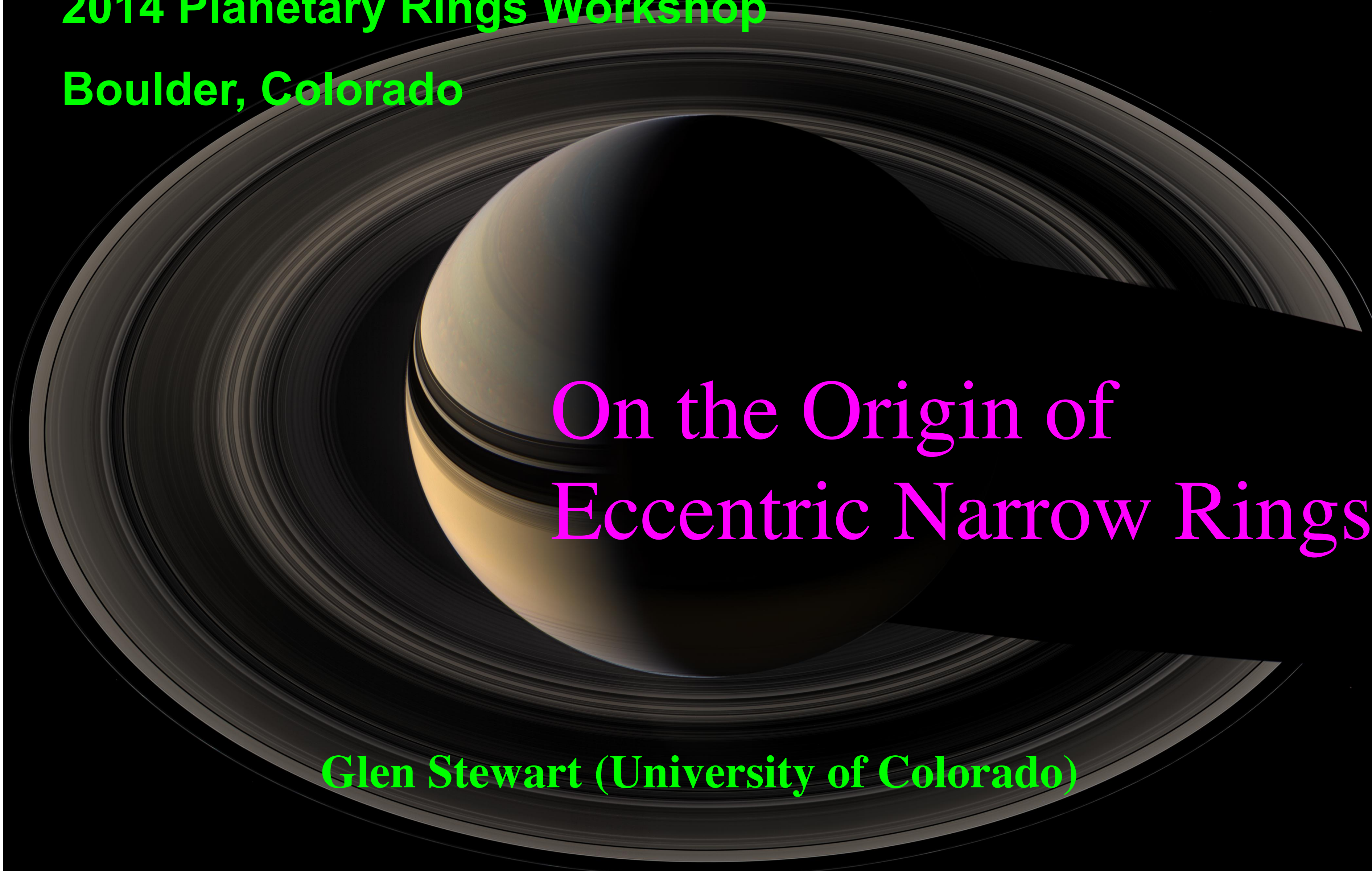
2014 Planetary Rings Workshop

Boulder, Colorado



On the Origin of Eccentric Narrow Rings

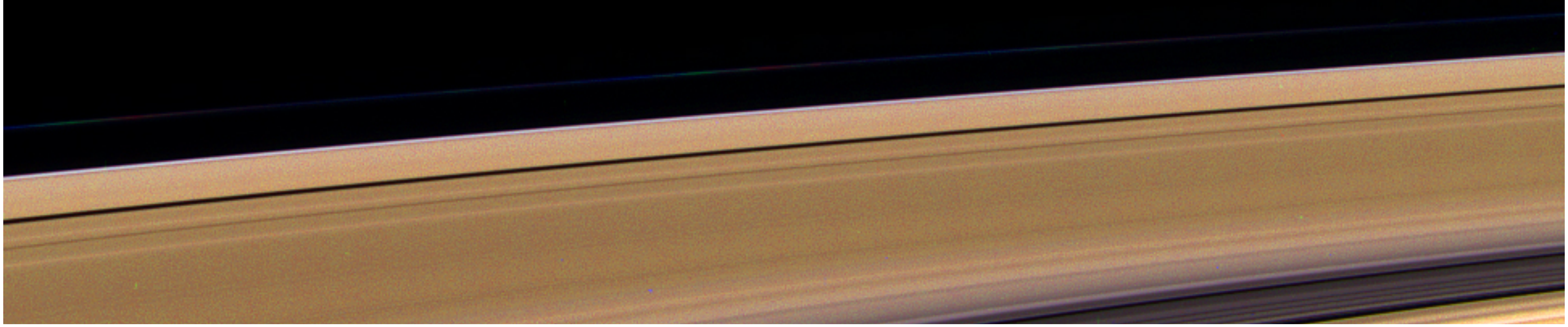
Glen Stewart (University of Colorado)

Why are narrow planetary rings usually eccentric?

(1) Ring is resonantly perturbed by a satellite

(2) Ring is resonantly perturbed by interior mode of Saturn

(3) Ring is subject to a viscous overstability



Why are narrow planetary rings usually eccentric?

(1) Ring is resonantly perturbed by a satellite

(2) Ring is resonantly perturbed by interior mode of Saturn

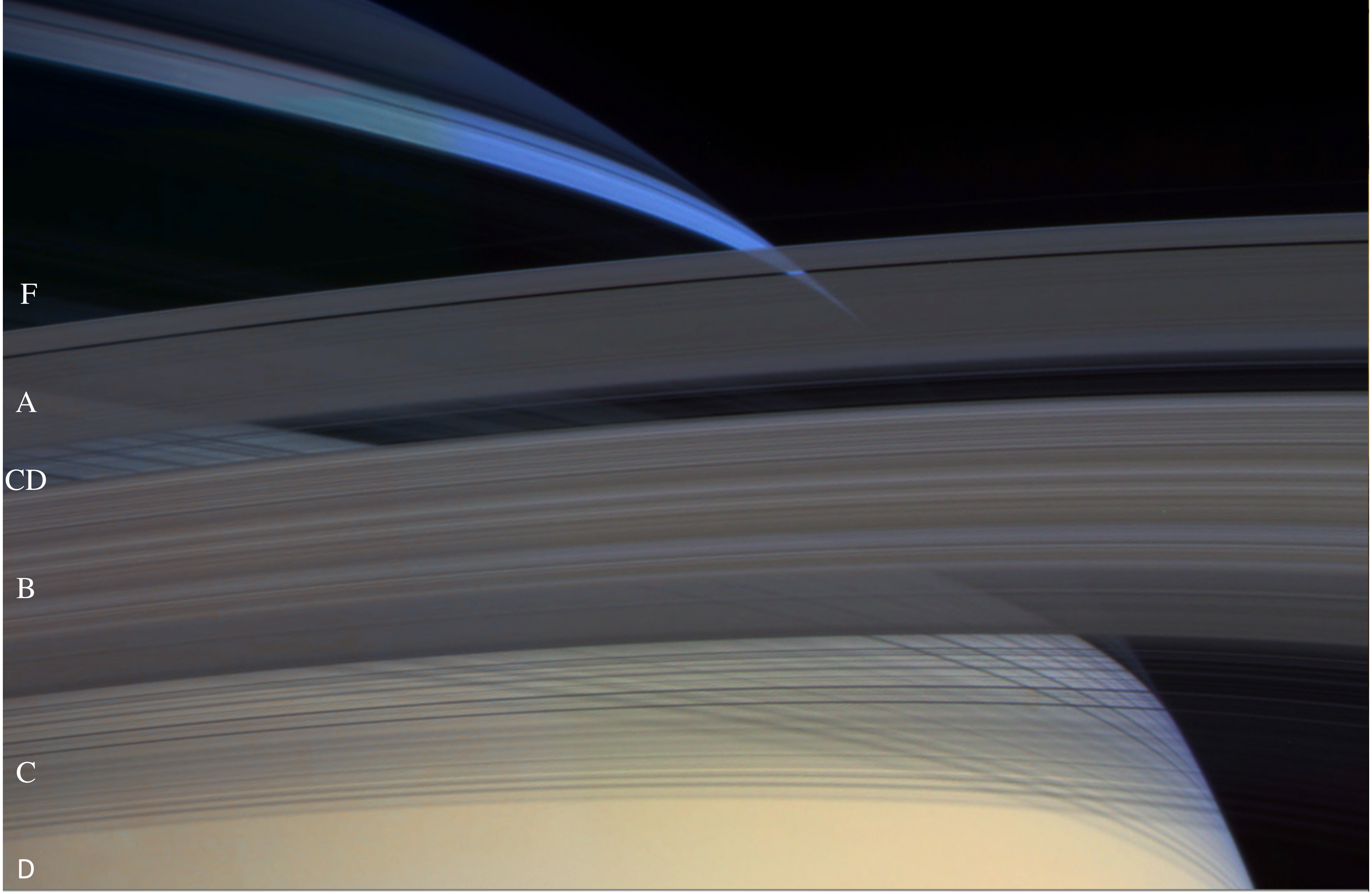
(3) Ring is subject to a viscous overstability

(4) Even if (1) or (2) is true, is a viscous overstability still required to generate the slowly precessing m = 1 mode?

(5) Is (3) also required to explain m = 1 mode on B ring edge?

A Brief History of Viscous Stress in Planetary Rings

GT 1978: Low density hard sphere collisions



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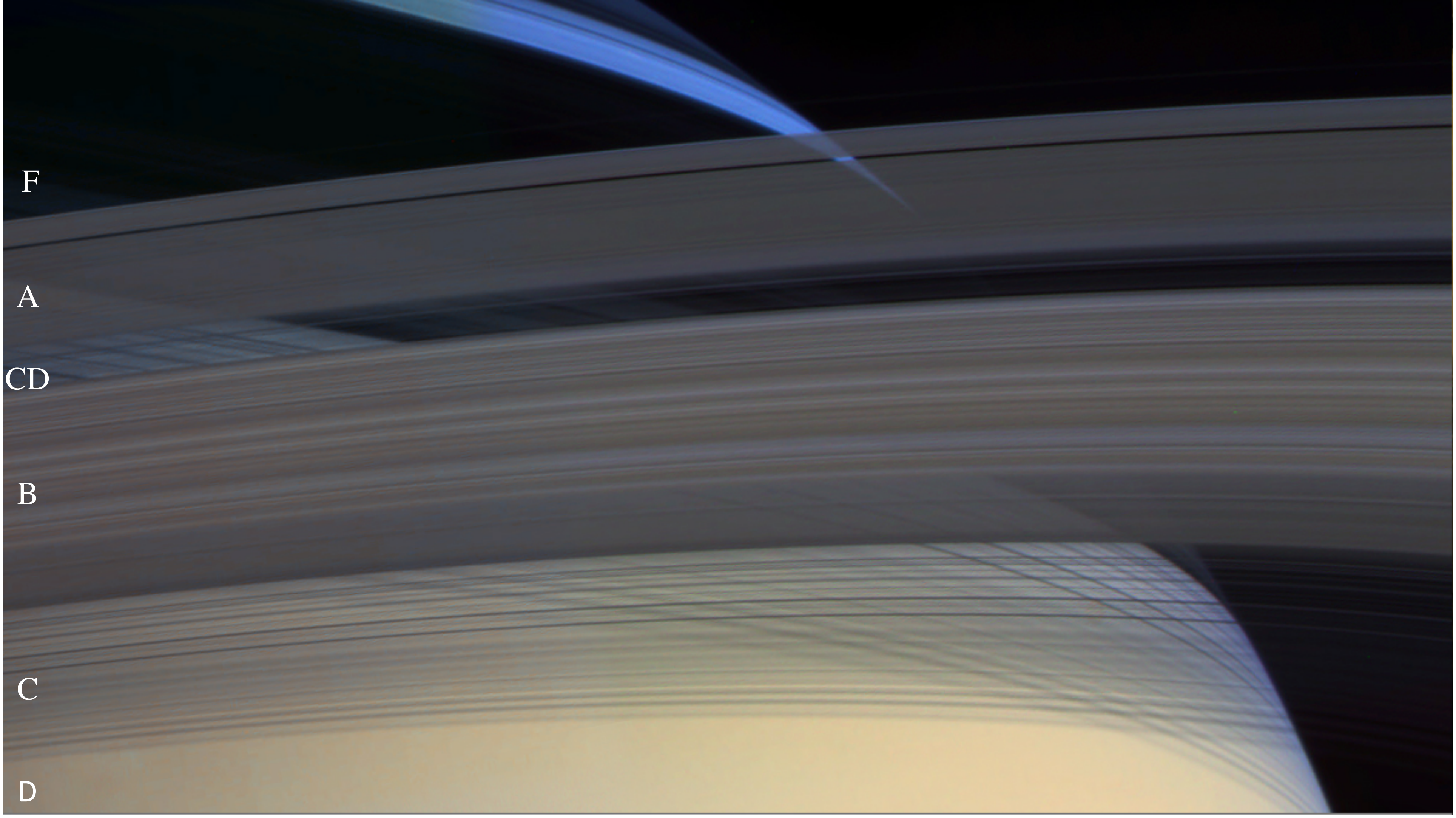

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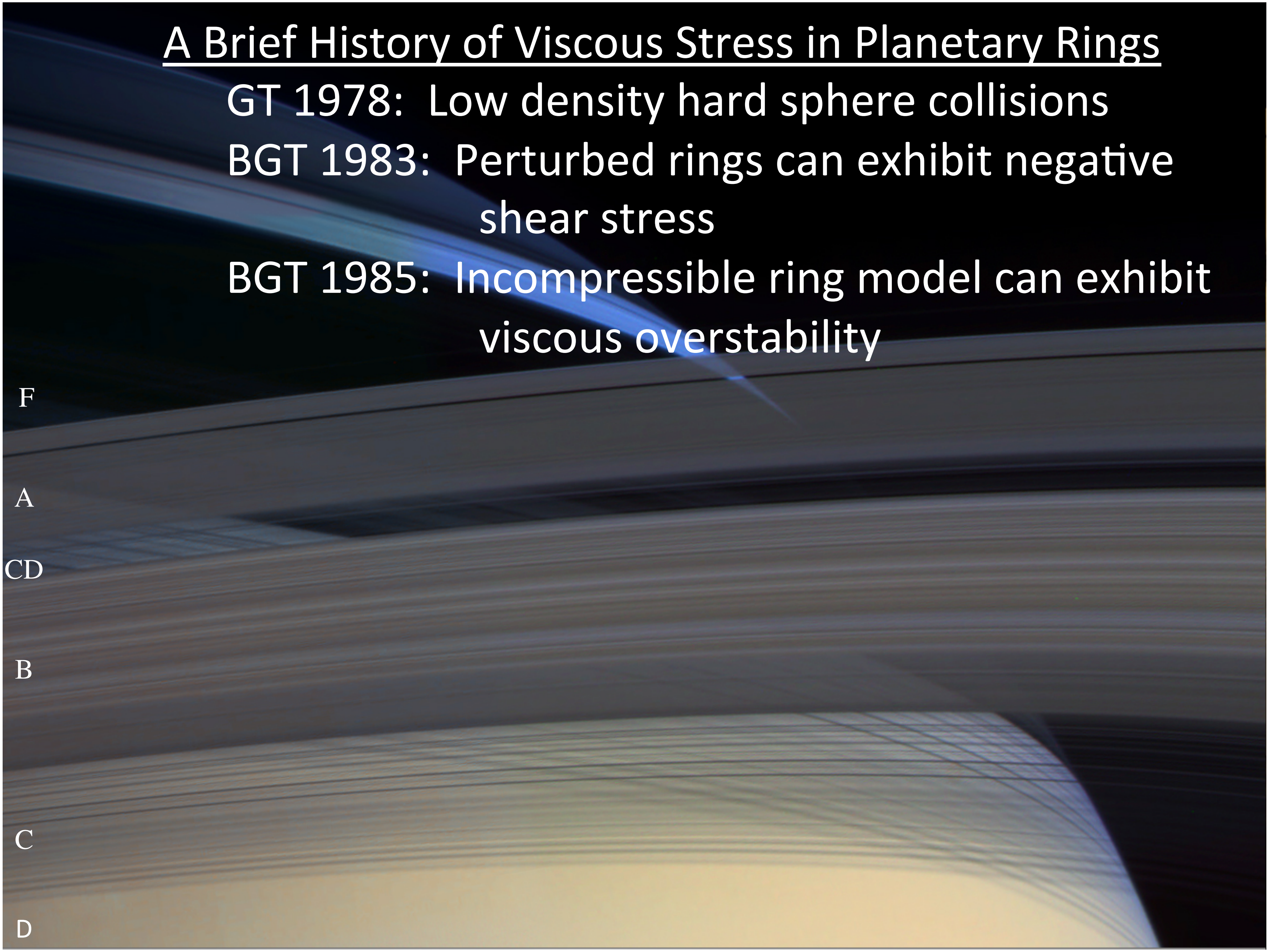

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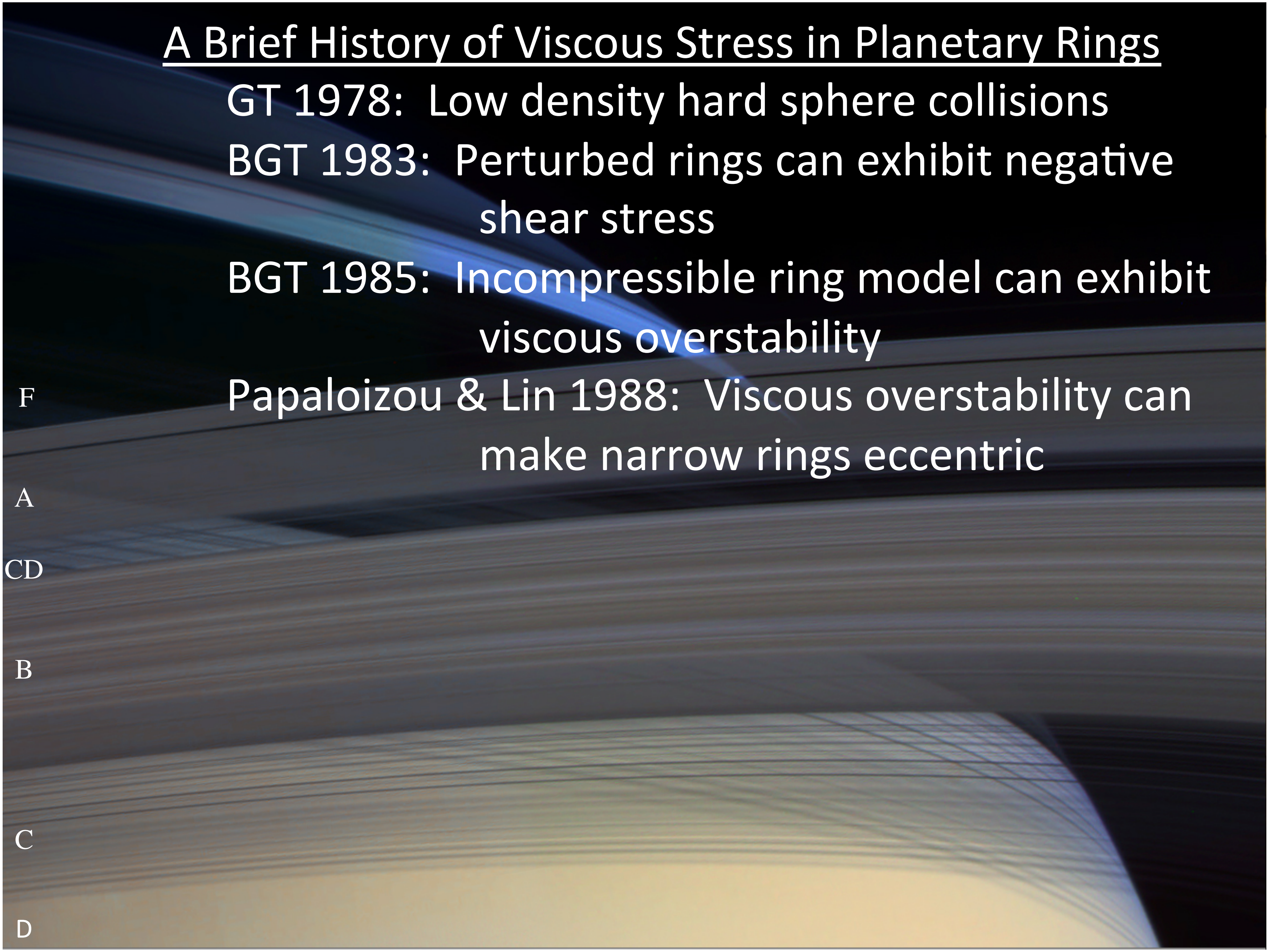

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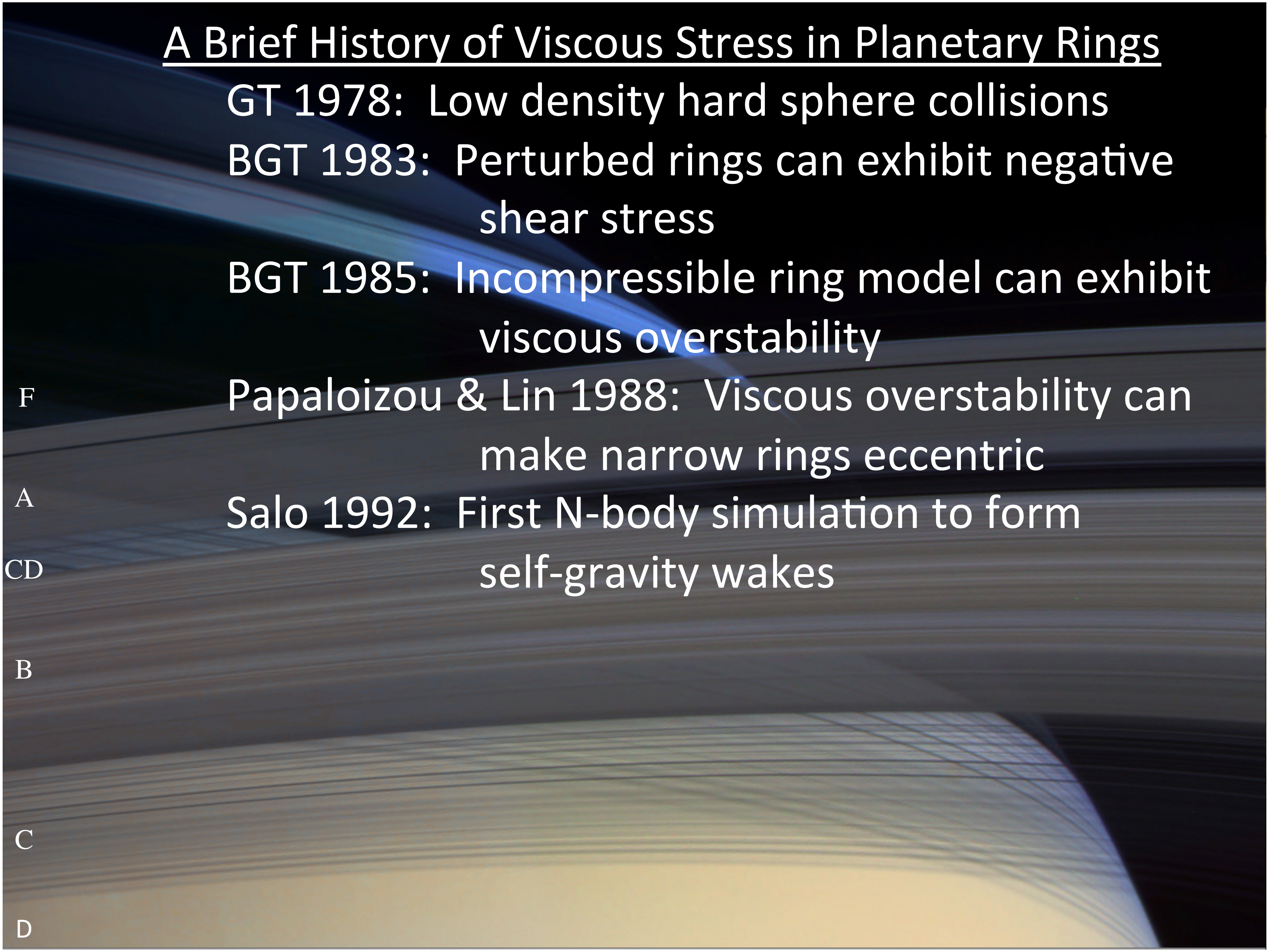

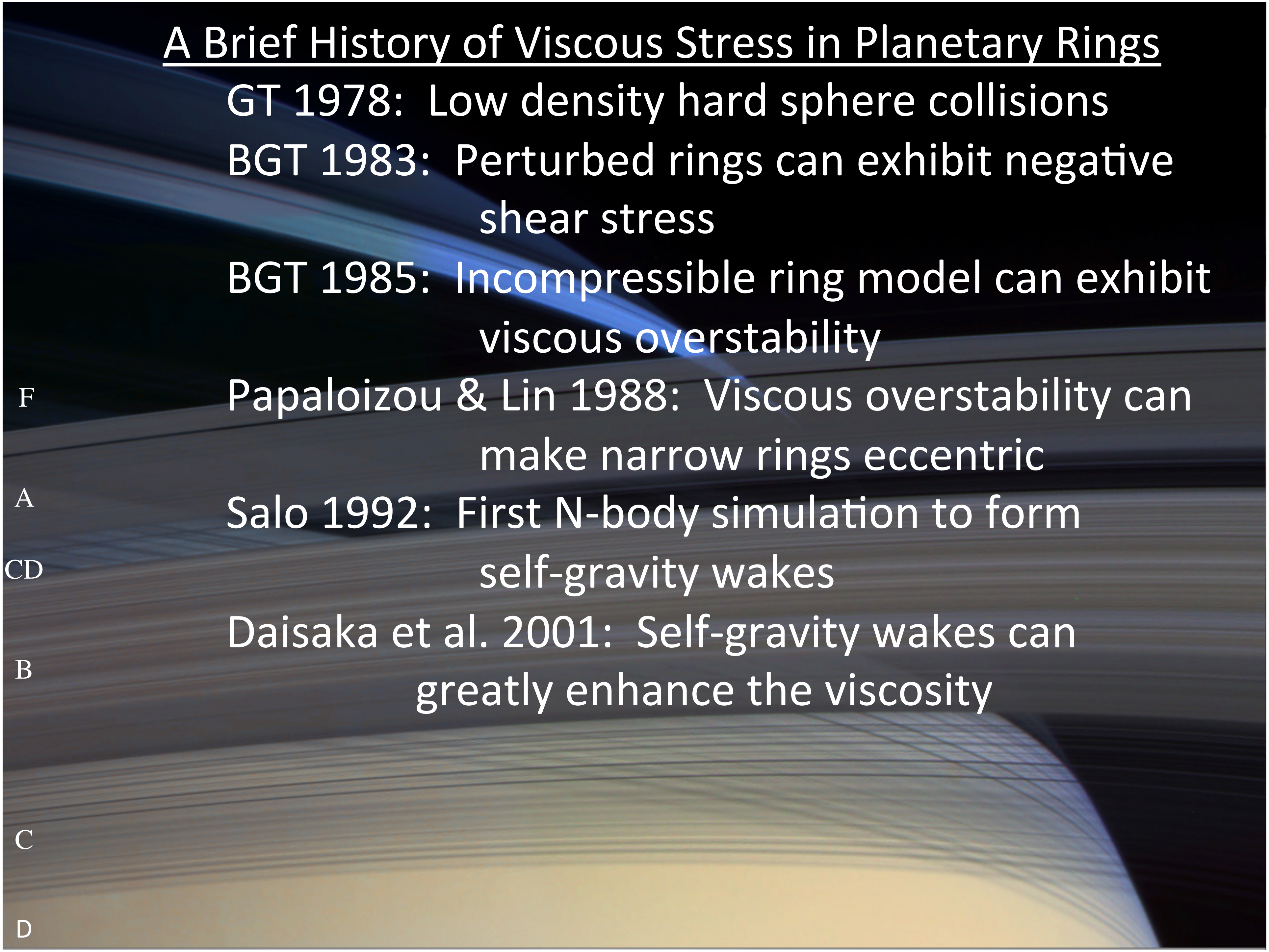

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Question: Do self-gravity wakes undermine BGT's incompressible model for viscous overstability?

F

A

CD

B

C

D

Viscous Stress in Perturbed Rings

(BGT 1995 Icarus)

$$\left(\frac{de}{dt}\right)_{\text{visc}} = -\frac{v^2}{\Omega a|\Delta a|(1-q^2)^{1/2}} [\mathcal{T}_1 \cos\gamma + \mathcal{T}_2 \sin\gamma], \quad (21)$$

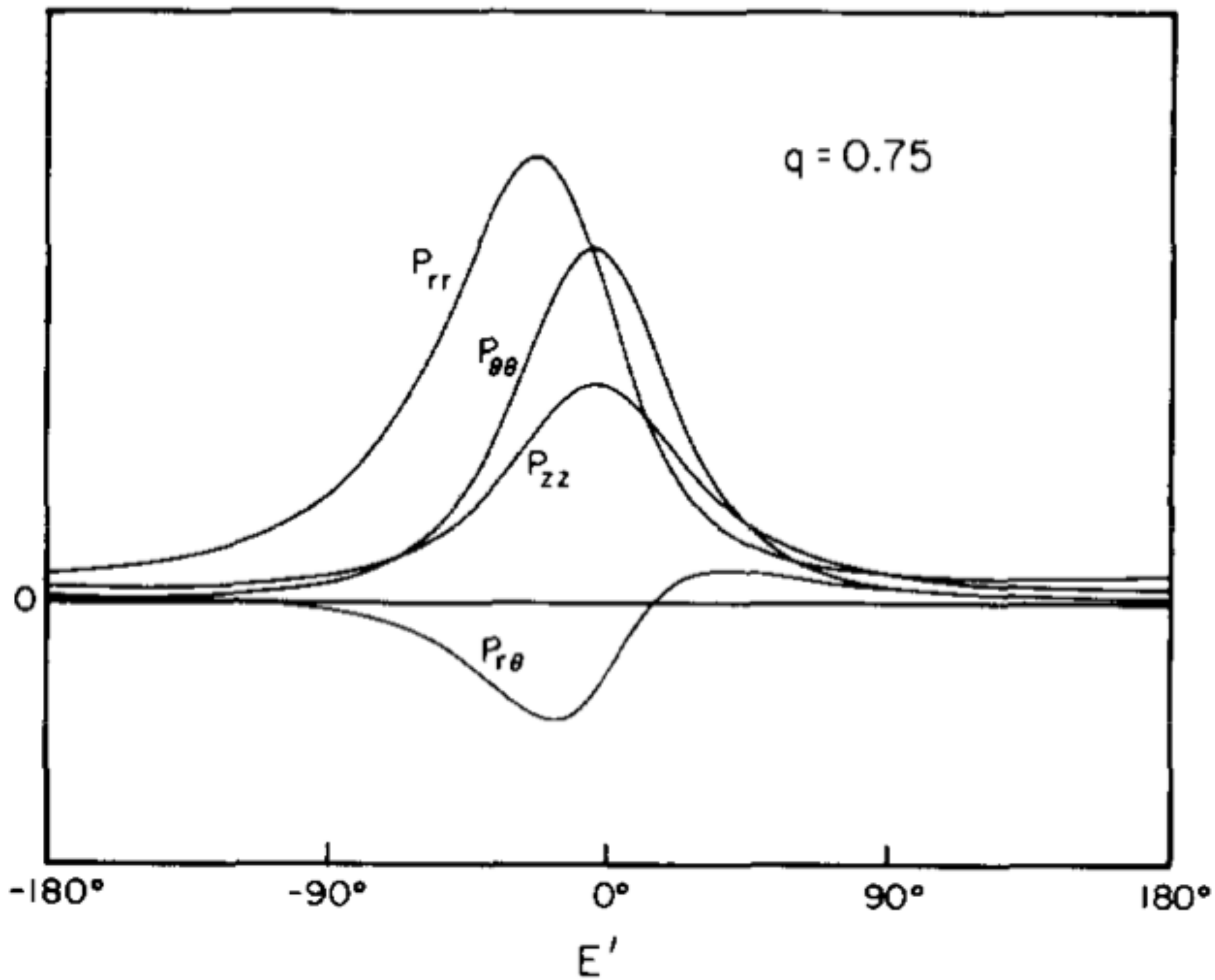
$$\left(\frac{d\varpi}{dt}\right)_{\text{visc}} = \frac{v^2}{e\Omega|\Delta a|(1-q^2)^{1/2}} [\mathcal{T}_1 \sin\gamma - \mathcal{T}_2 \cos\gamma],$$

$$\frac{\Sigma_0 v^2}{(1-q^2)^{1/2}} \begin{pmatrix} \mathcal{C}_{ij} \\ \mathcal{S}_{ij} \end{pmatrix} = \frac{1}{2\pi} \int_0^{2\pi} dE' P_{ij}^h \begin{pmatrix} \cos E' \\ \sin E' \end{pmatrix}, \quad (22)$$

$$\mathcal{T}_1 = \mathcal{S}_{rr} + 2\mathcal{C}_{r\phi}, \qquad \mathcal{T}_2 = 2\mathcal{S}_{r\phi} - \mathcal{C}_{rr}, \quad (23)$$

Pressure Tensor in a Perturbed Ring

(BGT 1983 Icarus, compressible)



Pressure Tensor in a Perturbed Ring

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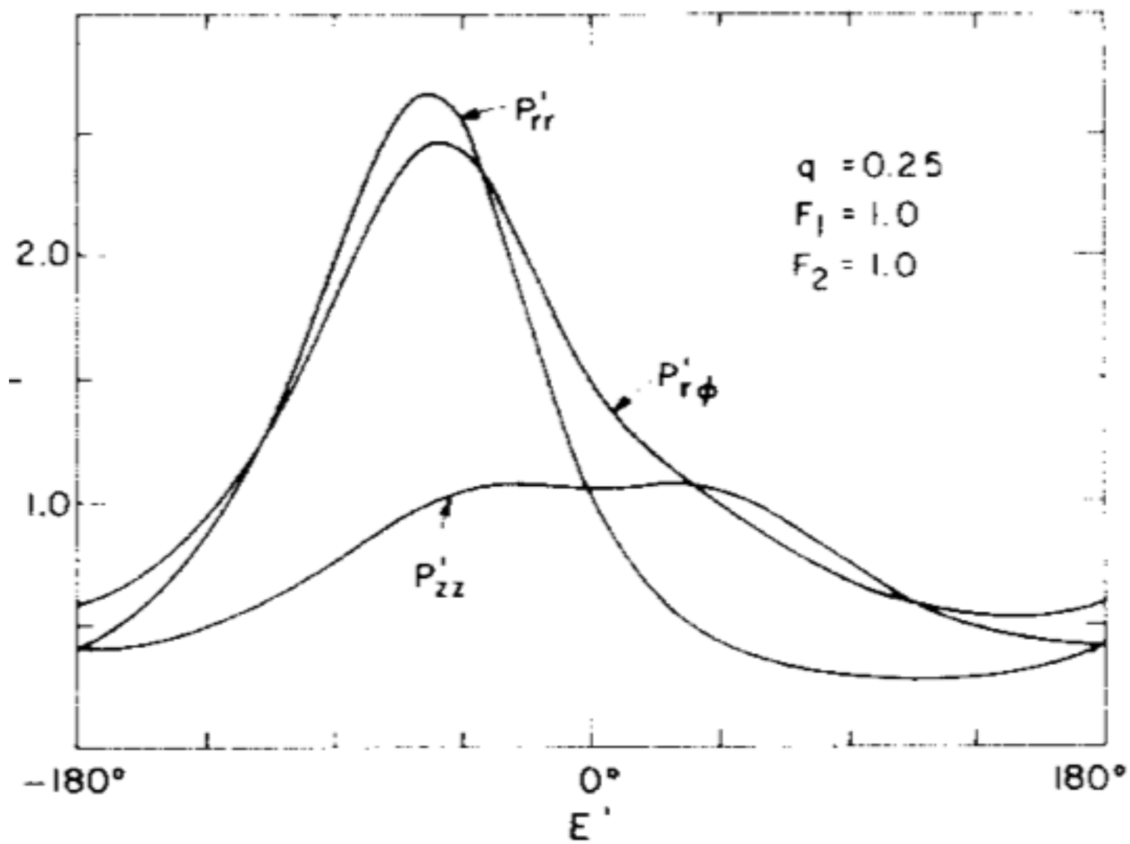

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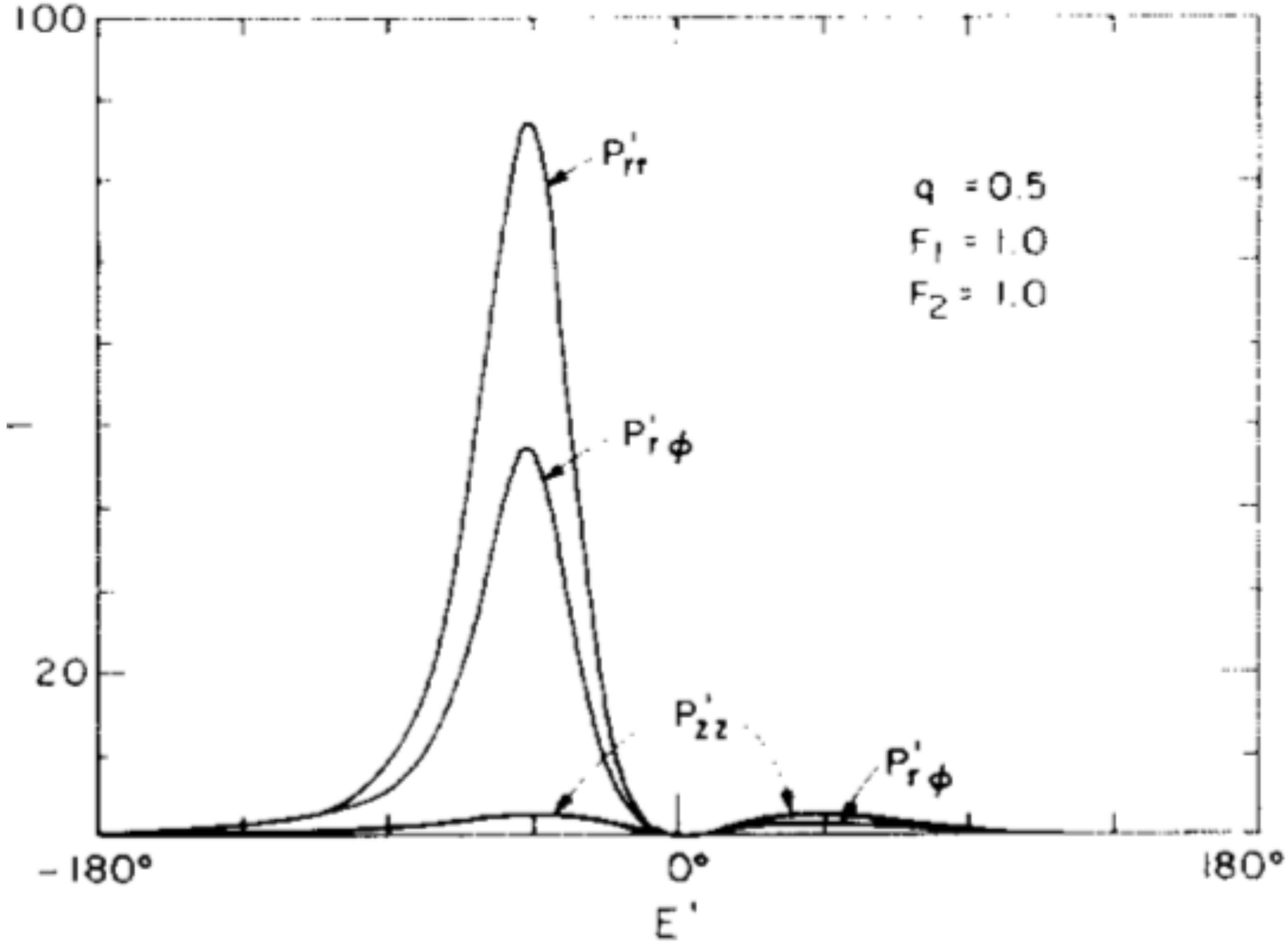
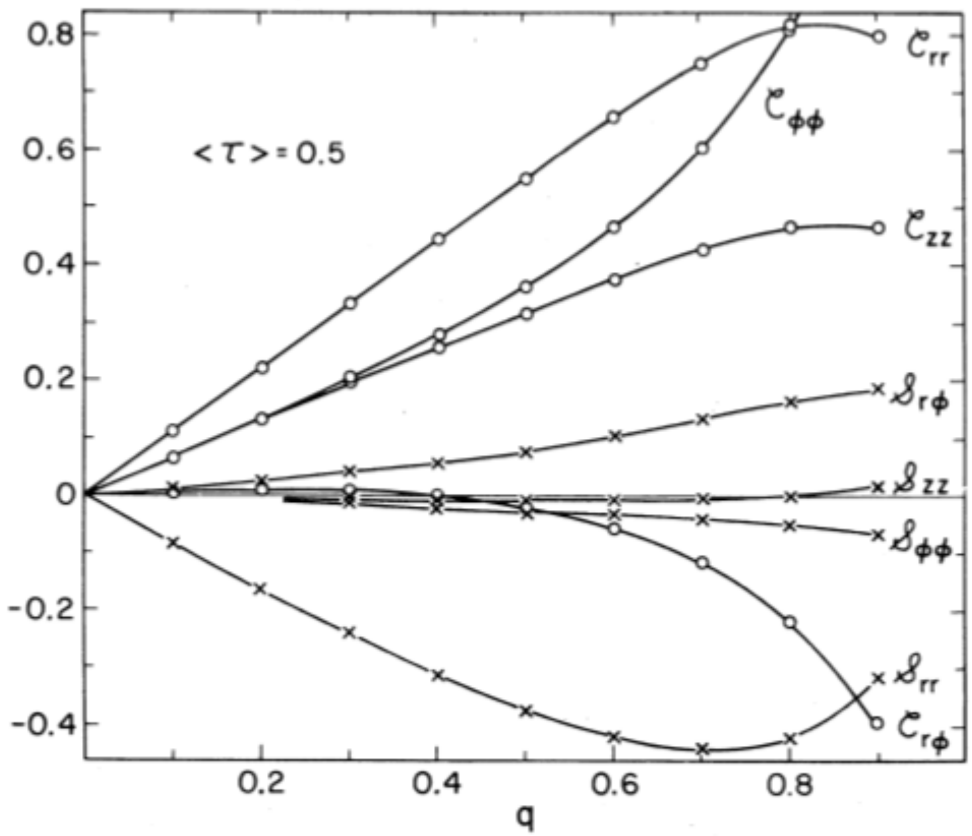


Viscous stress in a compressible ring

(BGT 1983 Icarus)

$$t_1 = S_{rr} + 2C_{r\phi} < 0$$



$t_1 = S + 2C$

Viscous stress in an incompressible ring

(BGT 1985 Icarus)

$$t_1 = S_{rr} + 2C_{r\phi}$$

Changes sign, leading to viscous overstability!

$$t_2 = 2S_{r\phi} - C_{rr}$$

Always < 0?

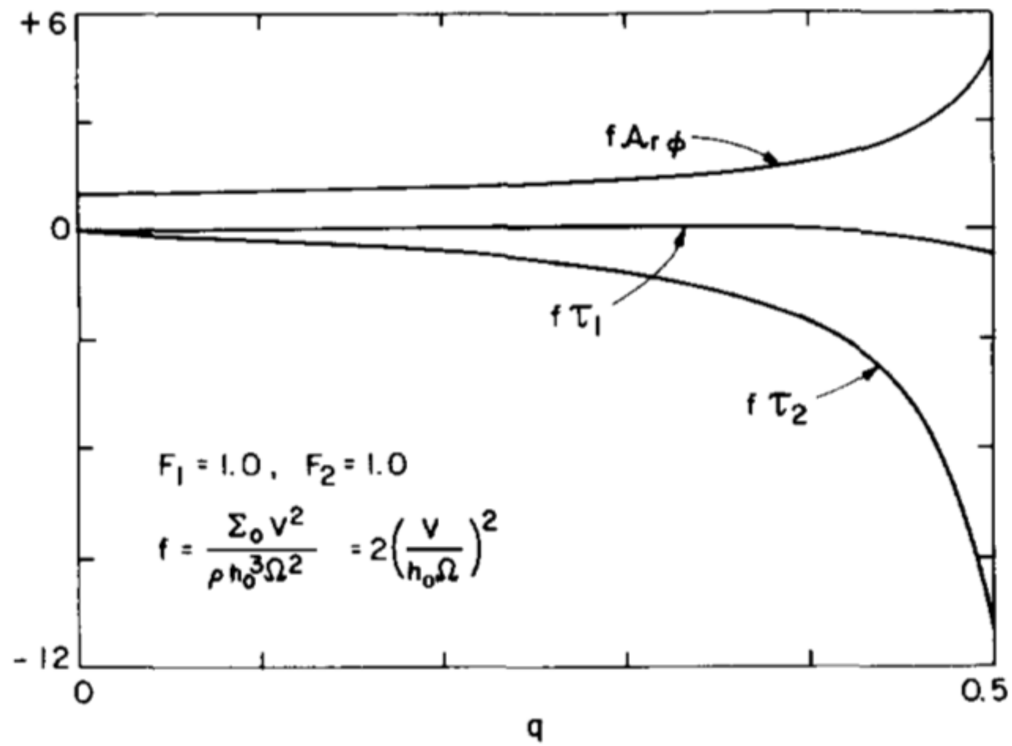
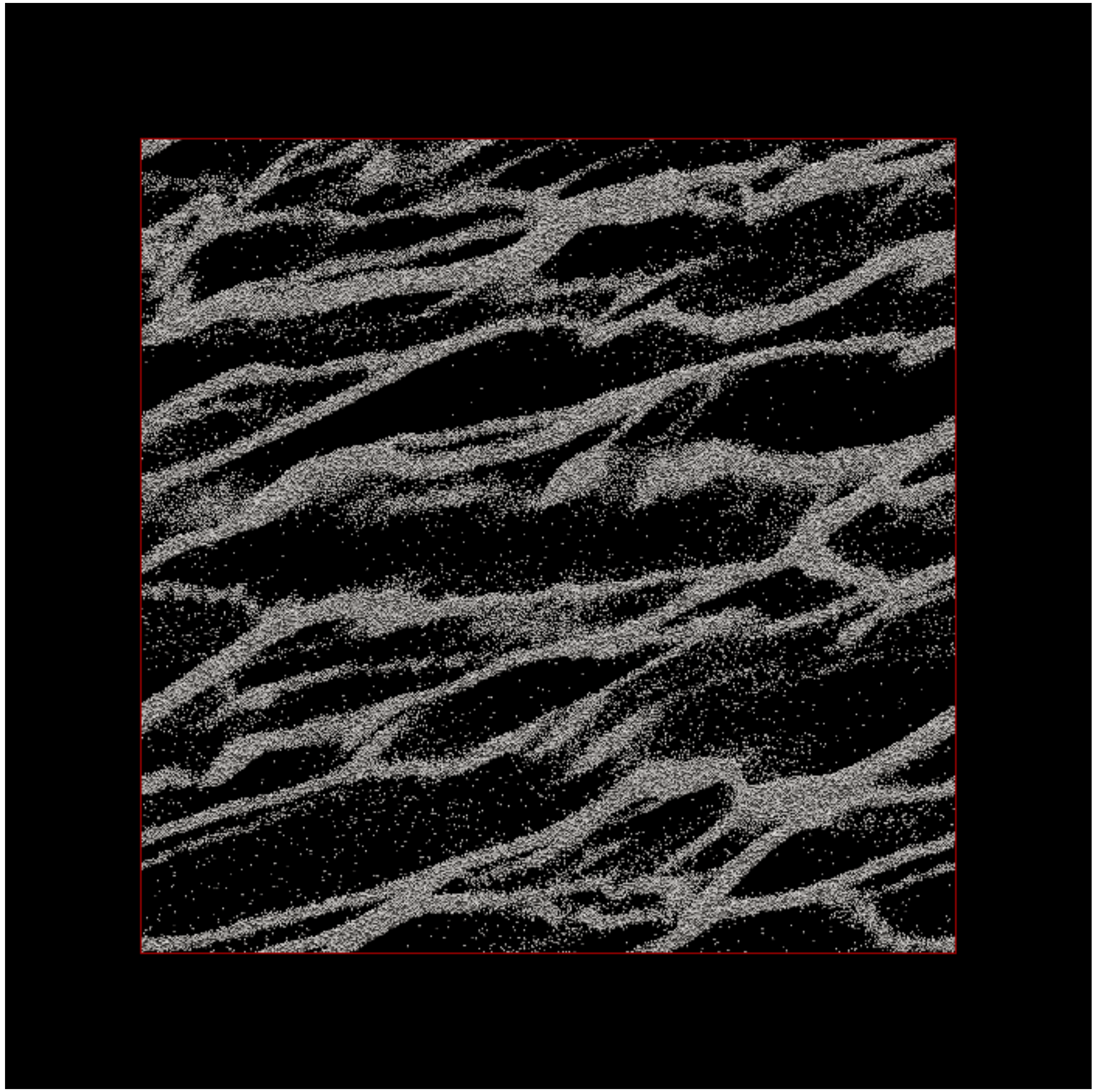


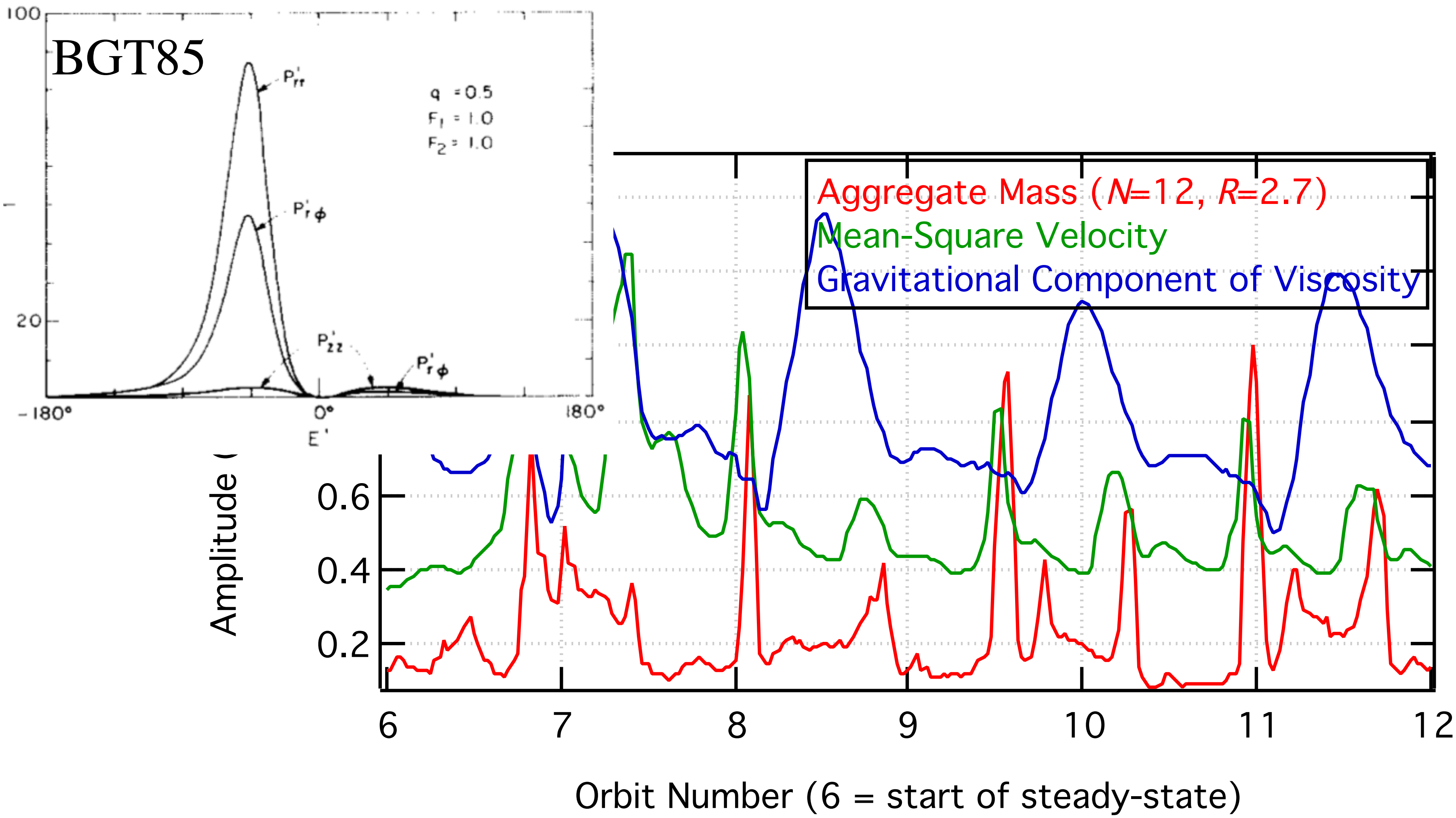
FIG. 4. Azimuthally averaged components $\mathscr{A}_{r\phi}$, $\mathscr{T}_1$, and $\mathscr{T}_2$ defined in Eq. (22), as functions of q, for $F_1 = F_2 = 1$. Although it is not apparent on this scale, $\mathscr{T}_1 > 0$ for $q \ll 1$ [cf. Eq. (92)]. For $q > 0.5$, p_0 is negative near $E' = 0$ and our model is no longer valid.

Maximum Stress Lags Maximum Density

Opposite of BGT Model!



In forced N-body simulations, gravitational stress peaks after maximum compression, so $t_2 = 2S_{r\phi} - C_{rr} > 0$??



Pulsational Overstability in Narrowly Confined Viscous Rings

Papaloizou and Lin (1988 Ap. J.)

$$\left[\kappa^2-(\Omega-\omega)^2\right]\xi(x)=-2G\int\frac{\Sigma_0(y)\xi(y)}{(y-x)^2}\,dy-3i\frac{\Omega}{\Sigma_0}\frac{\partial}{\partial x}\left[\Sigma_0\nu_0\frac{\partial\xi}{\partial x}Q\right]$$

$$\kappa^2-(\Omega-\omega)^2\approx 2\Omega_0\left(\omega-\Omega_p\right),\quad \text{where }\ \Omega_p=\frac{3}{2}\left(\frac{R_p}{r}\right)^2\Omega_0\ J_2$$

Sign of viscous term, Q, can be positive or negative, depending on the relative magnitude of P_{rr} and $P_{r\phi}$

Note that Q is similar to BGT's $t_2=2S_{r\phi}-C_{rr}$

Pulsational Overstability in Narrowly Confined Viscous Rings

Papaloizou and Lin (1988 Ap. J.)

$$x = r - r_0, \quad \Sigma_0(x) = \Sigma_c \sqrt{1 - \left(\frac{x}{w}\right)^2}, \quad \xi(x) = \sum_{n=0}^{N} c_n U_n\left(\frac{x}{w}\right)$$

$$\frac{w^2}{(y-x)^2} = -2\sum_{m=0}^{N}(m+1)\, U_m\left(\frac{x}{w}\right)\, U_m\left(\frac{y}{w}\right),$$ separable kernel!

$$n = 0 \;\rightarrow\; \xi(r,\phi) \sim e^{i\phi}$$ Radial displacement independent of r

$$n = 1 \;\rightarrow\; \xi(r,\phi) \sim (r - r_0)e^{i\phi}$$

Radial displacement linear in r, which causes radial compression or expansion

Keeping first two terms in the expansion yields

$$\frac{\omega - \Omega_{p0}}{\Omega_0} = \frac{3g + iQ\nu}{2} \pm \frac{1}{2}\sqrt{g^2 + p^2 - \nu^2 + 2iQ\nu g}$$

$$g = \frac{\pi G \Sigma_0}{w\Omega_0^2}, \quad p = -\frac{w}{\Omega_0}\frac{d\Omega_p}{dr}\bigg|_{r=r_0}, \quad \nu = \frac{9\nu_0}{2w^2\Omega_0}, \quad g \gg p, \nu$$

$$\frac{\omega_+ - \Omega_{p0}}{\Omega_0} \approx 2g + \frac{p^2}{4g} + iQ\nu\left(1 - \frac{p^2}{4g^2}\right)$$

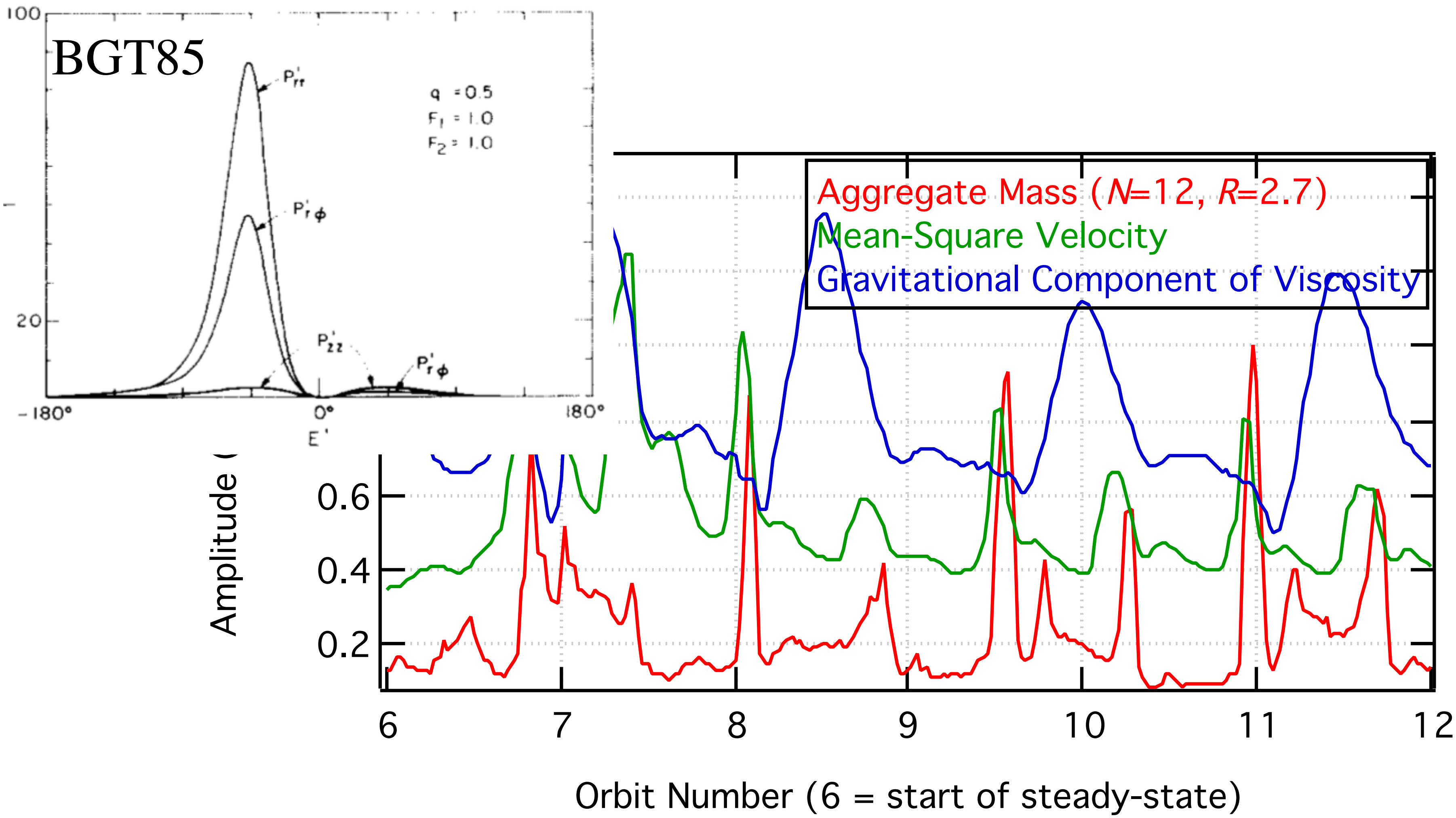
Unstable for $Q > 0$

$$\frac{\omega_- - \Omega_{p0}}{\Omega_0} \approx g - \frac{p^2}{4g} + \frac{iQ\nu p^2}{4g^2}$$

Unstable for $Q > 0$

Note that Q is similar to BGT's $t_2 = 2S_{r\phi} - C_{rr}$ and they find $t_2 < 0$!

In forced N-body simulations, gravitational stress peaks after maximum compression, so $t_2 = 2S_{r\phi} - C_{rr} > 0$??



Conclusions

- The BGT theory of viscous overstabilities in rings needs to be revised to account for self-gravity wakes. More resonantly forced N-body simulations are needed.

- The BGT incompressible fluid model probably only applies to localized regions in perturbed rings where streamline crowding is very strong. Not clear if orbit averaging is the best way to capture this localized effect.

- While BGT's model predicted that the maximum viscous stress by collisions would lead the region of maximum compression, the maximum viscous stress by self-gravity wakes lags the region of maximum compression. This phase change can drastically change when and where viscous instabilities can occur.