluster code (particles uniquely belong to one cluster)
  - needs to have a minimum number of adjustable parameters
  - already have a DBSCAN (Ester *et al*., 1996) implementation for craters (Robbins *et al*., 2014), adapted to use for rings particles!

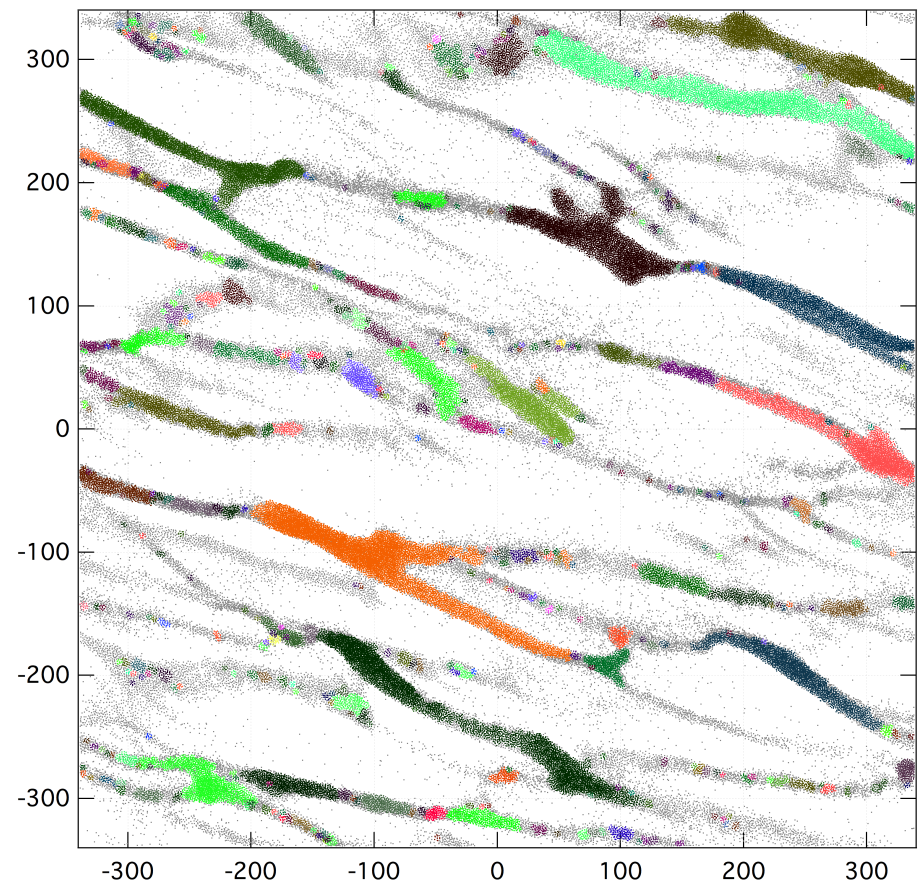
Mass Aggregates:
How DBSCAN Works (2 inputs)



Mass Aggregates: Examples (τ=1.0, ρ=0.45)

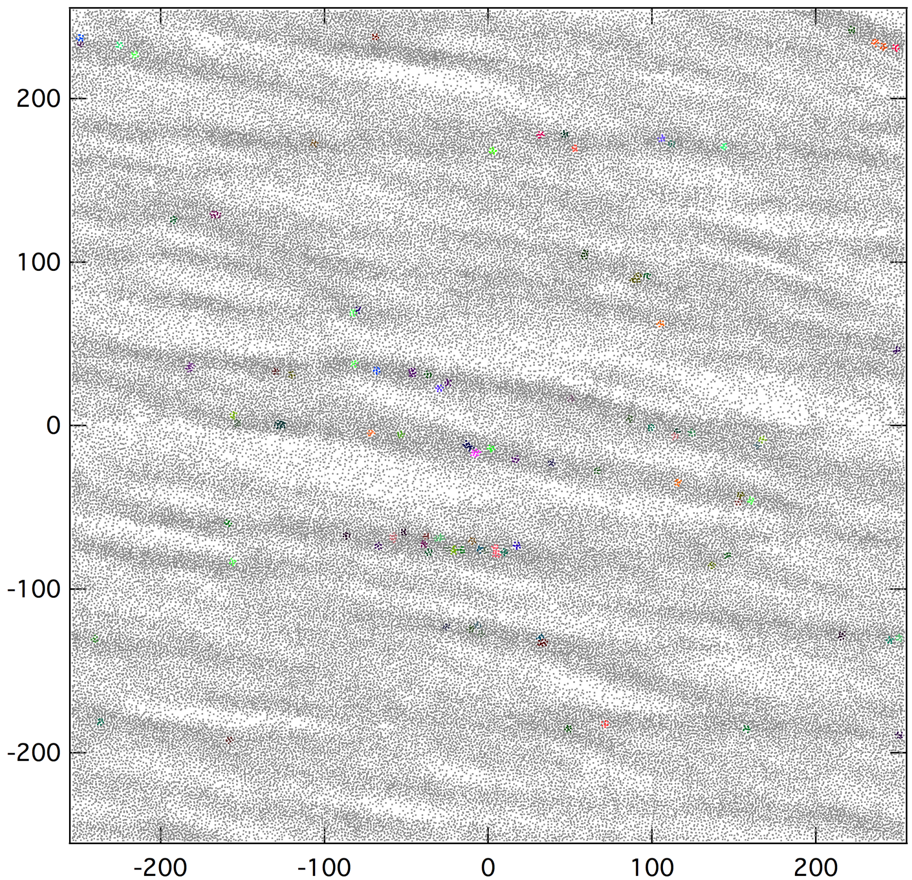


$R = 2.2r_p$ || $N = 10$

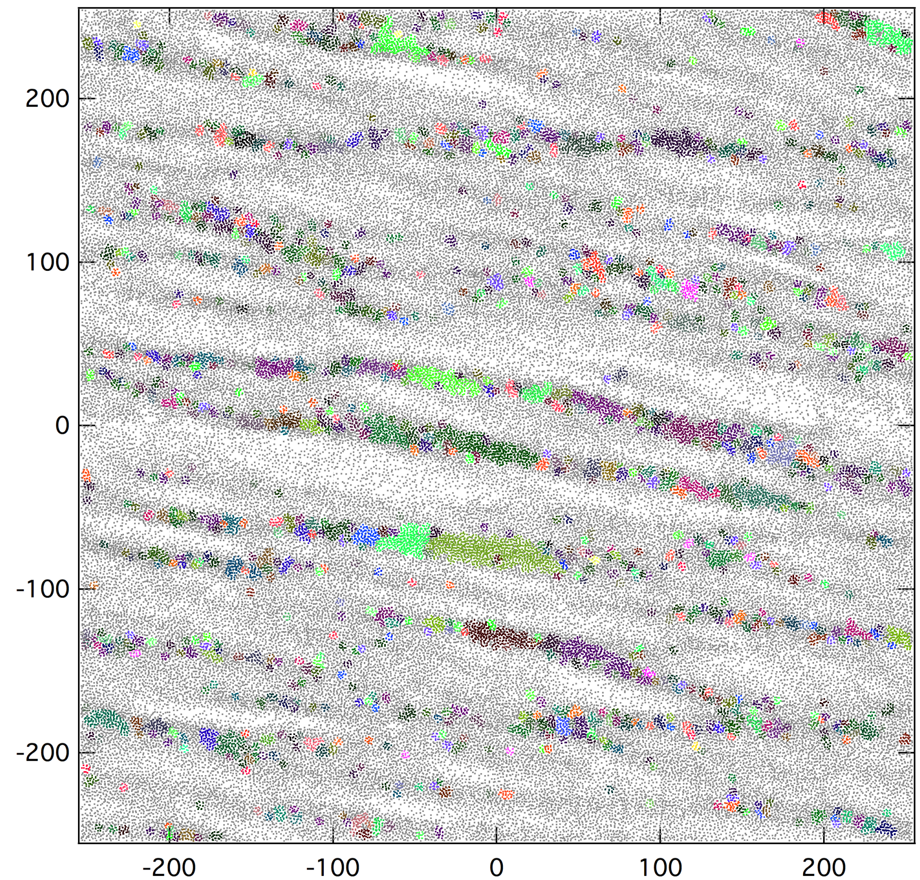


$R = 2.7r_p$ || $N = 12$

Mass Aggregates:
Examples (τ=1.5, ρ=0.225)

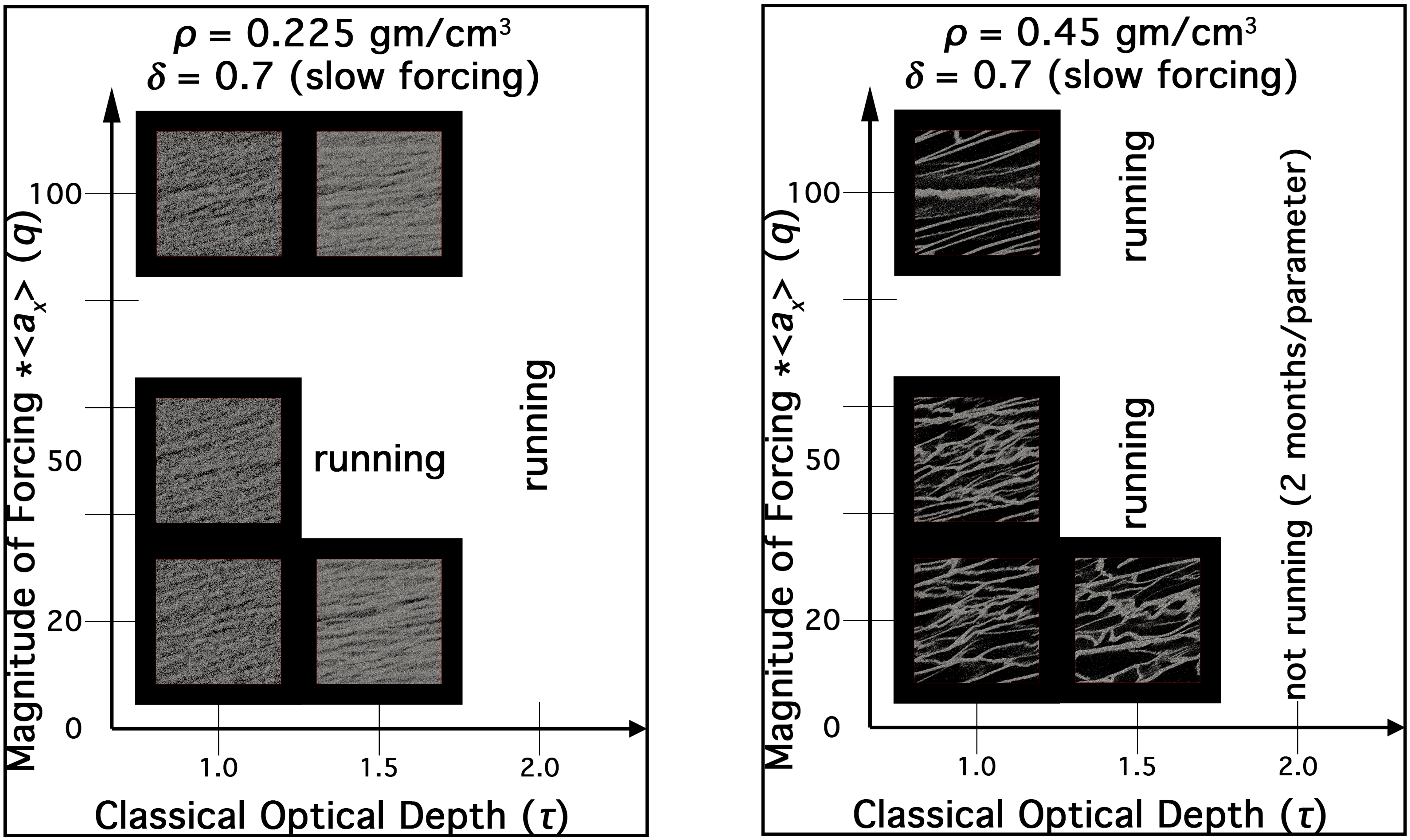


$R = 2.2r_p$ || $N = 10$

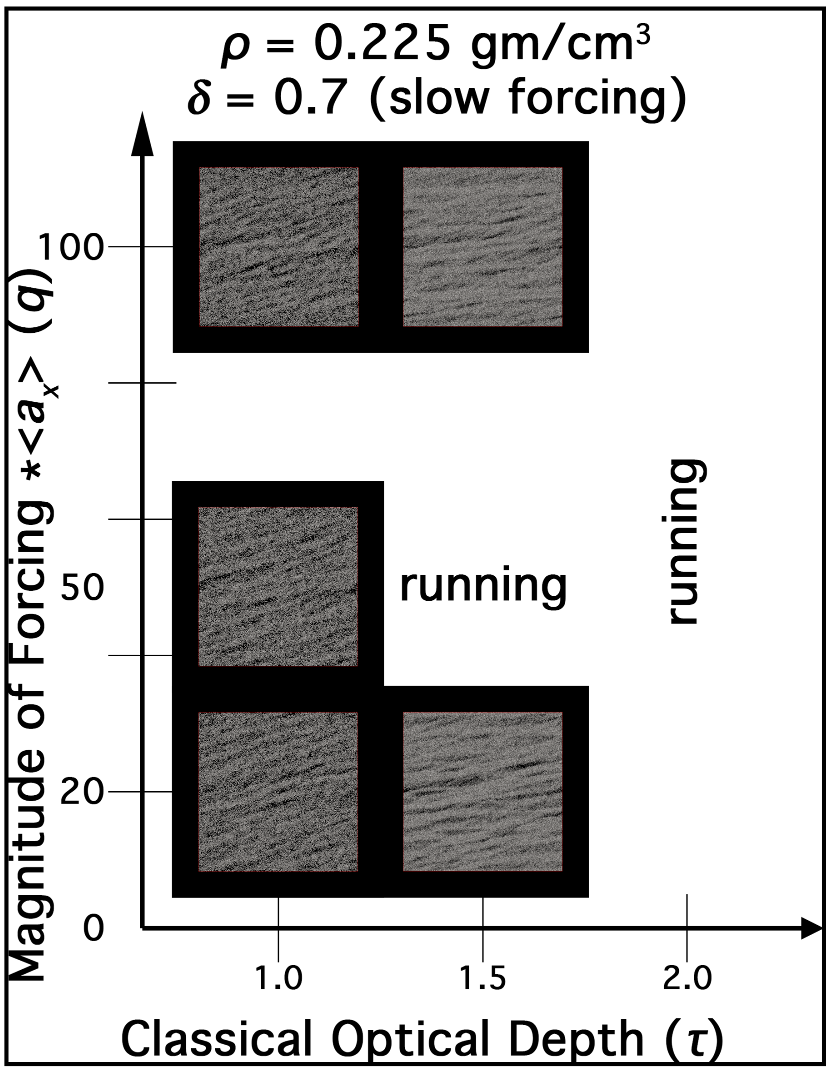


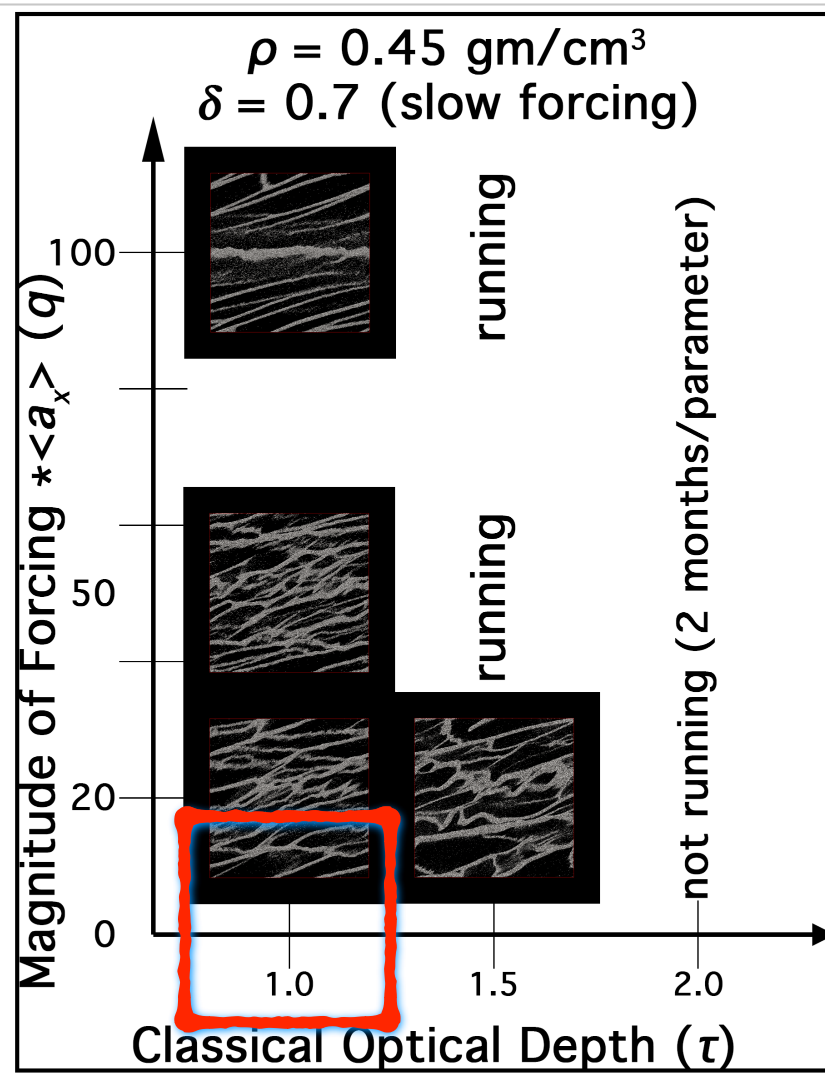
$R = 2.7r_p$ || $N = 12$

RESULTS:
PARAMETER SPACE NAVIGATION

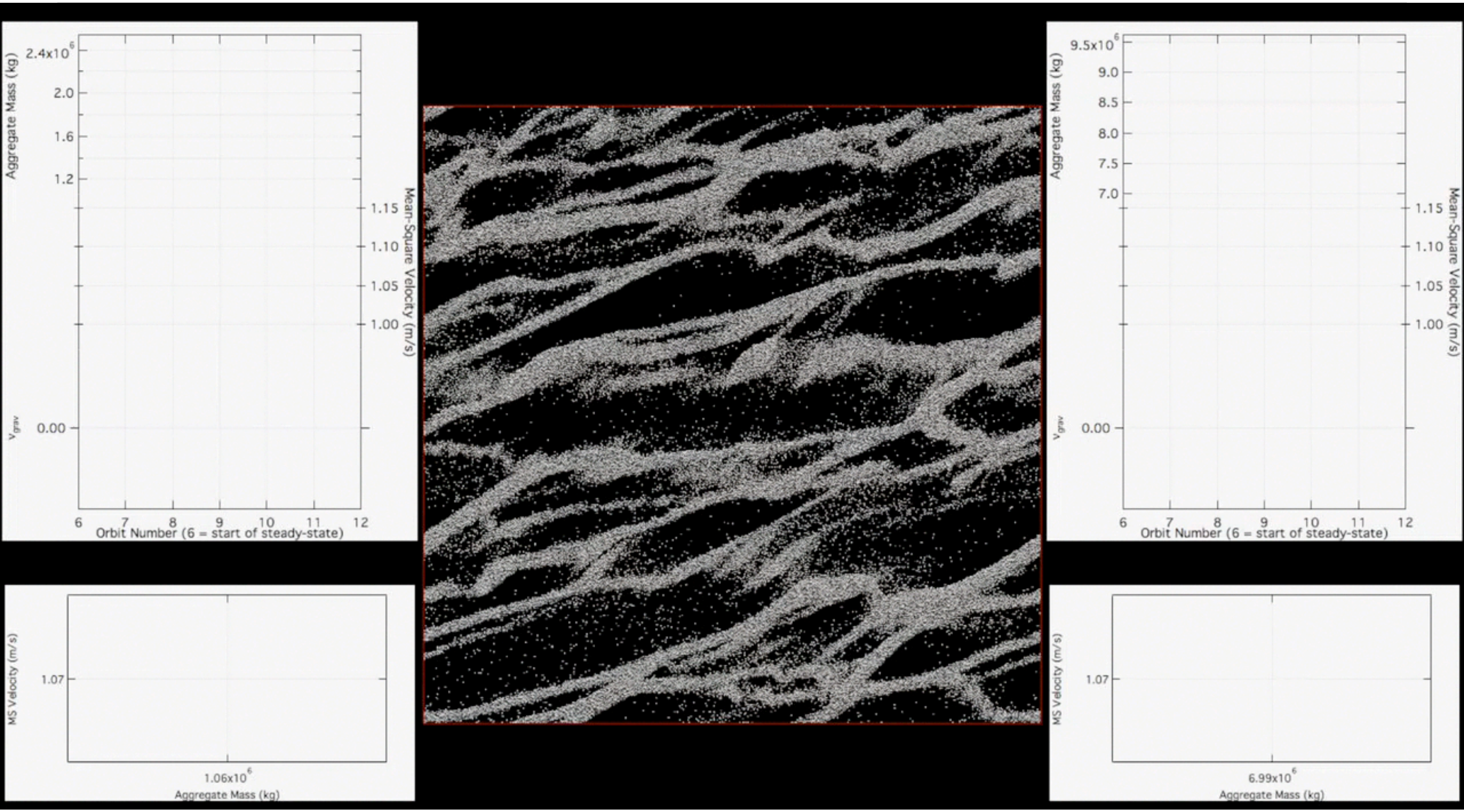


RESULTS:
PARAMETER SPACE NAVIGATION





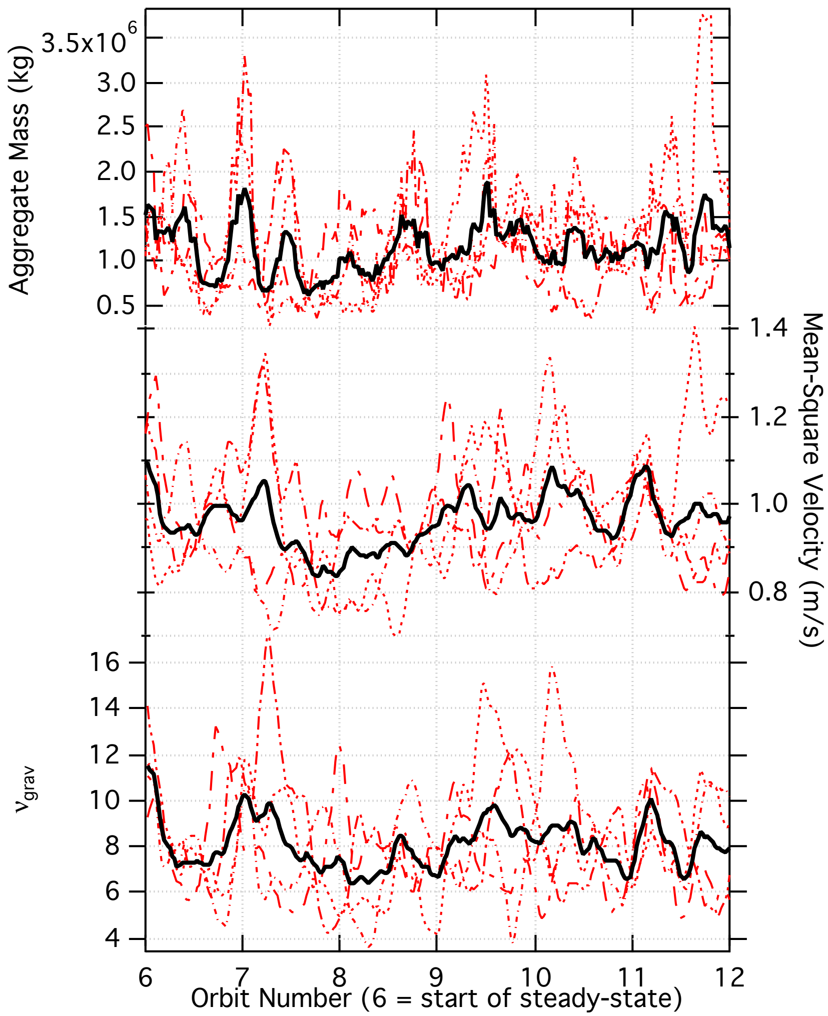
RESULTS:
τ=1.0, ρ=0.45, UNFORCED (STILL SEE VARIABILITY)

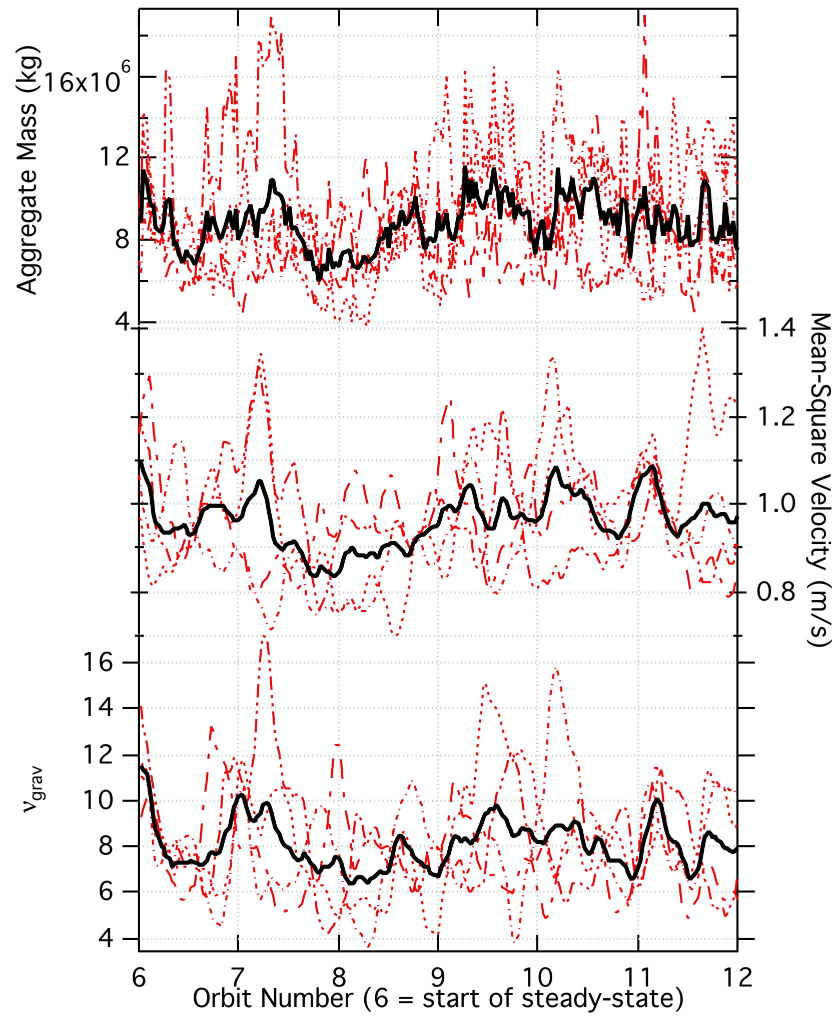


RESULTS:

τ=1.0, ρ=0.45, UNFORCED (STILL SEE VARIABILITY)

Unforced

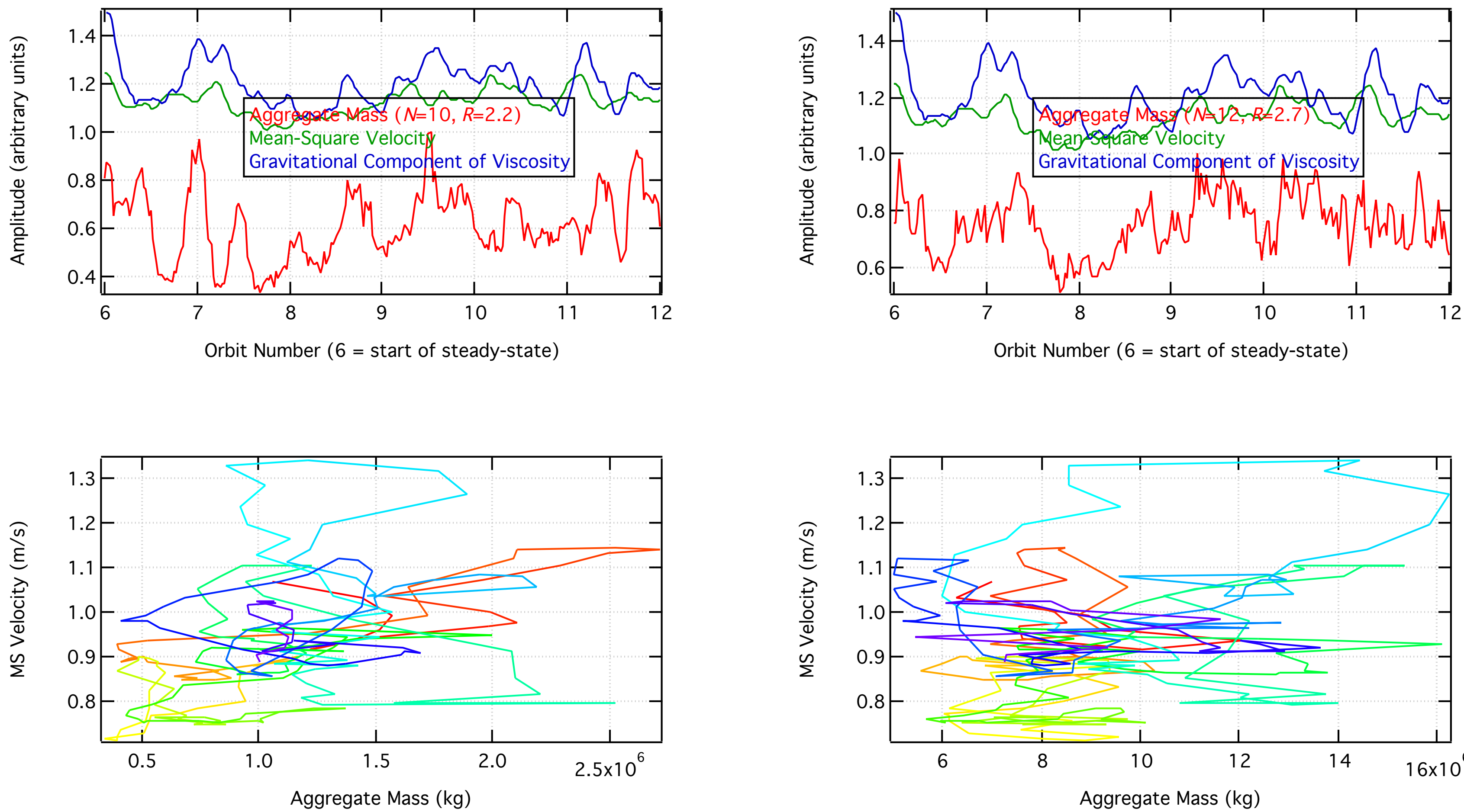




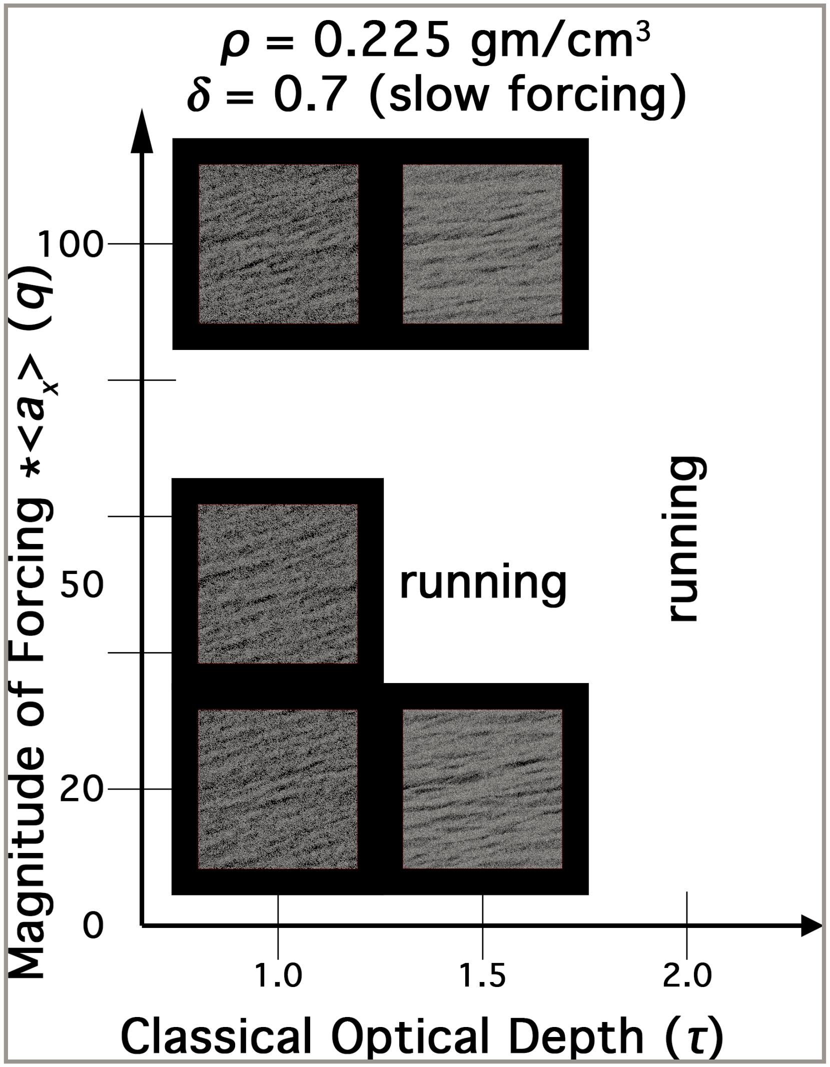
RESULTS:

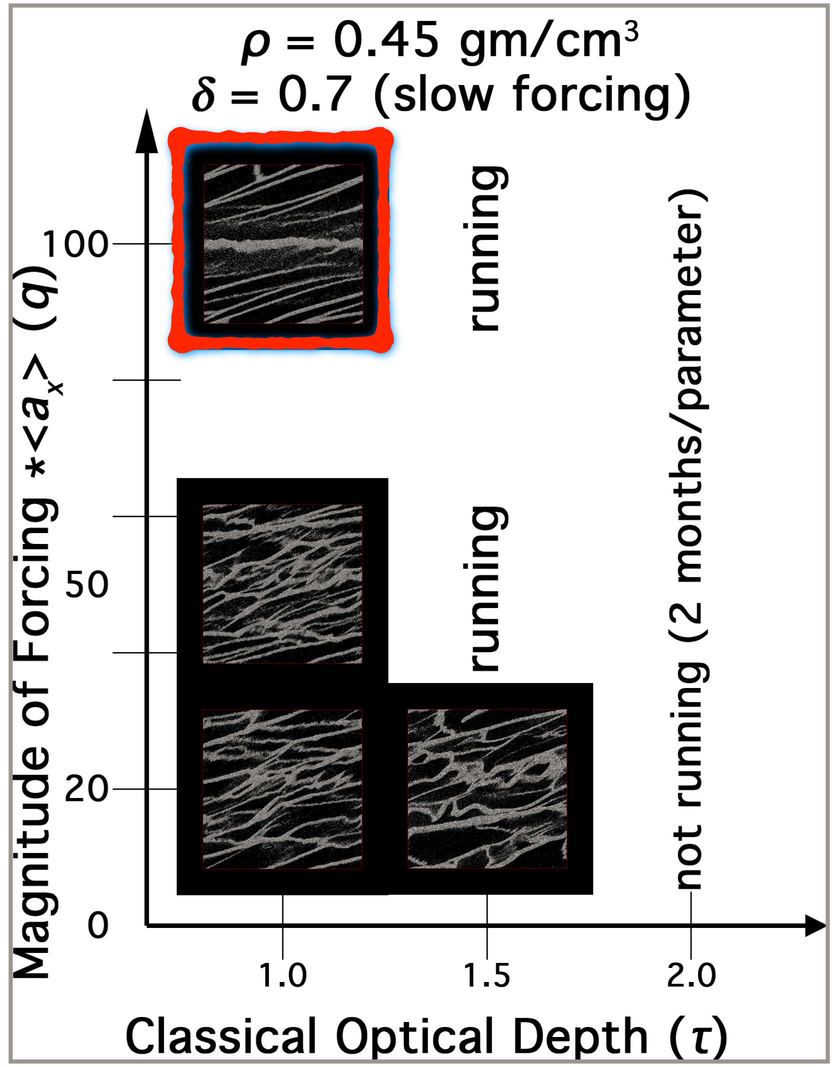
τ=1.0, ρ=0.45, UNFORCED (STILL SEE VARIABILITY)

Unforced



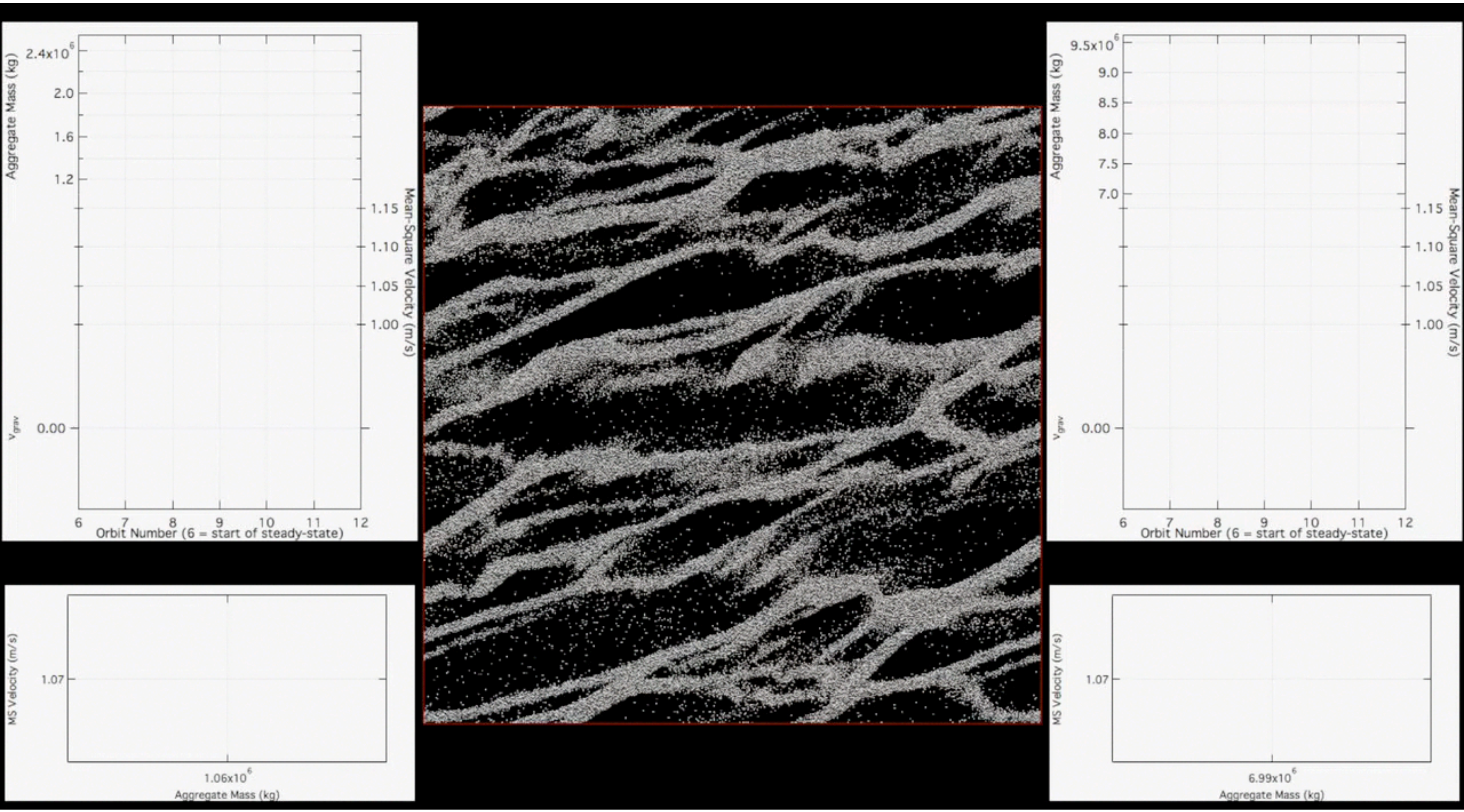
Results:
Navigational Chart





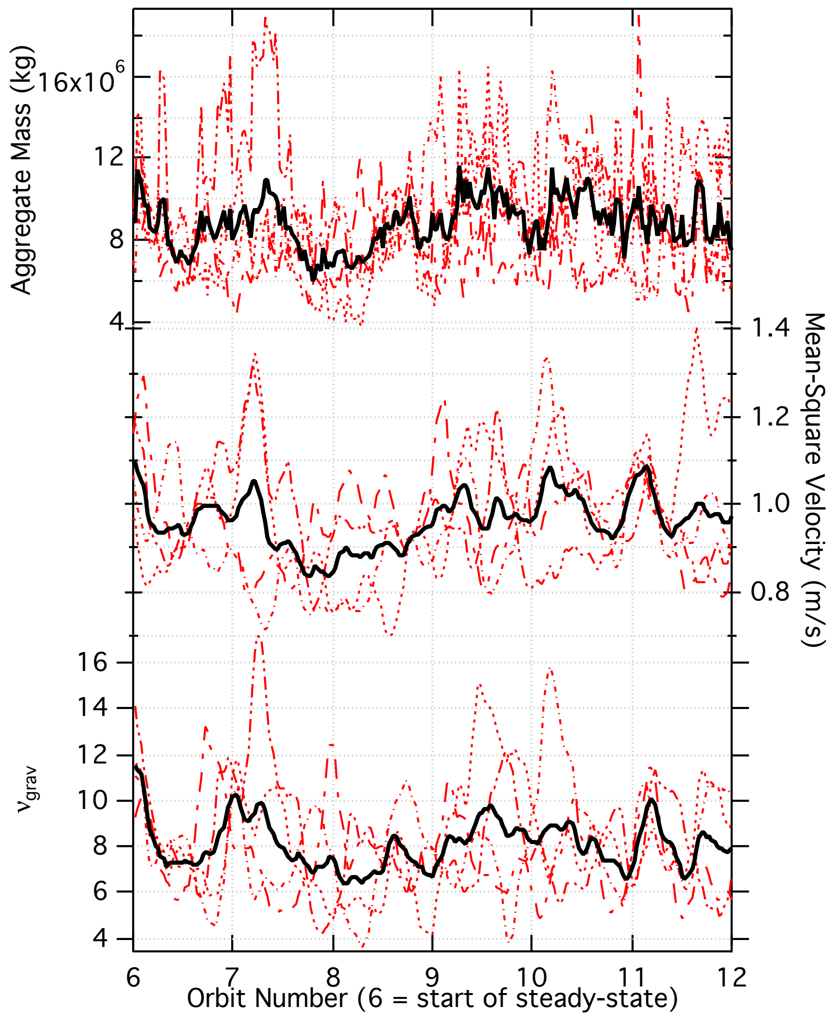
RESULTS (SIMILAR BEHAVIOR TO LEWIS & STEWART ENKE GAP SIMULATIONS):

τ=1.0, ρ=0.45, Q=100x, δ=0.7

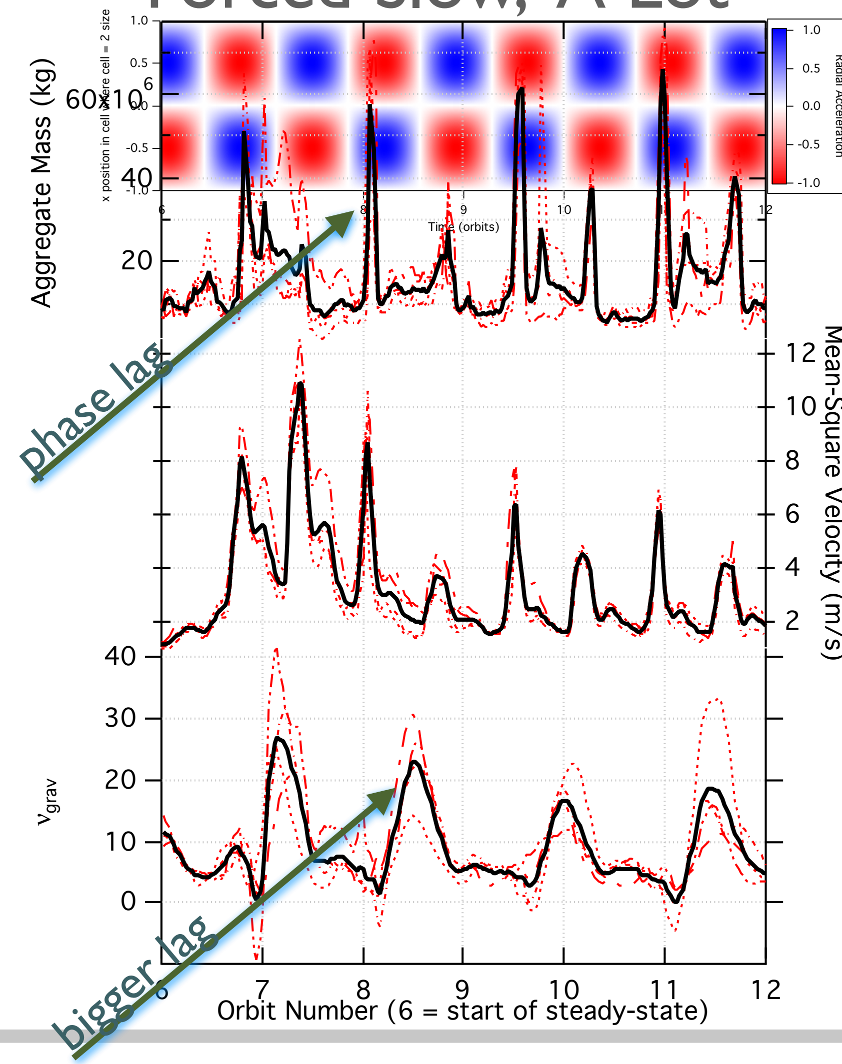


Results (Cluster Code "clumps" Parameters): τ=1.0, ρ=0.45, q=100x, δ=0.7

Unforced

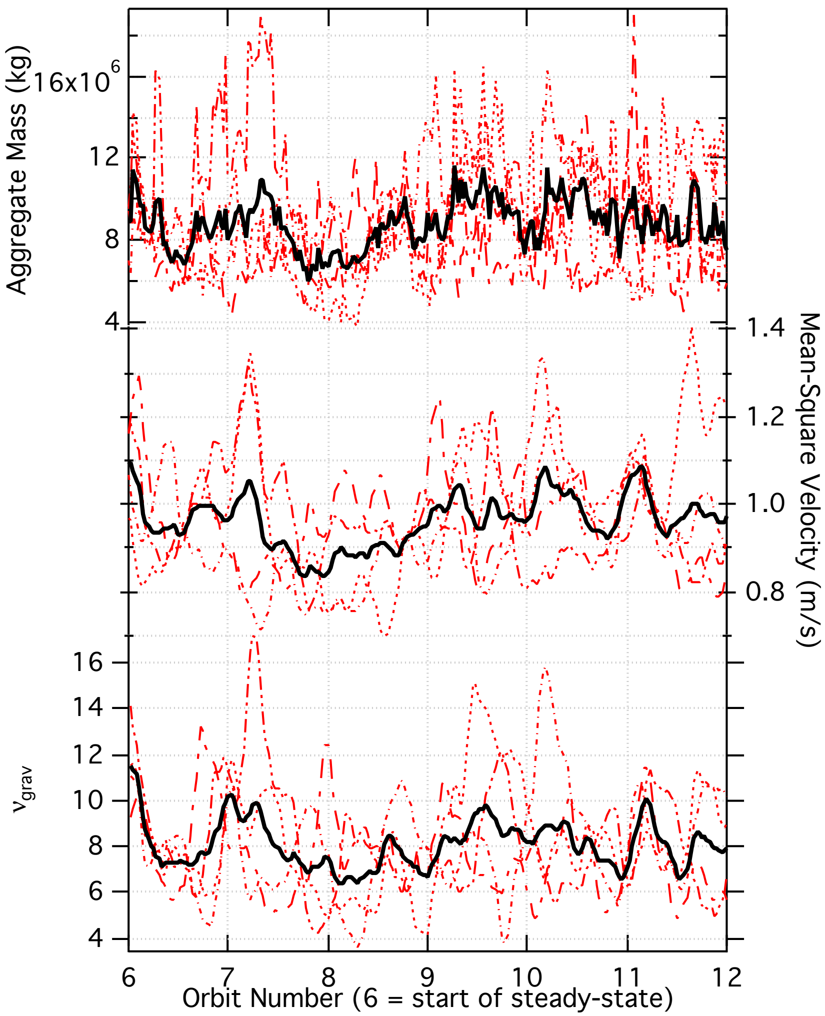


Forced Slow, A Lot

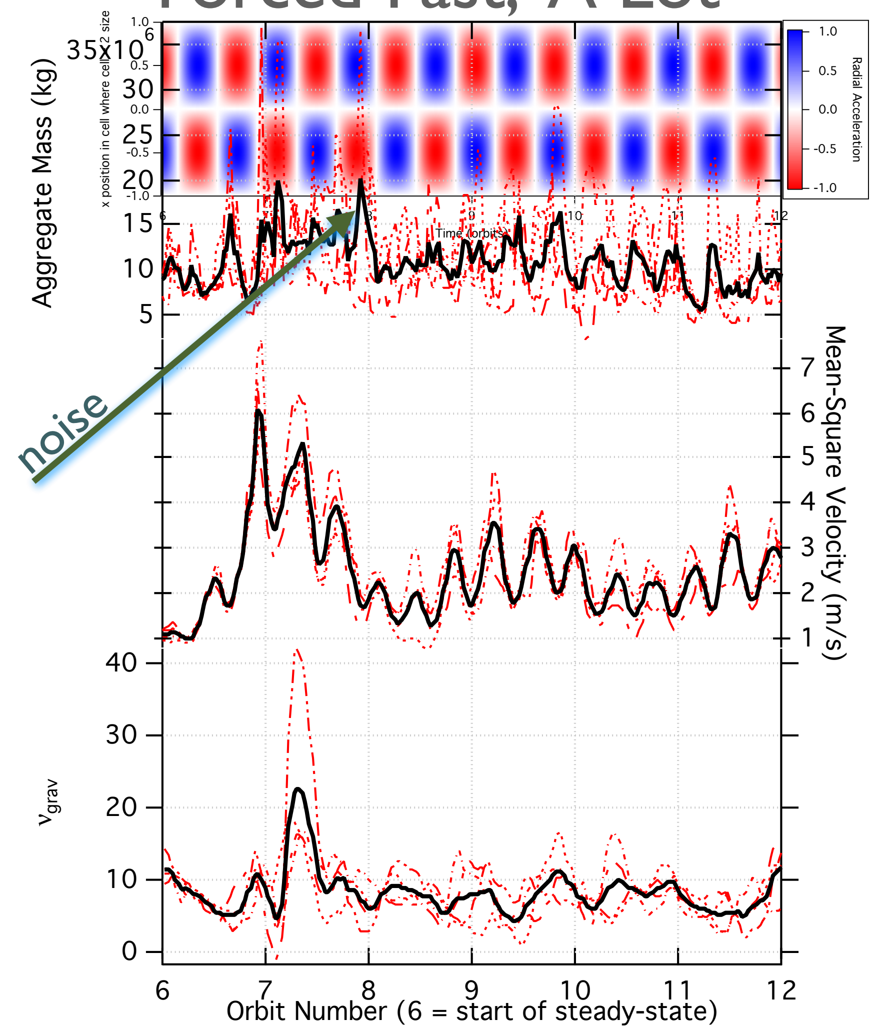


Results (Cluster Code "clumps" Parameters): τ=1.0, ρ=0.45, q=100x, δ=1.3

Unforced

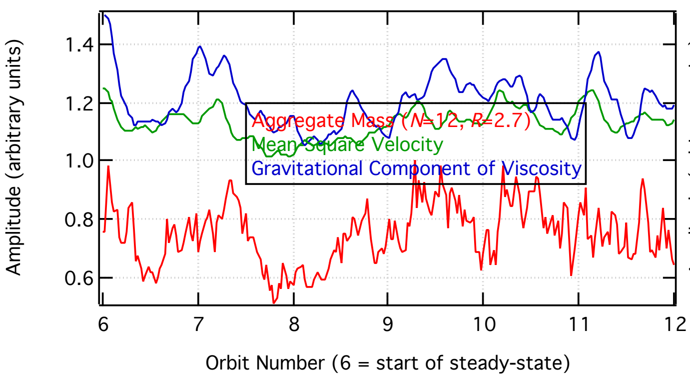


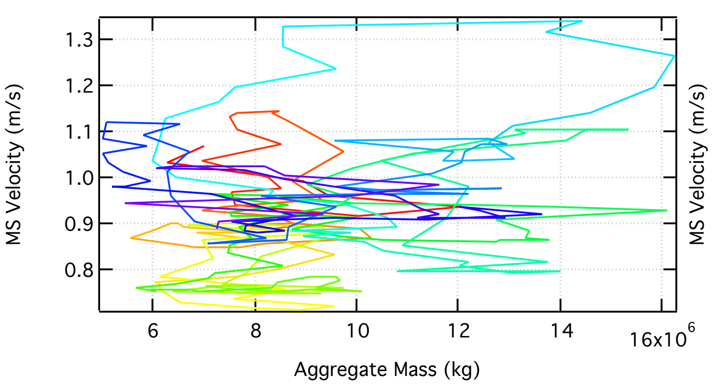
Forced Fast, A Lot



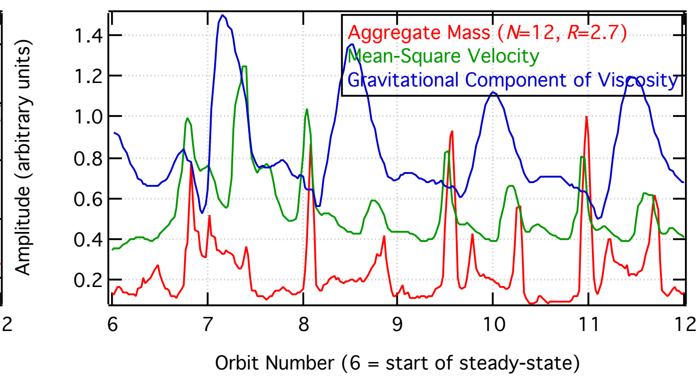
Results (Cluster Code "clumps" Parameters): $\tau=1.0$, $\rho=0.45$, $q=100$x

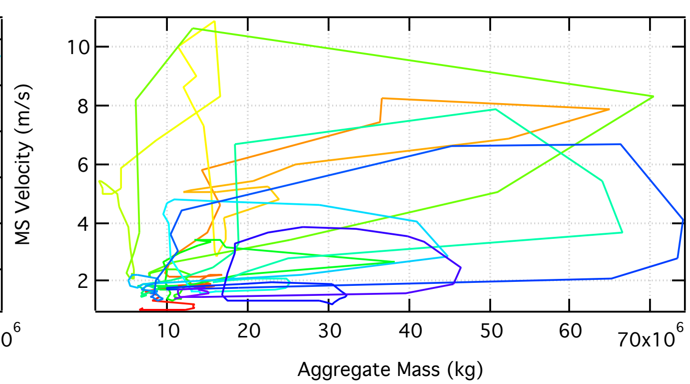
Unforced



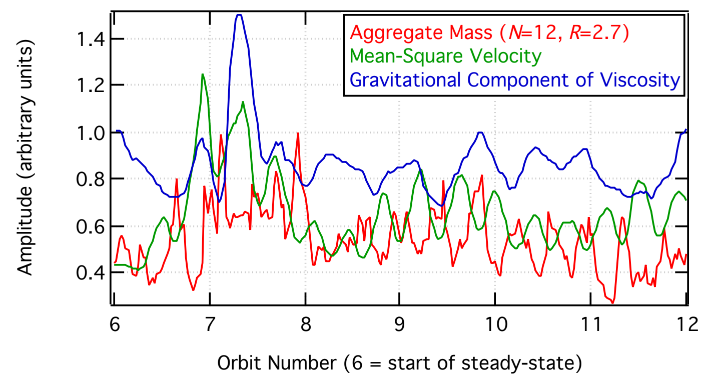


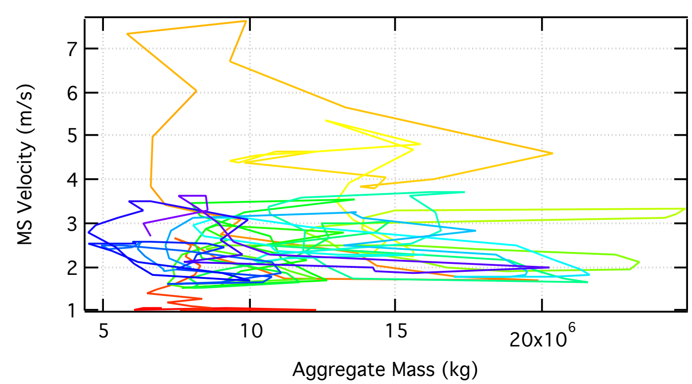
Forced Slow, A Lot



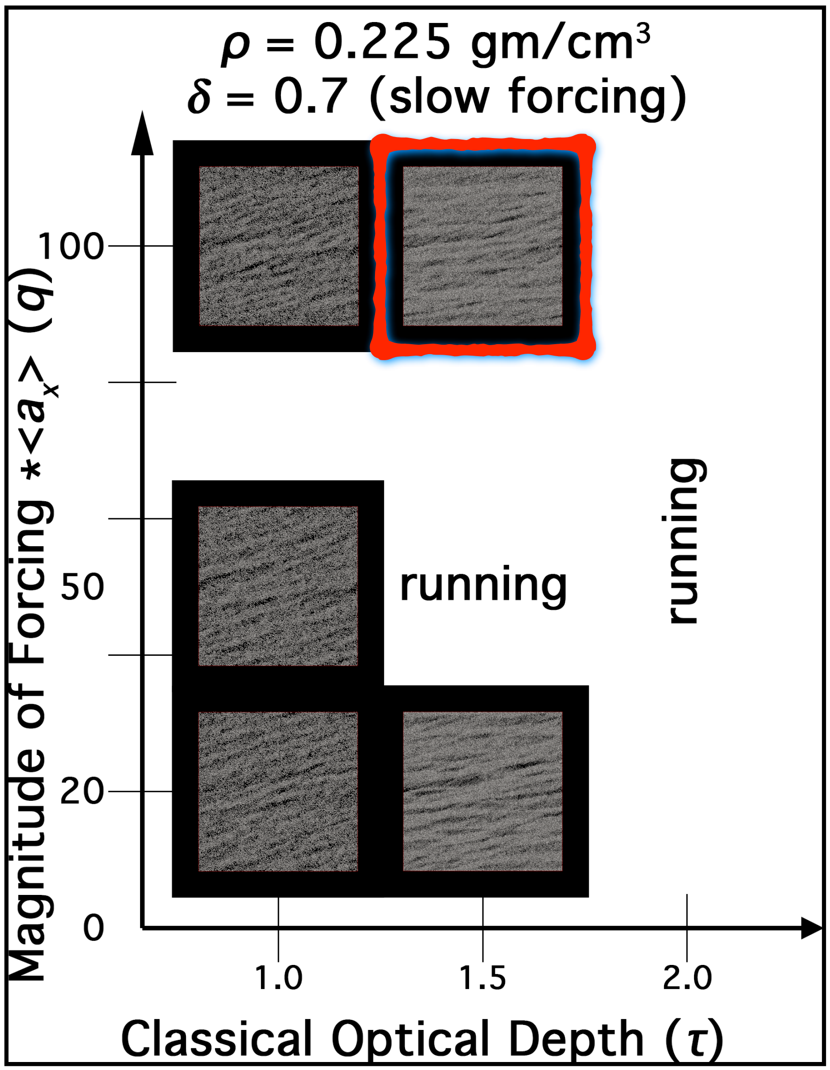


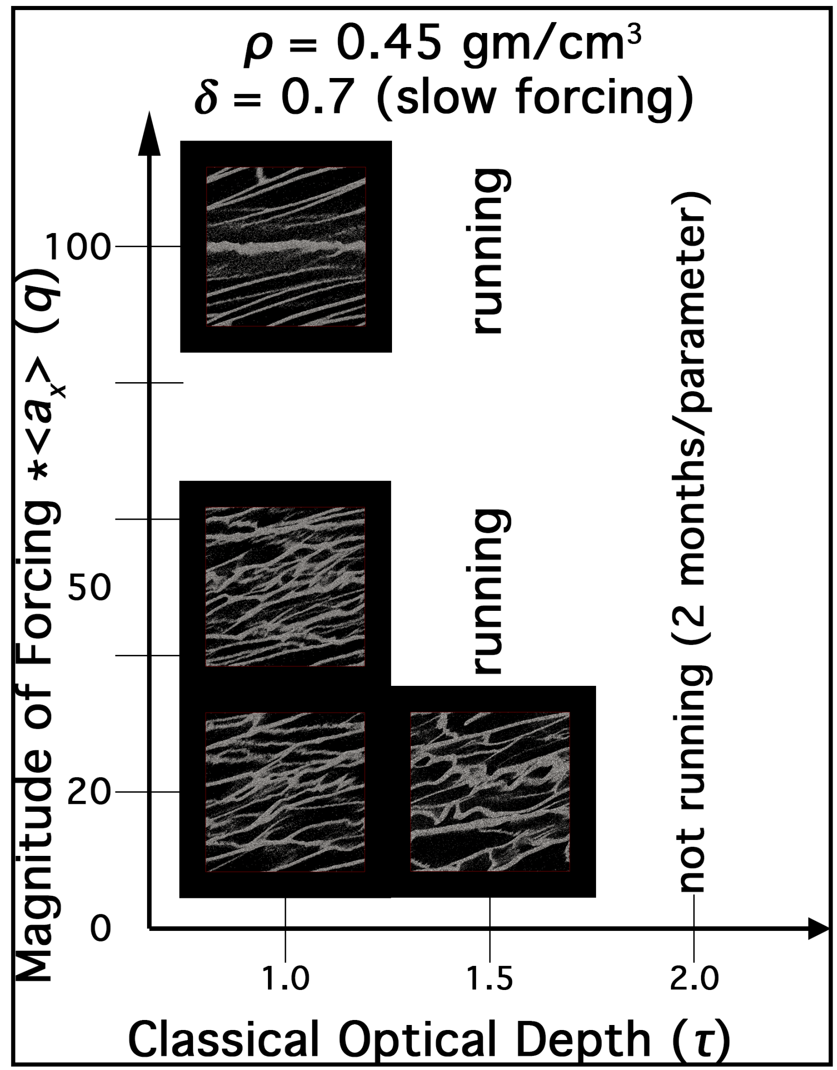
Forced Fast, A Lot



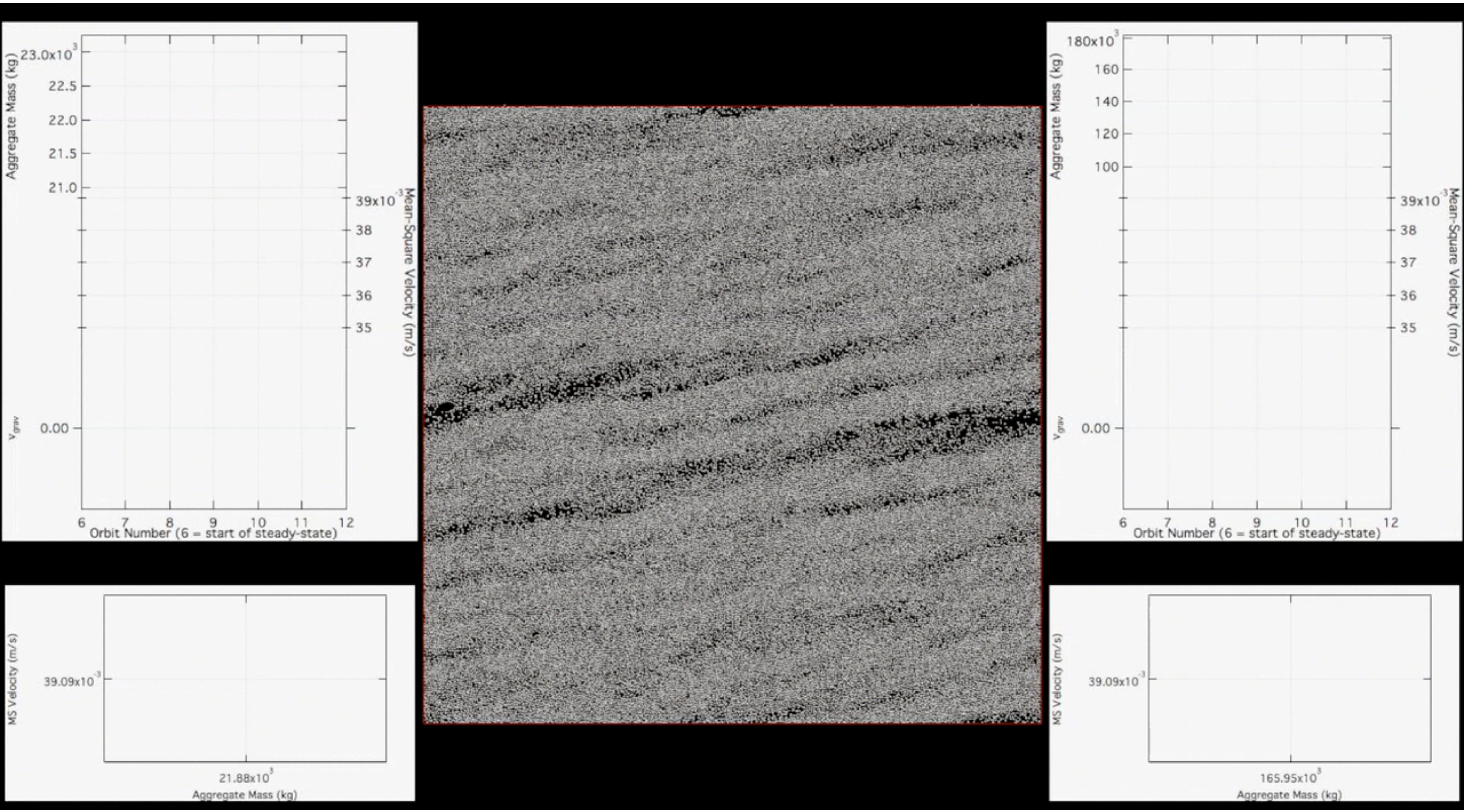


RESULTS: NAVIGATIONAL CHART



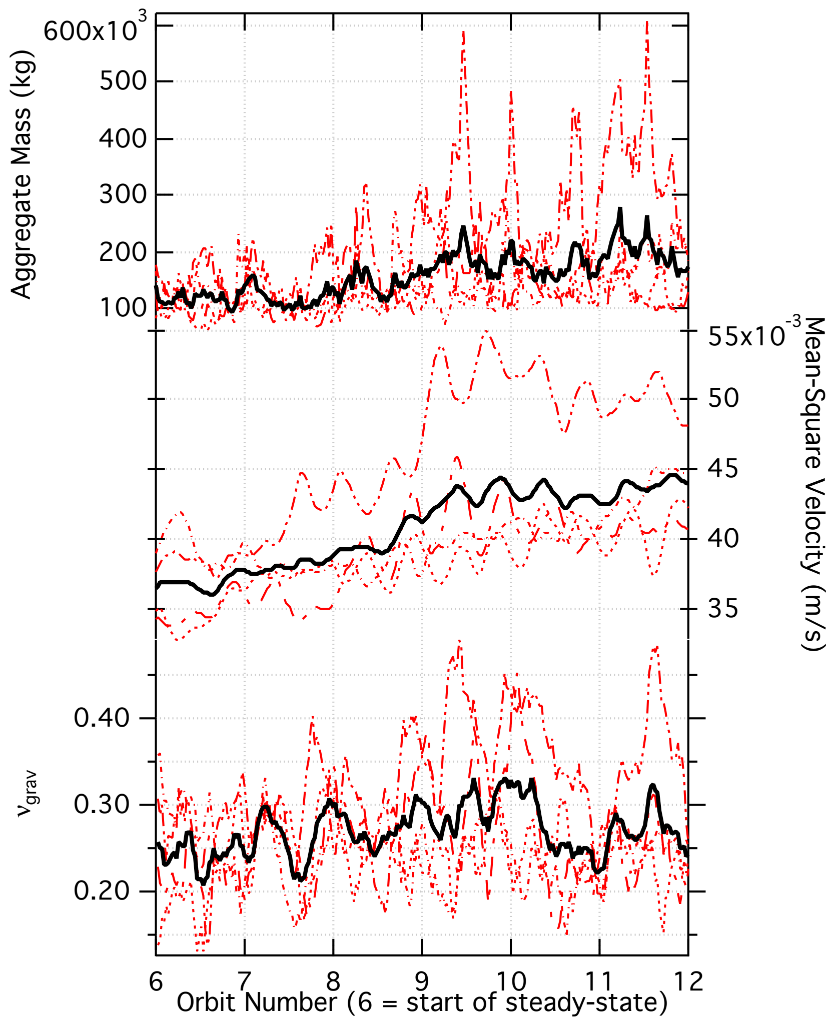


RESULTS (VISCOUS OVER-STABILITY?): τ=1.5, ρ=0.225, ϱ=100x, δ=0.7

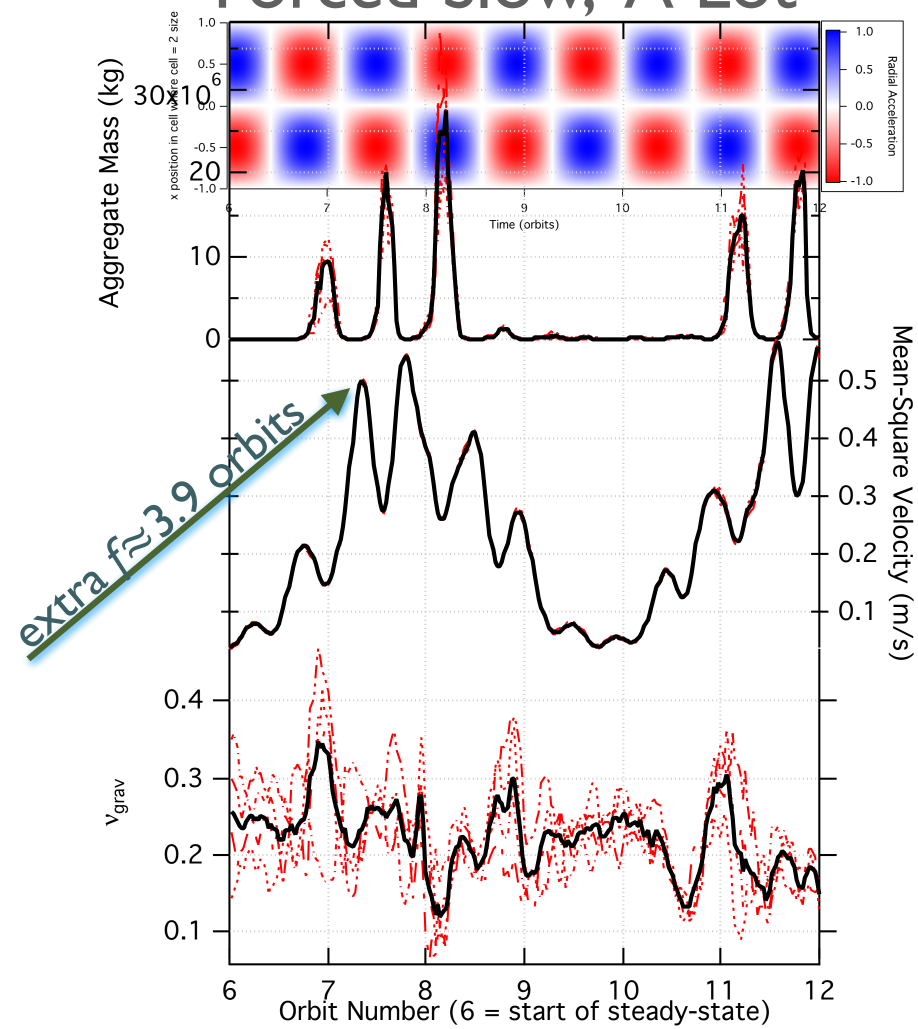


Results (Cluster Code "clumps" Parameters): τ=1.5, ρ=0.225, Q=100x, δ=0.7

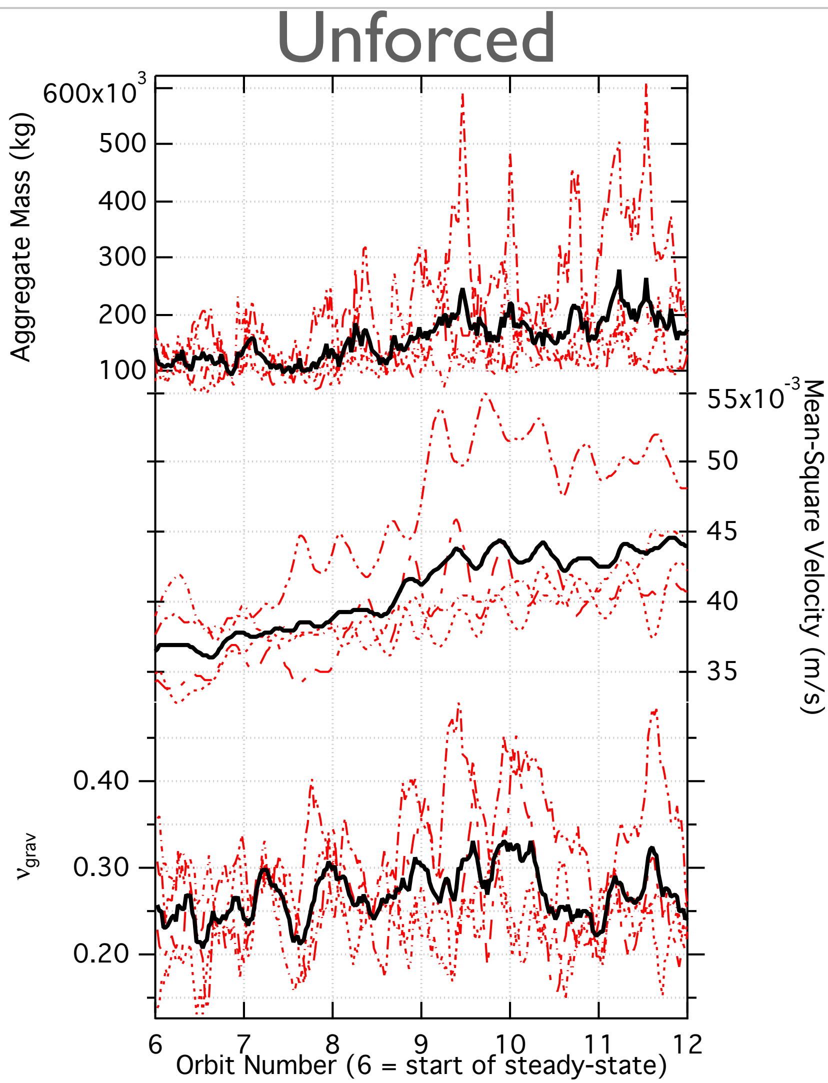
Unforced

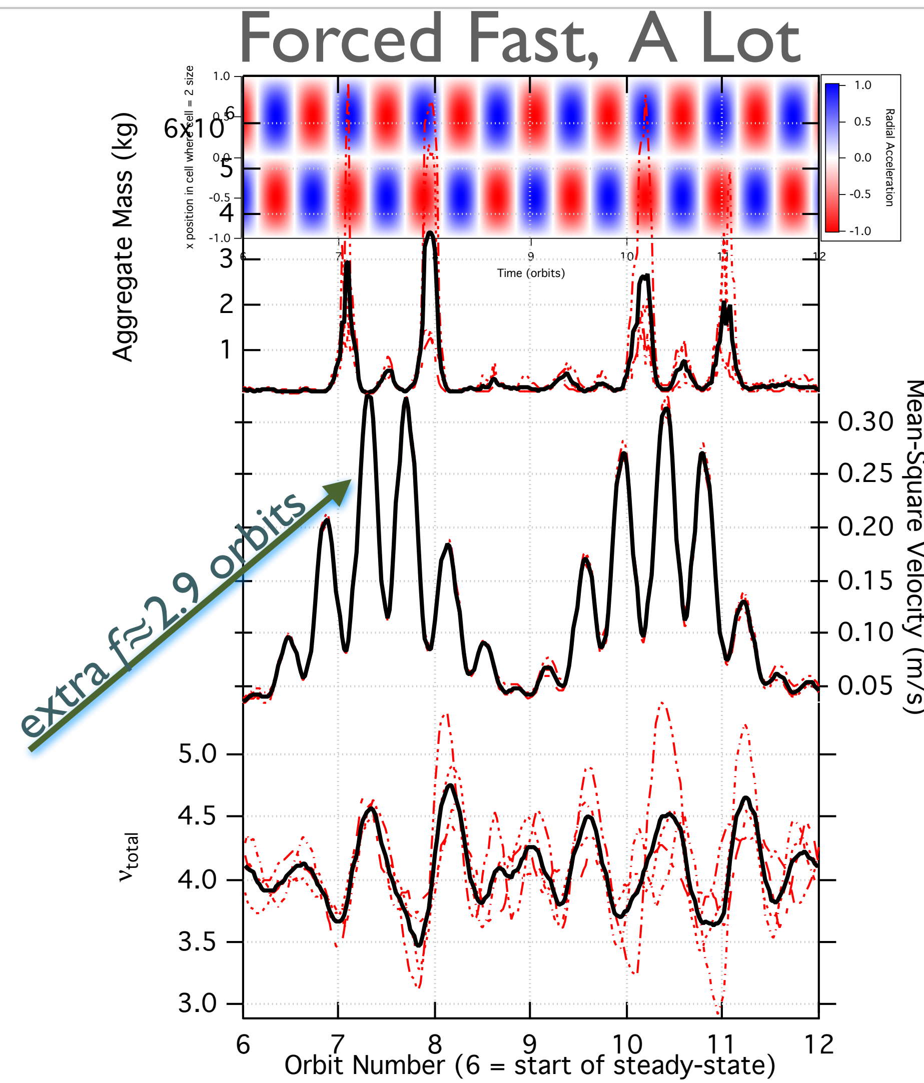


Forced Slow, A Lot



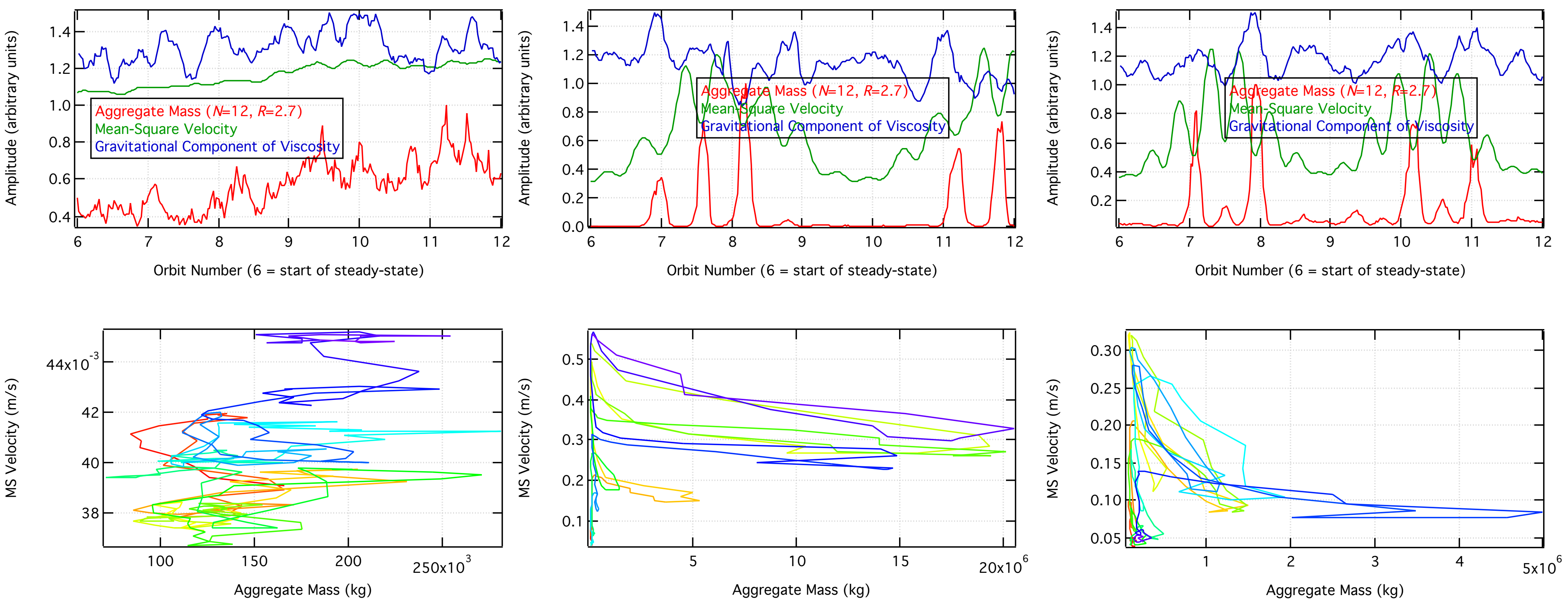
Results (Cluster Code "clumps" Parameters): τ=1.5, ρ=0.225, Q=100x, δ=1.3





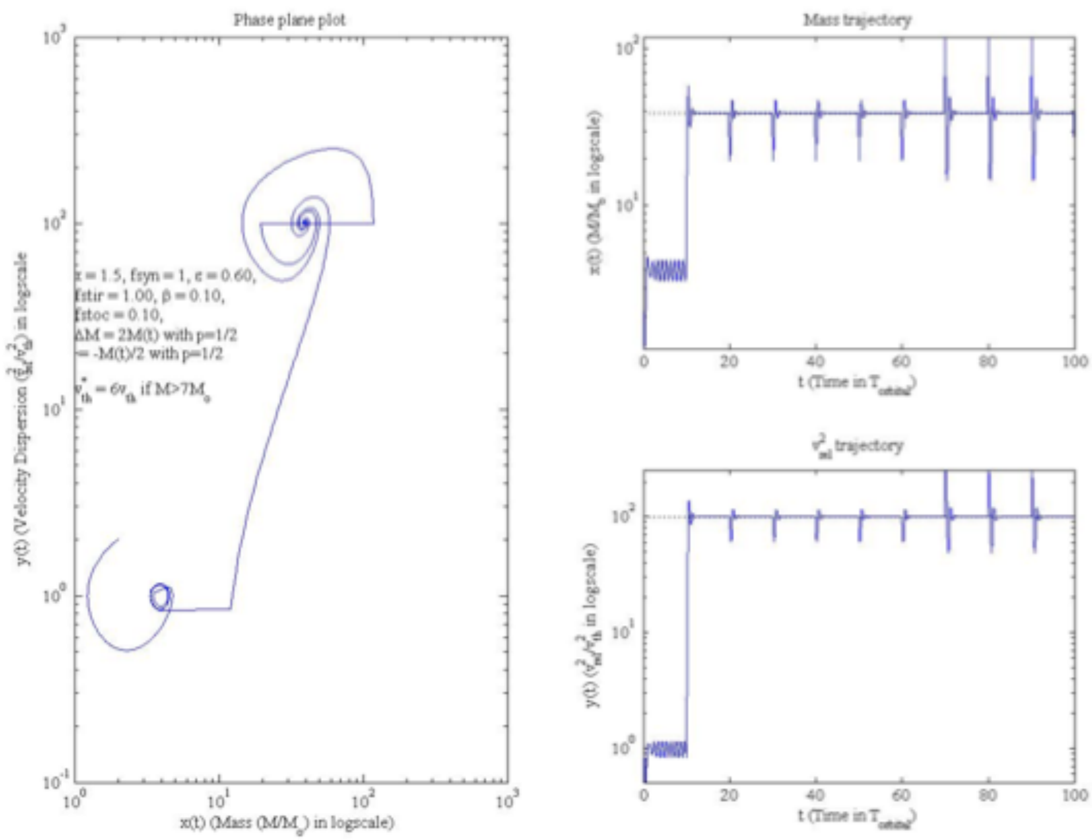
RESULTS (CLUSTER CODE "CLUMPS" PARAMETERS): τ=1.5, ρ=0.225, Q=100x

Unforced | Forced Slow, A Lot | Forced Fast, A Lot



SUMMARY OF PROGRESS

- *N*-body simulations to test predator-prey analytic model.
- Lots of simulations for large parameter space; several per parameter set.
- Several simulations do *not* show predator-prey characteristics (cycling in phase space and phase lags).
- But, some simulations do.



Gravity-dominated clumps give a new fixed point