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PLASMA ELECTRON ANALYSIS: VOYAGER PLASMA SCIENCE EXPERIMENT

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The exponential in Eq. 16 on page 9 should be replaced by

$$e^{-(\bar{v}_{nj} - V_n)^2 (1 + \varepsilon_B b_n^2)}$$

and Eq. 17 on page 9 by

$$G_{jk} = \frac{A_K \, e^{-(a_K^2 U_\perp^2 - H_{jk}^2 - I_{jk}^2)}}{\sqrt{(\beta_\perp \bar{v}_{nj}^2 + a_k^2)(\beta_\perp \bar{v}_{nj}^2 (1 + \varepsilon_B b_\perp^2) + a_k^2)}}$$

where

$$H_{jk}^2 = \frac{[\varepsilon_B \beta_\perp \bar{v}_{nj} (1 - U_{nj}) b_x b_n + a_K^2 U_{xj}]^2}{\beta_\perp \bar{v}_{nj}^2 (1 + \varepsilon_B b_x^2) + a_K^2}$$

$$I_{jk}^2 = \frac{[(\beta_\perp \bar{v}_{nj}^2 + a_k^2)(\varepsilon_B \beta_\perp \bar{v}_{nj}^2 (1-U_{nj}) b_y b_n + a_k^2 U_{yj}) + a_k^2 \varepsilon_B \beta_\perp \bar{v}_{nj}^2 b_x (b_x U_{yj} - b_y U_{xj})]^2}{(\beta_\perp \bar{v}_{nj}^2 + a_k^2)(\beta_\perp \bar{v}_{nj}^2 (1 + \varepsilon_B b_\perp^2) + a_k^2)}$$

PLASMA ELECTRON ANALYSIS:

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ABSTRACT

The Plasma Science Experiment (PLS) on the Voyager spacecraft have provided a wealth of data on the plasma ions and electrons in the interplanetary medium and the magnetospheres of the giant planets Jupiter and Saturn. This report presents a description of the analysis used to obtain electron parameters (density, temperature, etc.) from the PLS electron measurements which cover the energy range from 10 eV to 5950 eV. The electron sensor (D cup) and its transmission characteristics are described. A derivation of the fundamental analytical expression of the reduced distribution function F_e is given. This is followed by discussion showing how the electron distribution function f_e, used in the moment integrations, can be derived from F_e. Positive ions produce a correction current (ion feedthrough) to the measured electron current, which can be important to the measurements of the suprathermal electron component. In the case of Saturn, we show that this correction current, which can either add to or subtract from the measured electron current, is less than 20% of the measured signal at all times. Though not shown here, these feedthrough corrections are very important during the Voyager 1 inbound pass through Io's plasma torus. We then briefly comment about the corrections introduced by spacecraft charging to the Saturn encounter data, which can be important in regions of high density and shadow when the spacecraft can become negatively charged.

PLASMA ELECTRON ANALYSIS: VOYAGER PLASMA SCIENCE EXPERIMENT

by E. C. Sittler, Jr.

SECTION 1

INTRODUCTION

The Plasma Science Experiment (PLS) on Voyager is a collection of potential modulated Faraday cups which make both positive ion and electron measurements covering the energy per charge range from 10 eV to 5950 eV. The PLS instrumentation has successfully measured the plasma (ions and electrons) in the interplanetary medium (Sittler and Scudder (1980), Sittler et al. (1981a), Belcher et al. (1981), Gazis and Lazarus (1982), Lazarus and Gazis (1983)), and the magnetospheres of Jupiter (Bridge et al. (1979a,b), Scudder et al. (1981), McNutt et al. (1981), Bagenal and Sullivan (1981) and Saturn (Bridge et al. (1981, 1982), Sittler et al. (1981b), Hartle et al. (1982), Eviator et al., (1982, 1983), Goertz (1983), Sittler et al., (1983), and Lazarus and McNutt (1983)). The Voyager 2 PLS instrument has also played an important role in the detection of Jupiter's magnetotail beyond the orbit of Saturn (Kurth et al. (1981, 1982), Scarf et al. (1981, 1983), Lepping et al. (1982, 1983) and Desch (1983)). In this report we present a fairly detailed description of the electron analysis which has produced the electron parameters (density, temperature, etc.) used in many of the above studies. The analysis described herein is most descriptive of that used for the most recent publication by Sittler et al. (1983) on plasma electrons in Saturn's magnetosphere and is somewhat different from that used and briefly described in Scudder et al. (1981) for Jupiter. The ion analysis, which is fairly straightforward in the solar wind (Bridge et al. (1977), Belcher et al. (1981)) but can be considerably more difficult in the magnetospheres of Jupiter (McNutt et al., (1981) and Saturn (Lazarus and McNutt (1983)) will not be discussed here.

The paper is broken up into 8 sections with the introduction given in Section 1 and a description of the instrument and its operation given in Section 2 (a more complete description is given in Bridge et al. (1977)). Section 3 gives a schematic description of the D cup and its transmission characteristics, followed in Section 4 by a formal derivation of the analytical expression used in our fits to the measured electron spectra; all our parameter estimations, either directly or indirectly, are derived from this expression. We then mention in Section 5 the fitting procedure used in the analysis, which plays an important role in the moment integrations. In Section 6 we expand upon the discussion in Scudder et al. (1981) concerning the moment estimation of electron parameters. Section 7 describes the effect of ion feedthrough corrections upon the electron measurements, which in the case of Saturn, introduce a minor correction to the observed suprathermal component.

Finally, in Section 8 we comment about the effects of spacecraft charging upon the electron measurements.

SECTION 2

INSTRUMENT DESCRIPTION AND OPERATION

The PLS instrument shown in Figure 1 is composed of four potential modulated Faraday cups denoted by the letters A, B, C and D. The three main sensors A, B and C make only positive ion measurements and, except for rare brief spacecraft maneuvers, are always pointed nearly along the spacecraft-Sun line. The side sensor or D cup makes both positive ion and electron measurements and is normally oriented nearly at right angles to the solar direction. The angular response of the side sensor is cylindrically symmetric about its look direction, and provides a field of view with conical half angle $\sim 30^\circ$ (FWHM) about its normal. As shown in Section 3, the D sensor makes differential contiguous measurements of the electron distribution function along the sensor normal. Because the electron thermal speeds are much larger than flow speeds of the plasma, electron measurements are not very sensitive to sensor orientation, unless there are large pressure anisotropies. Since the instrument angular field of view is fairly broad, uncertainties due to pressure anisotropies are not expected to have an important effect upon the analysis.

For cold ions in the solar wind and magnetospheres of Jupiter and Saturn the mach numbers are usually greater than one and the measured currents are sensitive to sensor orientation. During the cruise phase of the mission only the main sensors are sensitive to the supersonic ion component of the solar wind, while the side sensor provides a one-dimensional view of the electrons at nearly right angles to the flow direction. During the encounters with the giant planets, Jupiter and Saturn, the D sensor was aligned to respond to the azimuthally flowing cold ions as much as possible. For most of the inbound portion of the encounter trajectories the D sensor was aligned to view the cold ions; while, except for the Voyager 1 Saturn encounter, the D sensor was not favorably aligned during the outbound passes. During the planetary encounters, the D sensor alignment was such that the center of its field of view generally looked at electrons with pitch angles between 45° and 135°.

Except for the Cruise 1 phase of the mission when electron measurements are made every 12 seconds and only the E1 mode is sampled, the side sensor completes a measurement cycle in 96 seconds during which it passes through the mode sequence M, E1, L and E2. M and L are the high and low resolution positive ion modes, respectively, while E1 and E2 (see Figure 4) are the low and high energy electron modes, respectively. The energy range for E1 is 10 eV to 140 eV while for E2 it is 140 eV to 5950 eV. Each electron mode is composed of 16 contiguously spaced energy channels; for E2 mode only the upper 12 channels are used (lower 4 channels not useful since suppressor is biased at −95 volts; see discussion in Section 3). The channels for E1 are nearly equally spaced in energy ($.099 < \Delta E/E < .37$), while for E2 they are more logarithmically spaced ($\Delta E/E = .29$). The sampling time for both energy modes is 3.84 seconds (0.96 second for Cruise 1) and E1 and E2 modes are separated in time by 45 seconds. This large time gap between low and high electron energy measurements can result in discontinuous changes in the composite energy spectrum across the 140 eV boundary joining the two energy modes. Fortunately, this happens only rarely, and the cold and hot components characterizing the electron distribution function within Jupiter's or Saturn's magnetosphere are usually measured by the low and high energy modes, respectively. The ion and electron measurements are never made simultaneously (the shortest time difference between ion and electron spectra is 25 seconds), which may lead to time aliasing problems whenever intercomparisons between ion and electron measurements are made.

SECTION 3
D SENSOR DESCRIPTION AND TRANSMISSION CHARACTERISTICS

The D cup or side sensor is schematically displayed in Figure 2. It has a cylindrical geometry with entrance aperture at one end and collector plate at the other end, and numerous grid meshes in between. The orientation of the side sensor normal relative to the spacecraft coordinate system is shown at the bottom of Figure 2. We have defined the sensor normal such that it points into the sensor and is thus oriented opposite to the sensor look direction. For added information about potential modulated Faraday cups we suggest reading the review article by Vasyliunas (1971) which gives an in-depth discussion of the use of potential modulated Faraday cups for space applications.

The Faraday cup sets up a one-dimensional potential barrier, aligned along the sensor normal, between the modulator grid (grid3) and the ground grid (grid 2) shown in Figure 2. This barrier is only effective for those charged particles having a charge of the same sign as the modulator voltage V_M relative to ground potential. The dc voltage $\overline{V}_M$ defines the energy or speed channel at which electrons are sampled; the superimposed 400 Hz square wave voltage, shown schematically in Figure 2 with amplitude $\Delta \overline{V}_M$, defines the energy or speed channel window size. Note that V_M is negative for electron measurements. The instantaneous current received by the collector is the integrated flux of electrons with velocity component ν_n aligned along sensor normal such that

$$\frac{1}{2} m_e \nu_n^2 > q V_M \tag{1}$$

with

$$V_M = \overline{V}_M \pm \Delta V_M/2 \tag{2}$$

and q = –e for electrons. A current with both a dc and 400 Hz component flows into the collector. Since the amplifiers are ac coupled to the collector, only those electrons satisfying the condition

$$\nu_{nj-} \leqslant \nu_n \leqslant \nu_{nj+} \tag{3}$$

where

$$\frac{1}{2} m_e \nu_{nj\pm}^2 = q(\overline{V}_{Mj} \pm \Delta V_{Mj}/2) \tag{4}$$

for speed channel j are sampled. This signal is then amplified, phase detected, and integrated before transmission. The role of the intermediary grids (4, 5 and 6) is mainly to reduce the capacitive coupling from modulator grid to collector plate. The suppressor grids (7 or 8 depending on instrument mode) main purpose is to return secondary electrons emitted by the collector back to the collector. For electron measurements in the E1 and E2 modes, the suppressor voltage V_s is –8 volts and –95 volts, respectively. In the normal suppressor grid configuration (grid 8 is grounded and grid 7 is biased at voltage V_s) the suppressor is not very effective in returning electrons back to the collector. Electron measurements are predominantly made in the normal grid configuration because the instrument is considerably quieter under these conditions (Lazarus, private communication). Preliminary estimates indicate that secondary electron corrections are only 10% for the thermal electrons (secondary electron yields are low), while ∿30% corrections are expected for suprathermals. At present these corrections have not yet been incorporated into the electron analysis. Because electrons with energies less than (V_s) cannot penetrate the potential barrier set up the suppressor grid, measurements are confined to energies greater than (V_s). It is for this reason the lower four E2 channels are not useable. We note that the D cup must be oriented at more than 75° from the solar direction, otherwise UV light striking the modulator grid will cause photoemission from the modulator grid, producing a contaminating signal that can swamp the measured current due to the plasma electrons. This problem was only intermittently present during the early phases of the cruise mission before the Jupiter encounters. Because the spacecraft is stabilized w.r.t. 3 axes, the electron measurements yield only a one-dimensional view of the electron distribution function.

Transmission Function $T(\vec{\nu}\,; \nu_{nj\pm})$

In order to make a quantitative estimate of electron parameters such as the electron temperature from the measured currents, one must have an accurate determination of the phase space sampled by the sensor. This is given by the transmisison function $T(\vec{\nu};\nu_{nj\pm})$ for which the index j specifies the speed channel. The transmission function is defined to be that fraction of a monoenergetic unidirectional beam of charged particles uniformly illuminating the entrance aperture which reach the collector plate. Because of cylindrical symmetry of the D cup the angular dependence of T is given solely by the angle of incidence relative to sensor normal θ (see Figure 2). We introduce the normalized response $R(\vec{\nu};\nu_{nj\pm})$ where

$$T(\vec{\nu};\nu_{nj\pm}) = T_N R(\vec{\nu};\nu_{nj\pm}) \tag{5}$$

The constant $T_N = 0.56$ is the normal transmission of the sensor (i.e., $\theta = 0°$) and it is equal to the product of the transparencies of all grid meshes times the ratio of the shielding ring (shown in Figure 2) and aperture cross-sectional areas. In Figure 3 we have plotted a family of curves for the angular response $R(\vec{\nu};\nu_{nj-})$ as a function of $\theta = \tan^{-1}(\nu_\perp/\nu_n)$. Each curve corresponds to a different speed along sensor normal $\hat{n}$ within the speed window of the lowest E1 speed channel (i.e., $\nu_{n1-} < \nu_n < \nu_{n1+}$). The angular response of R with half width $\sim 30°$ is principally caused by the common overlapping areas of the aperture and shielding ring projected upon the collector at angle θ. The dependence upon ν_n or electron energy, which can be seen to be small, results from the refraction of electron trajectories as they pass through regions of nonzero electric field (e.g., between modulator and ground grids 2 and 4). Furthermore, the angular and energy dependence is essentially independent of speed channel.

Electrons typically have large thermal speeds $\omega_e > 1000$ km/s relative to plasma flow speed $V < 600$ km/s. It follows that the angular width of the electron distribution function f_e seen by the sensor at the one thermal speed level (transverse direction, $\nu_\perp = \omega_e$) will be typically greater than

25° in the lower speed channels. Therefore, the angular width of f_e and R are comparable, resulting in a folding or convolution of instrument response with f_e in the transverse direction relative to sensor normal. This effect is taken into account in the analysis.

In contrast to the broadness of R relative to f_e in the transverse direction, the instrument samples differential slices of f_e along the sensor normal. To show this we present in Figure 4 a plot of the observed reduced distribution function F_e (similar in shape to f_e) measured within Saturn's extended plasma sheet at 1981 238 09:04 SCET by the Voyager 2 spacecraft. The E1 and E2 energy or speed ranges are denoted in the figure; the histogram format is used to indicate the width ($\Delta\nu_{nj}$) of each speed channel with mean speed $\overline{\nu}_{nj}$. The abrupt change in channel width at 140 eV occurs at the boundary separating the E1 and E2 energy ranges; the fractional window sizes ($\Delta\nu/\nu$) for E2 are about a factor of three larger than that for E1. The two component structure of the electron distribution function is clearly demonstrated by this figure. But most importantly, with regard to the analysis, the figure shows the differential character of the measurements in velocity component ν_n. Mathematically, this condition of differentiality, which is derived in the next section, is given by

$$\epsilon_j = \frac{1}{6}\left(\frac{\Delta\nu_{nj}}{\omega_e}\right)^2 << 1 \tag{6}$$

where

$$\Delta\nu_{nj} = \nu_{nj+} - \nu_{nj-} \tag{7}$$

is the speed width of speed channel j. For the speed channels at which the thermal electron measurements are confined (E1 mode), $\Delta\nu_{nj} \sim 300$ km/s. Referring to Eq. (6), along with the fact that $\omega_e > 1000$ km/s, one finds ϵ_j to be $< 2\%$ for all j; therefore, the E1 measurements are differential along $\hat{n}$. Thus, because the speed channels are so narrow, one can in principle measure electron temperatures less than $10^{4\circ}$ K or 1 ev. In reality this is not always possible, because of the contaminating signal introduced by the suprathermal electrons near the breakpoint energy E_{B1} (see Figure 4). Furthermore, because the measurements are confined above 10 eV, the signal in the lowest E1 channels for temperature $T_e < 1$ eV will be more than 10^{-4} below the peak flux level which occurs at $E < 1$ eV. Therfore, electron densities must be sufficiently high $n_e > 20$ to $300/cm^3$ (i.e., exact value depends on I_{TH} or I_N which are variable) with $\phi_{SC} = 0$ volts, if the cold electrons are to produce a signal greater than instrument threshold $I_{TH} \sim 10$ to 10^3 femptoamp or instrument noise $I_N \gtrsim 75$ femptoamps in the first few E1 channels. In the interplanetary medium where

ϕ_{SC} can exceed + 10 volts, a 1 eV low density ($n_e < 1/cm^3$) electron component is detectable whenever spacecraft potentials are this high. The electron measurements in the higher energy mode E2 can also be shown to be differential. At these higher energies ($E > 140$ eV) the thermal energy of the electrons usually scales with electron energy E (i.e., power law in electron energy or speed); therefore, at 1 keV the thermal speeds ω_e are $\sim$18,000 km/s. At 1 keV the channel widths $\Delta\nu_n$ are $\sim$3,000 km/s, so that ϵ_j estimated from Eq. (6) is less than 1%; hence, the E2 measurements are also differential in velocity component ν_n. With these facts in mind, we will now proceed to derive the analytical expression for the reduced distribution function F_e used in our fits.

SECTION 4

DERIVATION OF ANALYTICAL EXPRESSION FOR REDUCED DISTRIBUTION FUNCTION F_e

To begin it is necessary to write down the general expression relating the measured current ΔI_j (ac component) and the electron distriubtion function $f_e(v)$ we are trying to determine. Keeping in mind the definition of the transmission function $T(\vec{\nu};\nu_{nj\pm})$ and the fact that one can imagine $f_e(v)$ to be a weighted distribution of monoenergetic unidirectional beams of particles incident upon the sensor, it follows that the sampled current ΔI_j is given by

$$\Delta I_j = qAT_N \left\{ \int_{-\infty}^{\infty} d\nu_x \int_{-\infty}^{\infty} d\nu_y \int_{\nu_{nj-}}^{\infty} f_e(\vec{\nu}) R(\vec{\nu};\nu_{nj-})\, \nu_n d\nu_n \right.$$
$$\left. - \int_{-\infty}^{\infty} d\nu_x \int_{-\infty}^{\infty} d\nu_y \int_{\nu_{nj+}}^{\infty} f_e(\vec{\nu}) R(\vec{\nu};\nu_{nj+}) \nu_n d\nu_n \right\} \tag{8}$$

where $A = 100\ cm^2$ is the cross-sectional area of the entrance aperture, and $\nu_\perp^2 = \nu_x^2 + \nu_y^2$. It can be shown (see Sittler, 1978) that these two integrals can be combined to an accuracy better than 1% yielding

$$\Delta I_j = qAT_N \int_{-\infty}^{\infty} d\nu_x \int_{-\infty}^{\infty} d\nu_y \int_{\nu_{nj-}}^{\nu_{nj+}} f_e(\vec{\nu}) R(\vec{\nu};\nu_{nj-}) \nu_n d\nu_n \tag{9}$$

Since the measurements are differential along ν_n it is useful to use the mean value theorem for the ν_n integration. Doing this Eq. (9) becomes

$$\Delta I_j = qAT_N \bar{\nu}_{nj} \Delta\nu_{nj} \int_{-\infty}^{\infty} d\nu_x \int_{-\infty}^{\infty} d\nu_y f_e(\nu_x, \nu_y, \nu_{nj}{}^*) \bar{R}(\nu_\perp{}^2/\bar{\nu}_{nj}{}^2; \nu_{nj-}) \qquad (10)$$

where we have introduced the mean normalized response

$$\bar{R}(\nu_\perp{}^2/\bar{\nu}_{nj}{}^2; \nu_{nj-}) = \frac{1}{\nu_n \Delta \nu_n} \int_{\nu_{nj-}}^{\nu_{nj+}} R(\vec{\nu}; \nu_{nj-}) \nu_n d\nu_n \qquad (11)$$

The speed $\nu_{nj}{}^*$ has some value residing between $\nu_{nj\pm}$ where

$$\bar{\nu}_{nj} = \frac{1}{2}(\nu_{nj+} + \nu_{nj-}) \qquad (12)$$

is the mean speed for the jth speed channel. If the speed windows are sufficiently narrow, then one can further simplify Eq. (10) by setting $\nu_{nj}{}^* = \bar{\nu}_{nj}$. In order to estimate the error in making this approximation we made a Taylor series expansion of f_e about $\bar{\nu}_{nj}$ and substituted it into Eq. (9). Doing this, assuming a step function for R in ν_n (good approximation, see Figure 3), using the mean response $\bar{R}$ in place of R, and using a convected-Maxwellian for f_e one gets

$$\Delta I_j = qAT_N \bar{\nu}_{nj} \Delta \nu_{nj} \int_{-\infty}^{\infty} d\nu_x \int_{-\infty}^{\infty} d\nu_y f_e(\nu_x, \nu_y, \bar{\nu}_{nj}) \bar{R}(\nu_\perp{}^2/\bar{\nu}_{nj}{}^2; \nu_{nj-})$$
$$\left\{1 - \frac{1}{6}\left(\frac{\Delta\nu_{nj}}{\omega_e}\right)^2 \left(1 - \frac{V_n}{\bar{\nu}_{nj}}\right) + \ldots\right\} \qquad (13)$$

where $\vec{V}$ is the mean vector velocity of the electrons relative to the sensor coordinate system. Since $V_n < 600$ km/s, and $\bar{\nu}_{nj} > 2000$ km/s, the term $V_n/\bar{\nu}_{nj} << 1$ and can thus be dropped. The resulting correction term is identical to that in Eq. (6) where it was shown in general to be less than 2%. The smallness of this correction term means the measurements are differential in ν_n, and the approximation setting $\nu_{nj}{}^* = \bar{\nu}_{nj}$ has an accuracy better than 2%. Then by noting the fact that $\bar{R}$ is essentially identical for all speed channels, we get the following general expression for the measured currents

$$\Delta I_j = qAT_N \bar{\nu}_{nj} \Delta\nu_{nj} \int_{-\infty}^{\infty} d\nu_x \int_{-\infty}^{\infty} d\nu_y \, f_e(\nu_x, \nu_y, \bar{\nu}_{nj}) \, \bar{R}(\nu_\perp^2/\bar{\nu}_{nj}^2) \tag{14}$$

with an accuracy better than 2%.

In order to write down an analytical expression for Eq. (14), when for instance f_e is a bi-Maxwellian, we fit the following sum of Gaussians to $\bar{R}$,

$$\bar{R}(\nu_\perp^2/\bar{\nu}_{nj}^2) = A_1 \, e^{-a_1 \nu_\perp^2/\bar{\nu}_{nj}^2} + A_2 \, e^{-a_2 \nu_\perp^2/\bar{\nu}_{nj}^2} \tag{15}$$

The result of this fit is shown in Figure 5, where $A_1 = 0.929$, $a_1^2 = 1.11$, $A_2 = 0.0813$, and $a_2^2 = 19.09$. As can be seen this function gives a very accurate description of $\bar{R}$. Substituting Eq. (15) into (14) and integrating one gets

$$\Delta I_j = \left[qAT_N \pi \bar{\nu}_{nj}^3 \Delta\nu_{nj} \right] \left[\sum_{k=1}^{2} G_{jk} \right] \frac{n_e}{\pi^{3/2} \omega_{\parallel} \omega_\perp^2} \, e^{-\beta_\perp(\bar{\nu}_{nj} - V_n)^2 (1 - \epsilon_B b_n^2)} \tag{16}$$

where G_{jk} is the integrated response

$$G_{jk} = \frac{A_k \, e^{a_k^2 \left[U_{\perp j}^2 - \frac{a_k^2 U_{xj}^2}{\beta_\perp \bar{\nu}_{nj}^2 (1 + \epsilon_B b_x^2) + a_k^2} - \frac{a_k^2 U_{yj}^2}{\beta_\perp \bar{\nu}_{nj}^2 (1 + \epsilon_B b_y^2) + a_k^2} \right]}}{\sqrt{(\bar{\nu}_{nj}^2 \beta_\perp (1 + \epsilon_B b_x^2) + a_k^2)(\bar{\nu}_{nj}^2 \beta_\perp (1 + \epsilon_B b_y^2) + a_k^2)}} \tag{17}$$

and f_e, assumed to be a bi-Maxwellian, is given by the following expression

$$f_e(\vec{\nu}) = \frac{n_e}{\pi^{3/2} \omega_{\parallel} \omega_\perp^2} \, e^{-(\beta_\perp(\vec{\nu}_\perp - \vec{V})^2 + \Delta\beta(\nu_\parallel - V_\parallel)^2)} \tag{18}$$

See front page.

The parameters used in Eq. (16) are defined as follows: (1) $\beta_{\parallel,\perp} = 1/\omega_{\parallel,\perp}^2$, (2) $1/2\, m_e\, \omega_{\parallel,\perp}^2 = kT_{\parallel,\perp}$ where $T_{\parallel,\perp}$ is the electron temperature parallel and perpendicular to $\vec{B}$, respectively, (3) $\epsilon_B = \Delta\beta/\beta_\perp$ is the anisotropy parameter where $\Delta\beta = \beta_\parallel - \beta_\perp$, (4) $U_{\perp j}^2 = (V_\perp/\bar{\nu}_{nj})^2$, $U_{xj}^2 = (V_x/\bar{\nu}_{nj})^2$ and $U_{yj}^2 = (V_y/\bar{\nu}_{nj})^2$ (5) $\hat{b} = \vec{B}/B$ is a unit vector parallel to $\vec{B}$, and (6) n_e is the electron density. If we assume isotropy $\epsilon_B=0$ (good approximation for thermal electrons) Eqs. (16) and (17) reduce to

$$\Delta I_j = \left[qAT_N\pi\bar{\nu}_{nj}^3\Delta\nu_{nj}\right]\left[\sum_{k=1}^{2} A_k \frac{e^{-(a_k^2W_\perp^2)/(a_k^2+\bar{\nu}_{nj}^2/\omega_e^2)}}{a_k^2+\bar{\nu}_{nj}^2/\omega_e^3}\right]\frac{n_e}{\pi^{3/2}\omega_e^3}\, e^{-\frac{(\bar{\nu}_{nj}-V_n)^2}{\omega_e^2}} \tag{19}$$

where ω_e is the electron thermal speed and $W_\perp^2 = V_\perp^2/\omega_e^2$. For purposes of simplicity we have approximated the two Gaussian fit Eq. (15) with a single Gaussian $\bar{R} = e^{-a^2(\nu_\perp/\bar{\nu}_{nj})^2}$ shown by the dots in Figure 5 with $a^2 = 1.35$. By doing this Eq. (19) reduces to

$$\Delta I_j = \left[qAT_N\pi\bar{\nu}_{nj}^3\Delta\nu_{nj}\right]\left[\frac{e^{-(a^2W_\perp^2)/(a^2+\bar{\nu}_{nj}^2/\omega_e^2)}}{a^2+\bar{\nu}_{nj}^2/\omega_e^2}\right]\frac{n_e}{\pi^{3/2}\omega_e^3}\, e^{-\frac{(\bar{\nu}_{nj}-V_n)^2}{\omega_e^2}} \tag{20}$$

which has an accuracy better than 5% for all speed channels. Finally, dividing both sides of Eq. (20) by $qA\,T_N\,\bar{\nu}_{nj}^3\,\Delta\nu_{nj}$ one gets the reduced distribution function

$$F_{ej} = \frac{n_e}{\pi^{3/2}\omega_e^3}\, e^{-(\bar{\nu}_{nj}-V_n)^2/(\omega_e^2)}\left[\frac{e^{-(a^2W_\perp^2)/(a^2+\bar{\nu}_{nj}^2/\omega_e^2)}}{a^2+\bar{\nu}_{nj}^2/\omega_e^2}\right] \tag{21}$$

This expression can be generalized to include the energy shift correction introduced by the spacecraft potential ϕ_{SC} where most of the energy change is assumed to occur along the ν_n direction (see Sittler, 1978). Doing this, Eq. (21) has the more general form

$$F_{ej} = \frac{n_e}{\pi^{3/2}\omega_e^3}\, e^{-(\sqrt{\bar{\nu}_{nj}^2-u_{SC}^2}-V_n)^2/\,\omega_e^2}\left[\frac{e^{-(a^2W_\perp^2)/(a^2+\bar{\nu}_{nj}^2/\omega_e^2}}{a^2+\bar{\nu}_{nj}^2/\omega_e^2}\right] \tag{22}$$

where $1/2\, m_e\, u_{SC}^2 = e\phi_{SC}$ and $\bar{\nu}_{nj}^2 > u_{SC}^2$.