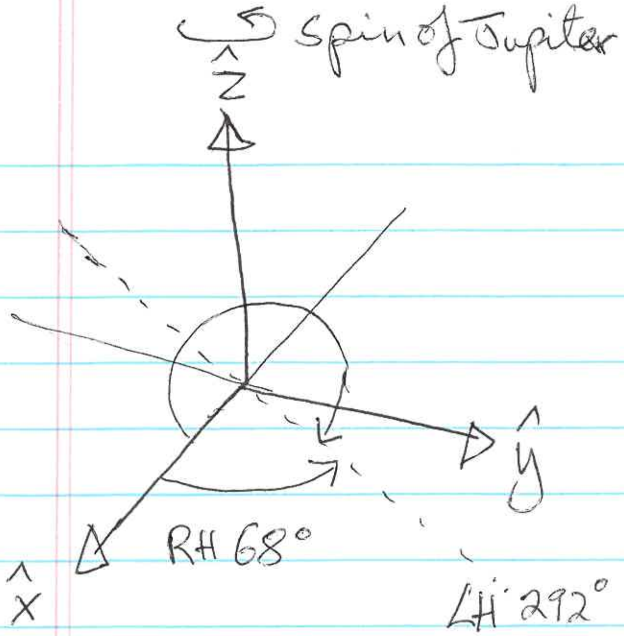
1



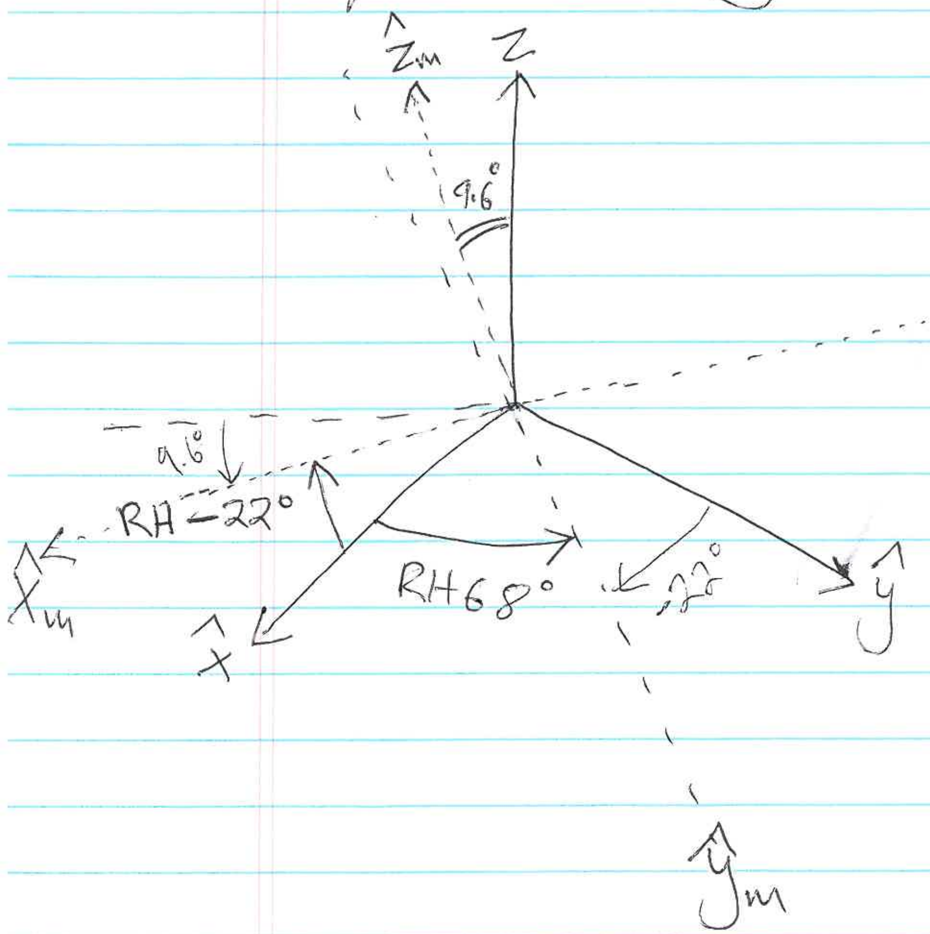
System III – Rotating w/Jupiter every 10 hrs

LH – increasing w/time when observed from Earth

RH = Right Hand longitude $= 360° - LH$

$(\hat{x}, \hat{y}, \hat{z})$ = RH geographic

at RH 68° & 248° the magnetic & geographic equators align.



$(\hat{x}_m, \hat{y}_m, \hat{z}_m)$ = (RH) Magnetic Coordinates

Magnetic dipole is tilted 9.6° towards RH −22°

(2)

Co-ordinate transformation from geographic RH System III to magnetic coordinates.

We seek the Matrix $M_{\ast}$ such that

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{mag.} = \begin{pmatrix} & & \\ & M_{\ast} & \\ & & \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\substack{RH \\ \text{System III} \\ \text{geographic}}}$$

Step 1: Rotate about $\hat{z}$ to align $\hat{y}$ and $\hat{y}_m$

$$x' = x \cos\alpha + y \sin\alpha \qquad \text{Angle } \alpha = -22^\circ$$
$$y' = -x \sin\alpha + y \cos\alpha$$
$$z' = z$$

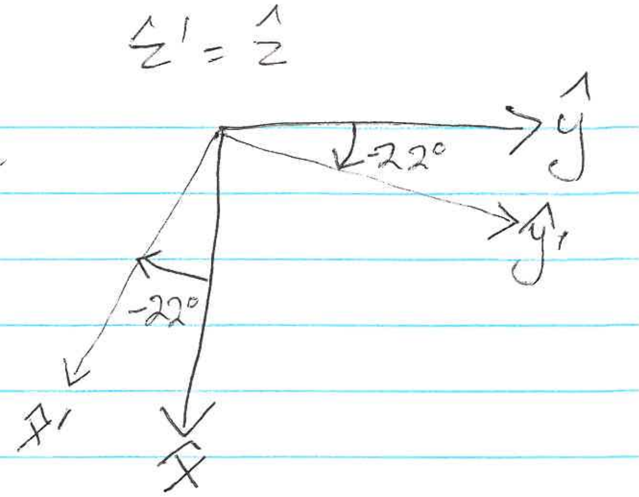
$$M_1 = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.927 & -0.375 & 0 \\ +0.375 & 0.927 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2: Rotate about $\hat{y}'$ by 9.6°

$$\beta = 9.6^\circ$$

$$M_2 = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} = \begin{bmatrix} 0.986 & 0 & -0.167 \\ 0 & 1 & 0 \\ 0.167 & 0 & 0.986 \end{bmatrix}$$

(3)



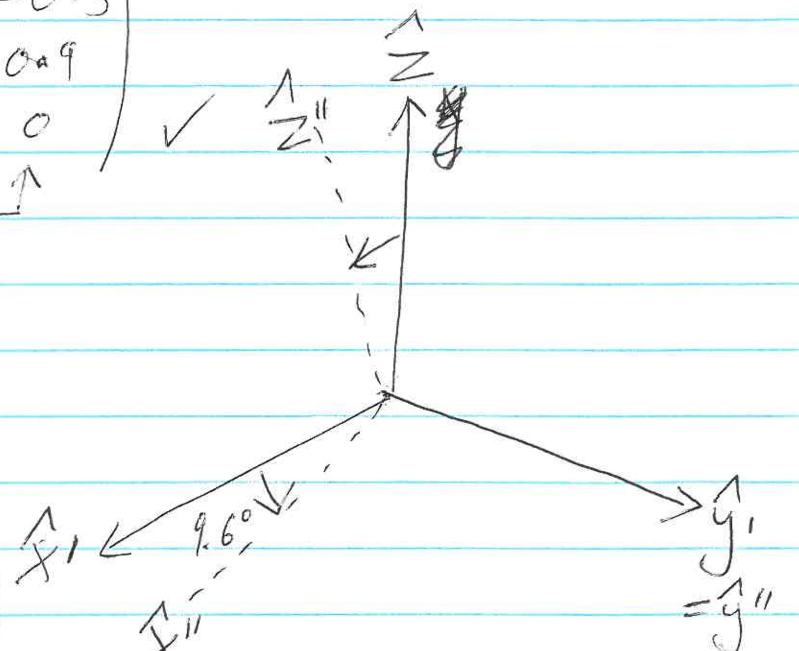
check Step 1

$$\begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{c} \hat{x} \\ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \end{array} = \begin{pmatrix} 0.9 \\ 0.3 \\ 0 \end{pmatrix} \checkmark$$

this is $\hat{x}$ in ' coordinates

$$\begin{pmatrix} 0.9 & -0.3 & 0 \\ +0.3 & 0.9 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.3 \\ 0.9 \\ 0 \end{pmatrix} \checkmark$$

this is $\hat{y}$ in $(x, y, z)'$ coords



check Step 2

$$\begin{pmatrix} .986 & 0 & -0.167 \\ 0 & 1 & 0 \\ 0.167 & 0 & 0.986 \end{pmatrix} \begin{array}{c} \hat{x}' \\ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \end{array} = \begin{pmatrix} 0.986 \\ 0 \\ 0.167 \end{pmatrix} \checkmark$$

This is $\hat{x}'$ in (x'', y'', z'') coords

$$\begin{pmatrix} .9 & 0 & -.2 \\ 0 & 1 & 0 \\ .2 & 0 & .9 \end{pmatrix} \begin{array}{c} \hat{y}' \\ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \end{array} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \checkmark$$

$$\begin{pmatrix} 0.9 & 0 & -.2 \\ 0 & 1 & 0 \\ .2 & 0 & .9 \end{pmatrix} \begin{array}{c} \hat{z} \\ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{array} = \begin{pmatrix} -0.2 \\ 0 \\ .9 \end{pmatrix} \checkmark$$

4

Step 3: Combine 2 rotations

$$M = M_2 M_1$$

$$\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \times \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\beta\cos\alpha & \cos\beta\sin\alpha & -\sin\beta \\ -\sin\alpha & \cos\alpha & 0 \\ \sin\beta\cos\alpha & \sin\beta\sin\alpha & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} 0.986 \times 0.927 & -0.986 \times 0.375 & -0.167 \\ +0.375 & 0.927 & 0 \\ 0.167 \times 0.927 & -0.167 \times 0.375 & 0.986 \end{bmatrix}$$

$$\boxed{\underbrace{= \begin{bmatrix} 0.914 & -0.369 & -0.167 \\ 0.375 & 0.927 & 0 \\ 0.155 & 0.062 & 0.986 \end{bmatrix}}_{M} \quad \text{So....} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{Mag} = \begin{bmatrix} M \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\substack{RH \\ \text{System III} \\ \text{geographic}}}}$$

5

Check

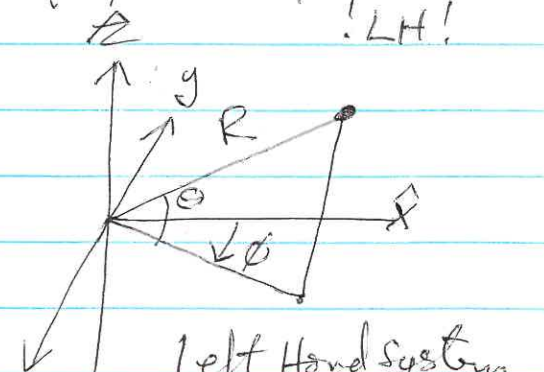
$$[M]\begin{array}{c}\hat{x}\\ \begin{pmatrix}1\\0\\0\end{pmatrix}_G\end{array} = \begin{array}{c}\hat{x}\\ \begin{pmatrix}0.914\\0.375\\0.155\end{pmatrix}_M\end{array} \quad \checkmark \quad |\;| = 1.00002 \;\checkmark$$

$$[M]\begin{array}{c}\hat{y}\\ \begin{pmatrix}0\\1\\0\end{pmatrix}_G\end{array} = \begin{pmatrix}-0.369\\0.927\\0.062\end{pmatrix}_M \quad \checkmark \quad |\;| = 0.99967 \;\checkmark$$

$$[M]\begin{array}{c}\hat{z}\\ \begin{pmatrix}0\\0\\1\end{pmatrix}_G\end{array} = \begin{pmatrix}-0.167\\0\\0.986\end{pmatrix}_M \;\checkmark \quad |\;| = 1.00004 \;\checkmark$$

= For location $(R, \theta, \phi)_{LH} = (R, S3LAT, S3LON)$!LH!

to $\rightarrow (R, \gamma, \lambda)_{RH\,system}$



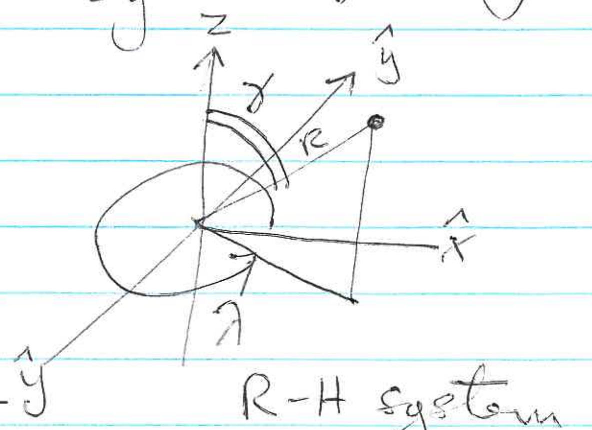
Left Hand system

$R = R$

Co-latitude $\gamma = 90 - \theta$

RH long. $\lambda = 360° - \phi$

$$\begin{pmatrix} x = R\cos\cancel{\phi}\sin\gamma \\ y = R\sin\cancel{\phi}\sin\gamma \\ z = R\cos\gamma \end{pmatrix}_{RH\ geog.}$$



R-H system

⑥ <u>Finally</u>,

If you want to go from

$(R, \gamma, \lambda)_{RH}$ to $(x, y, z)_{RH}$

then
$$x = R \cos\phi \sin\gamma$$
$$y = R \sin\phi \sin\gamma$$
$$z = R \cos\gamma$$

Going the other way....

$(x, y, z)_{RH}$ to $(R, \gamma, \lambda)_{RH}$

$$R = (x^2 + y^2 + z^2)^{1/2}$$

$$\lambda = \arctan(y/x)$$

$$\gamma = \arccos(z/R)$$

– So now you can go from

Juno's $(R, \theta, \phi)_{\text{system 3 L-H}}$ position

to $(x, y, z)_{\text{magnetic}}$ or $(R, \gamma_M, \lambda_M)_{\text{magnetic}}$