Appendix I

Analysis of Multi-Peaked Spectra

a) The Measurements

The Faraday cup is an electrostatic device which measures ion charge flux as a function of energy per charge. For positive ions with a distribution in velocity space described by the function $f(\vec{v})$ the currents measured by a Faraday cup are related to $f(\vec{v})$ in the following way (see Vasyliunas, 1971, and Belcher *et al.*, 1980). If $\hat{n}$ is the unit normal to a given cup aperture, a positively charged particle will reach the collector plate if V_n, its component of velocity parallel to $\hat{n}$, satisfies

$$\frac{1}{2} A m_p V_n^2 > e Z^* \phi \qquad (1)$$

where e and m_p are the charge and mass of a proton; Z^* and A are the charge and mass numbers of the ion and ϕ is the retarding potential between the modulator grid and the spacecraft (see Figure 34). Suppose $\{\phi_j\}$ represents the set of contiguous voltages of the K energy channels (j = 1 to K+1), with ϕ_j the lower voltage of the j-th channel. Let $\{v_j\}$ represent the corresponding set of velocities where

$$v_j = \left(\frac{2 e Z^* \phi_j}{A m_p} \right)^{1/2} \qquad (2)$$

The average and incremental velocities for the j-th channel are defined by

$$V_j = \left(\frac{v_j + v_{j+1}}{2} \right)$$

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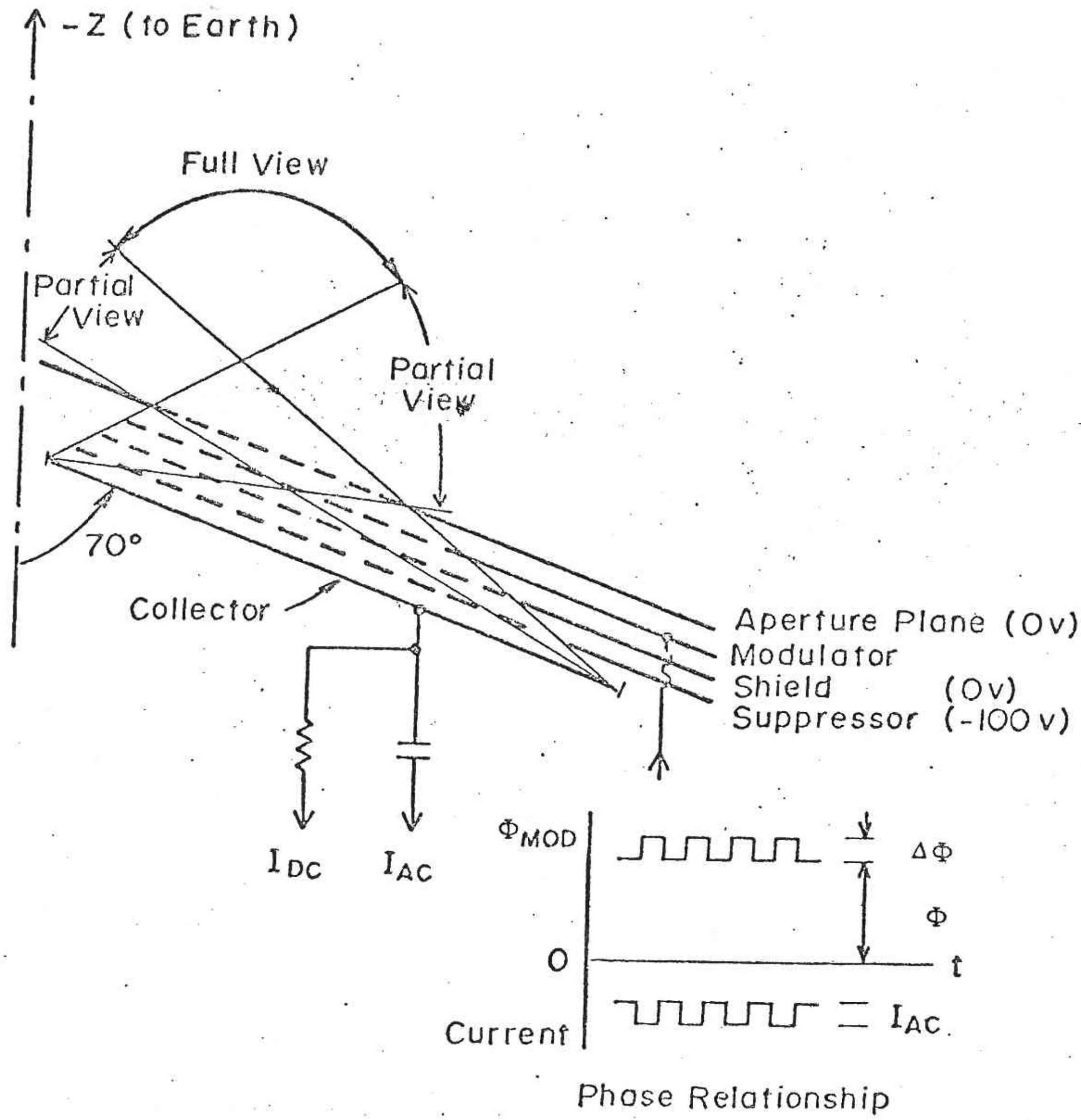

Figure 34. A cross-sectional view of the grid structure (schematic) for one of the three main sensors of the plasma detector. (From Bridge et al., 1977). For clarity only a single shield grid and a single modulator grid are shown. In the actual sensor six shield grids are used to obtain sufficient electrical isolation between the modulator and collector, and three grids are used for the modulator so that the retarding potential is defined to 0.5%.

and

$$\Delta v_j = v_{j+1} - v_j$$

Then the modulated AC current (I_{AC} in Figure 4) in the j-th energy channel corresponding to a modulator voltage varying between ϕ_J and ϕ_{j+1} is

$$I_j = Z^* e A_{eff} \int_{v_j}^{v_{j+1}} v_n dv_n \int_{-\infty}^{\infty} dv_{t1} \int_{-\infty}^{\infty} dv_{t2} f(\vec{v})\, G(v_n, \hat{n}) \tag{4}$$

where A_{eff} is the effective area of the aperture; $\hat{t}_1$ and $\hat{t}_2$ are mutually perpendicular unit vectors which are orthogonal to $\hat{n}$ and $G(\vec{v},\hat{n})$ is the response function of the sensor. In this preliminary analysis the response function has been taken as unity so that

$$I_j = Z^* e A_{eff} \int_{v_j}^{v_{j+1}} v_n\, F(v_n)\; dv_n \tag{5}$$

where $F(v_n)$ is the "reduced" one-dimensional distribution function

$$F(v_n) = \int_{-\infty}^{\infty} dv_{t1} \int_{-\infty}^{\infty} dv_{t2}\; f(v_n, v_{t1}, v_{t2}) \tag{6}$$

This means that each detector samples the full distribution function in a manner which is differential for the velocity component parallel to the detector normal and integral for the velocity components perpendicular to the detector normal. Figure 35 is a schematic illustration of this concept for a model proton distribution function with a pronounced heat flux.

In our present analysis the distribution functions of the positive ions are assumed to be characterized by a convected, isotropic Maxwellian function. Therefore, the "reduced" one-dimensional distribution function of each ionic species of density n_i, thermal speed w_i and bulk velocity $\vec{V}_i$ is

$$F_i(v_n) = \frac{n_i}{\omega_i\sqrt{\pi}} \exp\left[-\frac{(v_n - V_{in})^2}{\omega_i^2}\right] \tag{7}$$

where V_{in} is the component of the bulk flow along the cup normal $\hat{n}$. Thus for a multi-species plasma the current in the j-th energy channel becomes

$$I_j = e A_{eff} \sum_i \int_{v_j}^{v_{j+1}} v_n Z_i^* F_i(v_n)\, dv_n \tag{8}$$

Substituting $F_i(V_n)$ from above, this expression can be integrated to give

$$I_j = \frac{e A_{eff}}{2\sqrt{\pi}} \sum_i Z_i n_i \left[\omega_i \left\{ e^{-u_j^2} - e^{-u_{j+1}^2} \right\} + \sqrt{\pi}\, V_{ni} \left\{ \operatorname{erf}(u_{j+1}) - \operatorname{erf}(u_j) \right\} \right]$$

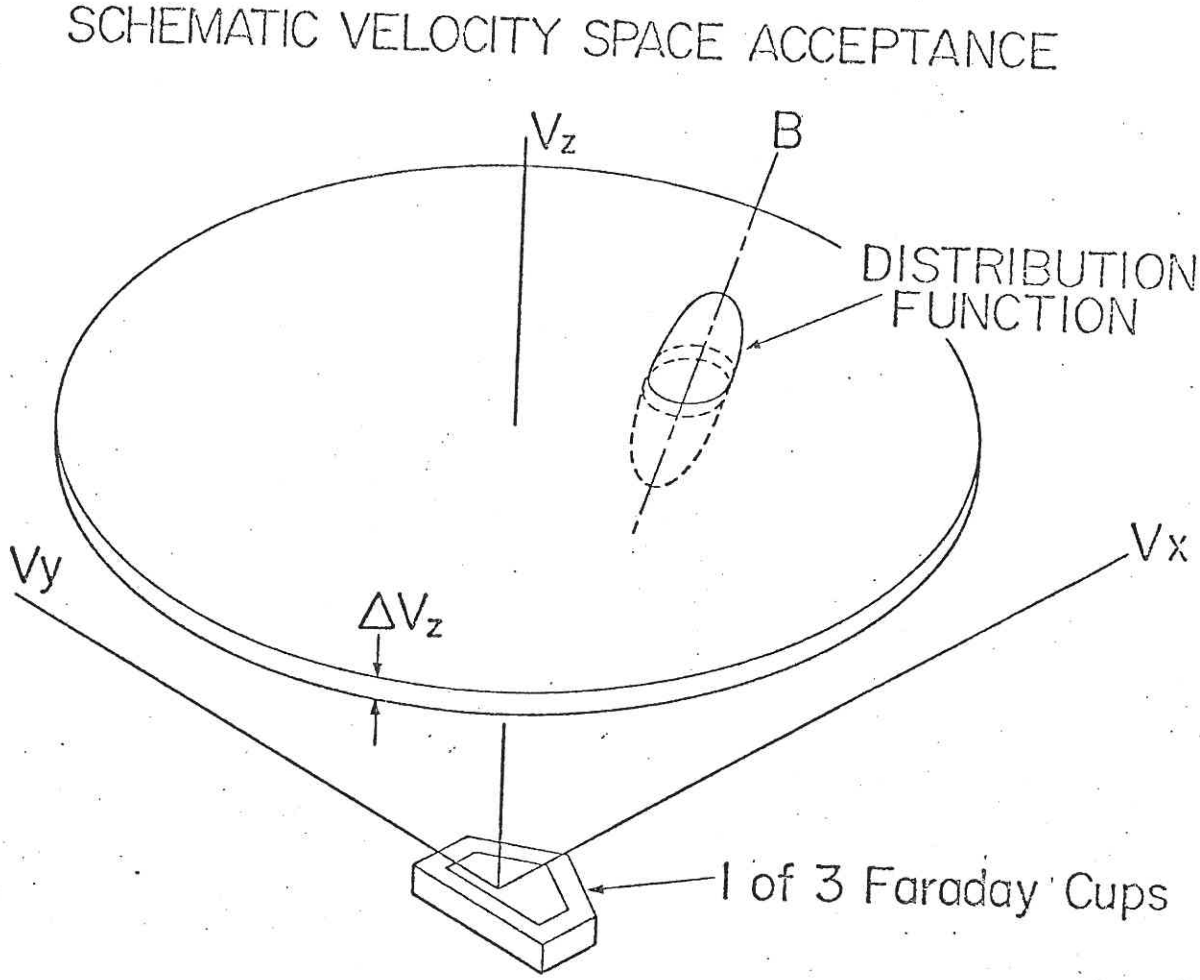

Figure 35. Schematic velocity space acceptance of the main sensors of the Voyager plasma instrument. Only particles within a narrow range of V_z (component of velocity normal to the cup aperture) are accepted into the sensor. The x and y components of velocity of the accepted particles can be within a wide range. This means the three-dimensional distribution of ions in velocity space is sliced differentially in the direction normal to the cup aperture and integrated in the x and y directions.

where

$$u_j = \left(\frac{v_j - V_{in}}{\omega_i}\right) \quad \text{and} \quad u_{j+1} = \left(\frac{v_{j+1} - V_{in}}{\omega_i}\right) \tag{9}$$

The above function is used to fit the measured currents (in a least squares sense) and hence find the plasma parameters n_i, w_i, V_{in} for each species.

Before the details of the full fitting procedure is described it should be realized that there are quantities that can be estimated independent of assumptions about composition. Because the velocity interval ΔV_j is much less than the average channel velocity v_j equation can be approximated by

$$I_j \cong e A_{eff} \sum_i Z_i^* v_j \Delta v_j F(v_j) \tag{10}$$

Summing over all the channels gives

$$\sum_j I_j \cong e A_{eff} \sum_i Z_i^* \left[\sum v_j \Delta v_j F(v_j)\right] \tag{11}$$

Using the fact that $\Delta v_j \ll v_j$ again means

$$\begin{aligned} \sum_j I_j &\cong e A_{eff} \sum_i Z_i^* \left(\int_{-\infty}^{\infty} v_n F_i(v_n) \, d v_n\right) \\ &= e A_{eff} \sum_i Z_i^* V_{in} n_i \end{aligned} \tag{12}$$