Bagenal May 2, 2012

Voyager PLS Spacecraft to Instrument Coordinates

Following notes from Stan Olbert and Alan Barnett (VoyPLSrespBarnett.pdf) the spacecraft to instrument transformation matrices are as below. These have been checked for unit flow vectors in the spacecraft (s/c) X, Y, and Z directions.

				Matrix from S/C to Cup coordinates								
DEGREES	azmuth1	polar2		1	2	3	4	5	6	7	8	9
A	30.00	20.00		0.5	-0.865	0	0.813	0.47	-0.342	0.296	0.171	0.94
B	150.00	20.00		0.5	0.865	0	-0.813	0.47	-0.342	-0.296	0.171	0.94
C	270.00	20.00		1	0	0	0	-0.94	-0.342	0	-0.342	0.94
D	223.00	88.00		-0.682	0.731	0	-0.026	-0.024	-0.999	-0.731	-0.682	0.035

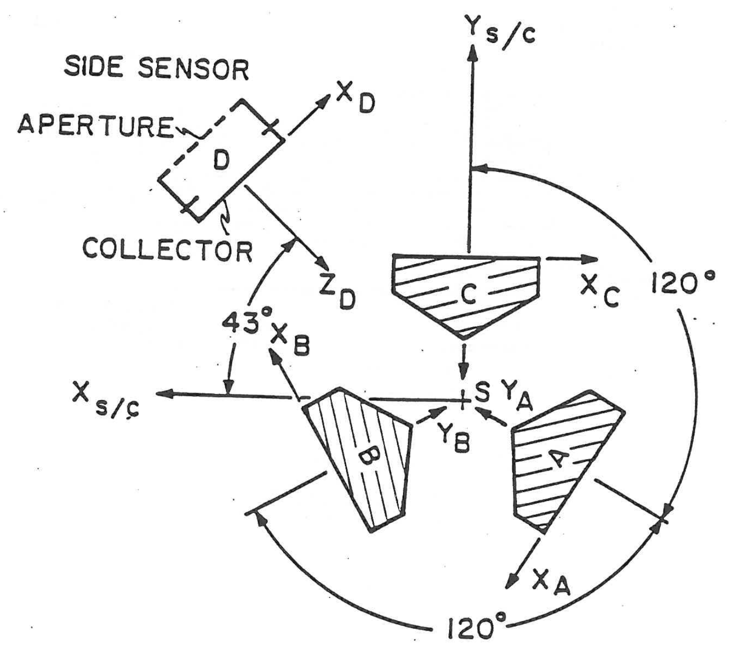
$$\begin{bmatrix}V_x\\V_y\\V_z\end{bmatrix}_{ABCD} = \begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}_{ABCD}\begin{bmatrix}V_x\\V_y\\V_z\end{bmatrix}_{S/C}$$

TESTS

Input-s/c	Vx	Vy	Vz
V1	0	0	1
A	0	-0.3420	0.9400
B	0	-0.3420	0.9400
C	0	-0.3420	0.9400
D	0	-0.9990	0.0350

	Vx	Vy	Vz
V1	0	1	0
A	-0.865	0.47	0.171
B	0.865	0.47	0.171
C	0	-0.94	-0.342
D	0.731	-0.024	-0.682

	Vx	Vy	Vz
V1	1	0	0
A	0.5	0.813	0.296
B	0.5	-0.813	-0.296
C	1	0	0
D	-0.682	-0.026	-0.731



Note that there are 2 previous Voyager memos (#20 Jim Sullivan and #29 Al Lazarus) that use different coordinate definitions. Both have slightly different angles and #20 has several typos.

2. Geometry and Operational Modes of PLS-Sensors.

(a) Location of PLS-Sensors in the space-craft coordinate system.

Figure 1 shows the sketch of the cluster of ~~the~~ four PLS-sensors. The drawing is a simplified version of a photograph; the scale is 1:5 of the real size. Note the agreed-upon labeling A, B, C, D for identifying each sensor. The D sensor is cylindrical in shape and is often referred to as the "D-cup"; the term "the side sensor" is also used frequently. The A, B, C sensors are identical in shape (pentagonal) and operation and ~~these~~ are called "the main sensors"; they form "the main cluster" which has one common axis of symmetry, S. The entire system is rigidly mounted on the "science boom" of the Voyager. ~~[illegible]~~ (There are, of course, two sets, one for each Voyager; there are no physical differences between the two sets, with one minor exception referring to the alignments of the grid meshes in the D-cup. For our purposes the two sets may be treated as identical.)

In discussing the response of each sensor it is convenient to make use of four distinct cartesian coordinate systems, (x_K, y_K, z_K), where $K=(A, B, C, D)$, one for each ~~four the~~ sensor. ~~under consideration~~. In addition one needs a common reference frame whose coordinates donot change in time ~~relative~~ with respect to the sensor coordinate systems. This common frame is called "the space-craft (s/c) coordinate system", $(x_{s/c}, y_{s/c}, z_{s/c})$, and is defined as follows:

$\hat{z}_{s/c}$ is parallel to the Earth-Voyager line and points away from the Earth, i.e., lies along the axis of the main antenna dish.

The PLS-sensors are mounted in such a way that the symmetry axis, S, of the main cluster is parallel to $\hat{z}_{s/c}$ and the x-axis of the C-sensor, $\hat{x}_C$, is antiparallel to $\hat{x}_{s/c}$. Figure 2 shows the relative configurations of the five coordinate systems. It is important to keep in mind the following facts:

1) each of $\hat{y}_A, \hat{y}_B, \hat{y}_C$ points toward S and each $\hat{y}_K$ lies in the plane of the collector of the K^{th} sensor, respectively.

2) each of $\hat{z}_A, \hat{z}_B, \hat{z}_C$ forms an angle of $20°$ with $\hat{z}_{s/c}$. and each $\hat{z}_K$ points into the K^{th} sensor at right angles to the collector plate resp

3) $\hat{x}_K$'s are defined by

$$\hat{x}_K = \frac{\hat{z}_K \times \hat{z}_{s/c}}{\sin\theta_K} \qquad K = (A, B, C, D)$$

where $\sin\theta_K = |\hat{z}_K \times \hat{z}_{s/c}|$

4) $\hat{z}_D$ is "anti-normal" to the collector plate of the D-cup, ~~and~~ forms an angle of 88° with $\hat{z}_{s/c}$, and points toward S.

If we introduce the polar and azimuth angles, $(\theta^{s/c}_{(K)}, \varphi^{s/c}_{(K)})$, in the s/c coordinate system for each $\hat{z}_K$, then we have the following representation:

$\hat{z}_K$	$\theta^{s/c}_{(K)}$	$\varphi^{s/c}_{(K)}$
$\hat{z}_A$	20°	30°
$\hat{z}_B$	20°	150°
$\hat{z}_C$	20°	270°
$\hat{z}_D$	88°	223°

To obtain the cartesian components of $\hat{z}_K$ in s/c system one simply recalls that

$$\hat{z}_{K,i} = \begin{cases} \cos\varphi^{s/c}_{(K)} \sin\theta^{s/c}_{(K)} \\ \sin\varphi^{s/c}_{(K)} \sin\theta^{s/c}_{(K)} \\ \cos\theta^{s/c}_{(K)} \end{cases} \qquad \left(i = (x_{s/c}, y_{s/c}, z_{s/c})\right)$$

The above information allows us to compute the polar and azimuth angles, (θ_K, φ_K), in the frame of the K^{th} sensor of any given unit vector if the polar and azimuth angles, $(\theta_{s/c}, \varphi_{s/c})$, of this vector are known in the s/c frame. Figures 3, 4, 5 and 6 give the graphical answer to this type of transformation. One often needs also to transform the cartesian ~~the cartesian representation~~ components of ~~of~~ a unit vector $\hat{u}$ ~~(u_1, u_2, u_3)~~ in the s/c frame into ~~four sets of~~ cartesian components of $\hat{u}$ in the frame of the k^{th} sensor; viz.:

$$u_i^K = T_{ij}^K u_j^{s/c} \qquad \begin{cases} i = 1,2,3 \\ j = 1,2,3 \end{cases} \quad K = A, B, C, D \qquad (2)$$

The four matrices T_{ij}^K are given by:

$$T_{ij}^K = \hat{x}_{iK} \cdot \hat{x}_{js/c} \qquad (3)$$

where $\hat{x}_{1K} \equiv \hat{x}_K$, $\hat{x}_{2K} \equiv \hat{y}_K$, $\hat{x}_{3K} \equiv \hat{z}_K$

and $\hat{x}_{1s/c} \equiv \hat{x}_{s/c}$, $\hat{x}_{2s/c} \equiv \hat{y}_{s/c}$, $\hat{x}_{3s/c} \equiv \hat{z}_{s/c}$.

The inverse transformation is obtainable from

$$u_i^{s/c} = \tilde{T}_{ij}^K u_j^K = u_j^K T_{ji}^K$$

Table 1 below gives the numerical values of all T_{ij}^K.

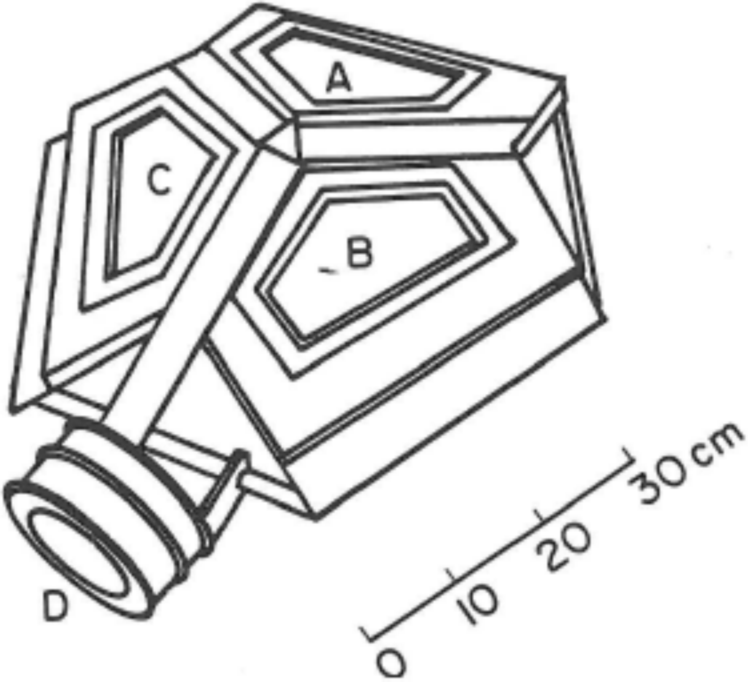
TABLE I

Transformation Matrix $T_{ij}^{K} = \hat{x}_{iK} \cdot \hat{x}_{j\,s/c}$ for u_i^K

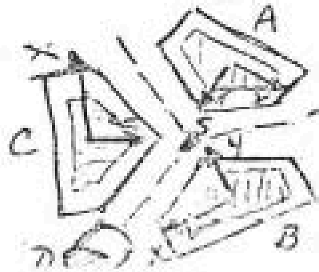
$u_i^K = T_{ij}^K u_j^{s/c}$

$(u_i^2 = 1)$

Sensor:	$u_i^K \backslash u_j^{s/c}$	$u_x^{s/c}$	$u_y^{s/c}$	$u_z^{s/c}$
A	u_x^A	.500	-.865	0
	u_y^A	.813	.470	-.342
	u_z^A	.296	.171	.940
B	u_x^B	.500	.865	0
	u_y^B	-.813	.470	-.342
	u_z^B	-.296	.171	.940
C	u_x^C	1.000	0	0
	u_y^C	0	-.940	.342
	u_z^C	0	-.342	.940
D	u_x^D	-.682	.731	0
	u_y^D	-.026	-.024	-.999
	u_z^D	-.731	-.682	.035


A
B
C
D
0
10
20
30 cm

2) Cup coordinates



S = sym. axis and S/c z-axis pointing into paper

z-axes for A, B, C cups point into paper

$$\cos\theta_{A,B,C} \equiv \hat{z}_{S/C} \cdot \hat{z}_{A,B,C}; \quad \theta_{A,B,C} = 20^\circ$$

$$\hat{x}_{A,B,C} \cdot \hat{z}_{S/C} = 0$$

"Negative cup normals", $\hat{z}_{A,B,C}$, in S/c coord. system:

$$(\hat{i}_{S/C},\ \hat{j}_{S/C},\ \hat{k}_{S/C})$$

$$\hat{z}_A = (.296, .171, .940)$$

$$\hat{z}_B = (-.296, .171, .940)$$

$$\hat{z}_C = (0, -.342, .940)$$

$$\left\{ \text{also } \hat{z}_D = (-.731, -.682, .035) \quad \text{just for your information} \right\}$$

From Memos #20 and #29

From Voyager Memos #20 and #29

Note that Lazarus & Sullivan use different coordinate systems to each other and to Barnett.

DEGREES	V1 az1	V1 pol2
A	120.70	19.99
B	-119.60	20.00
C	-0.30	19.47
D	-47.43	88.31

RADIANS	V1 az1	V1 pol2
A	2.1066	0.3489
B	-2.0874	0.3491
C	-0.0052	0.3398
D	-0.8278	1.5413

MATRIX ELEMENTS for AM - Voyager 1

	1	2	3	4	5	6	7	8	9
A	-0.5105	0.8599	0	-0.8080	-0.4798	0.3419	0.2939	-0.1745	0.9398
B	-0.4939	-0.8695	0	0.8171	-0.4642	0.3420	-0.2974	0.1689	0.9397
C	1.0000	-0.0052	0	0.0049	0.9428	0.3333	-0.0017	-0.3333	0.9428
D	0.6765	-0.7365	0	0.0217	0.0200	0.9996	-0.7361	-0.6762	0.0295

DEGREES	V2 az1	V2 pol2
A	120.30	19.97
B	-121.10	19.78
C	-0.58	19.60
D	-47.43	88.77

RADIANS	V2 az1	V2 pol2
A	2.0996	0.3485
B	-2.0874	0.3452
C	-0.0101	0.3421
D	-0.8278	1.5493

MATRIX ELEMENTS for AM - Voyager 2

	1	2	3	4	5	6	7	8	9
A	-0.5045	0.8634	0	-0.8115	-0.4742	0.3415	0.2949	0.1723	0.9399
B	-0.4939	-0.8695	0	0.8182	-0.4648	0.3384	-0.2942	0.1672	0.9410
C	0.9999	-0.0101	0	0.0095	0.9420	0.3355	-0.0034	-0.3354	0.9421
D	0.6765	-0.7365	0	0.0158	0.0145	0.9998	-0.7363	-0.6763	0.0215

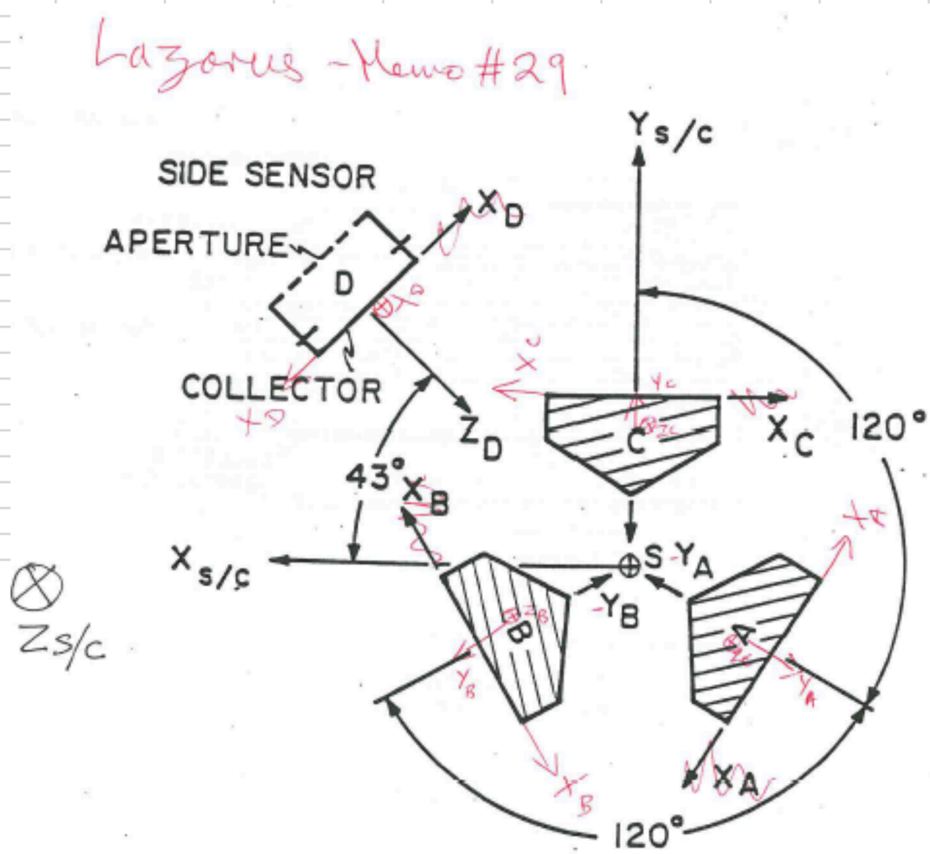
TESTS

Input-s/c	Vx	Vy	Vz
V1	0	0	1
A	0	0.3419	0.9398
B	0	0.3420	0.9397
C	0	0.3333	0.9428
D	0	0.9996	0.0295

	Vx	Vy	Vz
V1	0	1	0
A	0.8599	-0.4798	-0.1745
B	-0.8695	-0.4642	0.1689
C	-0.0052	0.9428	-0.3333
D	-0.7365	0.0200	-0.6762

	Vx	Vy	Vz
V1	1	0	0
A	-0.5105	-0.8080	0.2939
B	-0.4939	0.8171	-0.2974
C	1.0000	0.0049	-0.0017
D	0.6765	0.0217	-0.7361

These values make sense with coordinate system to right (red)



MIT Voyager Memo No. 29

To: Voyager Internal

From: A. Lazarus

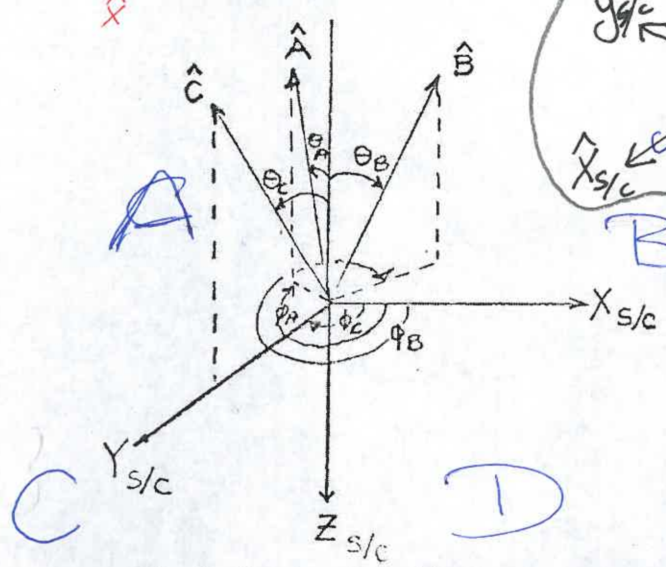
Date: January 25, 1978

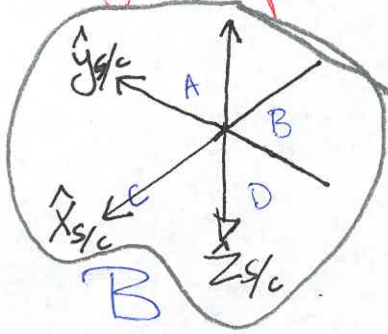
Subject: Relative cup transparencies.

This note describes a procedure for finding the relative cup transparencies which can then be used to correct the density calculations. The following calculations transform the solar wind velocity (given in "sensor" coordinates) to coordinate systems natural for each cup; find the flow angles realtive to the grid normal; and compute the transparency using optical grid transparencies. Nominal sensor orientations are used but the analysis herein can accept the correct orientations and can be modified to use more sophisticated transparencies.

The measured velocities are given in "sensor" coordinates as defined in Fig. 1. For any cup, we then define a "cup" coordinate system x_k, y_k, z_k, and normal n_k as follows

This is not consistent w/Memo #20 When $\hat{x}$ s/c is towards C – not y s/c | Also – Numbers for angles on p3 suggest this





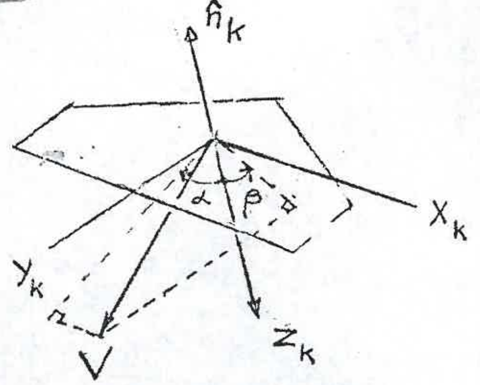


Fig.1- Sensor Coordinates

Fig.2- Cup Coordinates

BUT when Rotation Matrix AM is calculated – This seems to be right...

The steps we need to take are

1) find the rotation matrix giving V_{x_k}, V_{y_k}, V_{z_k} in terms of V_x, V_y, V_z (which are the wind velocity components in sensor coordinates);

2) knowing the cup velocity components, we can then calculate the cup transparency and then

3) correct the density measured by that cup.

Step 1

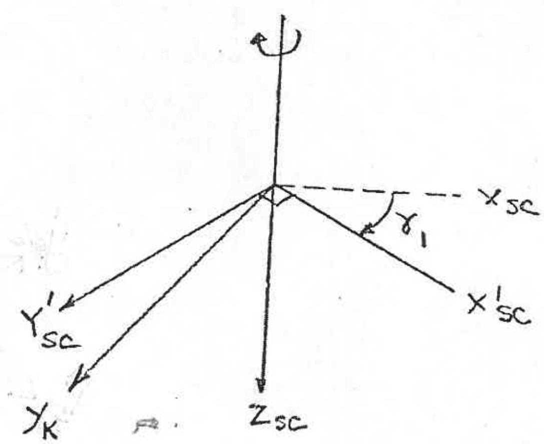
Use the Euler angle transformations as follows:

i) Rotate about z until x-axis is parallel to cup x-axis, x_k;

ii) rotate about new x-axis until new z-axis is in z_k, x_k plane;

iii) rotate about new y-axis until z-axes coincide.

i):



$$x' = x \cos \gamma_1 + y \sin \gamma_1$$

$$y' = -x \sin \gamma_1 + y \cos \gamma_1$$

$$z' = z$$

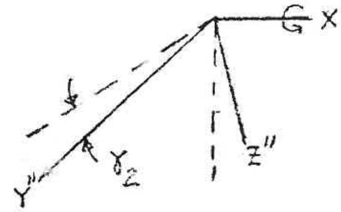
$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = M_1 \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\therefore M_1 = \begin{pmatrix} \cos \gamma_1 & \sin \gamma_1 & 0 \\ -\sin \gamma_1 & \cos \gamma_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ii) $x'' = x$

$y'' = y' \cos \gamma_2 + z' \sin \gamma_2$

$z'' = -y \sin \gamma_2 + z' \cos \gamma_2$

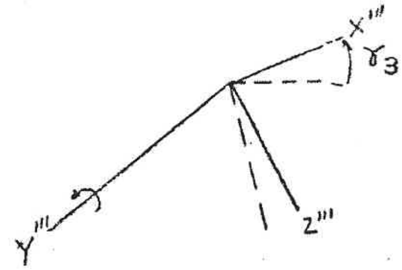


$$\therefore M_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_2 & +\sin\gamma_2 \\ 0 & -\sin\gamma_2 & \cos\gamma_2 \end{pmatrix}$$

iii) $x''' = x'' \cos \gamma_3 - z'' \sin \gamma_3$

$y''' = y''$

$z''' = z'' \sin \gamma_3 + z'' \cos \gamma_3$



$$M_{\cancel{2}\,3} = \begin{pmatrix} \cos\gamma_3 & 0 & -\sin\gamma_3 \\ 0 & 1 & 0 \\ \sin\gamma_3 & 0 & \cos\gamma_3 \end{pmatrix}$$

if $\gamma_3 = 0$, then $M_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ so $AM = M_2 M_1$

The overall transformation is $M_3 M_2 M_1 = AM$

$$AM = \begin{bmatrix} \cos\gamma_3 \cos\gamma_1 - \sin\gamma_3 \sin\gamma_2 \sin\gamma_1 & \cos\gamma_3 \sin\gamma_1 + \sin\gamma_3 \sin\gamma_2 \cos\gamma_1 & -\sin\gamma_3 \cos\gamma_2 \\ -\cos\gamma_2 \cancel{\sin\gamma_2} \sin\gamma_1 & \cos\gamma_2 \cos\gamma_1 & \sin\gamma_2 \\ \sin\gamma_3 \cos\gamma_1 + \cos\gamma_3 \sin\gamma_2 \sin\gamma_1 & \sin\gamma_3 \sin\gamma_1 - \cos\gamma_3 \sin\gamma_2 \cos\gamma_1 & \cos\gamma_3 \cos\gamma_2 \end{bmatrix}$$

Parameters for the sensors (ref. Voyager Memo. No. 20): Note that the axes are not the same* The axes in this memo are the same as used in the analysis program . WHICH??

Sensor	γ_1	γ_2	γ_3 = 0 ?	
A	+120.7°/+120.7°	19.99°/20.3°	?	.../...=
B	-119.6°/-121.2°	~~200.0°~~/19.76°	?	Voyager 1/2
C	-.3°/ -.56°	19.47°/19.5°	?	

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* $\phi = 90° - \phi_{old}$ – because $\hat{z} = -\hat{z}_{old}$; $\hat{x} = -\hat{x}_{old}$

$\therefore \gamma_1 = \cancel{-90° =} - \phi_{old}$; $\gamma_2 = \theta_{old}$

Using the nominal values,

$$
AM = \left\{ \begin{matrix} \begin{pmatrix} -.5 \\ -.5 \\ +1 \end{pmatrix} & \begin{pmatrix} .866 \\ -.866 \\ 0 \end{pmatrix} & 0 \\ \begin{pmatrix} -.813 \\ +.813 \\ 0 \end{pmatrix} & \begin{pmatrix} -.470 \\ -.470 \\ .940 \end{pmatrix} & .342 \\ \begin{pmatrix} .269 \\ -.269 \\ 0 \end{pmatrix} & \begin{pmatrix} -.171 \\ -.171 \\ +.342 \end{pmatrix} & .940 \end{matrix} \right\} \quad \text{and} \quad \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} = \begin{pmatrix} \text{cup A} \\ \text{cup B} \\ \text{cup C} \end{pmatrix}
$$

Wrong sign

Step 2

Transform the solar wind velocity components.

$$
\begin{pmatrix} V_{x_j} \\ V_{y_j} \\ V_{z_j} \end{pmatrix} = AM \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}
$$

Now given the velocity in the cup frame find $\cos\alpha$ and $\cos\beta$ where

$$\cos\alpha = \frac{V_z}{\sqrt{V_z^{2} + V_y^{2}}}$$

$$\cos\beta = \frac{V_z}{\sqrt{V_z^{2} + V_x^{2}}}$$

then $T = \left(1 - \frac{d}{s\cos\beta}\right)\left(1 - \frac{d}{s\cos\alpha}\right)$

where d is the diameter of a grid wire and

s is the spacing between the wire centers

MIT Voyager Memo No. 20

TO: Voyager Internal
H. Mertz

FROM: J.D. Sullivan

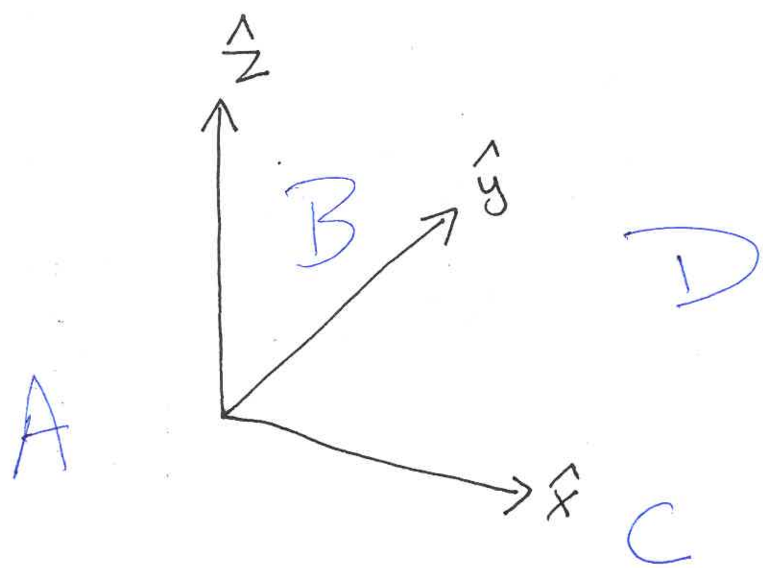
DATE: August 9, 1977

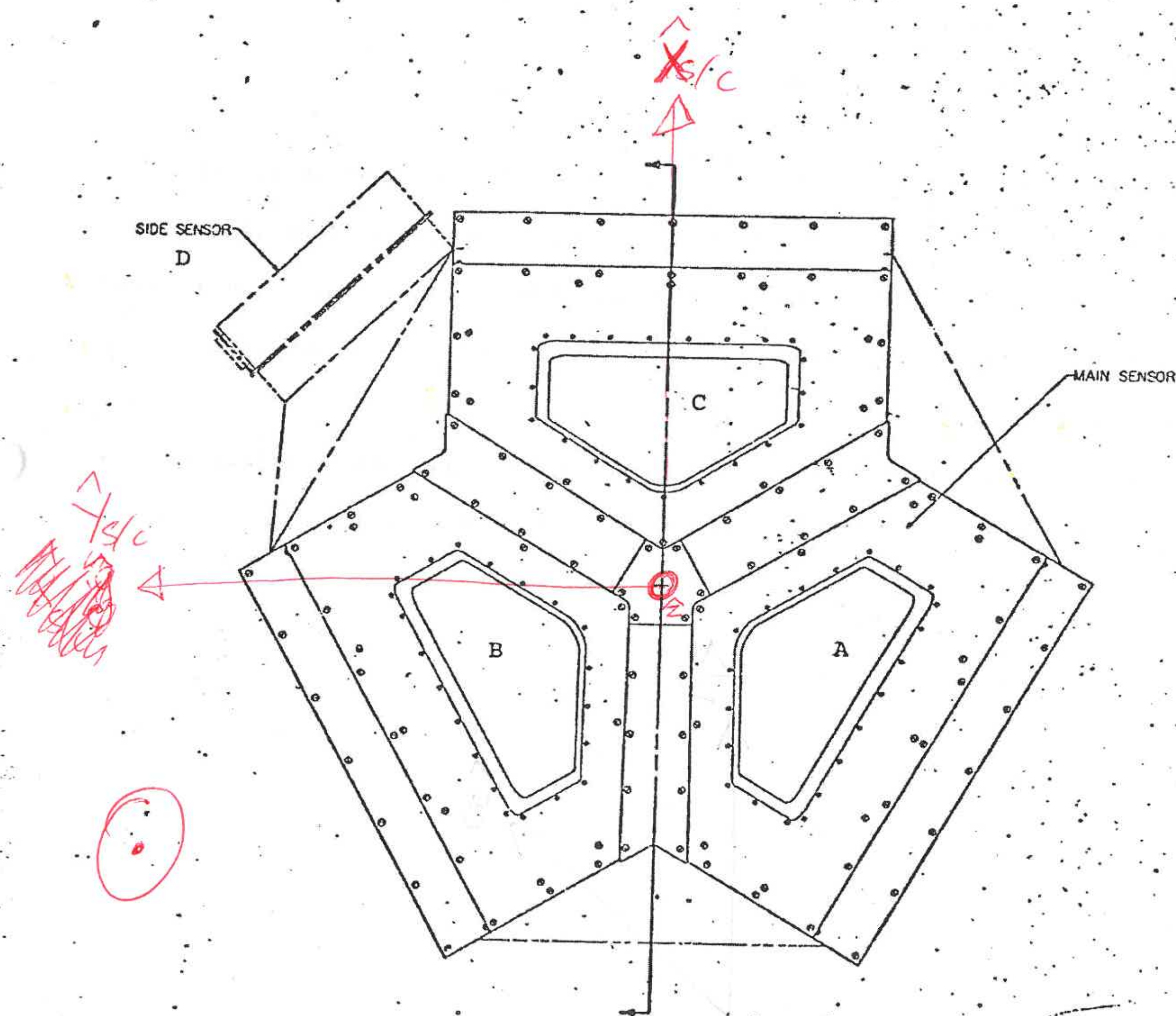
RE: Final sensor alignment date

// γ_2 of Memo#29

The angles are standard spherical coordinates (θ is the polar angle and

$-\gamma_1$ of Memo 29 = ϕ is the azimuth) of the outward normal to the respective sensors in a right handed coordinate system defined as follows: the z-axis is the outward normal to the front face of the reference cube (the nominal axis of symmetry for the main sensor); the x-axis is perpendicular to the z-axis in the general direction of the C sensor, and is in the plane which contains the outward normal to a side of the reference cube which is most nearly parallel to the long side of the C sensor; and the y-axis completes a right-handed coordinate system.




SIDE SENSOR
D
MAIN SENSOR
C
B
A
$\hat{X}_{s/c}$
$\hat{Y}_{s/c}$

FINAL PLS ALIGNMENTS

MIT Unit Serial Number

Sensor		PTM SN001	VOYAGER 1 FLT1 SN002	VOYAGER 2 ? FLT2 SN003
A	θ	20.3	19.99	19.97
	ϕ	-120.7	-120.7	-120.3
B	θ	19.78	20.0	19.78
	ϕ	121.2	119.6	121.1
C	θ	19.5	19.47	19.6
	ϕ	0.56	+0.3	0.58
D	θ	88.2	88.31	88.77
	ϕ	47.16	?	47.43
Date Measured		24 Mar'77	11 Apr'77	7 Mar'77
Reference Direction		90.84	90.92	90.35
		42.52	43.78	45.35
Linearity		0.999836(2)	0.999998(1)	0.999989(2)

Q: Are these values used in the analysis programs – eg. MJSANL?